Problem 8.1(L)

The electron configuration of Lithium is denoted $(1s)^2(2s)$. Write down a possible wavefunction for the ground state of Lithium based on this electron configuration.

Problem 8.2(L)

What would the atomic number of the three first inert elements (noble gases) be if the electrons happened to have spin-3/2?

Problem 8.3(H)

The electron configuration of carbon is $(1s)^2(2s)^2(2p)^2$.

a) Find the values of the total orbital angular momentum L, total values of spin S and values of the grand total angular momentum J that are allowed by the Pauli principle for carbon. Use the notation ${}^{2S+1}L_J$ to denote your results.

b) Use Hunds rules to determine the total orbital angular momentum L, the total spin S and the grand total angular momentum J in the ground state of carbon.

Problem 8.4(H)

A spin-1/2 fermion with mass m is constrained to move on a ring with circumference L. On the ring it experiences a periodic delta-function potential

$$V(x) = \alpha \sum_j \delta\left(x - ja\right)$$

where $\alpha \geq 0$, a is the distance between delta-functions, j is an integer and $L/a = N$ is a large integer which you can take to be $N = 100$ in this problem.

The energy eigenstates of this system are labelled $|K, n\rangle$ where $K = 2\pi l/Na$, $l \in \{0, 1, \ldots, N-1\}$ is known as the crystal momentum and $n \in \{1, 2, \ldots\}$ is the energy band index. The energy eigenvalues of these states are given by $E_{K,n} = (\hbar^2/2ma^2)z^2$, where z is found by solving the equation

$$\cos Ka = \cos z + \beta \frac{\sin z}{z}$$

where $\beta = ma\alpha/\hbar^2$.

a) Make a computer program (Python or Matlab or ...) that solves this equation numerically to find $E_{K,n}$ for a fixed value of β.

b) Set $\beta = 1$ and make a plot of $E_{K,1}$, $E_{K,2}$ and $E_{K,3}$ (in units of $\hbar^2/2ma^2$) as functions of Ka.

c) Find the fermi wave vector(s) K_f when there are 3 spin-1/2 fermions for each of the N sites. Find also the corresponding fermi energy E_f. Ignore any interactions between the fermions.

Problem 8.5(X)

Consider a two-dimensional harmonic oscillator potential with an oscillator frequency ω.

a) Write down the energies and degeneracies for the three lowest energy levels.

Now put n identical fermions, each with spin-1/2, into the two-dimensional harmonic oscillator potential.

b) Make a table that shows the lowest total energy per particle E_n/n as a function of n for $n \in \{1, 2, \ldots, 7\}$.

c) What are the atomic numbers for the *three* first inert elements (noble gases) in a hypothetical two-dimensional world where electrons are fermions with spin-1/2 and the $1/r$-potential from the nucleus is replaced by a two dimensional r^2-potential? Ignore the electron-electron interaction. Hint: Make a plot of the result in b) and look for the n's where the total energy per particle makes a jump.