# Oblig 8

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## Problem 8.3 (H)

### a)

The two lowest shells are completely filled, which in general means having $L = S = 0$. We therefore only look at the p-shells. They value $s = 1/2$ and $l = 2$

$$S_{\text{tot}} = \{0, 1\} \quad , \quad L_{\text{tot}} = \{0, 1, 2\}$$

Creating combinations with these values we discard the symmetrical ones, as electrons are fermions. These states $(L, S)$are therefore:

$$(0, 0), (1, 1), (2, 0)$$

This gives us three possible total spin $J \in \{0, 1, 2\}$. Using the term symbol notation to describe these states we get:

$$^1S_0 \ , \ ^3P_0 \ , \ ^3P_1 \ , \ ^3P_2 \ , \ ^1D_2$$

### b)

Hunds rules says the lowest energy state is the one with the highest $S$ value. This is all the $P$, states. Hunds rule also specifies that for a given degeneracy the lowest energy state belongs to the state with the lowest total angular momentum $J$. This is the $^3P_0$ state. :

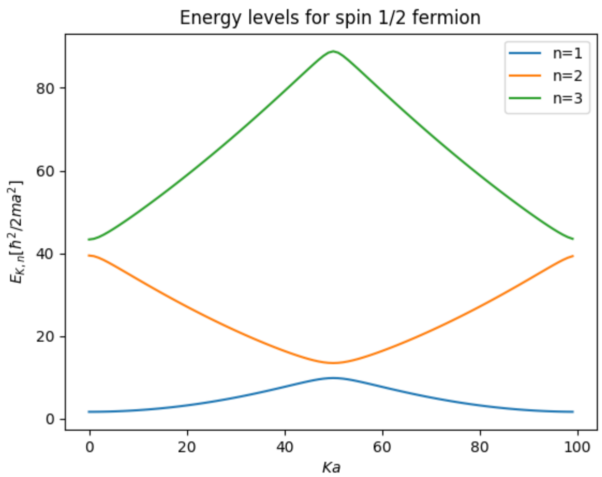
$$^3P_0$$

# Problem 8.4 (H)

## a

Plotting the equation for a range of $z$-values to find the values $z_0$. Using this we can solve the equation using scipy.

## b)




Figur 1

## c)

As there is space for $2N$ electrons in each shell, we know that with $3N$ electrons the lowest shell is filled, and the one above is half-filled. We find the lowest energy by filling the lowest energy level of the second shell. The fermi wave vectors are therefore $K_f = \pi/2$ and $K_f = 3\pi/2$. which occur at $l = 25$ and $l = 75$ respectively. The fermi energy is therefore $E_f \approx 24$

# Problem 8.5 (H)

## a)

The total energy of a harmonic oscillator is given by:

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \quad , \quad n = 0, 1, 2, 3...$$

With a two-dimensional harmonic oscillator we have:

$$E_n = E_{n_1} + E_{n_2} = \hbar\omega(n_1 + n_2 + 1)$$

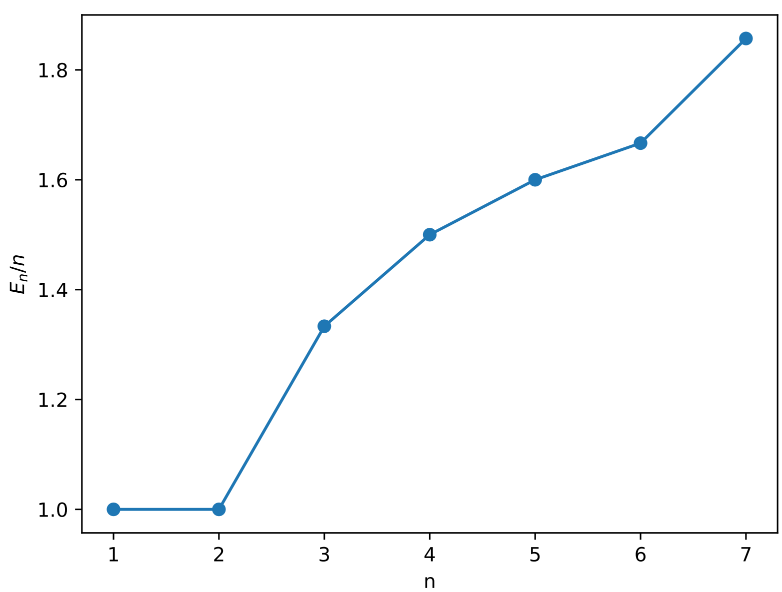
Representing a state using $|n_1 n_2\rangle$. We obviously see the lowest energy is in the state $|00\rangle$. Next we have $|10\rangle$ and $|01\rangle$, which have the same energy. The two lowest energy states are therefore $\hbar\omega$, $2\hbar\omega$ where the latter has a degeneracy of two. The third-lowest level energy levels belongs to $|11\rangle$, $|20\rangle$ and $|02\rangle$ with energy $3\hbar\omega$ and a degeneracy of three.

## b)

| $n$ | $E_n/n$ |
|---|---|
| 1 | 1 |
| 2 | 1 |
| 3 | 4 / 3 |
| 4 | 6 / 4 |
| 5 | 8 / 5 |
| 6 | 10 / 6 |
| 7 | 13 / 7 |

## c)

Inert elements are defined as an element where adding an electron makes the energy per particle to jump. We see this occurs at $n = 2, 6, 12, \ldots$ which continues to make jumps of 6.




Figur 2: Plot of the energy per number of particles in the two-dimensional harmonic oscillator.