Bras, Kets and Dirac-Delta

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1 Inner Product

There are many ways to write the inner product

- I.
$$(|u\rangle, |v\rangle) = (|v\rangle, |u\rangle)^*$$
- II. Second linearity makes first linearity impossible.
$$(|u\rangle, \alpha |v_1\rangle + \beta |v_2\rangle) = \alpha (|u\rangle, |v_1\rangle) + \beta (|u\rangle, |v_1\rangle)$$
- III.
$$(\alpha |v_1\rangle + \beta |v_2\rangle, |u\rangle) = \alpha^* ||v_1\rangle, |u\rangle\rangle + \beta^* (|v_2\rangle, |u\rangle)$$
- IV.
$$\underbrace{(|v\rangle, |v\rangle)}_{\mathbb{R}} \geq 0$$

1.1 Dirac Notation

We denote the inner product like this in Dirac-notation.

$$(|u\rangle, |v\rangle) = \langle u|v\rangle \in \mathbb{C}$$

- I
$$\langle u|v\rangle = (\langle v|u\rangle)^*$$
- II
$$|v'\rangle = \alpha |v_1\rangle + \beta |v_2\rangle$$
- III
$$\langle v|v\rangle \geq 0$$

1.2 Representation of Bras

The bra can be written as an operator operating on a ket, producing a number

$$\langle A| \simeq \int \mathrm{d}x A^*(x)$$

$$\langle A|B\rangle = \int \mathrm{d}x A^*(x) B(x)$$

$$\langle B|A\rangle = \int \mathrm{d}x B^*(x) A(x)$$

2 Sets of Kets

this is a ket $|u\rangle$

3 Discrete and Continuous Basis

3.1 Discrete

$$|f\rangle = \sum_{i=1}^{\infty} f_i \, |i\rangle$$

For orthonormal basis

$$\langle i|j\rangle = \delta_{ij} \rightarrow f_j = \langle j|f\rangle$$

3.2 Continuous

$$|f\rangle = \int_0^L f(x') \, |x'\rangle \, \mathrm{d}x$$

$$\langle x|f\rangle = f(x)$$

$$\langle x|f\rangle = \int_0^L \langle x| \, f(x) \, |x'\rangle \, \mathrm{d}x = \int_0^L f(x) \underbrace{\langle x|x'\rangle}_{\delta} \, \mathrm{d}x = f(x)$$

In a short interval $[-\epsilon, \epsilon]$ the function f becomes approximately constant. Using the definition of the Dirac-delta we get the following

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} f(x)\delta(x - x') \, \mathrm{d}x = f(x) \underbrace{\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \delta(x - x') \, \mathrm{d}x}_{1} = f(x)$$