Lecture 4

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Postulates of Quantum Mechanics

1. The ket representing the state is normalizable

$|\phi\rangle$ and $e^{i\theta}|\phi\rangle$ is the same state.

2. Observable quantities are represented by Hermitian operators

Erhenfest Theorem: Expectation values of measurements obey classical eq. of motion.

3. Measurement values of K is the eigenvalues of $\hat{K}$

$$\langle K\rangle_\psi = \sum_{i=1}^{D} \lambda_i P_{\lambda_i} = \sum_{i=1}^{D} \lambda_i \langle\psi|\lambda_i\rangle \langle\lambda_i|\psi\rangle$$

$$\langle\psi| \sum_{i=1}^{D} |\lambda_i\rangle \langle\lambda_i| \lambda_i = \langle\psi| \hat{K} |\psi\rangle$$

4. The probability of getting value* λ when measuring K in state $|\psi\rangle$ is $\langle\psi| \hat{P}_\lambda |\psi\rangle$ where $\hat{P}_\lambda = |\lambda\rangle\langle\lambda|$

Degenerate states:

$$\langle\psi| P_{\lambda_i} |\psi\rangle = \sum_{j=1}^{g} \left\langle\psi\middle|\lambda_i^j\right\rangle \left\langle\lambda_i^j\middle|\psi\right\rangle$$

where g is the degree of degeneracy.
Continuous eigenvalues λ:

$$P(\lambda, \lambda + \Delta\lambda) = \int_{\lambda}^{\lambda+\Delta\lambda} \langle\psi| \underbrace{|\lambda'\rangle\langle\lambda'|}_{\hat{P}_x} |\psi\rangle \, \mathrm{d}\lambda'$$

5. State after an ideal measurement which gave λ is $\propto \hat{P}_\lambda |pdi\rangle$

6. Time evolution: $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Interpretation

Minimal approach (where everyone agrees)

- QM makes statements about observable quantities
- Probabilities corresponds to relative number of occurrences in repeated experiments.

Going Further

Instrumentalist

- Inappropriate to consider an individual system to have definite values for all its physical properties.
- Measurement is fundamental and QM is only concerned with the outcome of measurements.

Realist

- An individual system has definite values for all its physical properties.
- Measurement is not fundamental.
- Probabilities reflect our ignorance of the system.
- QM is incomplete. There need to be an underlying theory explaining the hidden variables

John Bell: Created inequalities that must be be satisfied for such a theory, but are violated by QM.

Compatible Operators

Two operators are compatible if they share a common complete set of eigenkets (not necessarily eigenvalues). They will commute. If they do not commute, they are not compatible.

$$\hat{A}\,|\psi_n\rangle = a_n\,|\psi_n\rangle$$

$$\hat{B}\,|\psi_n\rangle = b_n\,|\psi_n\rangle$$

$$\{|\psi_n\rangle\}\ \text{spans the Hilbert space}$$

$$\hat{A}\hat{B}\,|\psi_n\rangle - \hat{B}\hat{A}\,|\psi_n\rangle = \hat{A}b_n\,|\psi_n\rangle - \hat{B}a_n\,|\psi_n\rangle = (a_n b_n - b_n a_n)\,|\psi_n\rangle = 0$$