Oblig 7

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Problem 1

a)

We know that
$$\vec{J} = \vec{L} + \vec{S} \rightarrow \vec{J}^2 = (\vec{L} + \vec{S})^2 = \vec{L}^2 + \vec{S}^2 + 2\vec{L} \cdot \vec{S}$$
We therefore know that:
$$\vec{L} \cdot \vec{S} = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$$
We begin with S^2
$$S^2 = \hbar^2 s(s+1) = \hbar^2 \frac{3}{4}$$
Next we have L^2:
$$L^2 = \hbar^2 l(l+1) = 6\hbar^2$$
We have j-values in integer steps in the range:
$$|l - s| \leq j \leq l + s \rightarrow \frac{3}{2} \leq j \leq \frac{5}{2}$$
Therefore we know j can be either $j_1 = 5/2$ or $j_2 = 3/2$ Next we have J^2:
$$J_1^2 = \hbar^2 j_1(j_1 + 1) = \hbar^2 \frac{35}{4}$$
$$J_2^2 = \hbar^2 j_2(j_2 + 1) = \hbar^2 \frac{15}{4}$$
Then we compute the dot product for j_1:
$$\mathbf{L} \cdot \mathbf{S} = \frac{1}{2}\left(\hbar^2 \frac{35}{4} - 6\hbar^2 - \frac{3}{4}\hbar^2\right) = \hbar^2$$
And for j_2:
$$\mathbf{L} \cdot \mathbf{S} = \frac{1}{2}\left(\hbar^2 \frac{15}{4} - 6\hbar^2 - \frac{3}{4}\hbar^2\right) = -\hbar^2 \frac{3}{2}$$
Therefore we know:
$$\hat{H}_{so_{j=5/2}} = \lambda \quad , \quad \hat{H}_{so_{j=3/2}} = -\frac{3}{2}\lambda$$
The full Hamiltonian is then dependent on the j-value:
$$\hat{H}_j = \hat{H}_0 + \hat{H}_{so_j}$$

There are two possible values for the m_s, and five possible values for m_l. If $j = 3/2$ then there are $2j + 1 = 4$ possible values for m_j meaning a total degeneracy of 40. If $j = 5/2$ there are $2j + 1 = 6$ possible values giving a total degeneracy of 60.