Overdispersion, and how to deal with it in R and JAGS

(requires R-packages AER, coda, lme4, R2jags, DHARMa/devtools)

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1 Introduction: what is overdispersion?

Overdispersion describes the observation that variation is higher than would be expected. Some distributions do not have a parameter to fit variability of the observation. For example, the *normal distribution* does that through the parameter σ (i.e. the standard deviation of the model), which is constant in a typical regression. In contrast, the *Poisson distribution* has no such parameter, and in fact the variance increases with the mean (i.e. the variance and the mean have the same value). In this latter case, for an expected value of E(y) = 5, we also expect that the variance of observed data points is 5. But what if it is not? What if the observed variance is much higher, i.e. if the data are overdispersed? (Note that it could also be lower, underdispersed. This is less often the case, and not all approaches below allow for modelling underdispersion, but some do.)

Overdispersion arises in different ways, most commonly through "clumping". Imagine the number of seedlings in a forest plot. Depending on the distance to the source tree, there may be many (hundreds) or none. The same goes for shooting stars: either the sky is empty, or littered with shooting stars. Such data would be overdispersed for a Poisson distribution. Also, overdispersion arises "naturally" if important predictors are missing or functionally misspecified (e.g. linear instead of non-linear).

Overdispersion is often mentioned together with zero-inflation, but it is distinct. Overdispersion also includes the case where none of your data points are actually 0. We'll look at zero-inflation later, and stick to overdispersion here.

2 Recognising (and testing for) overdispersion

May we should start with an example to get the point visualised. Note that we manually set the breaks to 1-unit bins, so that we can see the 0s as they are, not pooled with 1s, 2s, etc.

```
library(lme4)
data(grouseticks)
summary(grouseticks)
```

	INDEX	TICKS	BROOD	HEIGHT	YEAR	LOCATION
1	: 1	Min. : 0.00	606 : 10	Min. :403.0	95:117	14 : 24
2	: 1	1st Qu.: 0.00	602 : 9	1st Qu.:430.0	96:155	4 : 20
3	: 1	Median: 2.00	537 : 7	Median :457.0	97:131	19 : 20
4	: 1	Mean : 6.37	601 : 7	Mean :462.2		28 : 19
5	: 1	3rd Qu.: 6.00	643 : 7	3rd Qu.:494.0		50 : 17
6	: 1	Max. :85.00	711 : 7	Max. :533.0		36 : 16
(Other):397			(Other):356			(Other):287
	- HET AHT					

cHEIGHT

Min.:-59.241
1st Qu::-32.241
Median:-5.241
Mean:0.000
3rd Qu::31.759
Max.:70.759

INDEX is individual

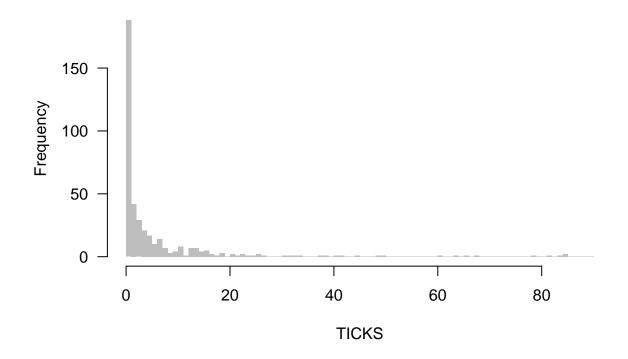
head(grouseticks)

	INDEX	TICKS	BROOD	HEIGHT	YEAR	LOCATION	cHEIGHT
1	1	0	501	465	95	32	2.759305
2	2	0	501	465	95	32	2.759305
3	3	0	502	472	95	36	9.759305
4	4	0	503	475	95	37	12.759305
5	5	0	503	475	95	37	12.759305
6	6	3	503	475	95	37	12.759305

attach(grouseticks)

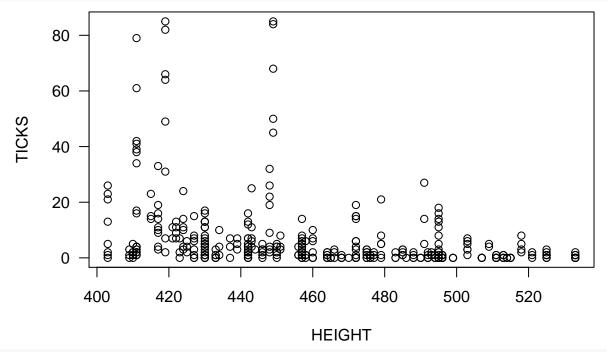
hist(TICKS, col="grey", border=NA, las=1, breaks=0:90)

Histogram of TICKS



The data are rich in 0s, but that does not mean they are 0-inflated. We'll find out about overdispersion by fitting the Poisson-model and looking at deviance and degrees of freedom (as a rule of thumb):

plot(TICKS ~ HEIGHT, las=1)



summary(fmp <- glm(TICKS ~ HEIGHT*YEAR, family=poisson))</pre>

Call:

glm(formula = TICKS ~ HEIGHT * YEAR, family = poisson)

Deviance Residuals:

Min 1Q Median 3Q Max -6.0993 -1.7956 -0.8414 0.6453 14.1356

Coefficients:

Estimate Std. Error z value Pr(>|z|)(Intercept) 27.454732 1.084156 25.32 <2e-16 *** -22.92 **HEIGHT** -0.058198 0.002539 <2e-16 *** -16.66 YEAR96 -18.994362 1.140285 <2e-16 *** YEAR97 -12.29 -19.247450 1.565774 <2e-16 *** HEIGHT: YEAR96 0.044693 0.002662 16.79 <2e-16 *** HEIGHT: YEAR97 0.040453 0.003590 11.27 <2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 5847.5 on 402 degrees of freedom Residual deviance: 3009.0 on 397 degrees of freedom

AIC: 3952

Number of Fisher Scoring iterations: 6

In this case, our residual deviance is 3000 for 397 degrees of freedom. The rule of thumb is that the ratio of deviance to df should be 1, but it is 7.6, indicating severe overdispersion. This can be done more formally, using either package **AER** or **DHARMa**:

```
library(AER)
dispersiontest(fmp)
```

Overdispersion test

```
data: fmp z = 4.3892, p-value = 5.69e-06 alternative hypothesis: true dispersion is greater than 1 sample estimates: dispersion 10.57844
```

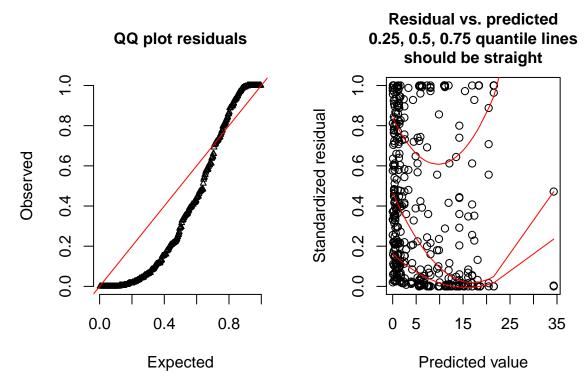
The value here is higher than 7.5 (remember, it was a rule of thumb!), but the result is the same: substantial overdispersion. Same thing in **DHARMa** (where we can additionally visualise overdispersion):

```
library(devtools) # assuming you have that
devtools::install_github(repo = "DHARMa", username = "florianhartig", subdir = "DHARMa")
library(DHARMa)
sim_fmp <- simulateResiduals(fmp, refit=T)
testOverdispersion(sim_fmp)</pre>
```

Overdispersion test via comparison to simulation under HO

```
data: sim_fmp
dispersion = 11.326, p-value < 2.2e-16
alternative hypothesis: overdispersion
plotSimulatedResiduals(sim_fmp)</pre>
```

DHARMa scaled residual plots



DHARMa works by simulating new data from the fitted model, and then comparing the observed data to those simulated (see DHARMa's nice vignette for an introduction to the idea).

3 "Fixing" overdispersion

Overdispersion means the assumptions of the model are not met, hence we cannot trust its output (e.g. our beloved P-values)! Let's do something about it.

3.1 Quasi-families

The quasi-families augment the normal families by adding a dispersion parameter. In other words, while for Poisson data $\bar{Y} = s_Y^2$, the quasi-Poisson allows for $\bar{Y} = \tau \cdot s_Y^2$, and estimates the overdispersion parameter τ (or underdispersion, if $\tau < 1$).

```
summary(fmqp <- glm(TICKS ~ YEAR*HEIGHT, family=quasipoisson, data=grouseticks))</pre>
```

```
Call:
glm(formula = TICKS ~ YEAR * HEIGHT, family = quasipoisson, data = grouseticks)
Deviance Residuals:
                   Median
              1Q
                                 3Q
                                         Max
-6.0993
        -1.7956
                  -0.8414
                                    14.1356
                             0.6453
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
               27.454732
                                       7.524 3.58e-13 ***
(Intercept)
                           3.648824
```

```
YEAR96
              -18.994362
                           3.837731
                                     -4.949 1.10e-06 ***
YEAR97
              -19.247450
                                     -3.652 0.000295 ***
                           5.269753
HEIGHT
               -0.058198
                           0.008547
                                     -6.809 3.64e-11 ***
YEAR96: HEIGHT
                0.044693
                           0.008959
                                      4.988 9.12e-07 ***
YEAR97: HEIGHT
                0.040453
                           0.012081
                                      3.349 0.000890 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for quasipoisson family taken to be 11.3272)
   Null deviance: 5847.5 on 402 degrees of freedom
                                   degrees of freedom
Residual deviance: 3009.0 on 397
AIC: NA
```

Number of Fisher Scoring iterations: 6

You see that τ is estimated as 11.3, a value similar to those in the overdispersion tests above (as you'd expect). The main effect is the substantially larger errors for the estimates (the point estimates do not change), and hence potentially changed significances (though not here). (You can manually compute the corrected standard errors as Poisson-standard errors $\cdot \sqrt{\tau}$.) Note that because this is no maximum likelihood method (but a quasi-likelihood method), no likelihood and hence no AIC are available. No overdispersion tests can be conducted for quasi-family objects (neither in **AER** nor **DHARMa**).

3.2 Different distribution (here: negative binomial)

Maybe our distributional assumption was simply wrong, and we choose a different distribution. For Poisson, the most obvious "upgrade" is the negative binomial, which includes in fact a dispersion parameter similar to τ above.

```
library(MASS)
summary(fmnb <- glm.nb(TICKS ~ YEAR*HEIGHT, data=grouseticks))</pre>
Call:
glm.nb(formula = TICKS ~ YEAR * HEIGHT, data = grouseticks, init.theta = 0.9000852793,
    link = log)
Deviance Residuals:
    Min
              1Q
                   Median
                                 3Q
                                         Max
        -1.0281 -0.5052
-2.3765
                            0.2408
                                      3.2440
Coefficients:
                Estimate Std. Error z value Pr(>|z|)
               20.030124
                           1.827525
                                     10.960 < 2e-16 ***
(Intercept)
YEAR96
              -10.820259
                                     -4.944 7.66e-07 ***
                           2.188634
YEAR97
              -10.599427
                                     -4.193 2.75e-05 ***
                           2.527652
               -0.041308
HEIGHT
                           0.004033 -10.242 < 2e-16 ***
YEAR96: HEIGHT
                0.026132
                           0.004824
                                       5.418 6.04e-08 ***
YEAR97:HEIGHT
                0.020861
                           0.005571
                                       3.745 0.000181 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for Negative Binomial(0.9001) family taken to be 1)
```

Null deviance: 840.71 on 402 degrees of freedom Residual deviance: 418.82 on 397 degrees of freedom

AIC: 1912.6

Number of Fisher Scoring iterations: 1

Theta: 0.9001 0.0867 Std. Err.:

2 x log-likelihood: -1898.5880

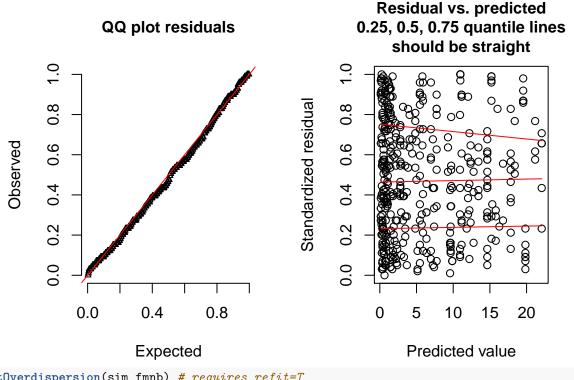
Already here we see that the ratio of deviance and df is near 1 and hence probably fine. Let's check:

try(dispersiontest(fmnb))

That's a bit disappointing. Well, we'll use **DHARMa** then.

```
sim_fmnb <- simulateResiduals(fmnb, refit=T, n=99)</pre>
plotSimulatedResiduals(sim_fmnb)
```

DHARMa scaled residual plots



testOverdispersion(sim_fmnb) # requires refit=T

Overdispersion test via comparison to simulation under HO

data: sim_fmnb

dispersion = 1.322, p-value < 2.2e-16 alternative hypothesis: overdispersion These figures show what it should look like!

3.3 Observation-level random effects (OLRE)

The general idea is to allow the expectation to vary more than a Poisson distribution would suggest. To do so, we multiply the Poisson-expectation with an overdispersion parameter (larger 1), along the lines of

$$Y \sim Pois(\lambda = e^{\tau} \cdot E(Y)) = Pois(\lambda = e^{\tau} \cdot e^{aX+b}),$$

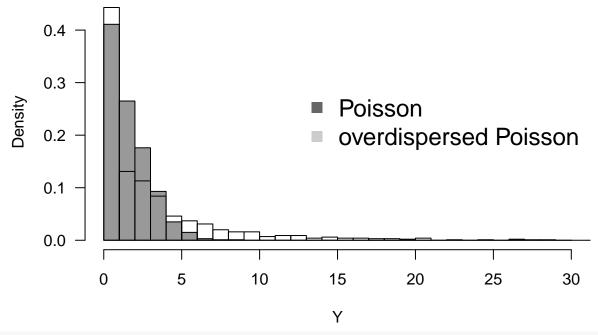
where expectation E(Y) is the prediction from our regression. Without overdispersion, $\tau = 0$. We use e^{τ} to force this factor to be positive.

You may recall that the Poisson-regression uses a log-link, so we can reformulate the above formulation to

$$Y \sim Pois(\lambda = e^{\tau} \cdot e^{aX+b}) = Pois(\lambda = e^{aX+b+\tau}).$$

So the overdispersion multiplier at the response-scale becomes an overdispersion summand at the log-scale.

That means, we can add another predictor to our model, one which changes with each value of Y, and which we do not really care for: a random effect. Remember that a (Gaussian) random effect has a mean of 0 and its standard deviation is estimated from the data. How does that work? Well, if we expected a value of, say, 2, we add noise to this value, and hence increase the range of values realised.



var(Y1); var(Y2)

[1] 2.074954

[1] 23.52102

We see that with an overdispersion modelled as observation-level random effect with mean= 0 and an innocent-looking sd= 1, we increase the spread of the distribution substantially. In this case both more 0s and more high values, i.e. more variance altogether.

So, in fact modelling overdispersion as OLRE is very simple: just add a random effect which is different for each observation. In our data set, the column INDEX is just a continuously varying value from 1 to N, which we use as random effect.

```
library(lme4)
summary(fmOLRE <- glmer(TICKS ~ YEAR*HEIGHT + (1|INDEX), family=poisson, data=grouseticks))</pre>
Generalized linear mixed model fit by maximum likelihood (Laplace Approximation) ['glmerMod']
Family: poisson (log)
Formula: TICKS ~ YEAR * HEIGHT + (1 | INDEX)
  Data: grouseticks
     AIC
             BIC
                   logLik deviance df.resid
                   -944.5
  1903.0
          1931.0
                            1889.0
Scaled residuals:
   Min
            1Q Median
                            3Q
                                   Max
-1.2977 -0.5020 -0.0659 0.2241 1.9138
Random effects:
Groups Name
                   Variance Std.Dev.
 INDEX (Intercept) 1.132
                            1.064
Number of obs: 403, groups: INDEX, 403
Fixed effects:
               Estimate Std. Error z value Pr(>|z|)
              (Intercept)
YEAR96
             -10.435391 2.274773 -4.587 4.49e-06 ***
YEAR97
              -9.469675 2.668900 -3.548 0.000388 ***
HEIGHT
              -0.040492  0.004149  -9.760  < 2e-16 ***
YEAR96:HEIGHT 0.025381
                          0.005018 5.058 4.24e-07 ***
YEAR97:HEIGHT 0.018682
                          0.005885 3.175 0.001500 **
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Correlation of Fixed Effects:
            (Intr) YEAR96 YEAR97 HEIGHT YEAR96:
YEAR96
            -0.825
YEAR97
           -0.703 0.581
           -0.997 0.822 0.701
HEIGHT
YEAR96:HEIG 0.824 -0.997 -0.580 -0.825
YEAR97:HEIG 0.702 -0.581 -0.997 -0.703 0.582
convergence code: 0
Model failed to converge with max|grad| = 0.0175048 (tol = 0.001, component 1)
Model is nearly unidentifiable: very large eigenvalue
 - Rescale variables?
Model is nearly unidentifiable: large eigenvalue ratio
 - Rescale variables?
Oops! What's that? So it converged ("convergence code: 0"), but apparently the algorithm is "unhappy".
Let's follow its suggestion and scale the numeric predictor:
height <- scale(grouseticks$HEIGHT)</pre>
summary(fmOLRE <- glmer(TICKS ~ YEAR*height + (1|INDEX), family=poisson, data=grouseticks))</pre>
```

Generalized linear mixed model fit by maximum likelihood (Laplace Approximation) ['glmerMod']

```
Family: poisson (log)
Formula: TICKS ~ YEAR * height + (1 | INDEX)
  Data: grouseticks
     AIC
             BIC
                   logLik deviance df.resid
  1903.0
           1931.0
                   -944.5
                             1889.0
                                         396
Scaled residuals:
     Min
               10
                   Median
                                 3Q
                                         Max
-1.29773 -0.50197 -0.06591 0.22414 1.91377
Random effects:
Groups Name
                   Variance Std.Dev.
 INDEX (Intercept) 1.132
                             1.064
Number of obs: 403, groups: INDEX, 403
Fixed effects:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)
                           0.1426
                                   2.395 0.01663 *
               0.3414
YEAR96
                1.2967
                           0.1705
                                   7.607 2.81e-14 ***
                           0.2006 -4.157 3.23e-05 ***
YEAR97
               -0.8340
height
               -1.4559
                           0.1494 -9.746 < 2e-16 ***
YEAR96:height
               0.9126
                           0.1807
                                   5.050 4.42e-07 ***
YEAR97:height
               0.6717
                           0.2117
                                   3.173 0.00151 **
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Correlation of Fixed Effects:
            (Intr) YEAR96 YEAR97 height YEAR96:
YEAR96
            -0.813
YEAR97
            -0.668 0.562
height
            0.312 -0.258 -0.216
YEAR96:hght -0.251
                   0.298 0.180 -0.826
YEAR97:hght -0.206 0.177 0.291 -0.704 0.583
```

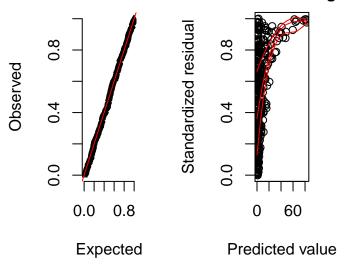
In the random-effects output, we see that the standard deviation for the random effect is around 1.06, i.e. similar to what we have simulated above. The overdispersion is thus substantial. Note that the estimates for intercept, YEAR96 and YEAR97 are *substantially* different (as is height, but then that has been re-scaled).

Here's the diagnostic plot (only **DHARMa**):

```
sim_fmOLRE <- simulateResiduals(fmOLRE, refit=T, n=250) # takes a while, about 10 minutes or so
plotSimulatedResiduals(sim_fmOLRE)</pre>
```

DHARMa scaled residual plots

Residual vs. predict QQ plot residuals 0.25, 0.5, 0.75 quantile should be straight



testOverdispersion(sim_fmOLRE) # requires refit=T

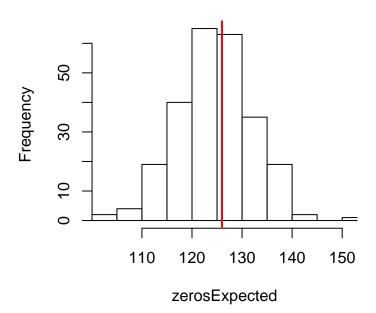
Overdispersion test via comparison to simulation under HO

data: sim_fmOLRE

dispersion = 1.1028, p-value = 0.1
alternative hypothesis: overdispersion

testZeroInflation(sim_fmOLRE) # no zero-inflation

Histogram of zerosExpected



Zero-inflation test via comparison to expected zeros with simulation under HO

```
data: sim_fmOLRE
ratioObsExp = 1.0075, p-value = 0.432
alternative hypothesis: more
```

Hm. The QQ-plot looks great, but the residual-predicted-plot is miserable. This may be due to a misspecified model (e.g. missing important predictors), leading to underfitting of the high values (all large values have high quantiles, indicating that these residuals (O-E) are all positive and large).

The overdispersion, and just for fun also the zero-inflation test, are negative. So overall I guess that the OLRE-model is fine.

We can finally compare the actual fit of all models:

```
AIC(fmp, fmnb, fm0LRE)

df AIC
fmp 6 3951.963
fmnb 7 1912.588
fm0LRE 7 1902.998

And the winner is: OLRE!
```

4 Overdispersion in JAGS

library(R2jags)

nc <- 3; nt <- 10

prepare data for JAGS:

In JAGS, we follow the OLRE-approach (we could also fit a negative binomial, of course, but the illustration of the OLRE is much nicer for understanding the workings of JAGS).

First, we need to prepare the data for JAGS, define which parameters to monitor and the settings for sampling, and write an (optional) inits-function. Then we define the actual overdispersion model.

```
# There is a convenient function to do this for us, and it includes interactions, too!
Xterms <- model.matrix(~ YEAR*height, data=grouseticks)[,-1]</pre>
head(Xterms)
  YEAR96 YEAR97
                   height YEAR96:height YEAR97:height
1
              0 0.0767401
                                       0
2
              0 0.0767401
                                       0
                                                      0
       0
3
       0
              0 0.2714198
                                       0
                                                      0
4
       0
              0 0.3548539
                                       0
                                                      0
5
       0
              0 0.3548539
                                       0
                                                      0
              0 0.3548539
6
                                       0
# The "[,-1]" removes the intercept that would automatically be produced.
grouseticksData <- list(TICKS=grouseticks$TICKS, YEAR96=Xterms[,1], YEAR97=Xterms[,2], HEIGHT=Xterms[,3]
                         INT96=Xterms[,4], INT97=Xterms[,5], N=nrow(grouseticks))
parameters <- c("alpha", "beta", "tau") # which parameters are we interested in getting reported?, "la
```

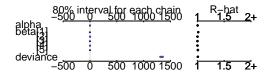
inits <- function(){list(OLRE=rnorm(nrow(grouseticks), 0, 2), tau=runif(1, 0,.005),</pre>

ni <- 1E4; nb <- ni/2 # number of iterations; number of burnins

number of chains; thinning

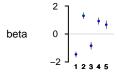
```
alpha=runif(1, 0, 2), beta = rnorm(5))}
OLRE <- function() {</pre>
  for(i in 1:N){ # loop through all data points
   TICKS[i] ~ dpois(lambda[i])
   log(lambda[i]) <- alpha + beta[1]*HEIGHT[i] + beta[2]*YEAR96[i] + beta[3]*YEAR97[i] +
      beta[4]*INT96[i] + beta[5]*INT97[i] + OLRE[i]
    # "OLRE" is random effect for each individual observation
    # alternatively, multiply lambda[i] by exp(OLRE[i]) in the ~ dpois line.
  # priors:
  for (m in 1:5){
   beta[m] ~ dnorm(0, 0.01)
                             # Linear effects
 alpha ~ dnorm(0, 0.01) # overall model intercept
 for (j in 1:N){
   OLRE[j] ~ dnorm(0, tau) # random effect for each nest
 tau ~ dgamma(0.001, 0.001) # prior for mixed effect precision
}
Now we can run JAGS, print and plot the results:
OLREjags <- jags(grouseticksData, inits=inits, parameters, model.file = OLRE, n.chains = nc,
                 n.thin = nt, n.iter = ni, n.burnin = nb, working.directory = getwd())
Compiling model graph
  Resolving undeclared variables
   Allocating nodes
Graph information:
  Observed stochastic nodes: 403
  Unobserved stochastic nodes: 410
  Total graph size: 4183
Initializing model
plot(OLREjags)
```

ders/cc/3jfhfx190rb2ptxnqrqxj94m0000gp/T//Rtmpt9GLu7/model954f4b52291b.txt", fit using jags, 3 chains, each with 10000 iteration:



medians and 80% intervals







OLREjags

Inference for Bugs model at "/var/folders/cc/3jfhfx190rb2ptxnqrqxj94m0000gp/T//Rtmpt9GLu7/model954f4b52
3 chains, each with 10000 iterations (first 5000 discarded), n.thin = 10
n.sims = 1500 iterations saved

	mu.vect	${\tt sd.vect}$	2.5%	25%	50%	75%	97.5%	Rhat	n.eff
alpha	0.322	0.150	0.027	0.226	0.328	0.418	0.597	1.005	470
beta[1]	-1.466	0.172	-1.770	-1.570	-1.459	-1.360	-1.167	1.018	370
beta[2]	1.309	0.187	0.969	1.188	1.306	1.423	1.647	1.004	580
beta[3]	-0.841	0.210	-1.229	-0.979	-0.841	-0.710	-0.432	1.003	700
beta[4]	0.925	0.199	0.565	0.796	0.924	1.040	1.283	1.004	770
beta[5]	0.663	0.250	0.224	0.509	0.665	0.809	1.088	1.011	1500
tau	0.853	0.106	0.669	0.787	0.849	0.918	1.056	1.003	1500
deviance	1337.698	97.692	1285.103	1315.542	1333.755	1350.141	1387.885	1.010	1500

For each parameter, n.eff is a crude measure of effective sample size, and Rhat is the potential scale reduction factor (at convergence, Rhat=1).

DIC info (using the rule, pD = var(deviance)/2) pD = 4778.0 and DIC = 6115.7

DIC is an estimate of expected predictive error (lower deviance is better).

OLREjags\$BUGSoutput\$mean # just the means

\$alpha

[1] 0.3215037

\$beta

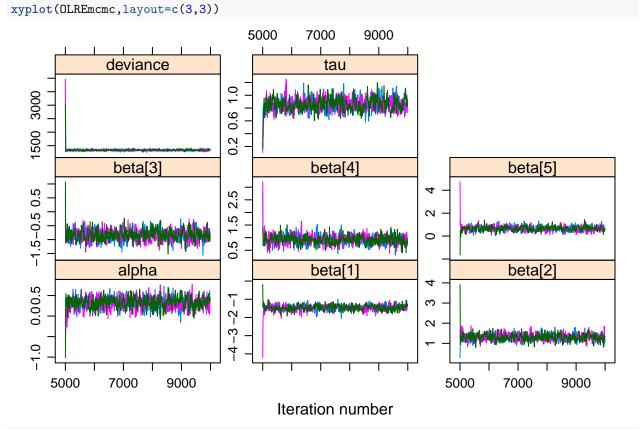
[1] -1.4660684 1.3087484 -0.8405339 0.9247887 0.6632121

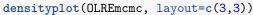
```
$deviance [1] 1337.698
```

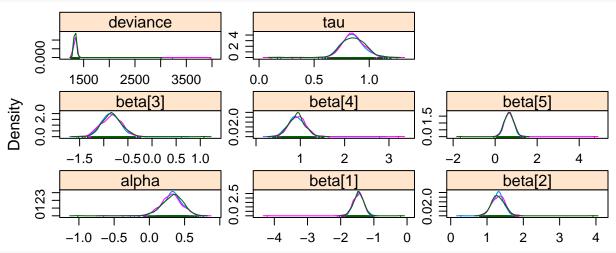
\$tau

[1] 0.85284

OLREmcmc <- as.mcmc.list(OLREjags\$BUGSoutput)
library(lattice)</pre>







gelman.diag(OLREmcmc)

Potential scale reduction factors:

	Point	est.	Upper	C.I.
alpha		1.01		1.02
beta[1]		1.02		1.03
beta[2]		1.01		1.02
beta[3]		1.00		1.01
beta[4]		1.01		1.01
beta[5]		1.01		1.01
deviance		1.03		1.03
tau		1.00		1.00

Multivariate psrf

1.01

This JAGS-object is not directly amenable to overdispersion-diagnostics with **DHARMa** (but see the experimental function createDHARMa). We can do something manually ourselves, but it is not identical to the output we had looked at before. I shall therefore leave it out here.