This is the first problem set. The problems are marked with different categories:

- (L) are problems that are referred to in the lectures and are meant to supplement the lecture material. Do them to get more out of the lectures!
- (H) are typical homework exercise problems that illustrate several concepts.
- (X) are exam type problems. At least one of the subproblems on the final exam will be taken, perhaps in a slightly altered form, from these (X) problems.
- (E) are extra problems. Do them if you feel the need.

In order to get an OK (get it approved) on your problem set you MUST

- Do a decent attempt on all the (H) and (X) problems

  and

- Evaluate/review and make comments on the problem set of another student.

The evaluation is done either by

- Participating in the feedback session the 1st hour on Wednesdays: Bring your answer sheets with you to the feedback session and evaluate another student's problem set there.

  or

- Hand in your answer on Canvas within the DUE time. Then wait until Canvas assigns you a problem set to review (a short while after the DUE time). You must provide your feedback on Canvas before Friday evening.

Written answers to the problem sets will be posted on the web at the beginning of the Wednesday feedback session.

To be eligible for the final exam you must get an OK on at least six assignments, three of them must be among the first six weekly assignments.

Problem 1.1(L)

The complex inner product $\langle u|v\rangle$ is linear in its *second* factor, which means: Given $|v\rangle = \alpha|v_1\rangle + \beta|v_2\rangle$ where $\alpha, \beta \in \mathbb{C}$, then $\langle u|v\rangle = \alpha\langle u|v_1\rangle + \beta\langle u|v_2\rangle$. Find the corresponding relation for the first factor of the complex inner product. I.e. Define $|u\rangle = \alpha|u_1\rangle + \beta|u_2\rangle$ and use the defining properties of the complex inner product to write out the inner product $\langle u|w\rangle$ for an arbitrary $|w\rangle$.

Problem 1.2(L)

Let $|w\rangle = \alpha|w_1\rangle + \beta|w_2\rangle$ where $\alpha, \beta \in \mathbb{C}$. Use the properties of the complex inner product to show that $\langle w| = \alpha^*\langle w_1| + \beta^*\langle w_2|$. Hint: Consider the inner product $\langle w|u\rangle$ where $|u\rangle$ is an arbitrary ket. Is your answer consistent with what you got in Problem 1.1?

Problem 1.3(L)

Two different kets are represented by two different complex functions of one variable x which take values in $\mathbb{R}$: $|f\rangle \simeq f(x)$ and $|g\rangle \simeq g(x)$. We use the symbol $\simeq$ to mean represented by. (You may replace it by $=$ if you like.) Write down the expressions for the inner products $\langle f|g\rangle$ and $\langle f|f\rangle$ in terms of $f(x)$ and $g(x)$. What is $f(x)$ if $\langle f|f\rangle = 0$?

Problem 1.4(L)

You are given an orthonormal basis set $|a_i\rangle$ where $i \in \{1, \dots, N\}$. A ket $|V\rangle = \sum_{i=1}^{N} v_i|a_i\rangle$. Show that $v_i = \langle a_i|V\rangle$.

Problem 1.5(L)

A basis set can be either discrete or continuous. For a continuous basis set $|x'\rangle$ the index x' which labels the basis vector takes values in a continuum, and an arbitrary ket is expressed as $|f\rangle = \int_0^L dx' f(x')|x'\rangle$ where the expansion coefficients become a function $f(x')$. We have put limits 0 and L on the values that the continuous variable takes, but these depends on the problem in question.
Show that the Dirac delta-normalization of the basis vectors: $\langle x|x'\rangle = \delta(x - x')$ implies that $\langle x|f\rangle = f(x)$. From this show that $\int_0^L dx'|x'\rangle\langle x'|$ is an operator (a mathematical object that maps a ket onto a ket) that maps a given ket onto itself (i.e. it is the identity operator).

Problem 1.6(H)

Consider the functions $\delta_\epsilon(x) = \frac{1}{\pi}\frac{\epsilon}{\epsilon^2+x^2}$ where ϵ is a parameter.
a) Show that $\int_{-\infty}^{\infty} dx\delta_\epsilon(x) = 1$
b) Verify that $\delta_{\epsilon/k}(x) = k\delta_\epsilon(kx)$ for a positive number k.

Such a sequence of functions is called a δ-sequence and constitutes one possible formal definition of a Dirac delta-function.

$$\int dx' \delta(x - x') f(x') \equiv \lim_{\epsilon \to 0} \int dx' \delta_\epsilon(x - x') f(x') = f(x)$$

c) Make numerical plots of $\delta_\epsilon(x)$ on the interval $x \in [-1, 1]$ for three values of ϵ, pick $\epsilon = \{0.01, 0.1, 1\}$.
d) Assume that you are given a table of numerical arguments x and values $y \equiv \delta_1(x)$ of the function δ_1. Explain how you can obtain a similar table of function arguments and values of the function $\delta_{0.1}$ using the numbers in the given table.

Problem 1.7(H)

Consider an orthonormal basis set $|b_i\rangle$ where $i \in 1, 2, \ldots, N$ and where an arbitrary ket is expressed as $|A\rangle = \sum_{i=1}^{N} \alpha_i |b_i\rangle$ where $\alpha_i \in \mathbb{C}$. Let $\hat{P}_1 = |b_1\rangle\langle b_1|$. Compute $\hat{P}_1|A\rangle$ and $\hat{P}_1\hat{P}_1|A\rangle$. Justify in words why $\hat{P}_1$ is called a projection operator.

Problem 1.8(X)

What is a quantum state? In your answer, explain how a quantum state is different from a classical state and how it is represented mathematically.

Problem 1.9(E)

Two different kets are represented by two different column matrices, each with two entries: $|u\rangle \simeq \left(\begin{smallmatrix} u_1 \\ u_2 \end{smallmatrix}\right)$ and $|v\rangle \simeq \left(\begin{smallmatrix} v_1 \\ v_2 \end{smallmatrix}\right)$. Compute $\langle u|v\rangle$. Also compute $\langle u|\alpha|v\rangle$ where $\alpha \in \mathbb{C}$.