Lecture Notes

Oskar Idland

Contents

Lecture 1

1 The Meaning of "Classical"

Meaning non-quantum.

1.1 Analytical Mechanics

- Abstraction of Newton's mechanics, where everything is vectors
- Reduces a vector problem to an energy problem

1.2 Lagrange - Hamiltonian formalism

Lagrangian

$$L = K - V$$

Hamiltonian

$$H = K + V$$

- Algorithmic methods for solving problems
  1. Find the generalized coordinates
  2. Express the kinetic energy in terms of the generalized coordinates
  3. Derive equations of motion (E.O.M) by a recipe
  4. Solve the differential equations.

1.3 Find the Generalized Coordinates

1.3.1 Example: Pendulum in a Plane

- Has length l and mass m
- x/y-coordinates are not the best choice.
- Newton: $m\vec{a} = m\vec{g} + \vec{\downarrow}$ and $|\vec{r}| = l$ with $\vec{r} = (x, y)$

Step 1: Find the generalized coordinates .
There is a **constraint equation**:

$$f(\vec{r}) = |\vec{r}| - l = 0$$

The rope has a **constraint force** $\vec{\downarrow}$, which makes the equation true. We can rewrite $\vec{r}$ using a single coordinate θ.

$$\vec{r} = (x, y) = (l \sin\theta, -l\cos\theta)$$

Step 2: Find the Kinetic Energy .
This lets us define the kinetic energy as follows:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$K = \frac{1}{2}m\left((l\cos\theta \cdot \dot{\theta})^2 + (l\sin\theta \cdot \dot{\theta})^2\right) = \frac{1}{2}ml^2\dot{\theta}^2$$

$$V = mgy = -mgl\cos\theta$$

Generalized Coordinates: Remaining coordinates (degrees of freedom) after the constraint equation has been applied. In a 3D system with N bodies with coordinates $\vec{r}_i$ where $i \in [1, N]$, with M **holonomic constraints** (velocity independent).

$$f_j(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t) = 0 \quad j \in [1, M]$$

The degrees of freedom are $d = 3N - M$ where the number 3 comes from the 3 dimensions. The generalized coordinates are written as q_k with $k \in [1, 2, \ldots, d]$, and the Cartesian coordinates written as:

$$\vec{r}_i = \vec{r}_i(q_1, q_2, \ldots, q_d, t) \quad | \quad \vec{r}_i = \vec{r}_i(q, t) \quad , q = \{q_1, q_2, \ldots q_d\}$$

Notice how the generalized coordinates are not vectors, but the Cartesian coordinates are.

Configuration Space The generalized coordinates $q = \{q_1, q_2, \ldots q_d\}$ of a d-dimensional space (manifold) called **configuration space**, With N bodies which position are given by the $3N$-dimenstional vector $\vec{R} = (\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N)$. In generalized coordinates $\vec{R} = \vec{R}(q_1, q_2, \ldots q_d, t)$. This is a hyper-surface of dimension $d = 3N - M$ in the $3N$-dimensional space with $q = \{q_1, q_2, \ldots q_d\}$.

Time independent constraints

- Time independent constraint equations
- The surface is fixed
- The time evolution of $\vec{R}$ is a curve on the surface $\vec{R}(t) = \vec{R}(q(t))$ and $\vec{V} = \vec{R} = \frac{\mathrm{d}\vec{R}}{\mathrm{d}t}$
- At a fixed point in time, moving the coordinates q a small amount gives a vector tangent to the surface.

The Cartesian coordinates are dependent on the general coordinates which are time dependent. This is solved by differential equations.