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Introduction

Electricity is considered an unusual commodity since its non-storability has a profound effect on the infrastructure and the organization of its market compared with other commodity markets, such as oil, coal, metals and agriculture.

Some consequences of the lack of storability are strong seasonality and possible spikes in prices. This volatile behaviour of spot electricity prices is a stylised feature of these markets and appears when, for example, temperature drops significantly.

All contracts regarding electricity guarantee the delivery of a given amount of power for a specified future time period. Thus, modelling of this market consists in three tasks: spot price modelling, derivation of futures/swaps and pricing of options. For the first two, one approach is establishing the connection between the spot and futures/forwards price dynamics by introducing a market price of risk. Alternatively, one may adopt the Heath-Jarrow-Morton (HJM) approach from interest rate theory in order to directly assume a dynamics for the forward and swap price evolution. The last step is pricing option, using either analytical or numerical methods.

The objective of this project is to calibrate an HJM model on French electricity swaps and price structured payoffs options, given market data.

(i) HJM model driven by a NIG process

Firstly, let us introduce the Normal Inverse Gaussian (NIG) process. It is a pure-jump Lévy process with infinite variation, often used in finance to model asset returns and price movements. It has three parameters:

- θ, the drift of the Brownian motion (the subordinated process)
- σ, the volatility of the Brownian motion
- κ, the volatility of the subordinator, which is an Inverse Gaussian process.

We will use this process as driver for an HJM model for the 2025 French power swap.

The electricity markets trade in swaps, hence forward contracts having a delivery period. The owner of a swap contract with delivery over the time interval $[T_1, T_2]$ would receive a constant flow of the commodity over this period, against a fixed payment per unit. Through the HJM approach we can derive directly a price dynamics for such contracts:

$$
\begin{aligned}
F(t,T_1,T_2) = F(0,T_1,T_2)\exp\Bigg(&\int_0^t A(u,T_1,T_2)\,du \\
&+\sum_{k=1}^{p}\int_0^t \Sigma_k(u,T_1,T_2)\,dW_k(u)+\sum_{j=1}^{n}\int_0^t \Upsilon_j(u,T_1,T_2)\,dJ_j(u)\Bigg)
\end{aligned}
\tag{1}
$$

where $A(u,T_1,T_2)$, $\Sigma_k(u,T_1,T_2)$ and $\Upsilon_j(u,T_1,T_2)$ are real-valued continuous functions. The three terms at the exponent are the drift, the Brownian component and the additive process component respectively.

In our case we have $p=0$ (no Brownian component), $n=1$ and constant $\Upsilon(u,T_1,T_2)=\Upsilon$. Thus, (1) becomes:

$$
F(t,T_1,T_2) = F(0,T_1,T_2)\exp\left(\int_0^t A(u,T_1,T_2)\,du+\int_0^t \Upsilon\,dJ(u)\right) \tag{2}
$$

Empirical results suggest that the logreturns of electricity futures prices are far from being normally distributed, especially because they are heavy-tailed. For this reason NIG models are more appropriate to describe these dynamics: J in (2) represents for us a Lévy process with increments being NIG(θ,σ,κ).

As a result, the parameters of our model, excluding the drift, are four: θ, σ, κ and Υ. The parameters σ and κ must be positive, since they are volatilities.

(ii) Drift condition

In order to avoid arbitrage opportunities, the dynamics of the swap has to be set under the risk-neutral probability $\mathbb{Q}$. By imposing that the price has to be a martingale, we obtain the following condition on the the drift A:

$$
\int_0^t A(u,T_1,T_2)\,du+\int_0^t \Upsilon\,d\gamma(u)+\int_0^t\int_{\mathbb{R}}\left(e^{\Upsilon z}-1-\Upsilon z\mathbb{1}_{|z|<1}\right)\nu(dz,du)=0 \tag{3}
$$

Using that the last two integrals in (3) constitute the Lévy-Kintchine representation of the NIG Lévy process, we obtain that:

$$
\begin{aligned}
\int_0^t A(u,T_1,T_2)\,du &= -\Psi_{NIG}(-\Upsilon i) \\
&= -\frac{1}{\kappa}+\frac{1}{\kappa}\sqrt{1-\Upsilon^2\sigma^2\kappa-2\Upsilon\theta\kappa}
\end{aligned}
\tag{4}
$$

(iii) Calibration 2025

Given a set of options with the underlying asset being a swap with a tenor of 1 year, maturing in 2025, and quoted with their implied volatilities, our objective is to calibrate our model to estimate the four parameters. Initially, we need to obtain the option prices by inversely applying the Black-Scholes formula.

The market options have strikes ranging from 200 to 400, a time to maturity of less than 1 year, and the swap price as of 04/11/2023 is 284.8775127.

Once we have the option market prices, we observe arbitrage opportunities in the market: certain option prices increase as the strikes rise. This phenomenon, though not uncommon for commodities, is likely attributed to the illiquidity of the market.

To obtain the model prices, we use the Carr-Madan formula, starting from an initial guess for the parameters.

To perform the calibration, we aim to minimize the Mean Squared Error (MSE) between market and model prices across all strikes and maturities, using the `fmincon` function in MATLAB.

The obtained parameters are shown in Table 1.

Parameter	**Calibrated Value**
σ	0.5478
κ	2.4042
θ	0.0182
Υ	1.1182

Table 1: Calibration of parameters of the model for 2025 option prices.

The Root Mean Squared Error (RMSE) for the calibration is 22.8896. This value is primarily influenced by significant discrepancies between model and market prices for deep In-The-Money (ITM) and Out-of-The-Money (OTM) strikes, as shown in Figure 1. These disparities are responsible for the observed arbitrage opportunities.

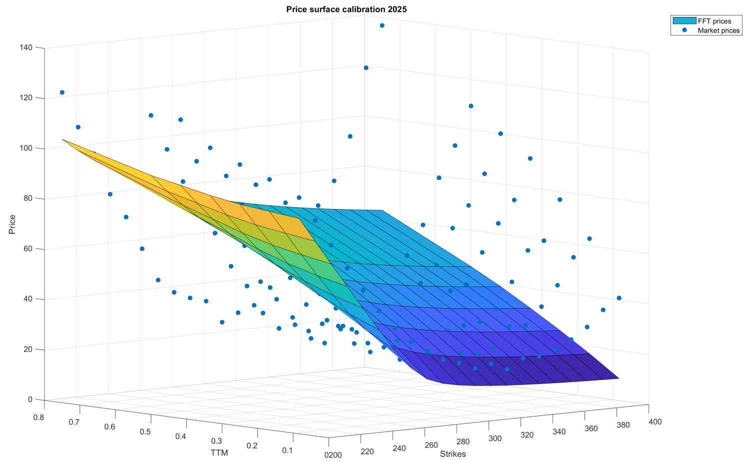


Figure 1: Calibration 2025 Swap option - all Strikes: the surface represents the model prices, the dots are the market data.

To enhance the calibration quality, we opt to calibrate the model using a constrained set of strike prices. Specifically, we choose strikes ranging from 230 to 330, encompassing those closest to the At-The-Money (ATM) strikes in terms of moneyness.

The output parameters in the optimization are in Table 2, while the plot is shown in Figure 2.

Parameter	**Calibrated Value**
σ	0.3117
κ	2.0766e-07
θ	-0.07481
Υ	1.6024

Table 2: Calibration of the parameters for 2025 option prices after strikes cut.

These values are slightly different from the ones obtained in the calibration performed on all strikes and the RMSE drops to 7.9894, consistently with our assumptions.

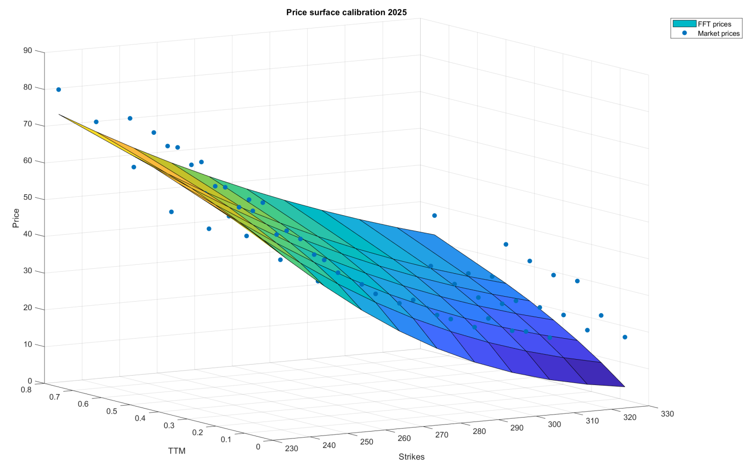


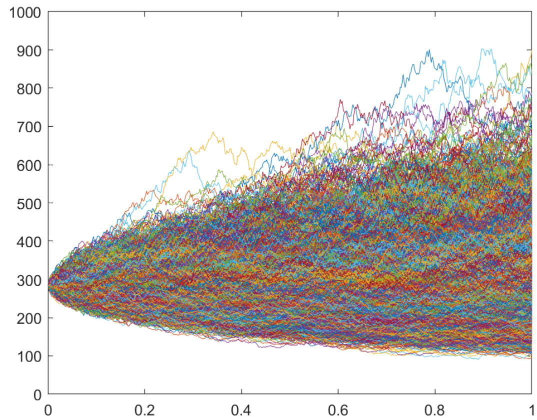
Figure 2: Calibration 2025 Swap option - most ATM Strikes

(iv) Lookback Pricing

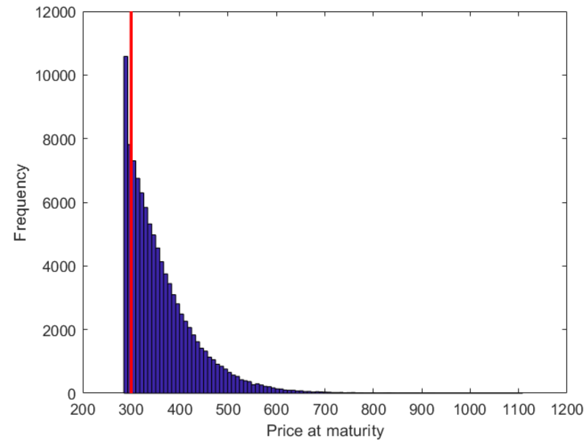
Our task involves pricing an option on 2025 Swap prices with a specified exotic payoff, utilizing the previously calibrated model as the dynamics of the underlying asset.

The particular option in question is a fixed-strike lookback call option with a monitoring period of one year. To evaluate such strongly path-dependent options, we opt for a Monte Carlo simulation. Consequently, the price is determined as the sample mean of the discounted payoffs, as illustrated in Figure 3a. In the plot, the maxima during the monitoring period are aggregated into a histogram, revealing that 83% of the options are in the money.

Assuming continuous monitoring, we discretize time and conduct simulations, as detailed in Figure 3b. The resulting calculated price is 61.9502, with a 95% confidence interval of $[60.6105, 63.2898]$.



(a) Simulation of underlying dynamics



(b) Histogram of the maxima

Figure 3: Pricing of lookback option.

(v) Calibration 2025 and 2027

We are asked to calibrate the model on both 2025 and 2027 1-year options. There are two approaches to do this: the first assumes that the NIG process is different for each swap, the second considers the dynamics for the underlying the same. The parameter Υ remains the same for both, assuming it is constant in time. Thus, in the first framework we need to calibrate 7 parameters (3 for the NIG of the first swap, 3 for the NIG of the second and Υ), instead in the second 4 parameters (3 for the NIG of both the swaps and Υ). In both cases we restrict the calibration only to the most ATM options, as discussed before.

The obtained parameters for the first case are in Table 3, while the plot of the surfaces, together with the market data, are in Figure 4. The RMSE is 6.4902.

Parameter	**Calibrated Value**
σ_{2025}	0.5051
κ_{2025}	3.9885e-07
θ_{2025}	-0.0407
σ_{2027}	0.6389
κ_{2027}	1.8281e-08
θ_{2027}	-5.0223
Υ	0.9892

Table 3: Calibration of parameters for both 2025 and 2027 separately - 7 parameters.

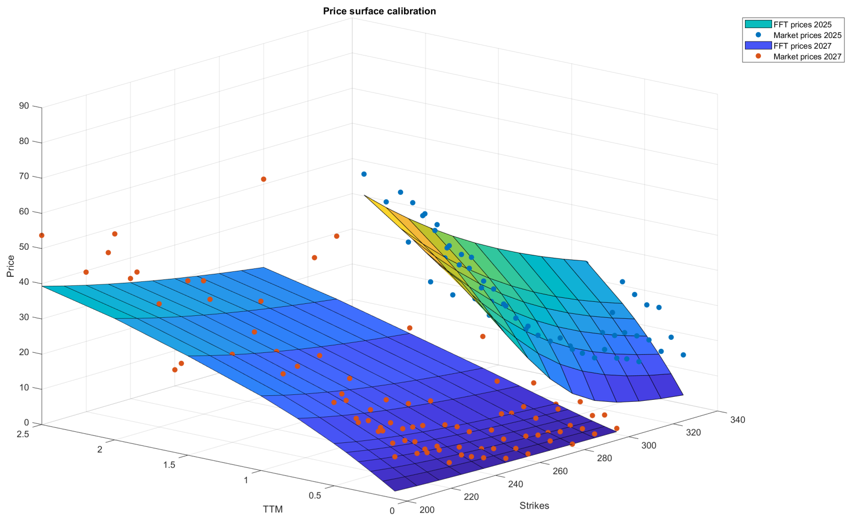


Figure 4: Calibration for 2025 and 2027 separately (7 parameters): model and market prices.

Instead, for the second case the parameters are in Table 4, while the plot is in Figure 5.

Parameter	**Calibrated Value**
σ	3.1867
κ	0.6490
θ	0.5768
Υ	0.1657

Table 4: Calibration of parameters for 2025 and 2027 together - 4 parameters.

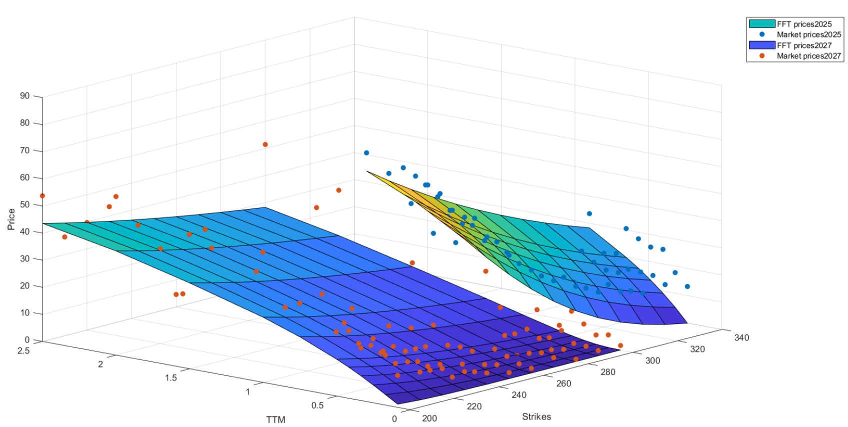


Figure 5: Calibration of 2025 and 2027 (4 parameters): model and market prices.

The RMSE is 10.9151. As expected, the error is higher in the case of less parameters, since there are less degrees of freedom.

(vi) Exchange option pricing

The final task is to price an option with payoff $(\max(F(1, T_1, T_2), F(1, T_3, T_4)) - K)^+$. We assume that the two underlyings have the same dynamics, hence we focus on the second approach in the previous point. In this case the process that models the 2027 swap price is always dominated by the 2025 one. For this reason our exotic option becomes a call option with the 2025 swap as underlying (payoff $(F(1, T_1, T_2) - K)^+$), and it can be priced with an explicit formula, exploiting the FFT algorithm.

On the other hand, in the first case (7 parameters) it is necessary to use Monte Carlo. The obtained price for the exchange option, varying the strike K, is shown in Figure 6.

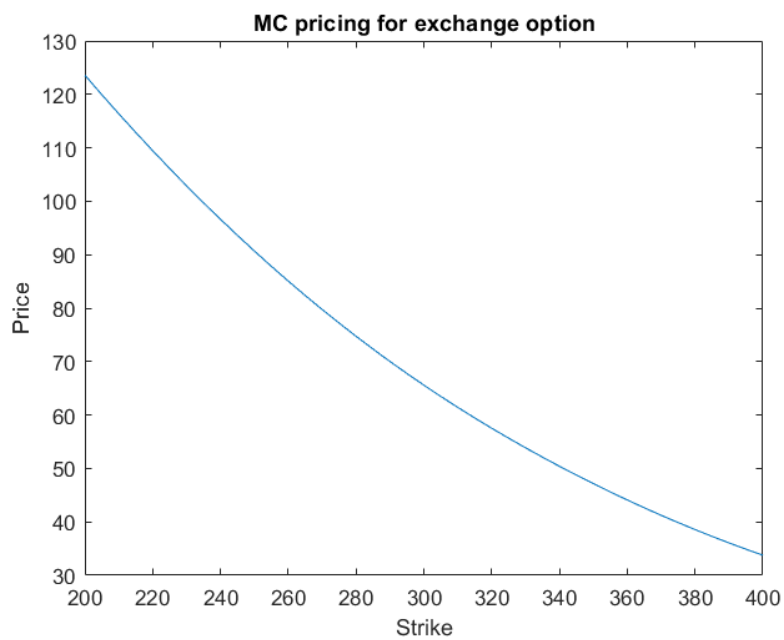


Figure 6: Monte Carlo-simulated price as a function of the strike.

References

[1] Benth, Fred Espen, Jurate Saltyte Benth, and Steen Koekebakker. Stochastic modelling of electricity and related markets. Vol. 11. World Scientific, 2008.