Computational Fluid Dynamics

Introduction & Recap

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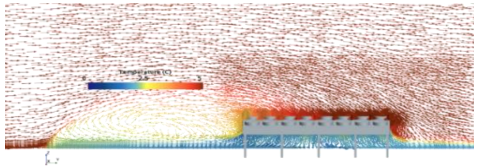
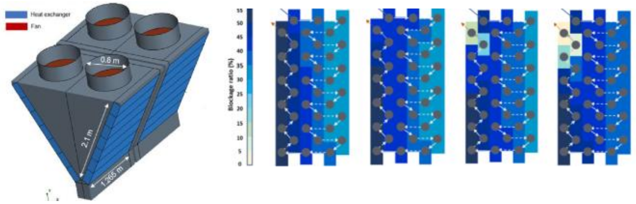
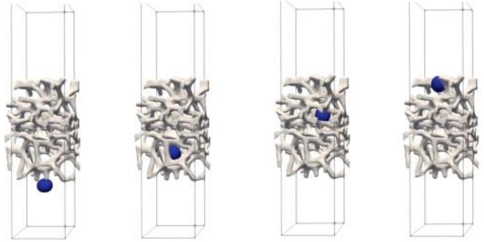
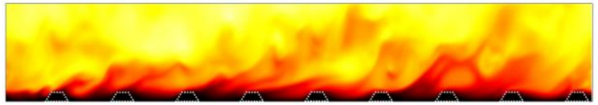
Research:

- Fluid mechanics and turbulence
- Heat transfer
- Power-to-hydrogen









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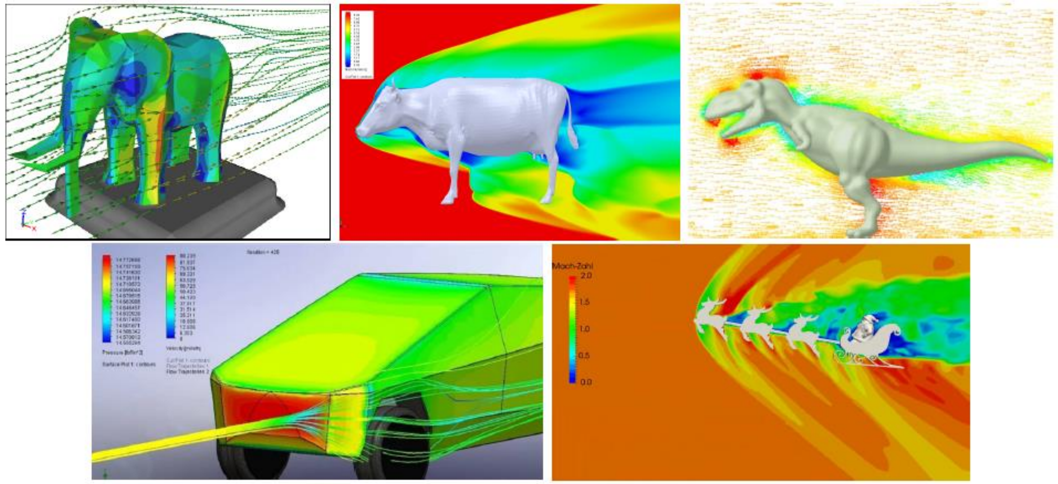
What is CFD?

- Solving the Navier-Stokes equations (and transport equations in fluids) numerically.

$$\nabla.\vec{\mathrm{v}} = 0 \qquad \frac{\partial \rho \vec{\mathrm{v}}}{\partial t} + \nabla.(\rho \vec{\mathrm{v}}\vec{\mathrm{v}}) = -\nabla p + \nabla.(\mu \nabla \vec{\mathrm{v}}) + f_B \qquad \frac{\partial \rho c_p T}{\partial t} + \nabla.\left(\rho c_p T \vec{\mathrm{v}}\right) = \nabla(k \nabla T) + q_{\mathrm{gen.}}$$

Why do we need CFD?

~~Because managers like colorful pictures~~!



CFD (Color For Directors)

Not the subject of this course!

What is CFD?

- Solving the Navier-Stokes equations (and transport equations in fluids) numerically.

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Why do we need CFD?

To calculate quantities related to fluid flow (lift/drag/thrust force, temperature, fluid velocity, heat or mass flux, pressure, mixing, sedimentation, …) with a reasonable cost and reliability.

Example applications:

- Turbomachinery
- Wind energy
- Automotive
- Aerospace
- Thermal engineering
- Pollution transport (air and see)
- Cardiovascular system
- Tribology
- Wind loading on buildings
- Marine engineering
- Astrophysics
- …

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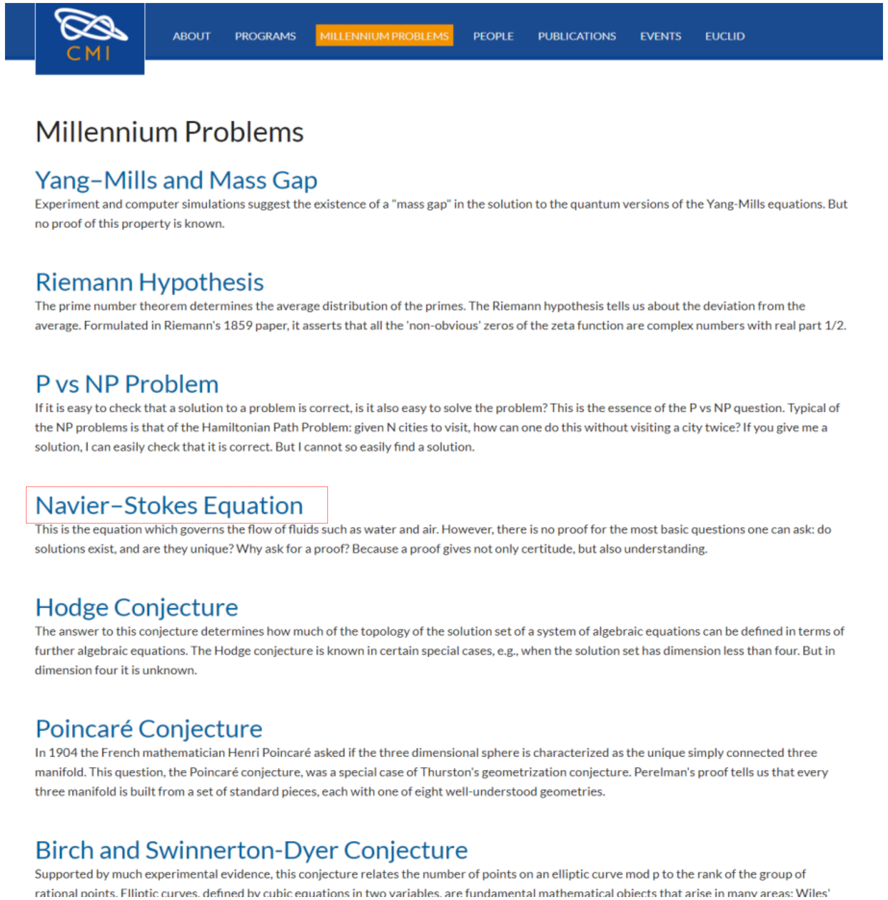
Why do we need CFD?

Analytical solutions generally do not exist.

1,000,000$ for the first mathematician to solve the NS!!



Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

Hodge Conjecture

The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

Poincaré Conjecture

In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

Birch and Swinnerton-Dyer Conjecture

Supported by much experimental evidence, this conjecture relates the number of points on an elliptic curve mod p to the rank of the group of rational points. Elliptic curves, defined by cubic equations in two variables, are fundamental mathematical objects that arise in many areas: Wiles'

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Why do we need CFD?

Experiments often cannot be the only option.

- Expensive (not suitable for iterative design and optimization or brainstorming)
- Often require down-scaling (dynamic similarity not always guaranteed).
- One can only measure certain quantities at certain locations with a certain resolution.
- Not suitable for study of hypothetical scenarios (e.g. no gravity!)

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Why do we need CFD?

CFD complements experimental measurements in the industry (to reduce costs) and in research (to provide physical insight).

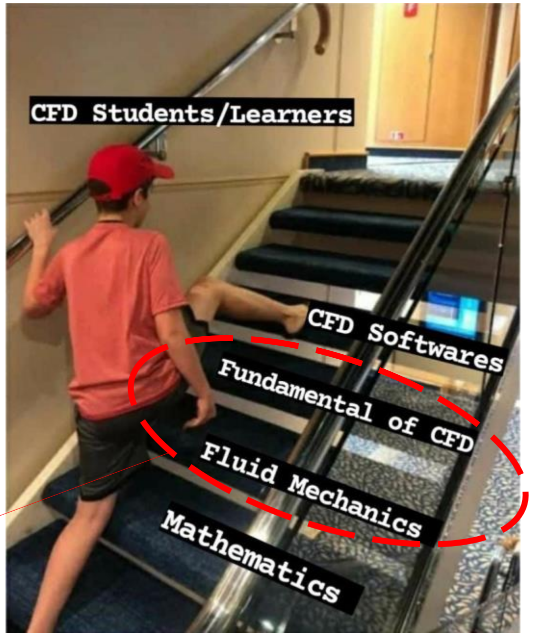
But CFD is prone to errors!

Expertise is required to:

- Set-up simulations in the right way (minimize the error)
- Interpret the results in the right way (explain the errors)

✓ Knowledge on fundamentals of CFD and flow physics is needed

→ **The goal of this CFD course**



Course content

Fundamentals of CFD – numerical methods

- Focus on Finite Volume Method (FVM)
- We stick to the most important methods and simplest version of governing equations

Use of a general-purpose software (openFoam)

- We will start with simple laminar problems, then will cover some topics in turbulence modeling and multi-phase flows.
- Flow physics is discussed as long as necessary for understanding the numerical solution

We have 14 sessions! The CFD course will put you on the right track. But you need to learn more (i.e. by doing projects) if you want to become an expert.

Course content

Computational Fluid Dynamics (CFD) Spring 2025 The program is subject to modification during the semester – check for updates			
Wk.	*Date*	*Part 1*	*Part 2*
5	Wed., Jan. 29	Introduction	Introduction, continued
6	Wed., Feb. 5	Finite Volume Method 1	
7	Wed., Feb. 12	OpenFoam intro1	Work on Assignment1
8	Wed., Feb. 17	Finite Volume Method 2	OF tutorial1
9	Wed., Feb. 26	Linear equation systems	Exercise
10	Wed., Mar. 5	Unsteady problems 1	OF intro2
11	Wed., Mar. 12	Unsteady problems 2	Work on Assignment2
12	Wed., Mar. 19	Navier Stokes solution 1	
13	Wed., Mar. 26	Navier Stokes solution 2	
14	Wed., Apr. 2	Turbulence modeling 1	
15	Wed., Apr. 9	OF intro 3	OF tutorial2
16	Wed., Apr. 16	break	
17	Wed., Apr. 23	Turbulence modeling 2	OF tutorial3
18	Wed., Apr 30	Multi-phase flows 1	OF tutorial4
19	Wed., May 7	Multi-phase flows 2	OF tutorial5
20	Wed., May 14	Backup	Work on Assignment3

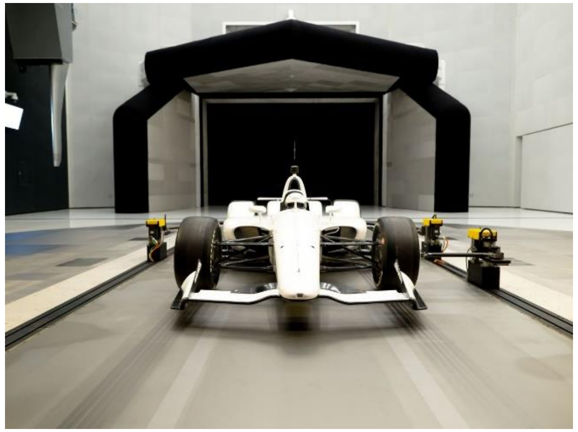
- Part 1 of each session starts at 8:15
- Part 2 of each session starts roughly at 10:15

First part of each session is (often) spent on the theory lecture.

Second part of each session is (often) spent on class activity or openFoam tutorial sessions.

Sources of error in CFD

The actual physical problem



Mathematical model

A set of governing equations

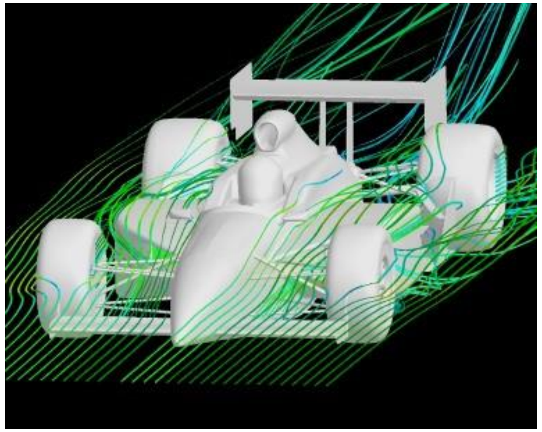
$$\nabla.\vec{\mathrm{v}} = 0$$

$$\frac{\partial \rho \vec{\mathrm{v}}}{\partial t} + \nabla.(\rho \vec{\mathrm{v}}\vec{\mathrm{v}}) = -\nabla p + \nabla.(\mu \nabla \vec{\mathrm{v}}) + f_B$$

$$\frac{\partial \rho c_p T}{\partial t} + \nabla.(\rho c_p T \vec{\mathrm{v}}) = \nabla(k \nabla T) + q_{\text{gen.}}$$

Numerical method

The numerical solution of the equations



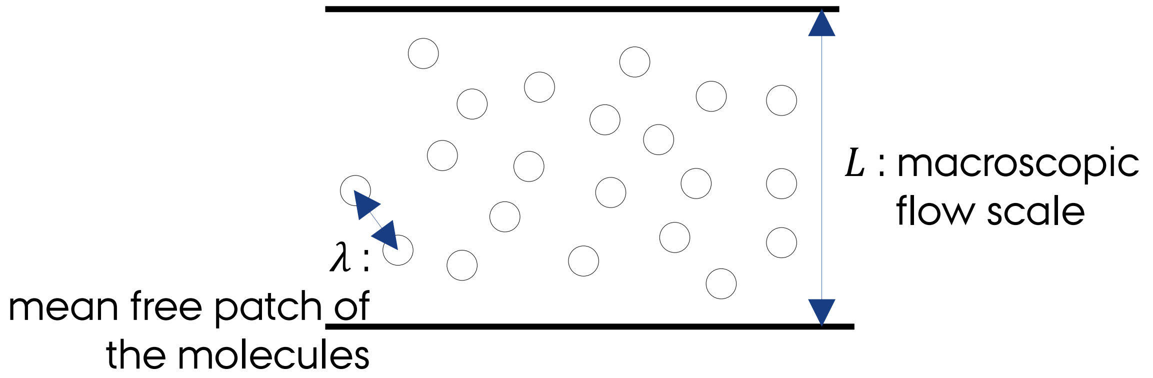
$$\phi^{\text{actual}} + \epsilon^{\text{model}} + \epsilon^{\text{numerical}} = \phi^{\text{computed}}$$

(modeling error) (numerical error)

Modeling error in fluid mechanics

Examples of 'modeling' in Fluid Mechanics

- Navier Stokes equation is a model itself! It relies on the continuum assumption.



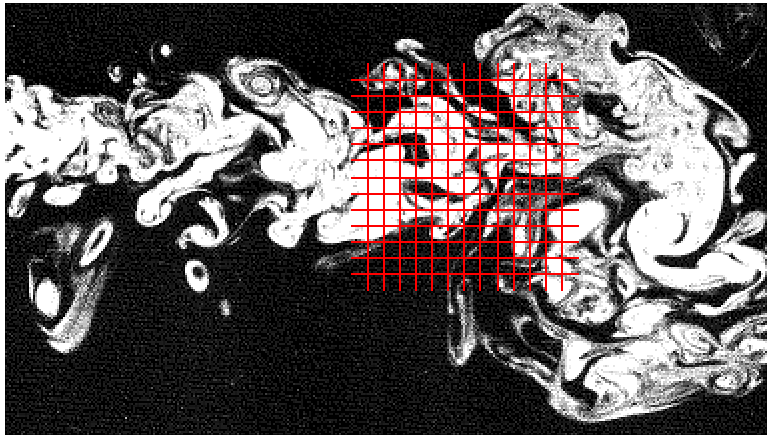
In most (but not all) problems $L \gg \lambda$

→ **Navier Stokes equation is almost exact!**

Modeling error in fluid mechanics

Examples of ‘modeling’ in Fluid Mechanics

- Navier Stokes equation is a model itself! It relies on the continuum assumption.
- Turbulence modeling is often necessary since cost of resolving all scales of turbulence is too high.



$$\frac{\partial \rho \overline{\vec{v}}}{\partial t} + \nabla.\left(\rho \overline{\vec{v}}\ \overline{\vec{v}}\right) = -\nabla p + \nabla.\left(\mu \nabla \overline{\vec{v}}\right) + \nabla.\tau_{\mathrm{turb}}$$

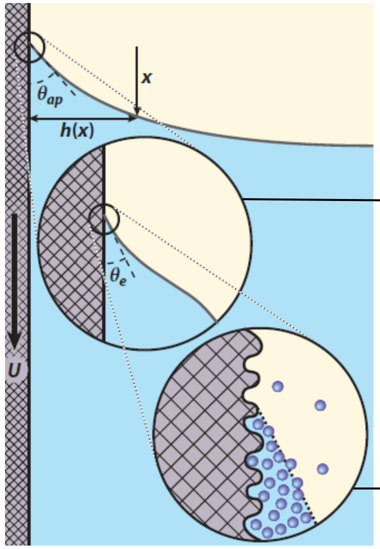
Solve for the time averaged variable on a course grid

Add a ‘modelled term’ to mimic the effect of turbulence

Modeling error in fluid mechanics

Examples of 'modeling' in Fluid Mechanics

- Navier Stokes equation is a model itself! It relies on the continuum assumption.
- Turbulence modeling is often necessary since cost of resolving all scales of turbulence is too high.
- Contact line modeling is often necessary since cost of resolving the contact region is too high.



Apparent contact line is calculated using a 'contact-line model' in CFD

Actual contact line dynamics is determined at nano-scale

** Figure taken from Snoeijer & Andreotti, Annu Rev. Fluid Mech. 45 (2013)*

Modeling error in fluid mechanics

Examples of ‘modeling’ in Fluid Mechanics

- Navier Stokes equation is a model itself! It relies on the continuum assumption.
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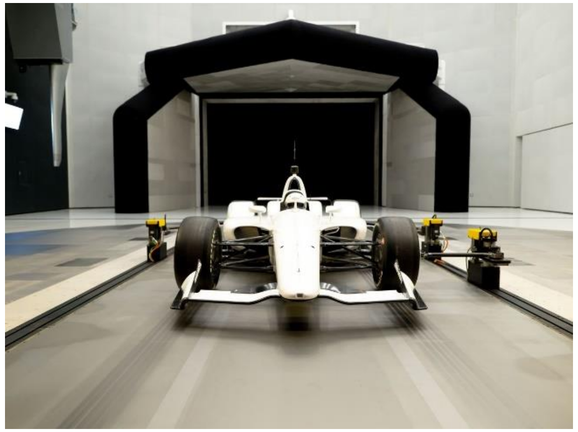
- Contact line modeling is often necessary since cost of resolving the contact line dynamics is too high.
- …..

Sources of error in CFD

The actual physical problem



Mathematical model

A set of governing equations

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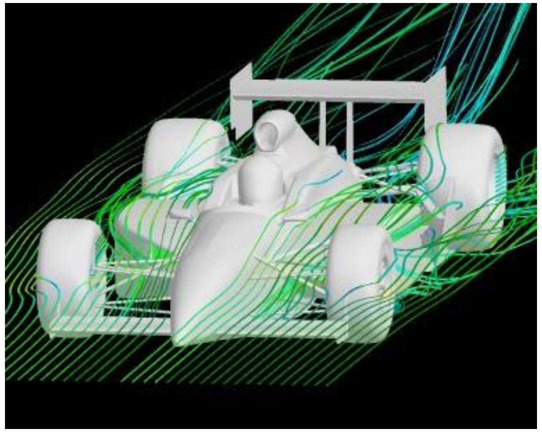
Numerical method

The numerical solution of the equations



$$\phi^{\text{actual}} + \epsilon^{\text{model}} + \epsilon^{\text{numerical}} = \phi^{\text{computed}}$$

(modeling error) (numerical error)

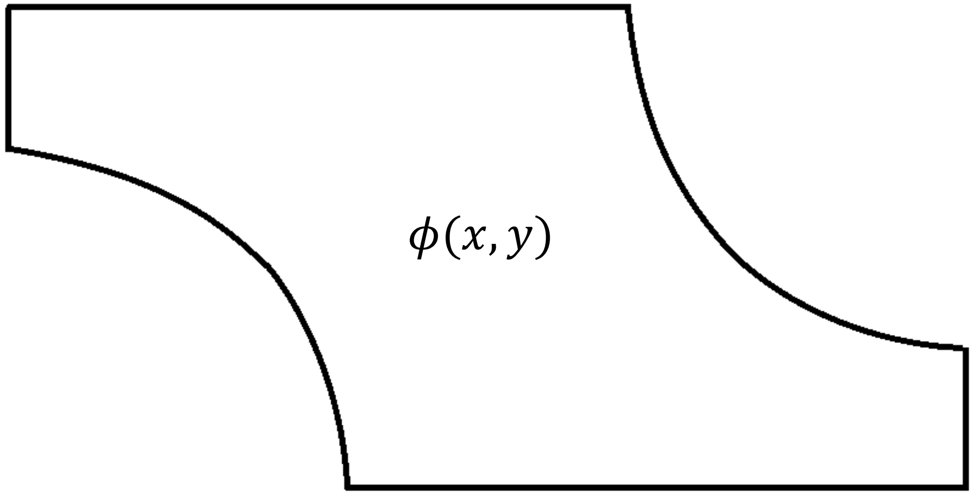
Numerical error in CFD

Numerical error is sum of 3 main contributors:

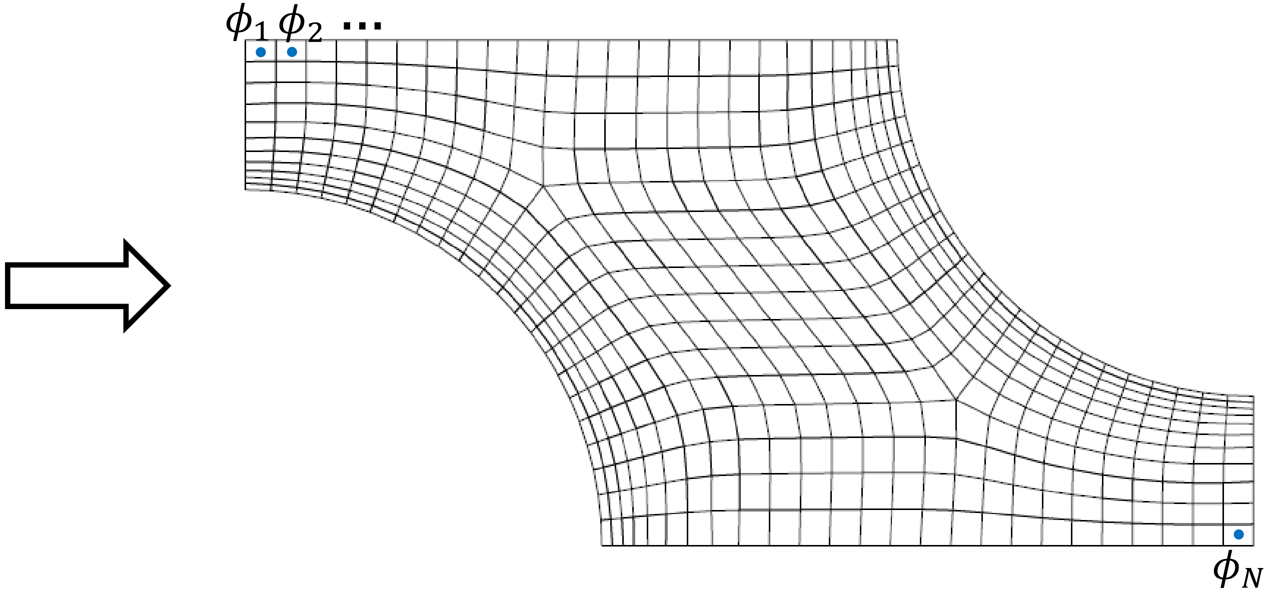
- Discretization error

  Physical quantities are continuous but numerical methods deal with discrete quantities.

  The process of converting PDEs for the ‘continues variables’ to a system of numerically soluble equations for the ‘discrete variables’ is called **discretization**. This process involves approximation and therefore produces error.



1 continues variable



N discrete variables (ϕ_i)

Numerical error in CFD

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- Iteration error

  Numerical methods in CFD are often iterative. Reaching the exact solution requires an infinite number of iterations. → more in lecture 5.

- Round-off error

  Computers can store numbers with a certain precision. Accuracy of a solution cannot be better than the precision of stored numbers (machine precision).

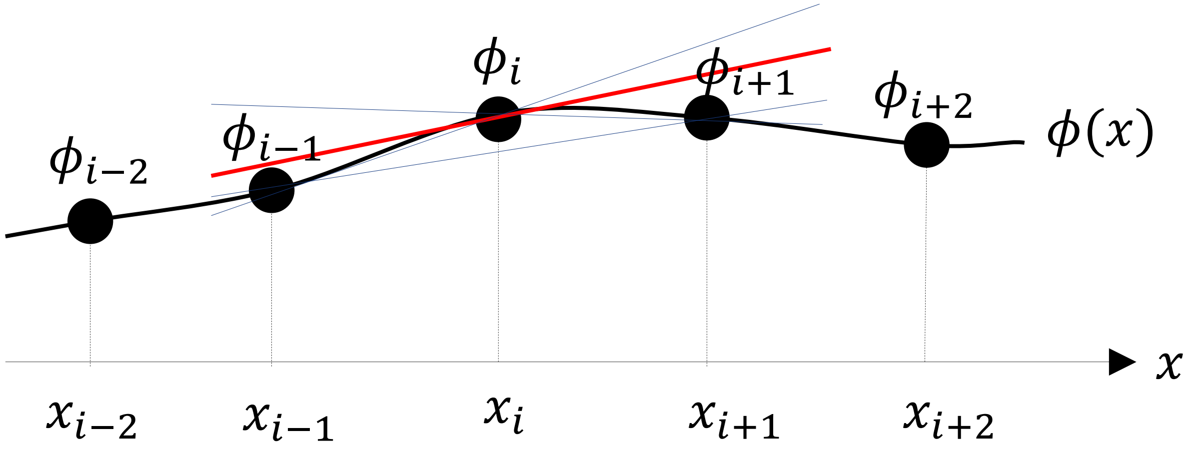
$$\epsilon^{\text{numerical}} = \epsilon^{\text{discret.}} + \epsilon^{\text{iter.}} + \epsilon^{\text{roundoff}}$$

Discretization error

- *A clarifying example* : approximate derivative of a function ϕ using discrete points.

Continues function

$$\left(\frac{\partial \phi}{\partial x}\right)_i$$



Possible discrete approximation(s)

$$\frac{\phi_i - \phi_{i-1}}{x_i - x_{i-1}}$$

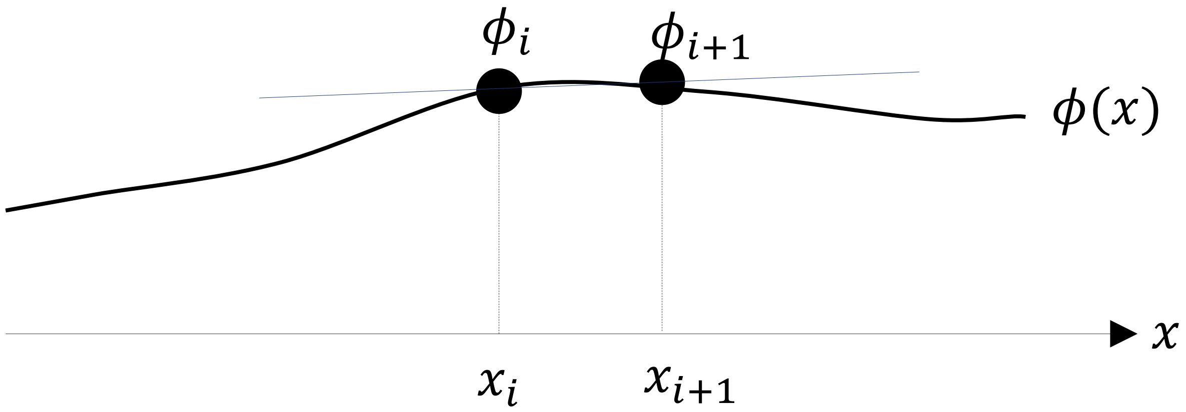
or $$\frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i}$$

or $$\frac{\phi_{i+1} - \phi_{i-1}}{x_{i+1} - x_{i-1}}$$

- The function and its disceretized approximation are not exactly the same. The difference is error!

Discretization error

Approximation of first derivative $\left(\frac{\partial \phi}{\partial x}\right)_i$



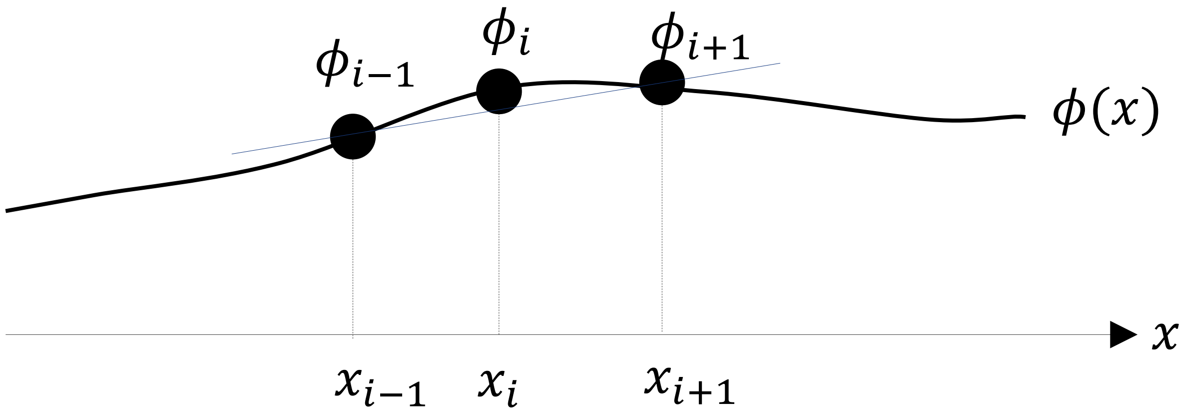
Taylor series:

$$\phi(x) = \phi(x_i) + (x - x_i)\left(\frac{\partial \phi}{\partial x}\right)_i + \frac{(x-x_i)^2}{2!}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_i + \frac{(x-x_i)^3}{3!}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_i + \cdots + \frac{(x-x_i)^n}{n!}\left(\frac{\partial^n \phi}{\partial x^n}\right)_i + \cdots$$

$x \rightarrow x_{i+1}$:

$$\underbrace{\boxed{\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i}}}_{\substack{\text{Approximation} \\ \text{(forward difference formula)}}} \underbrace{- \frac{x_{i+1} - x_i}{2}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_i - \frac{(x_{i+1} - x_i)^2}{6}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_i \cdots}_{\epsilon_\tau \text{ : truncation error}}$$

Discretization error



Approximation of first derivative $\left(\frac{\partial\phi}{\partial x}\right)_i$

Taylor series:

$$\phi(x) = \phi(x_i) + (x - x_i)\left(\frac{\partial\phi}{\partial x}\right)_i + \frac{(x-x_i)^2}{2!}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i + \frac{(x-x_i)^3}{3!}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i + \cdots + \frac{(x-x_i)^n}{n!}\left(\frac{\partial^n\phi}{\partial x^n}\right)_i + \cdots$$

$(x \to x_{i+1}) - (x \to x_{i-1})$:

$$\boxed{\left(\frac{\partial\phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{x_{i+1} - x_{i-1}}} \underbrace{- \frac{(x_{i+1} - x_i)^2 - (x_i - x_{i-1})^2}{2\,(x_{i+1} - x_{i-1})}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i - \frac{(x_{i+1} - x_i)^3 + (x_i - x_{i-1})^3}{6\,(x_{i+1} - x_{i-1})}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i + \cdots}_{\epsilon_\tau \text{ : truncation error}}$$

Approximation
(central difference formula)

ϵ_τ : truncation error

Discretization error

- If the grid is uniform ($\Delta x = \text{const.}$) :

central difference

$$\boxed{\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}} \overbrace{- \frac{\Delta x^2}{6}\left(\frac{\partial^3 \phi}{\partial x^3}\right)}^{\epsilon_\tau \;= O(\Delta x^2)} + \cdots$$

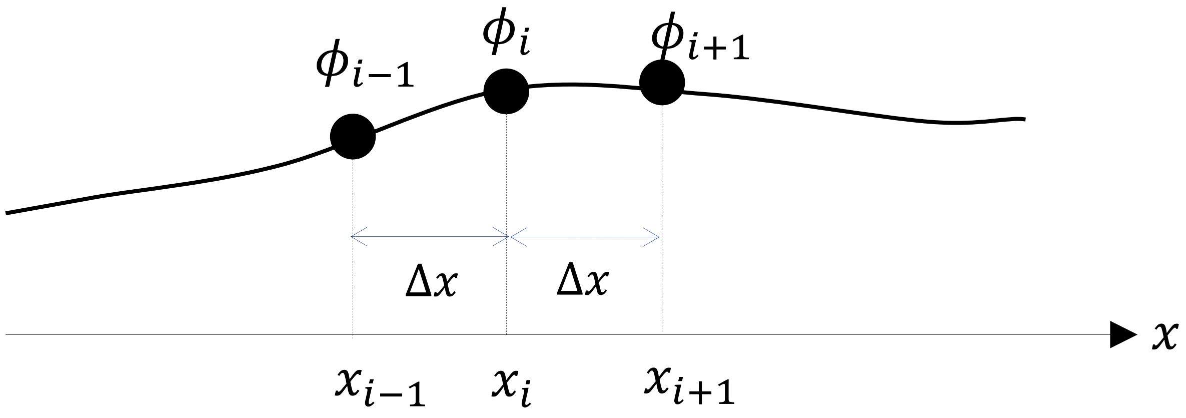
forward difference

$$\boxed{\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_i}{\Delta x}} \overbrace{+ \frac{\Delta x}{2}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_i - \frac{\Delta x^2}{6}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_i + \cdots}^{\epsilon_\tau \;= O(\Delta x)}$$

backward difference

$$\boxed{\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_i - \phi_{i-1}}{\Delta x}} \overbrace{+ \frac{\Delta x}{2}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_i - \frac{\Delta x^2}{6}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_i + \cdots}^{\epsilon_\tau \;= O(\Delta x)}$$

The leading term dominates the error



$$\epsilon_\tau \sim (\Delta x)^m$$

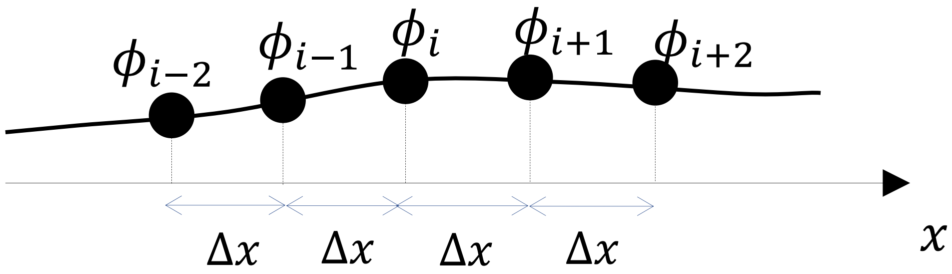
- Error can be reduced by:
  (1) Decreasing Δx (grid refinement)
  (2) Increasing m (higher order approximation)
- Central difference formula is 2nd order accurate while forward/backward difference are 1st order.

* Central difference on non-uniform grids is formally less accurate but for moderately varying grids it behaves similar to second order

Discretization error

- Higher order approximations require larger number of points (more complicated and computationally more expensive), e.g.,

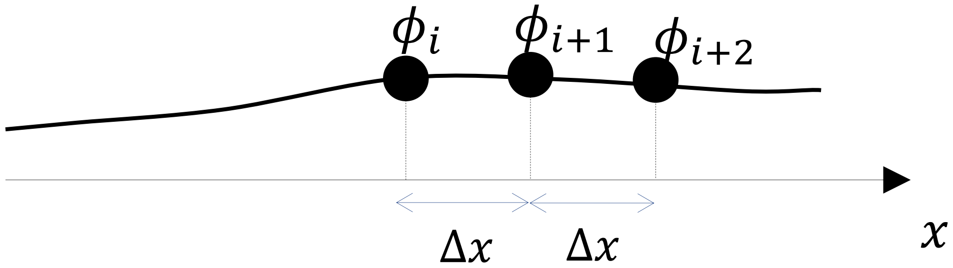
$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{-\phi_{i+2} + 8\phi_{i+1} - 8\phi_{i-1} + \phi_{i-2}}{12\Delta x} + O(\Delta x^4)$$



- Other examples

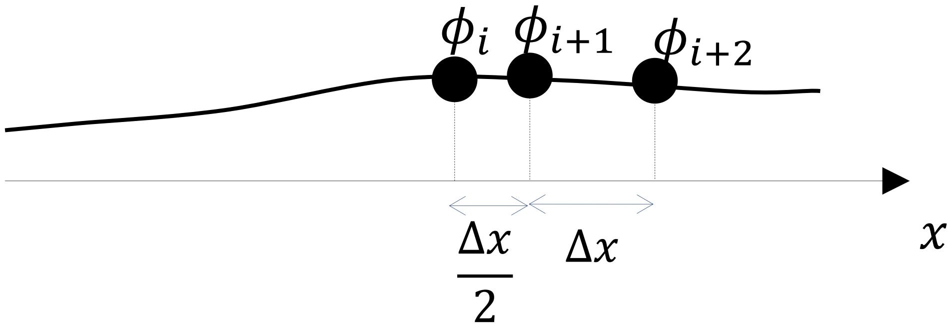
One sided 2nd order approximation

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{-\phi_{i+2} + 4\phi_{i+1} - 3\phi_i}{2\Delta x} + O(\Delta x^2)$$



One sided 2nd order approximation (alternative grid)

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{-\phi_{i+2} + 9\phi_{i+1} - 8\phi_i}{3\Delta x} + O(\Delta x^2)$$



Discretization error

- The discretization error of a numerical method is sum of all truncation errors resulting from different approximations employed for discretization of different terms.

$$\epsilon^{\text{discret.}} \sim \Delta^{M}$$

Overall order of the method

Representative grid size

Governing equations

$\vec{\mathrm{v}} = (\mathrm{v}_x , \mathrm{v}_y , \mathrm{v}_z) = (u , v , w) = (u_1 , u_2 , u_3)$

Integral form of

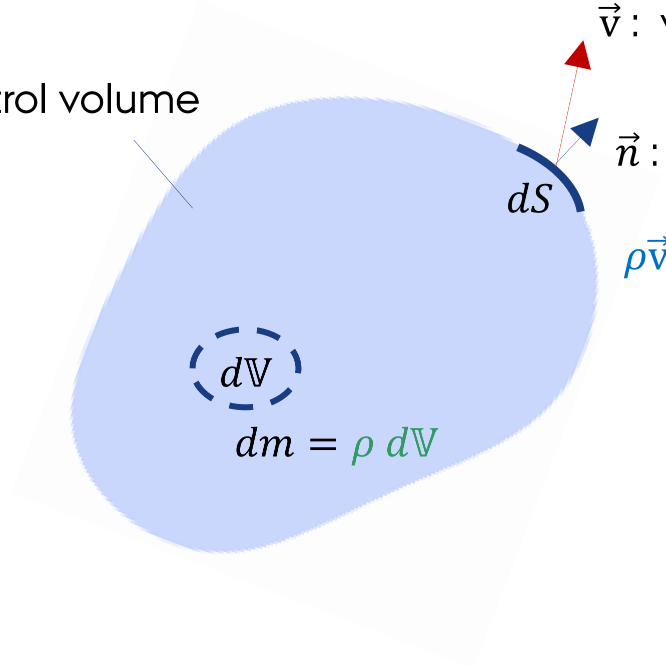
- Mass conservation

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho \, d\mathbb{V} + \int_{S} \rho\vec{\mathrm{v}}.\vec{n}\, dS = 0$$

- Momentum conservation (Force balance)

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho\vec{\mathrm{v}}\, d\mathbb{V} + \int_{S} \rho\vec{\mathrm{v}}\,\vec{\mathrm{v}}.\vec{n}\, dS = \underbrace{\int_{S} \mathcal{T}.\vec{n}\, dS}_{\text{stress tensor}} + \underbrace{\int_{\mathbb{V}} f_B\, d\mathbb{V}}_{\text{body force (e.g. gravity)}}$$

Control volume



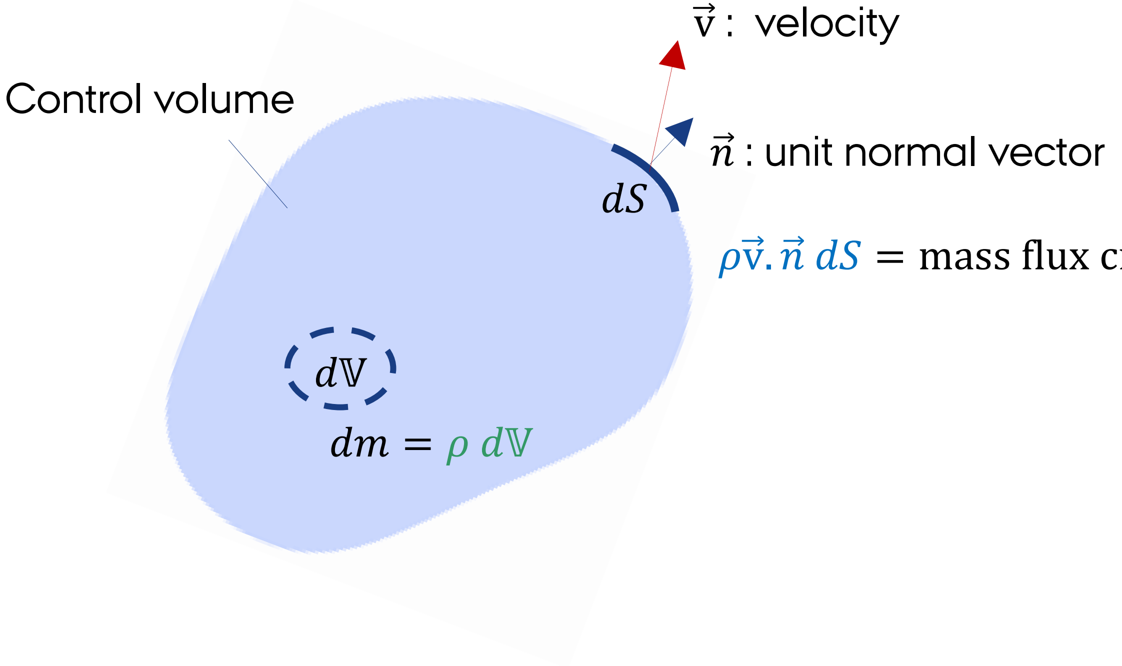
$\rho\vec{\mathrm{v}}.\vec{n}\, dS =$ mass flux crossing area dS

$$\mathcal{T} = \underbrace{-pI}_{\text{pressure}} + \underbrace{\tau}_{\text{viscous stress tensor}} \quad , \quad \tau = \underbrace{\mu}_{\text{viscosity}} (\nabla\vec{\mathrm{v}} + \nabla\vec{\mathrm{v}}^T)\ ^*$$

* compressibility effect ignored

Governing equations

$\vec{\mathrm{v}} = (\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z) = (u, v, w) = (u_1, u_2, u_3)$



$\rho \vec{\mathrm{v}}.\vec{n}\, dS$ = mass flux crossing area dS

Integral form of

- Mass conservation

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho\, d\mathbb{V} + \int_{S} \rho \vec{\mathrm{v}}.\vec{n}\, dS = 0$$

- Momentum conservation (Force balance)

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho \vec{\mathrm{v}}\, d\mathbb{V} + \int_{S} \rho \vec{\mathrm{v}}\, \vec{\mathrm{v}}.\vec{n}\, dS = \int_{S} \mathcal{T}.\vec{n}\, dS + \int_{\mathbb{V}} f_B\, d\mathbb{V}$$

$$\mathcal{T} = -pI + \tau \quad , \quad \tau = \mu\left(\nabla \vec{\mathrm{v}} + \nabla \vec{\mathrm{v}}^T\right)$$

- Energy conservation

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho c_p T\, d\mathbb{V} + \int_{S} \rho c_p T\, \vec{\mathrm{v}}.\vec{n}\, dS = -\int_{S} \vec{q}.\vec{n}\, dS + \int_{\mathbb{V}} q_{\mathrm{gen.}}\, d\mathbb{V}$$

$$\vec{q} = -k\, \nabla T$$

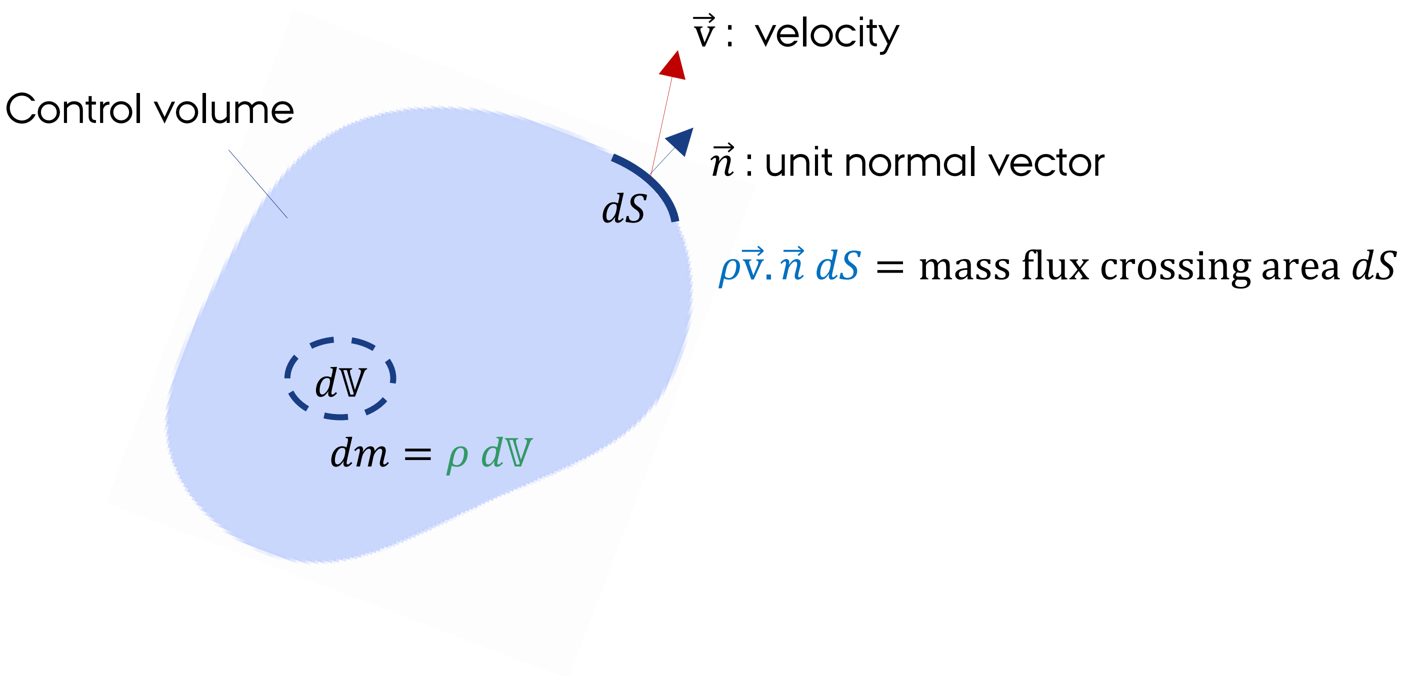
thermal conductivity

heat flux

heat generation (e.g. due to reaction or viscous heating)

Governing equations

$\vec{\mathrm{v}} = (\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z) = (u, v, w) = (u_1, u_2, u_3)$



Integral form of

- Mass conservation

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho\, d\mathbb{V} + \int_S \rho\vec{\mathrm{v}}.\vec{n}\, dS = 0$$

- Momentum conservation (Force balance)

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho\vec{\mathrm{v}}\, d\mathbb{V} + \int_S \rho\vec{\mathrm{v}}\,\vec{\mathrm{v}}.\vec{n}\, dS = \int_S \mathcal{T}.\vec{n}\, dS + \int_{\mathbb{V}} f_B d\mathbb{V}$$

- Energy conservation

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho c_p T d\mathbb{V} + \int_S \rho c_p T\, \vec{\mathrm{v}}.\vec{n}\, dS = \int_S k\, \nabla T.\vec{n}\, dS + \int_{\mathbb{V}} q_{\text{gen.}} d\mathbb{V}$$

- Generic scalar transport equation (variable: ϕ)

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho\phi d\mathbb{V} + \int_S \rho\phi\vec{\mathrm{v}}.\vec{n}\, dS = -\int_S \Gamma\nabla\phi.\vec{n}\, dS + \int_{\mathbb{V}} q_\phi d\mathbb{V}$$

Governing equations

$\vec{\mathrm{v}} = (\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z) = (u, v, w) = (u_1, u_2, u_3)$

Differential form of

- Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{\mathrm{v}}) = 0$$

- Momentum conservation (Force balance)

$$\frac{\partial \rho\vec{\mathrm{v}}}{\partial t} + \nabla.(\rho\vec{\mathrm{v}}\vec{\mathrm{v}}) = -\nabla p + \nabla.\tau + f_B$$

- Energy conservation

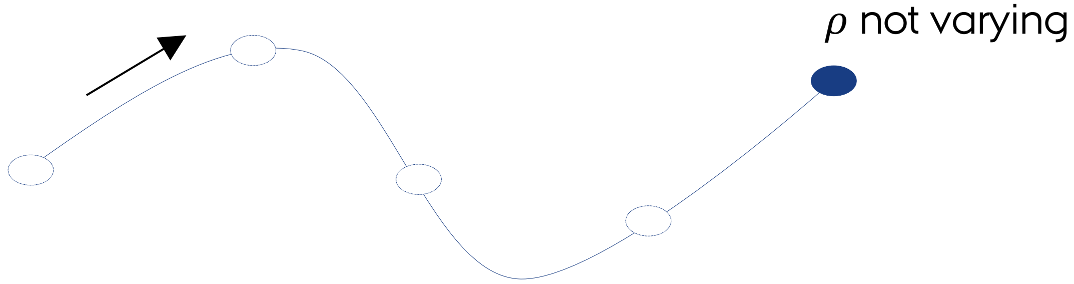
$$\frac{\partial \rho c_p T}{\partial t} + \nabla.(\rho c_p T\vec{\mathrm{v}}) = \nabla.(k\nabla T) + q_{\mathrm{gen.}}$$

- Generic scalar transport equation (variable: ϕ)

$$\frac{\partial \rho\phi}{\partial t} + \nabla.(\rho\phi\vec{\mathrm{v}}) = \nabla.(\Gamma\nabla\phi) + q_\phi$$

Compressibility

- If variation in density of fluid elements in the flow is negligible the flow is called **incompressible**.



- It can be shown that variation in ρ for a fluid element is proportional to Mach number.

$$\mathrm{M} = \frac{\mathrm{v}}{c}$$

v → Flow characteristic velocity

c → Speed of sound

- In practice, a flow can be approximated as incompressible when $\mathbf{M < 0.3}$.
- At a low Mach number, it is preferable to solve a problem as incompressible as it reduces the computational time (stiffness issue).

Compressibility

- For incompressible flow the mass conservation can be simplified as: $\nabla.\vec{\mathrm{v}} = 0$

Governing equations for incompressible flow

Differential form

$$\nabla.\vec{\mathrm{v}} = 0$$

$$\frac{\partial \rho \vec{\mathrm{v}}}{\partial t} + \nabla.(\rho \vec{\mathrm{v}}\vec{\mathrm{v}}) = -\nabla p + \nabla.(\mu \nabla \vec{\mathrm{v}}) + f_B$$

$$\frac{\partial \rho \phi}{\partial t} + \nabla.(\rho \phi \vec{\mathrm{v}}) = \nabla.(\Gamma \nabla \phi) + q_\phi$$

Integral form

$$\int_S \vec{\mathrm{v}}.\vec{n}\, dS = 0$$

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho \vec{\mathrm{v}}\, d\mathbb{V} + \int_S \rho \vec{\mathrm{v}}\, \vec{\mathrm{v}}.\vec{n}\, dS = -\int_S p\, \vec{n}\, dS + \int_S \mu\, \nabla \vec{\mathrm{v}}.\vec{n}\, dS + \int_{\mathbb{V}} f_B d\mathbb{V}$$

$$\frac{\partial}{\partial t}\int_{\mathbb{V}} \rho \phi d\mathbb{V} + \int_S \rho \phi \vec{\mathrm{v}}.\vec{n}\, dS = -\int_S \Gamma \nabla \phi.\vec{n}\, dS + \int_{\mathbb{V}} q_\phi d\mathbb{V}$$

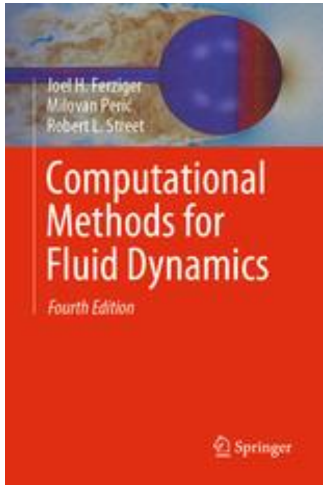
- In the present course, the main focus is on incompressible CFD

Practical information

- Final grade based on 3 group assignments and 1 individual assignment.
- Group assignments should be done in project groups – each group consisting of 4 students.
- **Form your project group before 5 Feb. in Brightspace. Otherwise, you will be assigned to a random group.** If your group has less than 4 members, random members will be added.
- openFoam tutorials and other class activities (except assignments) do not directly contribute to the grade but are crucial to your learning.
- Slides and lecture notes are the main learning material.
- Reference for CFD fundamentals:

  Computational Methods for Fluid Dynamic" J. H. Fziger, M. Peric, R. L. Street, 4th Ed.

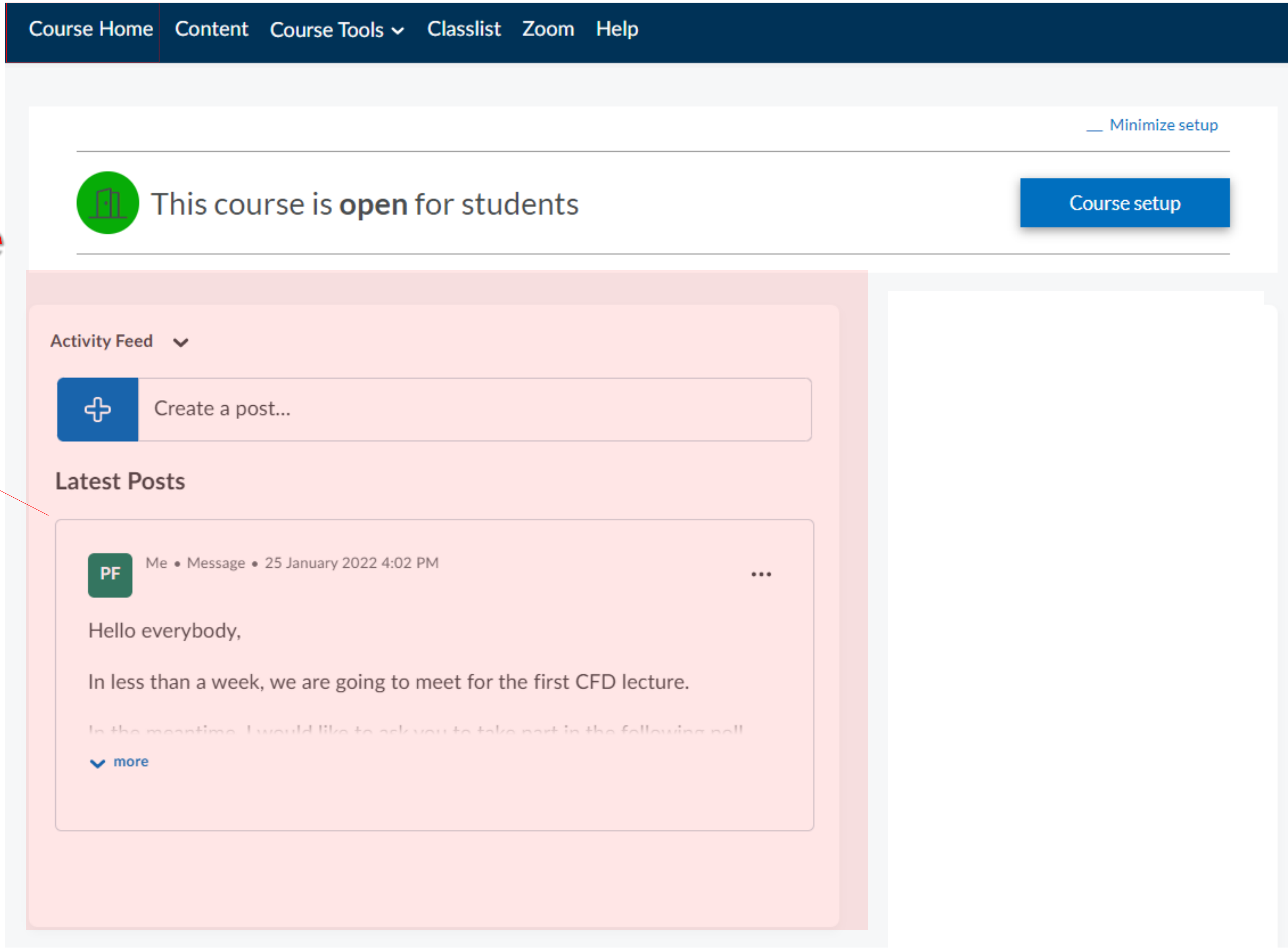
  (selected topics)
- Textbook is not mandatory – only if you like to learn more.



Practical information

All important announcements will be posted here.

Keep notifications on!

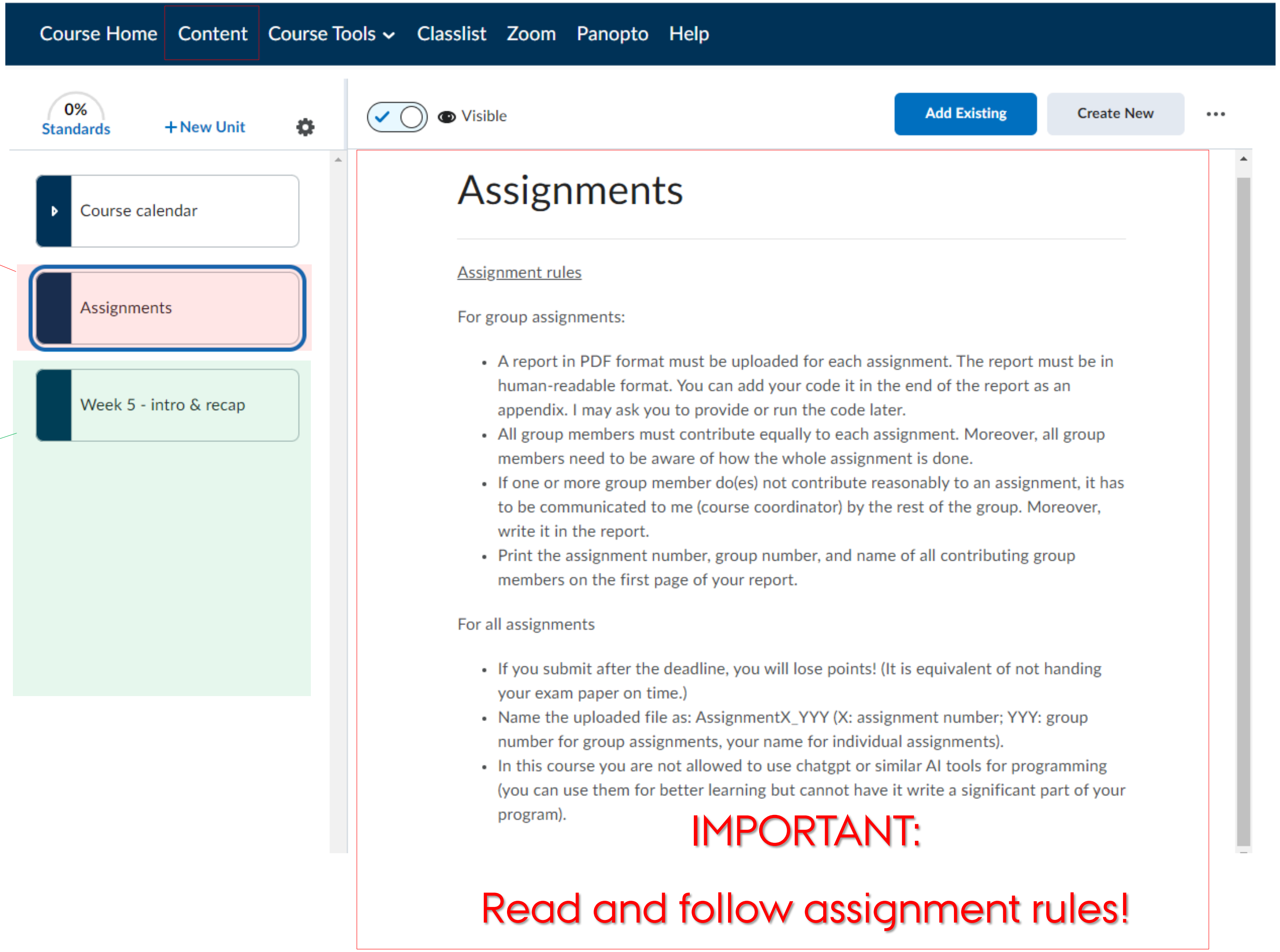


Practical information

Assignments will be announced here.

Keep notifications on!

Before each session, I upload the new slides and other material in the right module folder.



IMPORTANT:

Read and follow assignment rules!

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