MAT1120 Eksamen 2013

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Oppgave 1

a)

$$A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix}, \quad C = \text{rref}(A) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

From C we know that only the three first columns in A are linearly independent which means its rank is 3. The Null space must therefore be 1-dimensional. To find the basis for the null space we can use the following equation:

$$A\mathbf{x} = \mathbf{0}$$

With the help of matrix C we can solve this equation easily

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = x_4 \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

The basis of the Null space is therefore

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

b)

To find and orthogonal basis of the Column space we can use the Gram-Schmidt process. We start by finding the first vector in the basis. This is the first column in A which is

$$\mathbf{v_1} = \mathbf{x_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \mathbf{x_2} - \text{proj}_{\mathbf{x_2}}(\mathbf{v_1}) = \mathbf{x_2} - \frac{\mathbf{x_2} \cdot \mathbf{v_1}}{\mathbf{v_1}, \mathbf{v_1}} \mathbf{v_1} = \mathbf{x_2} - \frac{0}{3}\mathbf{v_1} = \mathbf{x_2} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{v_3} = \mathbf{x_3} - \text{proj}_{\mathbf{x_3}}(\mathbf{v_1}) - \text{proj}_{\mathbf{x_3}}(\mathbf{v_2}) = \mathbf{x_3} - \frac{\mathbf{x_3} \cdot \mathbf{v_1}}{\mathbf{v_1} \cdot \mathbf{v_1}} \mathbf{v_1} - \frac{\mathbf{x_3} \cdot \mathbf{v_2}}{\mathbf{v_2} \cdot \mathbf{v_2}} \mathbf{v_2} = \mathbf{x_3} - \frac{1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{0}{3} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{v_3} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix}$$

$$\mathbf{v_3}' = 3\mathbf{v_3} = \begin{pmatrix} -1 \\ 2 \\ -1 \\ 3 \end{pmatrix}$$

$$\mathbf{v_4} = \mathbf{x_4} - \mathrm{proj}_{\mathbf{x_4}}(\mathbf{v_1}) - \mathrm{proj}_{\mathbf{x_4}}(\mathbf{v_2}) - \mathrm{proj}_{\mathbf{x_4}}(\mathbf{v_3}')$$

$$\mathbf{v_4} = \mathbf{x_4} - \frac{\mathbf{x_4} \cdot \mathbf{v_1}}{\mathbf{v_1} \cdot \mathbf{v_1}}\mathbf{v_1} - \frac{\mathbf{x_4} \cdot \mathbf{v_2}}{\mathbf{v_2} \cdot \mathbf{v_2}}\mathbf{v_2} - \frac{\mathbf{x_4} \cdot \mathbf{v_3}'}{\mathbf{v_3}' \cdot \mathbf{v_3}'}\mathbf{v_3}'$$

$$\mathbf{v_4} = \mathbf{x_4} - \frac{4}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{3}{3}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} - \frac{5}{15}\begin{pmatrix} -1 \\ 2 \\ -1 \\ 3 \end{pmatrix}$$

$$\mathbf{v_4} = \mathbf{x_4} - \frac{1}{3}\begin{pmatrix} -4-1+1 \\ -4+1-2 \\ -4+0+1 \\ 0-1-3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} -4 \\ -5 \\ -3 \\ -3 \end{pmatrix}$$

$$\mathbf{v_4} = \frac{1}{3}\begin{pmatrix} 6+4 \\ 3+5 \\ 3+3 \\ 6+3 \end{pmatrix}$$

$$\mathbf{v_4} = \frac{1}{3}\begin{pmatrix} 10 \\ 8 \\ 5 \\ 9 \end{pmatrix}$$