Problem Set 2

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Problem 2.1 (L)

$$|\psi\rangle = \sum_{i=1}^{2} \psi_i \, |a_i\rangle = \sum_{i=1}^{2} \psi_i' \, |a_i'\rangle$$

As the two bases are orthonormal, we can express ψ_i in the following manner:

$$\sum_{i=1}^{2} \psi_i = \sum_{i=1}^{2} \langle a_i| \, \psi_i' \, |a_i'\rangle$$

We can then express ψ_i' in terms of ψ_i:

$$\sum_{i=1}^{2} \psi_i' = \sum_{i=1}^{2} \langle a_i'| \, \psi_i \, |a_i\rangle$$

Problem 2.2 (L)

We begin by creating a matrix to convert from the basis $B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ into the new basis of $B' = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$. By getting the combined matrix of the basis vectors into row reduced echelon form.

$$\begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 1 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

The last two columns creates the transformation matrix T.

$$\hat{O}' = T\hat{O}$$

Problem 2.3 (L)

We can see that $\hat{O}^\dagger = \hat{O}$

Problem 2.4 (L)

When an operator acts on its eigenstate, it returns the eigenvalue times the eigenstate.

$$\therefore \langle\lambda| \, \hat{K} \, |u\rangle = \lambda \, \langle\lambda|u\rangle$$