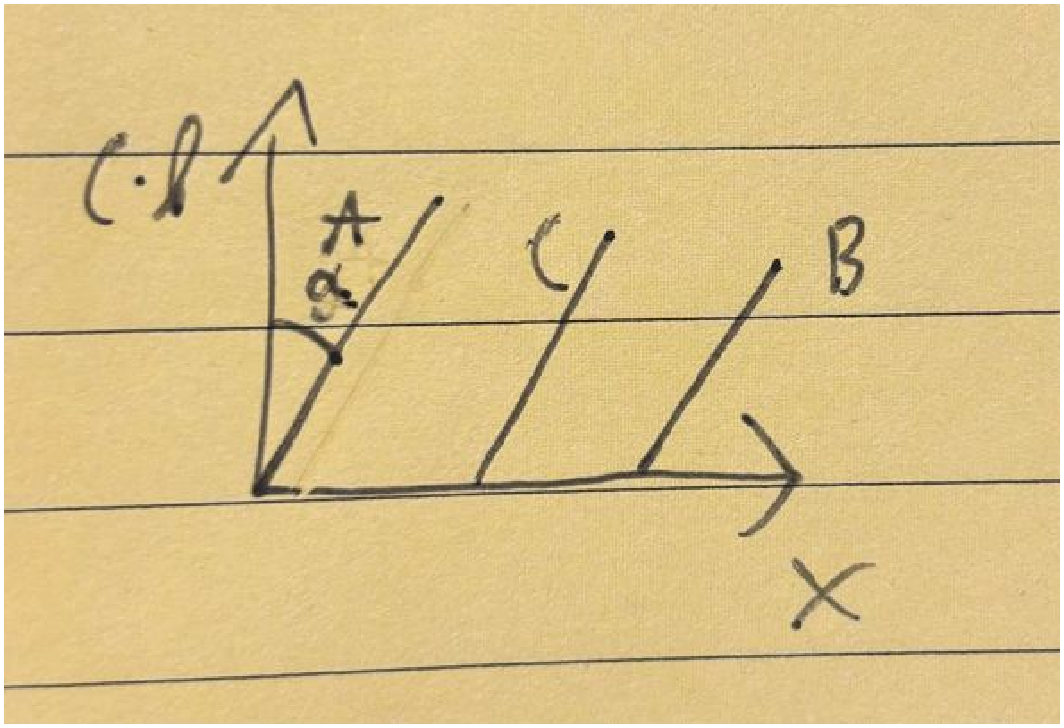
Oblig 5

Oskar Idland

Problem 1

a)



Figur 1: Minkowski diagram

The velocity is represented by the angle between the y-axis and the world line. We have constant velocity v and can use the fact that the world line passes through the origin. Therefore, the velocity is given by $v = x_A(t)/t$. We can represent the slope of the world line as $\tan\alpha = v/c$.

b)

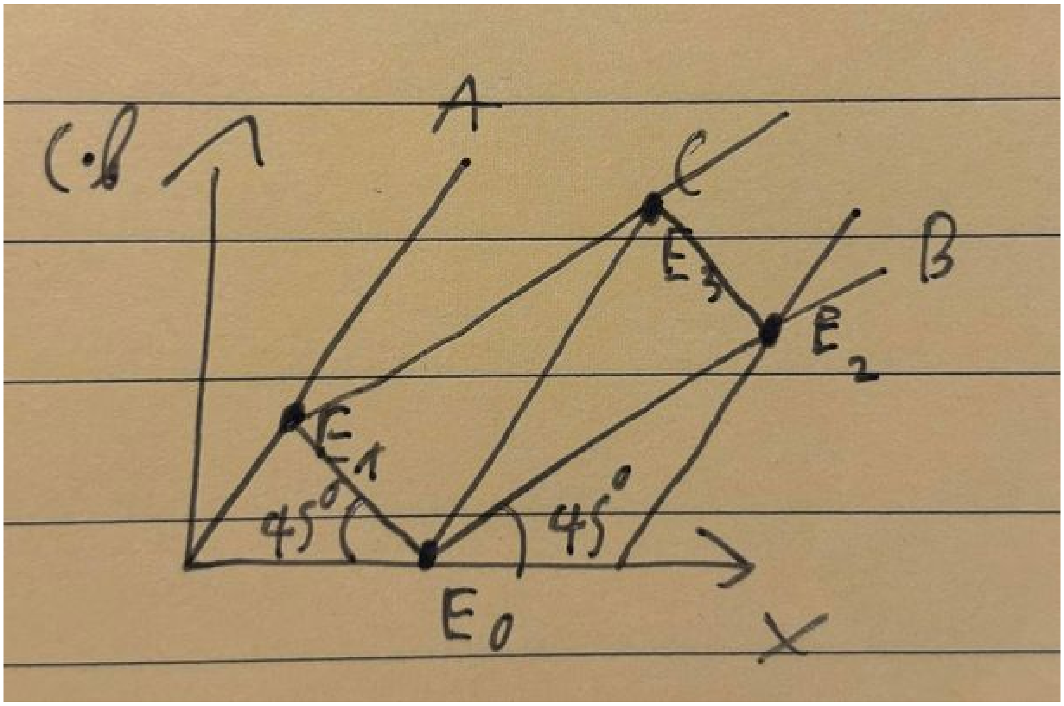
As light travels at the speed of light, it will appear as a 45° angle.

c)

From the perspective of S', it would seem like none of the points where moving. The light travels an equal distance in event E_1 and E_2 and will therefore appear simultaneous. The drawing from problem b, is not correct from this POV.

d)

See figure 2.



Figur 2: Minkowski diagram with events

e)

Problem 2

a)

As the rod is moving along the y-axis, there will not be any length contraction in the x-direction. Point B is located at the end of the rod, with a distance to point A of $L_0/2$. This gives us $x'_B = L_0/2$. Both y coordinates move at velocity u. It is only represented by a different time coordinate. We therefore get $y'_B = ut'_B$. As we neither move in the z-direction nor have different values in each point, their coordinates are the same, meaning $z'_B = z'_A = 0$.

b)

Using the Lorentz transformation, we get the following:

$$x_A = \gamma(x'_a + vt'_A) = \gamma v t'_A$$

$$x_B = \gamma(x'_b + vt'_B) = \gamma(L_0/2 + vt'_B)$$

$$y_A = ut'_A$$

$$y_B = ut'_B$$

$$z_A = z_B = 0$$
$$t_A = \gamma(t'_A + vx'_A/c^2) = \gamma t'_A$$
$$t_B = \gamma(t'_B + vx'_B/c^2) = \gamma(t'_B + v(L_0/2)/c^2)$$

Now we must convert from the primed to the unprimed system. We can use the following:

$$t'_A = t_A/\gamma$$

$$t'_B = t_B/\gamma - vL_0/(2c^2)$$

We can now insert these into the equations for x_A and x_B:

$$x_A = \gamma v(t_A/\gamma) = vt_A$$

$$x_B = \gamma(L_0/2 + v(t_B/\gamma - vL_0/(2c^2))) = \gamma L_0/2 + vt_B - \gamma v^2 L_0/(2c^2)$$

And for y_A and y_B:

$$y_A = ut_A/\gamma$$
$$y_B = ut_B/\gamma - uvL_0/(2c^2)$$

c)

$$\tan\phi = \frac{y_B - y_A}{x_B - x_A}$$

We insert the expressions for x_A, x_B, y_A and y_B:

$$\tan\phi = -\gamma uv/c^2$$

As long as $v, u \neq 0$ we get an angle different from 0, meaning the rod is not parallel to the x-axis.
The length of the rod is just the absolute value of the difference between the start (x_A, y_A) and end (x_B, y_B) of the rod.

$$L/2 = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$L = 2\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$L = 2\sqrt{L_0^2/(4\gamma^2) + u^2v^2L_0^2/(4c^4\gamma^2)}$$
$$L = L_0\sqrt{1/\gamma^2 + u^2v^2/c^4}$$