Бобрович Николай 4136. Вариант 3.

**Задание:**

4136

Постоянный шаг

Что Вы можете сказать относительно сходимости метода при разных ? Насколько существенно начальное приближение , – как оно влияет на сходимость метода? на скорость сходимости? Исследуйте наискорейший спуск.

**Решение:**

f(X)=x1+0.3\*x2^2

**Итерация №1**.

X0=(1;-1).  
Вычислим значение функции в начальной точке f(X0) = 1.3.  
В качестве направления поиска выберем вектор градиент в текущей точке:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ▽ f(X) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | 0.6x2 | |  | |

Значение градиента в точке X0:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ▽ f(X0) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -0.6 | |  | |

Проверим критерий остановки:  
|▽f(X0)| < ε  
Имеем:  
IMG_256  
Сделаем шаг вдоль направления антиградиента.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X1 = X0 - λ1▽ f(X0) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -1 | |  | | - λ1 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -0.6 | |  | | = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1.0-1.0λ1 | | 0.6λ1-1.0 | |  | |

Вычислим значение функции в новой точке.  
f(X1) = (1.0-1.0\*λ1)+0.3\*(0.6\*λ1-1.0)2  
Найдём такой шаг, чтобы целевая функция достигала минимума вдоль этого направления. Из необходимого условия существования экстремума функции (f'(X)=0):  
0.216\*λ1-1.36 = 0  
Получим шаг: λ1 = 6.2963  
Выполнение этого шага приведёт в точку:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X1 = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -1 | |  | | - 6.2963 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -0.6 | |  | | = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -5.2963 | | 2.7778 | |  | |

**Итерация №2**.

X1=(-5.2963;2.7778).  
Вычислим значение функции в точке f(X1) = -2.982.  
Значение градиента в точке X1:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ▽ f(X1) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | 1.6667 | |  | |

Проверим критерий остановки:  
|▽f(X1)| < ε  
Имеем:  
|▽f(X1)| = 1.944>0.1  
Сделаем шаг вдоль направления антиградиента.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X2 = X1 - λ2▽ f(X1) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -5.2963 | | 2.7778 | |  | | - λ2 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | 1.6667 | |  | | = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -1.0λ2-5.2963 | | 2.7778-1.6667λ2 | |  | |

Вычислим значение функции в новой точке.  
f(X2) = (-1.0\*λ2-5.2963)+0.3\*(2.7778-1.6667\*λ2)2  
Найдём такой шаг, чтобы целевая функция достигала минимума вдоль этого направления. Из необходимого условия существования экстремума функции (f'(X)=0):  
1.6667\*λ2-3.7778 = 0  
Получим шаг: λ2 = 2.2667  
Выполнение этого шага приведёт в точку:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X2 = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -5.2963 | | 2.7778 | |  | | - 2.2667 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | 1.6667 | |  | | = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -7.563 | | -1 | |  | |

**Итерация №3**.

X2=(-7.563;-1).  
Вычислим значение функции в точке f(X2) = -7.263.  
Значение градиента в точке X2:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ▽ f(X2) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -0.6 | |  | |

Проверим критерий остановки:  
|▽f(X2)| < ε  
Имеем:  
|▽f(X2)| = 1.166>0.1  
Сделаем шаг вдоль направления антиградиента.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X3 = X2 - λ3▽ f(X2) = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -7.563 | | -1 | |  | | - λ3 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -0.6 | |  | | = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -1.0λ3-7.563 | | 0.6λ3-1.0 | |  | |

Вычислим значение функции в новой точке.  
f(X3) = (-1.0\*λ3-7.563)+0.3\*(0.6\*λ3-1.0)2  
Найдём такой шаг, чтобы целевая функция достигала минимума вдоль этого направления. Из необходимого условия существования экстремума функции (f'(X)=0):  
0.216\*λ3-1.36 = 0  
Получим шаг: λ3 = 6.2963  
Выполнение этого шага приведёт в точку:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X3 = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -7.563 | | -1 | |  | | - 6.2963 | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | 1 | | -0.6 | |  | | = | |  |  |  |  | | --- | --- | --- | --- | | |  | | --- | | -13.8593 | | 2.7778 | |  | |