

Statistics Café SoSe 2018

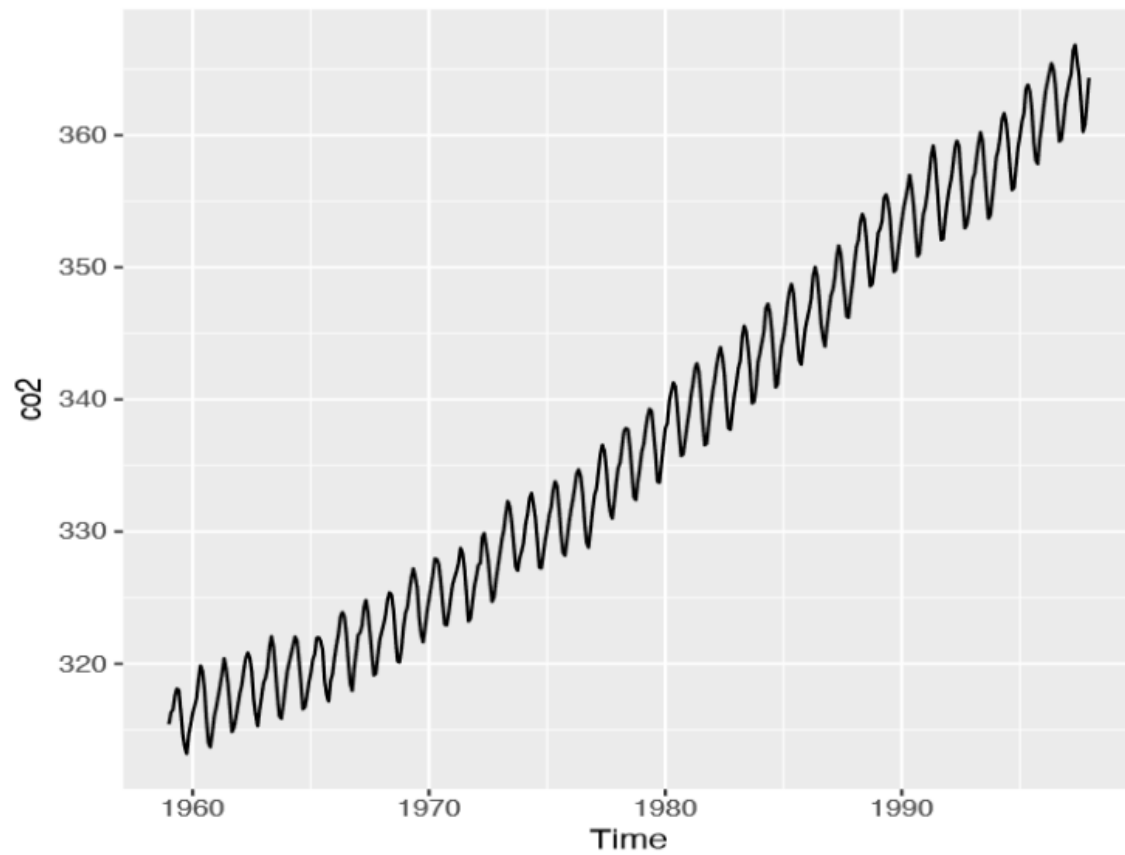
## **Time Series III**

**“Using equation-free methods to analyse and forecast ecological systems from time series data.”**

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Dormann (2018)

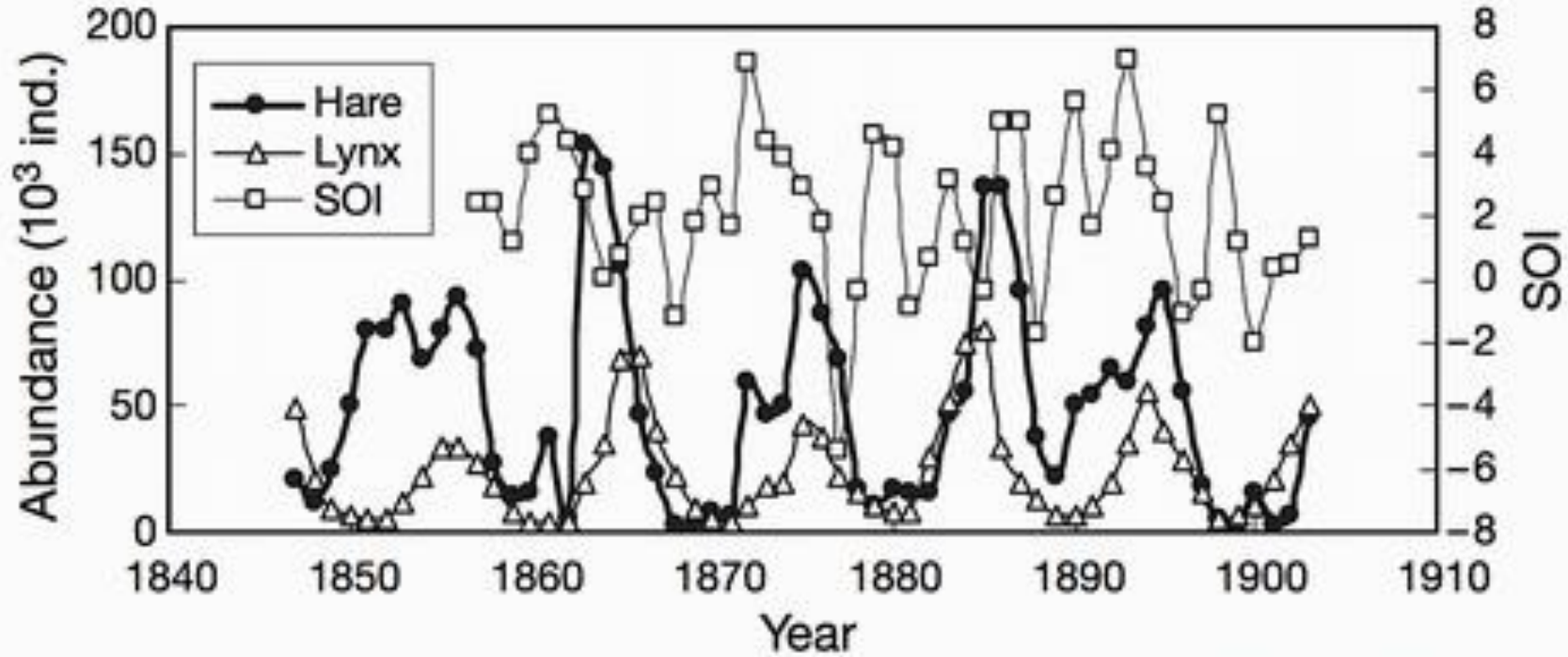
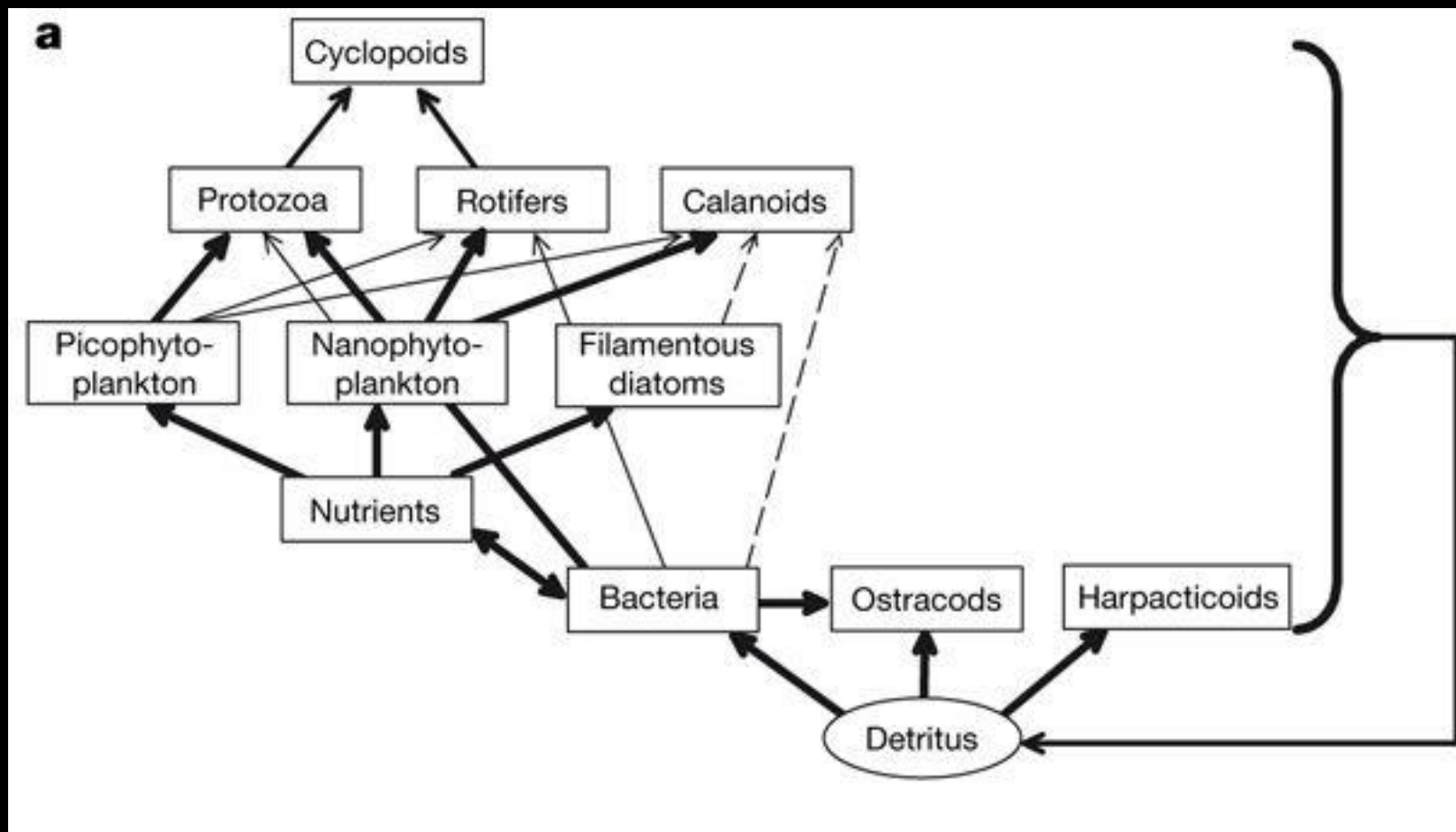
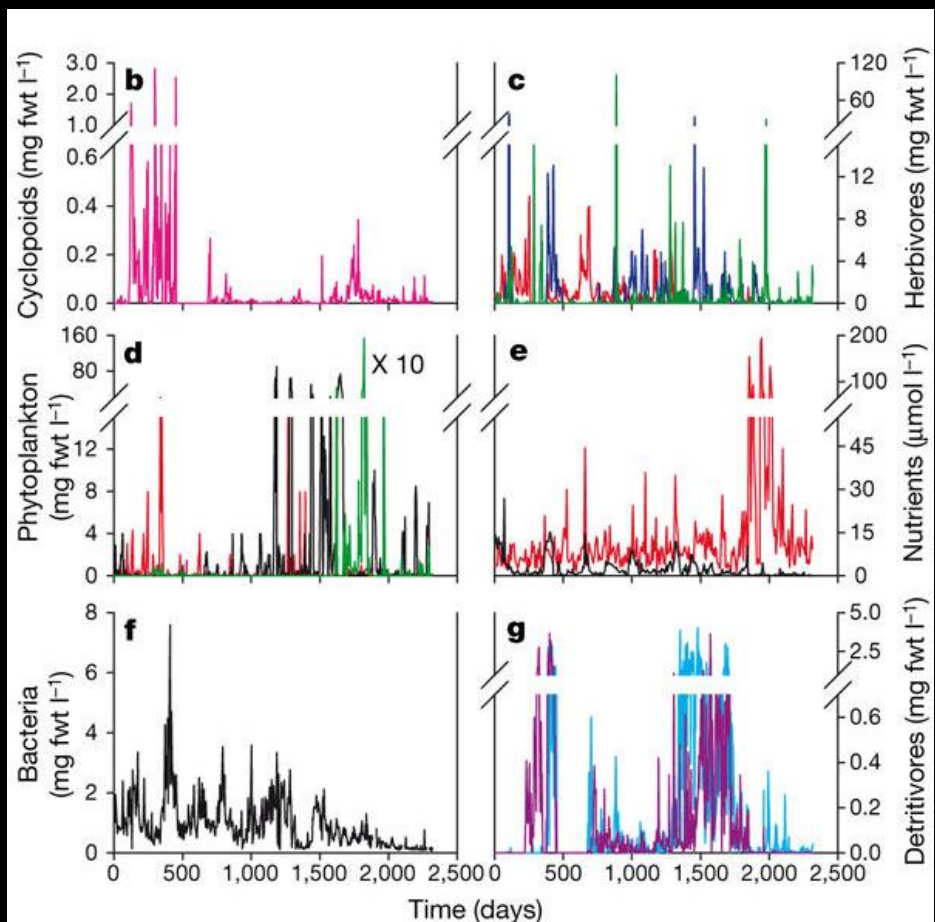
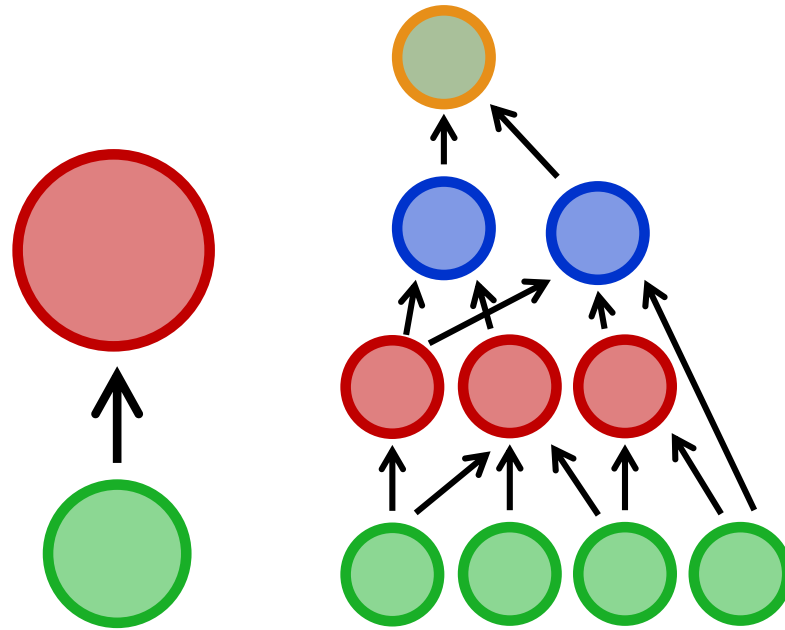


Fig. 1. *Lepus americanus* and *Lynx canadensis*. Hare and lynx abundances (redrawn from Leigh 1968) and Southern Oscillation Index (SOI) (data from Shi & Wang 1989) during 1847–1903



Benica et al (2008) *Nature* 451:822–825.

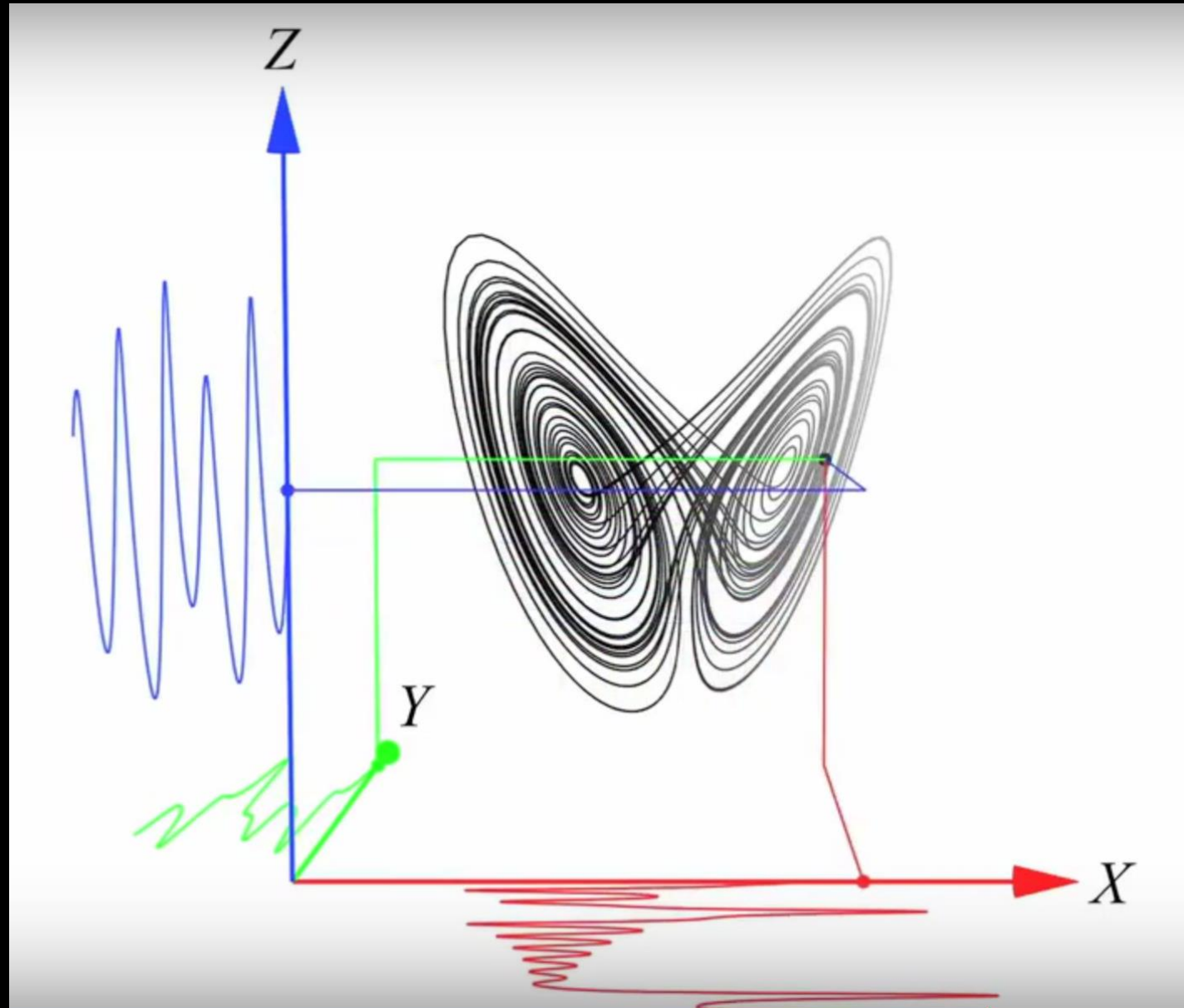


$$\frac{dX}{dt} = f(X, Y, Z, \dots)$$

$$\frac{dY}{dt} = f(X, Y, Z, \dots)$$

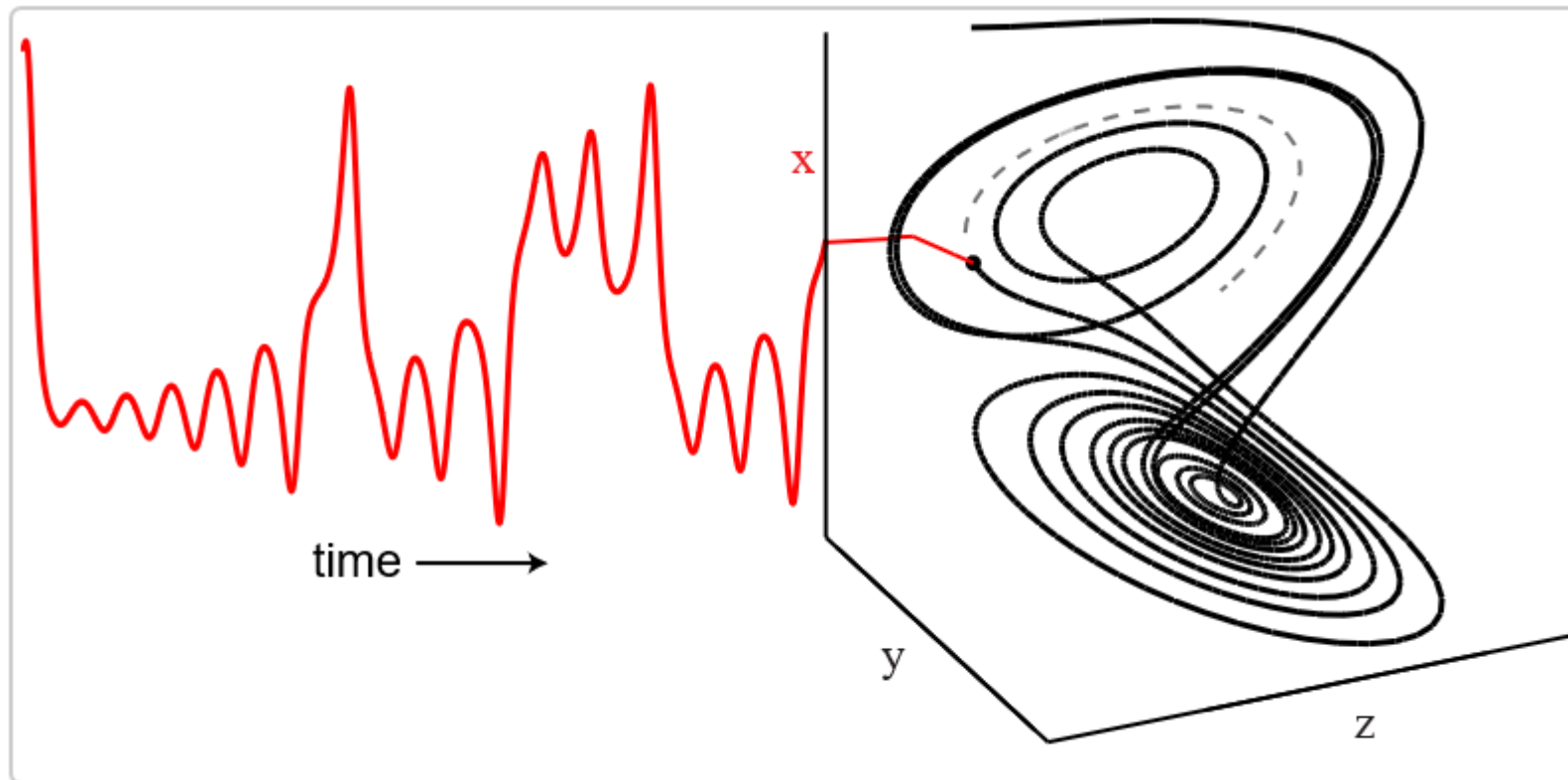
$$\frac{dZ}{dt} = f(X, Y, Z, \dots)$$

- Highly dynamic
- Non-linear
- State-dependent
- Time-lagged
- Fluctuating, chaotic, multi-stable



Sugihara et al (2012) *Science* 338:496–500.

## State-space or attractor reconstruction (after Takens' Theorem)

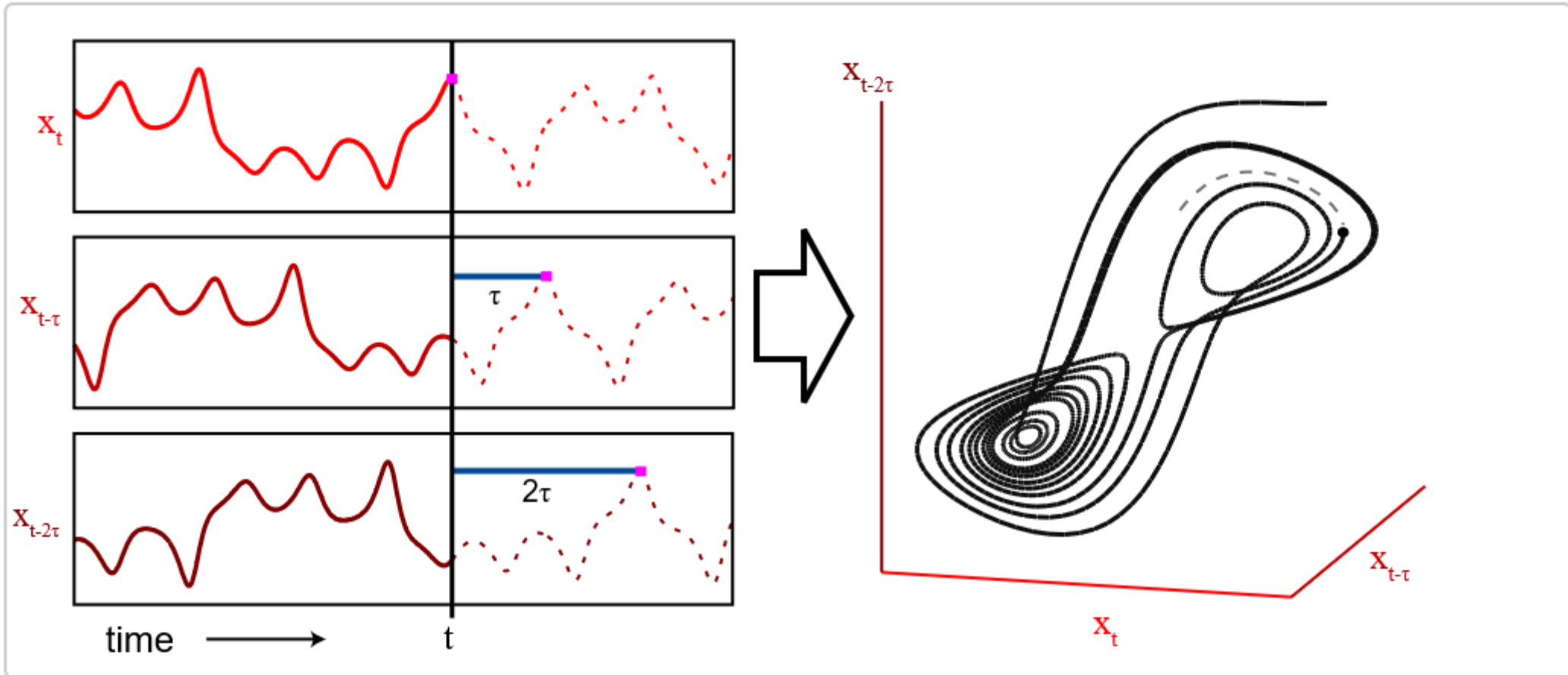


Time Series Projection from the Lorenz Attractor

Ye et al. (2017). <https://cran.r-project.org/web/packages/rEDM/vignettes/rEDM-tutorial.html>

## Univariate embedding using lags

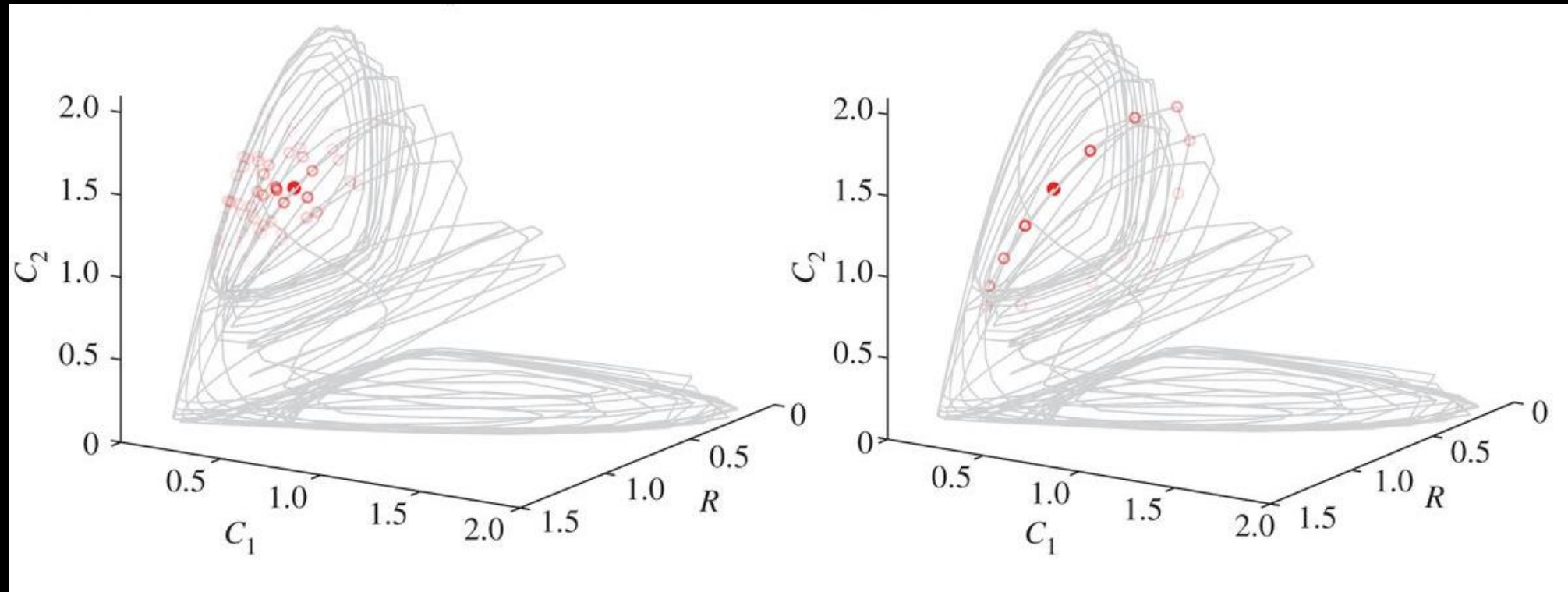
$$\vec{x}_t = \langle x_t, x_{t-\tau}, \dots, x_{t-(E-1)\tau} \rangle$$



Attractor Reconstruction from Lagged Coordinates

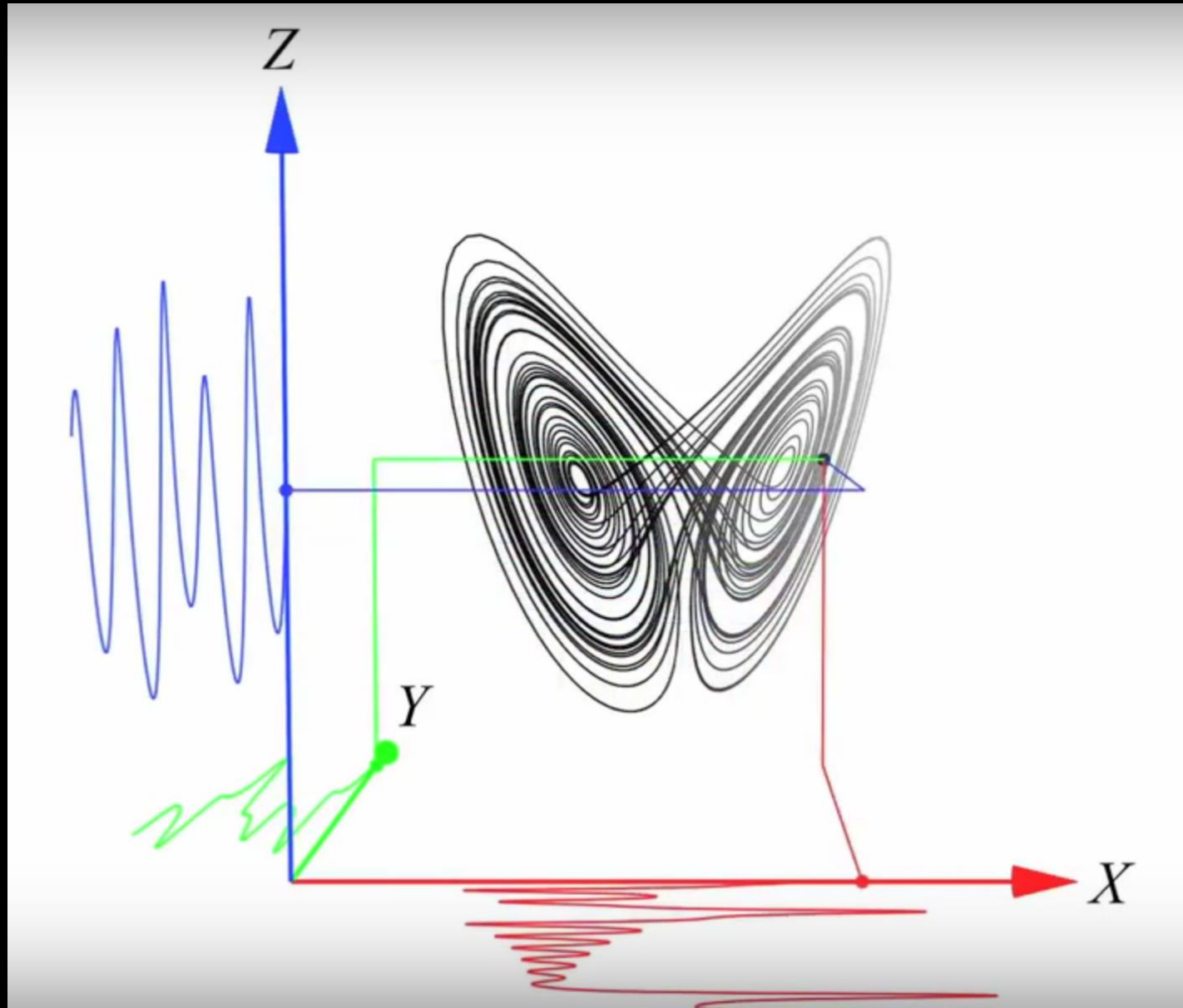


## Simplex Projection and S-maps: Forecasting using distance in state-space vs. distance in time



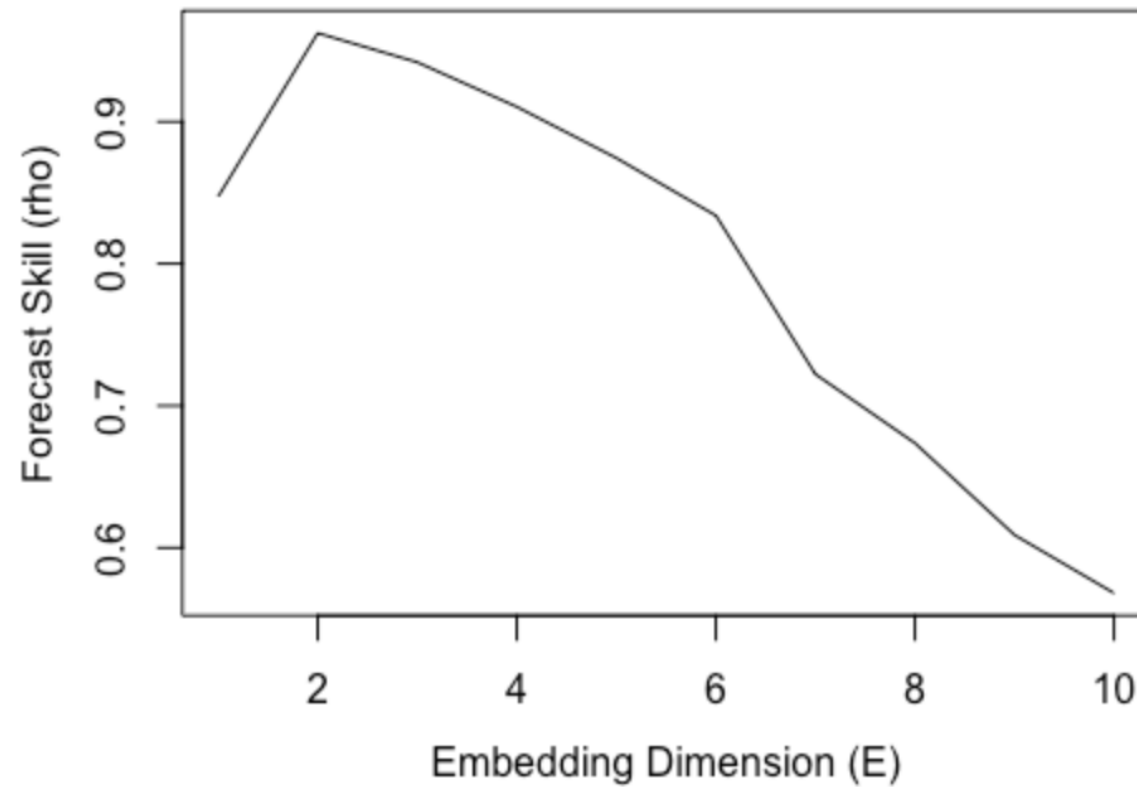
Ethan R. Deyle et al. Proc. R. Soc. B 2016;283:20152258

## Multivariate embedding



Sugihara et al (2012) *Science* 338:496–500.

## 1) Complexity of the system

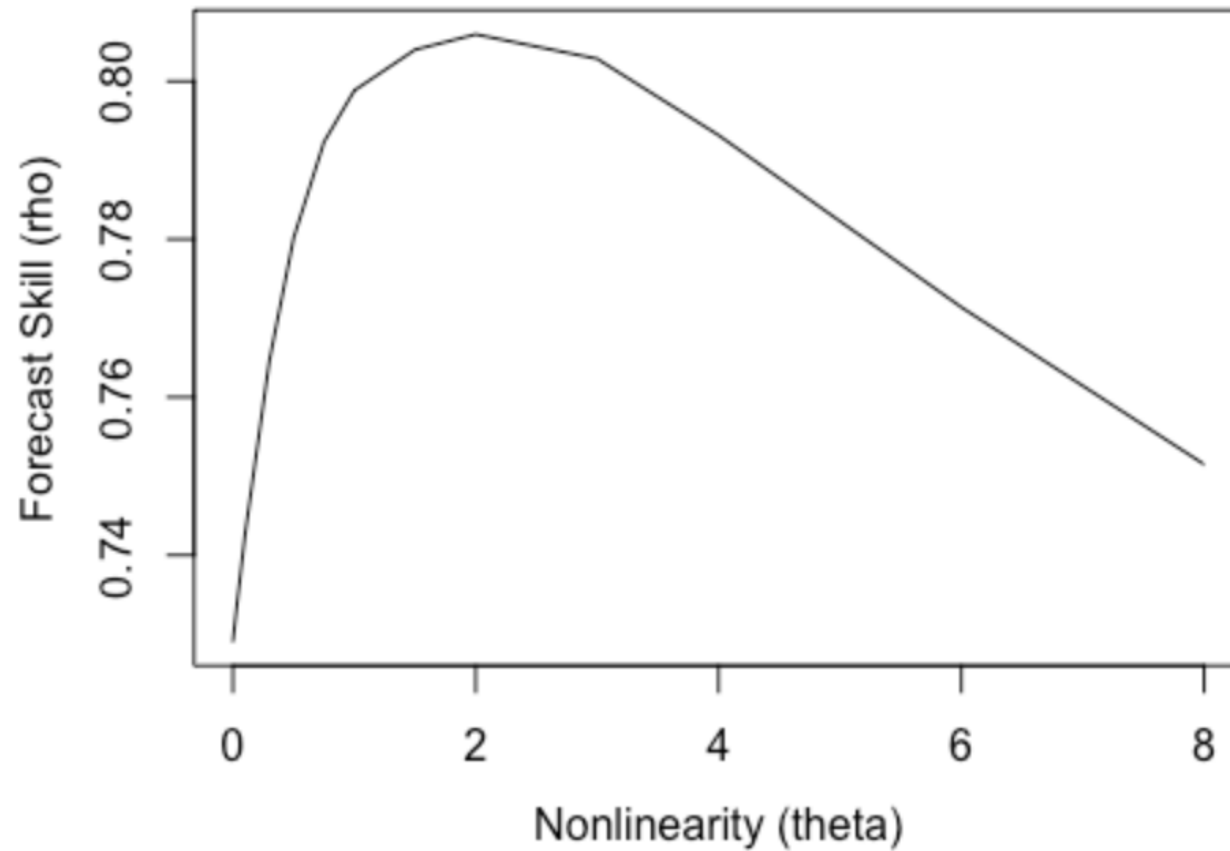


$$E < 2D + 1$$

D = true dimensions

Ye et al. (2017). <https://cran.r-project.org/web/packages/rEDM/vignettes/rEDM-tutorial.html>

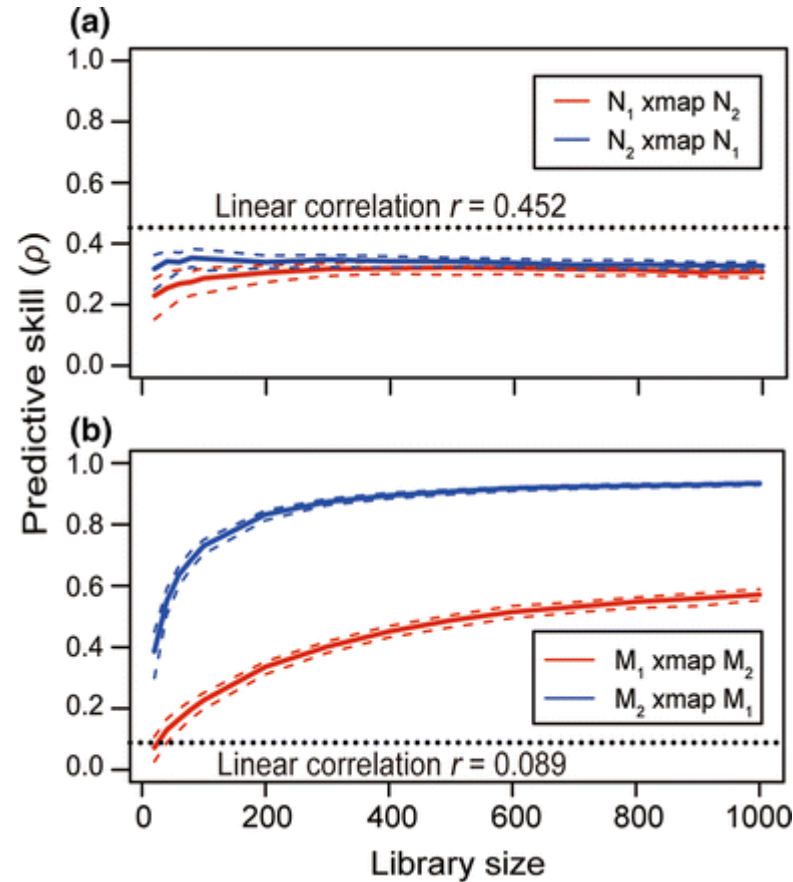
## 2) Nonlinearity of the system



Theta as a tuning variable

Ye et al. (2017). <https://cran.r-project.org/web/packages/rEDM/vignettes/rEDM-tutorial.html>

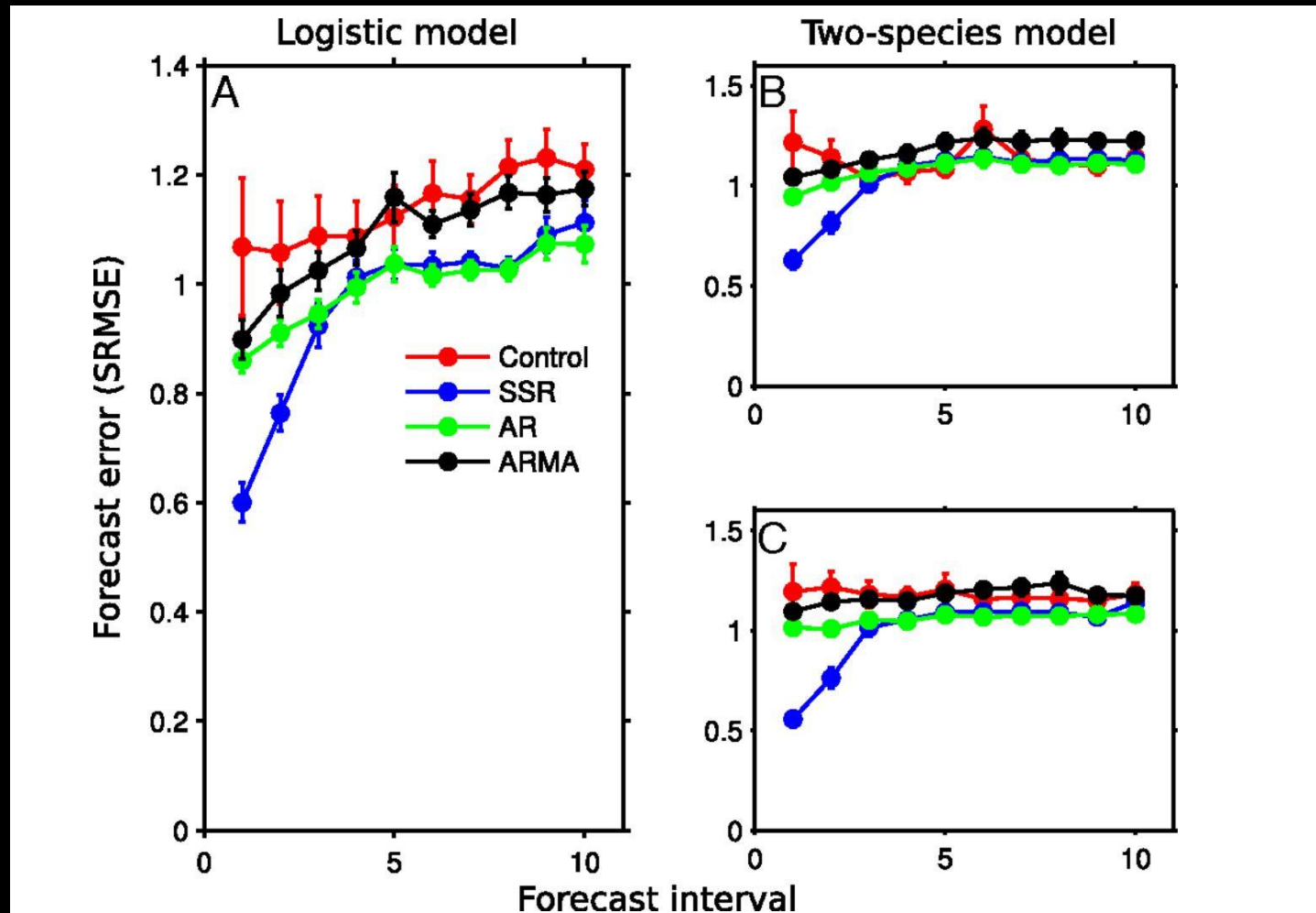
### 3) Causality



skill of forecasting one time series from a coupled one increases with information (= time series length used for attractor reconstruction = library size)

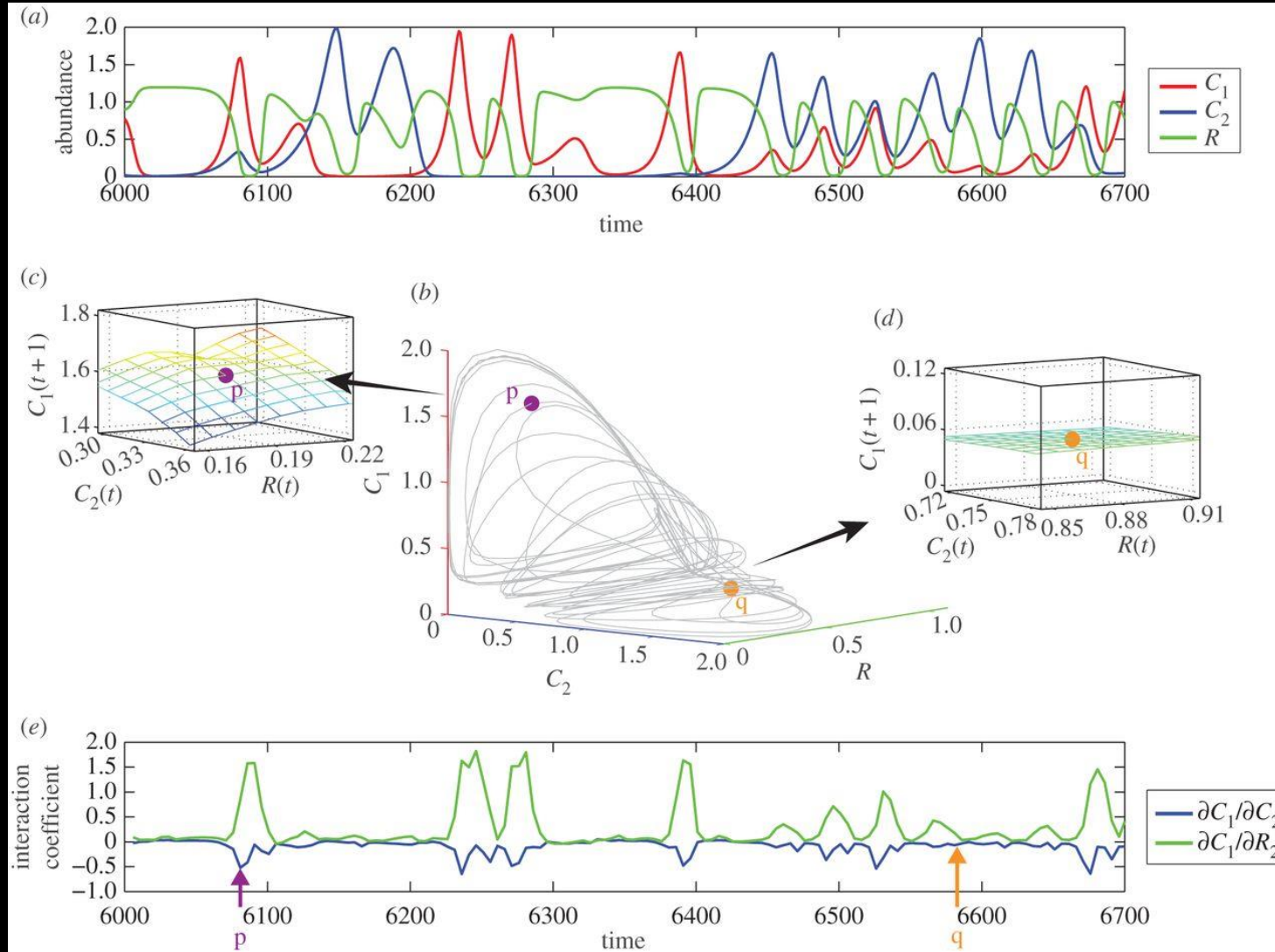
**Convergent cross-mapping**

## 4) Forecasting ahead



Perretti et al (2013) PNAS 110:13:5253-5257

## 5) Interaction signs and strengths



**S-map coefficients as approximations of the elements of the Jacobian interaction matrix**

## References and resources:

- Ye, Clark, Deyle & Sugihara (2017). rEDM: an R package for Empirical Dynamic Modelling. more on CRAN (<https://CRAN.R-project.org/package=rEDM>) and GitHub (<https://github.com/ha0ye/rEDM>)
- Deyle et al. (2016). Tracking and forecasting ecosystem interactions in real time. Proc. R. Soc. B 283:20152258.
- Chang, Ushio, Hsieh (2017). Empirical dynamic modelling for beginners. Ecol. Res. 32:785-796
- Sugihara et al. (2012). Detecting causality in complex ecosystems. Science 338:496-500
- Perretti et al. (2013). Model-free forecasting outperforms the correct mechanistic model for simulated and experimental data. PNAS 110:5253-5257
- Videos on <https://www.youtube.com/watch?v=8DikuwwPWsY> and in the Online Supplementary Material of Sugihara et al. (2012)
- Background and further reading on Takens Theorem: [https://en.wikipedia.org/wiki/Takens%27s\\_theorem](https://en.wikipedia.org/wiki/Takens%27s_theorem)

**Some antidote:** *(at least regarding the forecasting performance)*

Hartig & Dormann (2013). Does model-free forecasting really outperform the true model? PNAS 110:E3975



