TRAINING SET 1, FYS3140 spring 2020

Problem T1.1 (Refreshing some basics: Euler's formula, powers and roots)

a) Express the number $\sqrt{2}e^{5i\pi/4}$ in $x + iy$ form.

b) Express the number $\frac{(1+i)^{48}}{(\sqrt{3}-i)^{25}}$ in $x + iy$ form.

c) Express the number $\frac{\exp(1+3\pi i)}{\exp(-1+i\pi/2)}$ in $x + iy$ form.

d) Find all values of the root $(8i\sqrt{3} - 8)^{1/4}$.

e) Show that the sum of the three cube roots of 8 is zero. Then show that the sum of the n nth roots of *any* complex number is zero.

f) Find all roots of $i^{1/3}$ in polar form

g) Find the two roots w_1 and w_2 of $\sqrt{2 + 2i\sqrt{3}}$ in polar form. Then write them in $x + iy$ form and show that $w_2 = -w_1$ as expected.

Problem T1.2 (Complex power series)

a) Find the disk of convergence for $\sum_{n=0}^{\infty} n(n + 1)(z - 2i)^n$

b) *[Example from lecture]* Write down the power series representation of the complex exponential function e^z and show that its disk of convergence is the entire complex plane.

Problem T1.3 (Elementary functions)

In **a)** and **b)**, use the definitions of $\sin z, \cos z, \sinh z,$ and $\cosh z$ in terms of exponential functions to show that

a) $\sin 2z = 2 \sin z \cos z$.

b) $\cosh^2 z - \sinh^2 z = 1$.

c) Use a series you know to show that $\sum_{n=0}^{\infty} \frac{(1+i\pi)^n}{n!} = -e$