A Mild Introduction to Structural Equation Modeling Using `lavaan`

UseR! Workshop

28 May 2020

Contents

Data preparation

Install and load relevant R packages

```
# Install R packages (if needed)
# install.packages(c("lavaan", "semPlot", "MPsychoR", "corrplot"))

## Load relevant libraries
library(lavaan)
library(semPlot)
library(MPsychoR)
library(corrplot)
```

Data input

```
# Select the data
data("Bergh")
View(Bergh)
attach(Bergh)

# Sample size
nrow(Bergh)
```

```
## [1] 861
```

```
## Create mean scores per construct
Bergh$Open <- (O1+O2+O3)/3
Bergh$Agree <- (A1+A2+A3)/3
Bergh$Prejudice <- (EP+SP+DP+HP)/4
```

Model 1: Regression model with manifest variables only

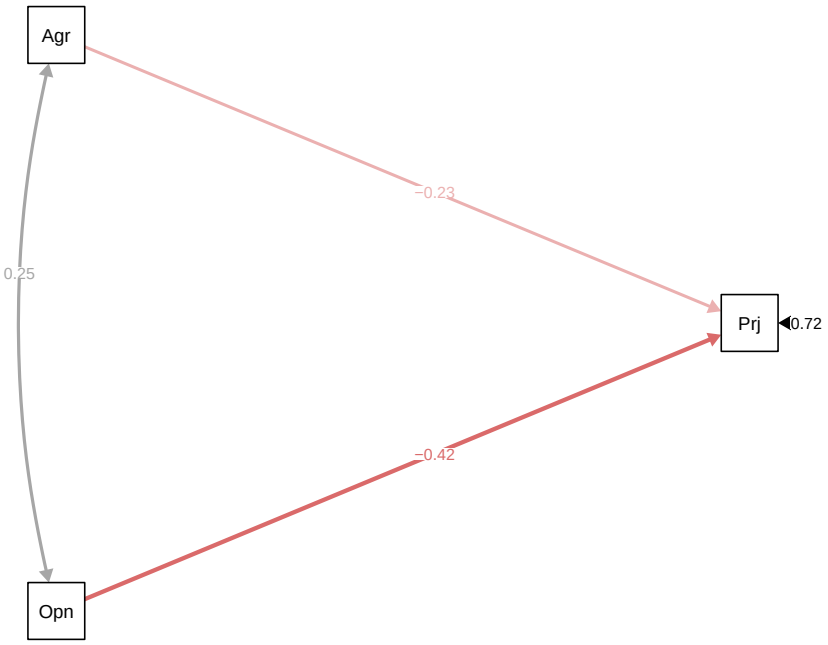
```
# Step 1: Model specification
model1 <- '
            # Structural model
            Prejudice ~ b1*Open + b2*Agree

            # Covariance structure of exogenous variables
            Open ~~ Open + Agree
            Agree ~~ Agree

'

# Step 2: Model estimation
model1.fit <- sem(model1,
                  data = Bergh,
                  meanstructure = FALSE,
                  estimator = "ML")

# Step 3: Evaluate the model
```

```r
# Summary
summary(model1.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-5 ended normally after 21 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                          6
##
##   Number of observations                           861
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Model Test Baseline Model:
##
##   Test statistic                               335.486
##   Degrees of freedom                                 3
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1689.786
##   Loglikelihood unrestricted model (H1)      -1689.786
##
##   Akaike (AIC)                                3391.572
##   Bayesian (BIC)                              3420.121
##   Sample-size adjusted Bayesian (BIC)         3401.066
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.000
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.000
##   P-value RMSEA <= 0.05                             NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
## Regressions:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   Prejudice ~
##     Open       (b1)   -0.612    0.043  -14.118    0.000   -0.612   -0.423
##     Agree      (b2)   -0.324    0.043   -7.522    0.000   -0.324   -0.225
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   Open ~~
##     Agree              0.049    0.007    7.148    0.000    0.049    0.251
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     Open               0.192    0.009   20.748    0.000    0.192    1.000
##     Agree              0.194    0.009   20.748    0.000    0.194    1.000
##    .Prejudice          0.291    0.014   20.748    0.000    0.291    0.723
##
## R-Square:
##                    Estimate
##     Prejudice          0.277
```

```
# Visualize the path model
semPaths(model1.fit,
         rotation = 2,
         layout = "tree2",
         what = "std",
         posCol = "black",
         edge.width = 0.5,
         style = "Lisrel",
         fade = T,
         edge.label.position = 0.55)
```




```
# Global fit indices
fitMeasures(model1.fit)
```

```
##                npar                fmin               chisq                  df
```

```
##               6.000               0.000               0.000               0.000
##              pvalue      baseline.chisq         baseline.df     baseline.pvalue
##                  NA             335.486               3.000               0.000
##                 cfi                 tli                nnfi                 rfi
##               1.000               1.000               1.000               1.000
##                 nfi                pnfi                 ifi                 rni
##               1.000               0.000               1.000               1.000
##                logl   unrestricted.logl                 aic                 bic
##           -1689.786           -1689.786            3391.572            3420.121
##              ntotal                bic2                rmsea      rmsea.ci.lower
##             861.000            3401.066               0.000               0.000
##      rmsea.ci.upper        rmsea.pvalue                 rmr          rmr_nomean
##               0.000                  NA               0.000               0.000
##               srmr        srmr_bentler srmr_bentler_nomean                crmr
##               0.000               0.000               0.000               0.000
##         crmr_nomean          srmr_mplus   srmr_mplus_nomean               cn_05
##               0.000               0.000               0.000               1.000
##               cn_01                 gfi                agfi                pgfi
##               1.000               1.000               1.000               0.000
##                 mfi                ecvi
##               1.000               0.014
```

```r
# Local fit measures
modificationindices(model1.fit)
```

```
## [1] lhs      op       rhs      mi       epc      sepc.lv  sepc.all sepc.nox
## <0 rows> (or 0-length row.names)
```

```r
resid(model1.fit, type = "raw")
```

```
## $type
## [1] "raw"
##
## $cov
##           Prejdc Open Agree
## Prejudice 0
## Open      0      0
## Agree     0      0    0
```

```r
resid(model1.fit, type = "standardized")
```

```
## $type
## [1] "standardized"
##
## $cov
##           Prejdc Open Agree
## Prejudice 0
## Open      0      0
## Agree     0      0    0
```

```r
# Fitted values of the covariance matrix and the mean vector
fitted(model1.fit)
```

```
## $cov
##           Prejdc Open   Agree
## Prejudice  0.402
## Open      -0.133  0.192
```

```
## Agree     -0.093  0.049  0.194
```

```
# List all parameter values
parameterEstimates(model1.fit)
```

```
##         lhs op       rhs label    est    se       z pvalue ci.lower ci.upper
## 1 Prejudice  ~      Open    b1 -0.612 0.043 -14.118      0   -0.697   -0.527
## 2 Prejudice  ~     Agree    b2 -0.324 0.043  -7.522      0   -0.408   -0.239
## 3      Open ~~      Open        0.192 0.009  20.748      0    0.174    0.210
## 4      Open ~~     Agree        0.049 0.007   7.148      0    0.035    0.062
## 5     Agree ~~     Agree        0.194 0.009  20.748      0    0.176    0.213
## 6 Prejudice ~~ Prejudice        0.291 0.014  20.748      0    0.263    0.318
```

```
# Check the model setup
inspect(model1.fit)
```

```
## $lambda
##           Prejdc Open Agree
## Prejudice      0    0     0
## Open           0    0     0
## Agree          0    0     0
##
## $theta
##           Prejdc Open Agree
## Prejudice 0
## Open      0      0
## Agree     0      0    0
##
## $psi
##           Prejdc Open Agree
## Prejudice 6
## Open      0      3
## Agree     0      4    5
##
## $beta
##           Prejdc Open Agree
## Prejudice      0    1     2
## Open           0    0     0
## Agree          0    0     0
```

```
# Check the starting values
inspect(model1.fit, what = "start")
```

```
## $lambda
##           Prejdc Open Agree
## Prejudice      1    0     0
## Open           0    1     0
## Agree          0    0     1
##
## $theta
##           Prejdc Open Agree
## Prejudice 0
## Open      0      0
## Agree     0      0    0
##
## $psi
```

```
##           Prejdc Open  Agree
## Prejudice 0.201
## Open      0.000  0.096
## Agree     0.000  0.000 0.097
##
## $beta
##           Prejdc Open Agree
## Prejudice      0    0     0
## Open           0    0     0
## Agree          0    0     0
```

```
# Check the estimates
inspect(model1.fit, what = "est")
```

```
## $lambda
##           Prejdc Open Agree
## Prejudice      1    0     0
## Open           0    1     0
## Agree          0    0     1
##
## $theta
##           Prejdc Open Agree
## Prejudice 0
## Open      0      0
## Agree     0      0    0
##
## $psi
##           Prejdc Open  Agree
## Prejudice 0.291
## Open      0.000  0.192
## Agree     0.000  0.049 0.194
##
## $beta
##           Prejdc   Open  Agree
## Prejudice      0 -0.612 -0.324
## Open           0  0.000  0.000
## Agree          0  0.000  0.000
```

```
# Step 4: Further hypothesis testing
# H0: b1=b2
lavTestWald(model1.fit, constraints = "b1==b2")
```

```
## $stat
## [1] 17.76479
##
## $df
## [1] 1
##
## $p.value
## [1] 2.499661e-05
##
## $se
## [1] "standard"
```

Model 1 with bootstrapping of standard errors

```
# Step 2: Model estimation with bootstrapping
set.seed(616)
model1.fit.boot <- sem(model1,
                       data = Bergh,
                       meanstructure = FALSE,
                       estimator = "ML",
                       se = "bootstrap",
                       bootstrap = 1000)

# Step 3: Evaluate the model
# Summary
summary(model1.fit.boot,
        rsquare = TRUE,
        fit.measures = TRUE,
        standardized = TRUE,
        ci = TRUE)
```

```
## lavaan 0.6-5 ended normally after 21 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                          6
##
##   Number of observations                           861
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Model Test Baseline Model:
##
##   Test statistic                               335.486
##   Degrees of freedom                                 3
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1689.786
##   Loglikelihood unrestricted model (H1)      -1689.786
##
##   Akaike (AIC)                                3391.572
##   Bayesian (BIC)                              3420.121
##   Sample-size adjusted Bayesian (BIC)         3401.066
##
## Root Mean Square Error of Approximation:
```

```
##
##   RMSEA                                          0.000
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.000
##   P-value RMSEA <= 0.05                             NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Standard errors                             Bootstrap
##   Number of requested bootstrap draws             1000
##   Number of successful bootstrap draws            1000
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|) ci.lower ci.upper
##   Prejudice ~
##     Open      (b1)   -0.612    0.044  -13.905    0.000   -0.704   -0.528
##     Agree     (b2)   -0.324    0.043   -7.576    0.000   -0.410   -0.238
##    Std.lv  Std.all
##
##    -0.612   -0.423
##    -0.324   -0.225
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|) ci.lower ci.upper
##   Open ~~
##     Agree             0.049    0.007    6.802    0.000    0.034    0.061
##    Std.lv  Std.all
##
##     0.049    0.251
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|) ci.lower ci.upper
##     Open              0.192    0.009   22.345    0.000    0.174    0.209
##     Agree             0.194    0.008   24.159    0.000    0.177    0.211
##    .Prejudice         0.291    0.017   17.440    0.000    0.258    0.323
##    Std.lv  Std.all
##     0.192    1.000
##     0.194    1.000
##     0.291    0.723
##
## R-Square:
##                    Estimate
##     Prejudice         0.277
```

```r
# List all parameter values
parameterEstimates(model1.fit.boot, ci = TRUE, boot.ci.type = "basic")
```

```
##         lhs op       rhs label    est    se       z pvalue ci.lower ci.upper
## 1 Prejudice  ~      Open    b1 -0.612 0.044 -13.905      0   -0.696   -0.520
## 2 Prejudice  ~     Agree    b2 -0.324 0.043  -7.576      0   -0.409   -0.238
## 3      Open ~~      Open        0.192 0.009  22.345      0    0.175    0.210
```

```
## 4      Open ~~     Agree        0.049 0.007   6.802      0    0.036    0.063
## 5     Agree ~~     Agree        0.194 0.008  24.159      0    0.178    0.212
## 6 Prejudice ~~ Prejudice        0.291 0.017  17.440      0    0.258    0.324
```

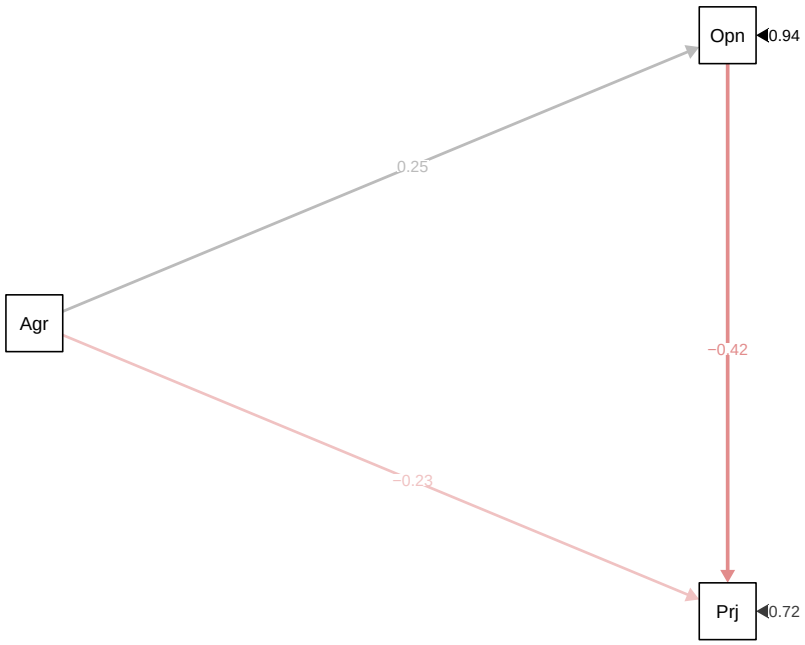
Model 2: Mediation model with manifest variables only (T)

```
# Step 1: Model specification
model2 <- '
            # Structural model
            Prejudice ~ b1*Open + b2*Agree
            Open ~ b3*Agree

            # Covariance structure of exogenous variables
            Agree ~~ Agree

            # New parameters (indirect and total effect)
            ind := b1*b3
            tot := ind + b2

'

# Step 2: Model estimation
model2.fit <- sem(model2,
                  data = Bergh,
                  meanstructure = FALSE,
                  estimator = "ML")

# Step 3: Evaluate the model
# Summary
summary(model2.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-5 ended normally after 19 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                          6
##
##   Number of observations                           861
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Model Test Baseline Model:
##
##   Test statistic                               335.486
##   Degrees of freedom                                 3
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
```

```
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       1.000
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1689.786
##   Loglikelihood unrestricted model (H1)      -1689.786
##
##   Akaike (AIC)                                3391.572
##   Bayesian (BIC)                              3420.121
##   Sample-size adjusted Bayesian (BIC)         3401.066
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.000
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.000
##   P-value RMSEA <= 0.05                             NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.000
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   Prejudice ~
##     Open      (b1)   -0.612    0.043  -14.118    0.000   -0.612   -0.423
##     Agree     (b2)   -0.324    0.043   -7.522    0.000   -0.324   -0.225
##   Open ~
##     Agree     (b3)    0.250    0.033    7.614    0.000    0.250    0.251
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     Agree              0.194    0.009   20.748    0.000    0.194    1.000
##    .Prejudice          0.291    0.014   20.748    0.000    0.291    0.723
##    .Open               0.180    0.009   20.748    0.000    0.180    0.937
##
## R-Square:
##                    Estimate
##     Prejudice          0.277
##     Open               0.063
##
## Defined Parameters:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     ind               -0.153    0.023   -6.701    0.000   -0.153   -0.106
##     tot               -0.477    0.046  -10.304    0.000   -0.477   -0.331
```

```r
# Visualize the path model
semPaths(model2.fit,
         rotation = 2,
         layout = "tree2",
         what = "std",
         posCol = "black",
         edge.width = 0.5,
         style = "Lisrel",
         fade = T,
         edge.label.position = 0.55)
```

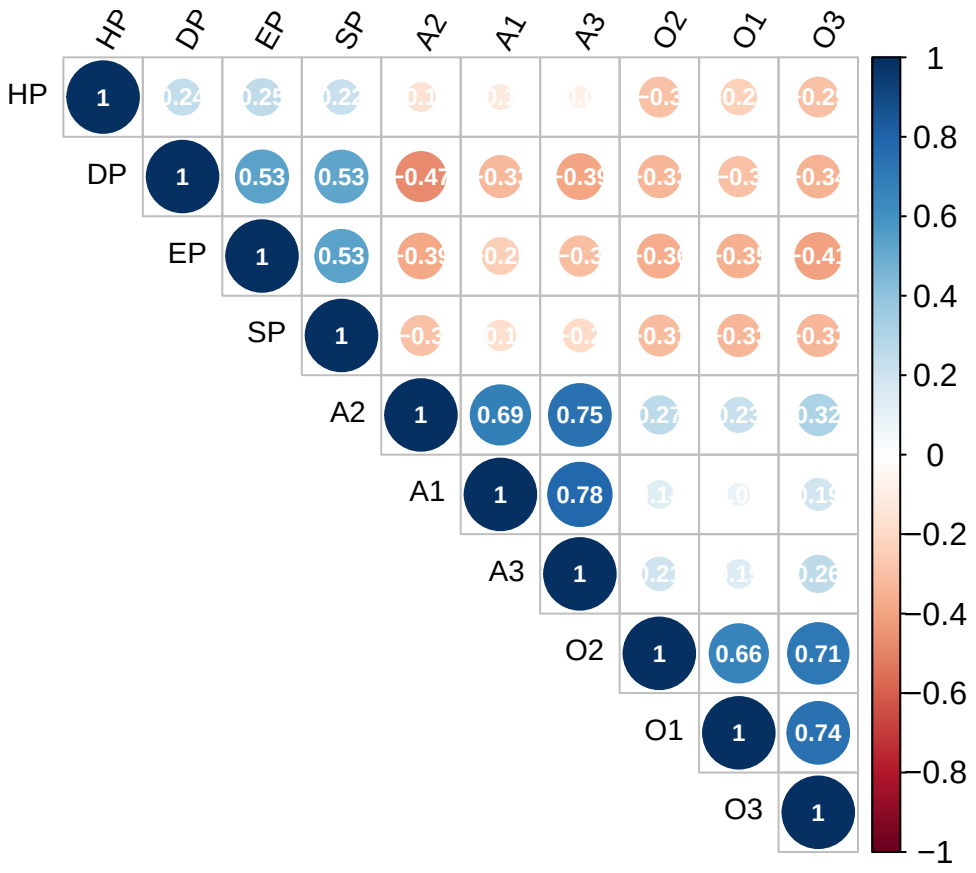


Correlation matrix

```r
# Extract the correlation matrix
Bergh.cor <- cor(Bergh[,1:10], method = "pearson", use = "pairwise.complete.obs")
Bergh.cor
```

```
##            EP         SP         HP         DP         A1         A2         A3
## EP  1.0000000  0.5328577  0.2545270  0.5314828 -0.2486889 -0.3889079 -0.3031269
## SP  0.5328577  1.0000000  0.2219292  0.5252140 -0.1710822 -0.2973829 -0.1987969
## HP  0.2545270  0.2219292  1.0000000  0.2415626 -0.1120012 -0.1510590 -0.0827062
## DP  0.5314828  0.5252140  0.2415626  1.0000000 -0.3292610 -0.4709318 -0.3936544
## A1 -0.2486889 -0.1710822 -0.1120012 -0.3292610  1.0000000  0.6867541  0.7835360
## A2 -0.3889079 -0.2973829 -0.1510590 -0.4709318  0.6867541  1.0000000  0.7453925
## A3 -0.3031269 -0.1987969 -0.0827062 -0.3936544  0.7835360  0.7453925  1.0000000
## O1 -0.3543605 -0.3317130 -0.2332906 -0.2994080  0.0861290  0.2293831  0.1488831
## O2 -0.3622272 -0.3127873 -0.2972669 -0.3327277  0.1393367  0.2698570  0.2082816
## O3 -0.4089230 -0.3300734 -0.2930209 -0.3407396  0.1904259  0.3178221  0.2584276
##            O1         O2         O3
## EP -0.3543605 -0.3622272 -0.4089230
## SP -0.3317130 -0.3127873 -0.3300734
## HP -0.2332906 -0.2972669 -0.2930209
## DP -0.2994080 -0.3327277 -0.3407396
## A1  0.0861290  0.1393367  0.1904259
## A2  0.2293831  0.2698570  0.3178221
## A3  0.1488831  0.2082816  0.2584276
## O1  1.0000000  0.6624692  0.7444363
## O2  0.6624692  1.0000000  0.7140617
## O3  0.7444363  0.7140617  1.0000000
```

```r
# Correlogram
corrplot(Bergh.cor, type = "upper", order = "hclust",
         tl.col = "black", tl.srt = 60,
         addCoef.col = "white",
         number.cex = 0.75,
         cl.cex = 1,
         tl.cex = 0.9)
```



Model 3: Measurement model (CFA)

```
# Step 1: Model specification
model3 <- '
            # Measurement models
            OP =~ O1 + O2 + O3
            AG =~ A1 + A2 + A3
            PR =~ EP + SP + HP + DP

            # Covariance structure
            OP ~~ OP + AG + PR
            AG ~~ AG + PR
            PR ~~ PR

'

# Step 2: Model estimation
model3.fit <- sem(model3,
                  data = Bergh,
                  meanstructure = FALSE,
                  estimator = "ML")

# Step 3: Evaluate the model
# Summary
```

```r
summary(model3.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-5 ended normally after 54 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         23
##
##   Number of observations                           861
##
## Model Test User Model:
##
##   Test statistic                               186.620
##   Degrees of freedom                                32
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              4270.205
##   Degrees of freedom                                45
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.963
##   Tucker-Lewis Index (TLI)                       0.949
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -5672.807
##   Loglikelihood unrestricted model (H1)      -5579.497
##
##   Akaike (AIC)                               11391.614
##   Bayesian (BIC)                             11501.050
##   Sample-size adjusted Bayesian (BIC)        11428.008
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.075
##   90 Percent confidence interval - lower         0.065
##   90 Percent confidence interval - upper         0.085
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.054
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
## Latent Variables:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP =~
##     O1                1.000                              0.400    0.827
##     O2                0.934    0.036   26.185    0.000    0.374    0.799
##     O3                1.149    0.040   28.900    0.000    0.460    0.898
##   AG =~
##     A1                1.000                              0.426    0.846
##     A2                0.910    0.032   28.812    0.000    0.388    0.823
##     A3                1.030    0.032   31.899    0.000    0.439    0.914
##   PR =~
##     EP                1.000                              0.530    0.746
##     SP                0.886    0.051   17.348    0.000    0.469    0.686
##     HP                1.030    0.112    9.160    0.000    0.545    0.350
##     DP                0.746    0.041   18.308    0.000    0.395    0.741
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP ~~
##     AG                0.049    0.007    7.105    0.000    0.286    0.286
##     PR               -0.122    0.011  -11.371    0.000   -0.573   -0.573
##   AG ~~
##     PR               -0.110    0.011  -10.241    0.000   -0.485   -0.485
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OP                0.160    0.011   14.156    0.000    1.000    1.000
##     AG                0.182    0.012   14.822    0.000    1.000    1.000
##     PR                0.281    0.025   11.385    0.000    1.000    1.000
##    .O1                0.074    0.005   14.555    0.000    0.074    0.317
##    .O2                0.079    0.005   15.837    0.000    0.079    0.361
##    .O3                0.051    0.005    9.630    0.000    0.051    0.194
##    .A1                0.072    0.005   14.461    0.000    0.072    0.284
##    .A2                0.072    0.005   15.697    0.000    0.072    0.322
##    .A3                0.038    0.004    9.152    0.000    0.038    0.165
##    .EP                0.224    0.016   14.198    0.000    0.224    0.444
##    .SP                0.248    0.015   16.146    0.000    0.248    0.530
##    .HP                2.137    0.107   20.052    0.000    2.137    0.878
##    .DP                0.128    0.009   14.376    0.000    0.128    0.451
##
## R-Square:
##                    Estimate
##     O1                0.683
##     O2                0.639
##     O3                0.806
##     A1                0.716
##     A2                0.678
##     A3                0.835
##     EP                0.556
##     SP                0.470
##     HP                0.122
##     DP                0.549
```

```
# Global fit indices
fitMeasures(model3.fit)
```

```
##                npar                fmin               chisq                  df
##              23.000               0.108             186.620              32.000
##              pvalue      baseline.chisq         baseline.df     baseline.pvalue
##               0.000            4270.205              45.000               0.000
##                 cfi                 tli                nnfi                 rfi
##               0.963               0.949               0.949               0.939
##                 nfi                pnfi                 ifi                 rni
##               0.956               0.680               0.964               0.963
##                logl   unrestricted.logl                 aic                 bic
##           -5672.807           -5579.497           11391.614           11501.050
##              ntotal                bic2               rmsea      rmsea.ci.lower
##             861.000           11428.008               0.075               0.065
##      rmsea.ci.upper        rmsea.pvalue                 rmr          rmr_nomean
##               0.085               0.000               0.025               0.025
##                srmr        srmr_bentler srmr_bentler_nomean                crmr
##               0.054               0.054               0.054               0.060
##         crmr_nomean          srmr_mplus   srmr_mplus_nomean              cn_05
##               0.060               0.054               0.054             214.124
##               cn_01                 gfi                agfi                pgfi
##             247.764               0.959               0.930               0.558
##                 mfi                ecvi
##               0.914               0.270
```

```r
# Local fit measures
modindices(model3.fit)
```

```
##    lhs op rhs     mi    epc sepc.lv sepc.all sepc.nox
## 27  OP =~  A1 19.169 -0.131  -0.052   -0.104   -0.104
## 28  OP =~  A2 34.333  0.169   0.068    0.144    0.144
## 29  OP =~  A3  1.024 -0.028  -0.011   -0.023   -0.023
## 30  OP =~  EP  2.036 -0.109  -0.044   -0.061   -0.061
## 31  OP =~  SP  0.446  0.048   0.019    0.028    0.028
## 32  OP =~  HP 22.203 -0.842  -0.337   -0.216   -0.216
## 33  OP =~  DP  8.316  0.165   0.066    0.124    0.124
## 34  AG =~  O1 15.221 -0.111  -0.047   -0.098   -0.098
## 35  AG =~  O2  0.098  0.009   0.004    0.008    0.008
## 36  AG =~  O3 10.358  0.094   0.040    0.078    0.078
## 37  AG =~  EP  0.373  0.038   0.016    0.023    0.023
## 38  AG =~  SP 22.259  0.283   0.121    0.176    0.176
## 39  AG =~  HP  2.920  0.258   0.110    0.070    0.070
## 40  AG =~  DP 32.769 -0.268  -0.114   -0.214   -0.214
## 41  PR =~  O1  2.227  0.047   0.025    0.052    0.052
## 42  PR =~  O2  1.570 -0.039  -0.021   -0.044   -0.044
## 43  PR =~  O3  0.077 -0.009  -0.005   -0.010   -0.010
## 44  PR =~  A1 19.206  0.121   0.064    0.127    0.127
## 45  PR =~  A2 68.775 -0.220  -0.117   -0.247   -0.247
## 46  PR =~  A3  9.718  0.080   0.042    0.088    0.088
## 47  O1 ~~  O2  0.123  0.002   0.002    0.031    0.031
## 48  O1 ~~  O3  1.554  0.012   0.012    0.192    0.192
## 49  O1 ~~  A1  3.654 -0.006  -0.006   -0.085   -0.085
## 50  O1 ~~  A2  0.940  0.003   0.003    0.042    0.042
## 51  O1 ~~  A3  2.244 -0.004  -0.004   -0.080   -0.080
## 52  O1 ~~  EP  0.021  0.001   0.001    0.006    0.006
## 53  O1 ~~  SP  5.349 -0.013  -0.013   -0.099   -0.099
## 54  O1 ~~  HP  0.749  0.014   0.014    0.034    0.034
```

```
## 55  O1 ~~  DP  1.246  0.005   0.005    0.050    0.050
## 56  O2 ~~  O3  2.451 -0.013  -0.013   -0.208   -0.208
## 57  O2 ~~  A1  1.256 -0.004  -0.004   -0.048   -0.048
## 58  O2 ~~  A2  0.448  0.002   0.002    0.028    0.028
## 59  O2 ~~  A3  0.109  0.001   0.001    0.017    0.017
## 60  O2 ~~  EP  0.169  0.002   0.002    0.018    0.018
## 61  O2 ~~  SP  0.109  0.002   0.002    0.014    0.014
## 62  O2 ~~  HP  7.969 -0.044  -0.044   -0.108   -0.108
## 63  O2 ~~  DP  0.908 -0.004  -0.004   -0.042   -0.042
## 64  O3 ~~  A1  0.261  0.002   0.002    0.027    0.027
## 65  O3 ~~  A2  0.121  0.001   0.001    0.018    0.018
## 66  O3 ~~  A3  1.840  0.004   0.004    0.085    0.085
## 67  O3 ~~  EP  1.762 -0.008  -0.008   -0.071   -0.071
## 68  O3 ~~  SP  4.455  0.012   0.012    0.108    0.108
## 69  O3 ~~  HP  3.172 -0.027  -0.027   -0.083   -0.083
## 70  O3 ~~  DP  2.594  0.007   0.007    0.086    0.086
## 71  A1 ~~  A2  9.573 -0.025  -0.025   -0.344   -0.344
## 72  A1 ~~  A3 69.141  0.088   0.088    1.688    1.688
## 73  A1 ~~  EP  2.134  0.008   0.008    0.066    0.066
## 74  A1 ~~  SP  1.050  0.006   0.006    0.044    0.044
## 75  A1 ~~  HP  2.147 -0.023  -0.023   -0.058   -0.058
## 76  A1 ~~  DP  0.414  0.003   0.003    0.029    0.029
## 77  A2 ~~  A3 19.619 -0.041  -0.041   -0.778   -0.778
## 78  A2 ~~  EP  4.065 -0.011  -0.011   -0.088   -0.088
## 79  A2 ~~  SP  1.918 -0.008  -0.008   -0.058   -0.058
## 80  A2 ~~  HP  0.368 -0.009  -0.009   -0.023   -0.023
## 81  A2 ~~  DP  9.809 -0.013  -0.013   -0.137   -0.137
## 82  A3 ~~  EP  0.513  0.004   0.004    0.039    0.039
## 83  A3 ~~  SP  8.962  0.015   0.015    0.155    0.155
## 84  A3 ~~  HP  7.936  0.038   0.038    0.133    0.133
## 85  A3 ~~  DP  1.665 -0.005  -0.005   -0.070   -0.070
## 86  EP ~~  SP  5.328  0.033   0.033    0.140    0.140
## 87  EP ~~  HP  0.132 -0.011  -0.011   -0.016   -0.016
## 88  EP ~~  DP  8.308 -0.035  -0.035   -0.205   -0.205
## 89  SP ~~  HP  0.805 -0.026  -0.026   -0.036   -0.036
## 90  SP ~~  DP  3.206  0.019   0.019    0.107    0.107
## 91  HP ~~  DP  1.034 -0.023  -0.023   -0.043   -0.043
```

```
resid(model3.fit)
```

```
## $type
## [1] "raw"
##
## $cov
##    O1     O2     O3     A1     A2     A3     EP     SP     HP     DP
## O1  0.000
## O2  0.000  0.000
## O3  0.001 -0.001  0.000
## A1 -0.028 -0.013 -0.007  0.000
## A2  0.008  0.018  0.026 -0.002  0.000
## A3 -0.016  0.000  0.006  0.002 -0.002  0.000
## EP  0.000 -0.007 -0.009  0.021 -0.030  0.009  0.000
## SP -0.002  0.000  0.008  0.038 -0.008  0.035  0.010  0.000
## HP -0.051 -0.100 -0.090  0.025 -0.008  0.054 -0.007 -0.019  0.000
## DP  0.013  0.002  0.011 -0.007 -0.044 -0.017 -0.008  0.006 -0.015  0.000
```

```
resid(model3.fit, type = "standardized")
```

```
## $type
## [1] "standardized"
##
## $cov
##    O1     O2     O3     A1     A2     A3     EP     SP     HP     DP
## O1  0.000
## O2  0.349  0.000
## O3  1.225 -1.598  0.000
## A1 -5.521 -2.482 -1.555  0.000
## A2  1.621  3.639  5.588 -3.240  0.000
## A3 -3.822 -0.018  1.838  7.510 -4.704  0.000
## EP -0.052 -1.051 -1.625  2.934 -4.450  1.638  0.000
## SP -0.330  0.074  1.307  5.177 -1.085  5.615  2.165  0.000
## HP -2.440 -4.947 -4.275  1.085 -0.421  2.595 -0.366 -0.904  0.000
## DP  2.764  0.352  2.630 -1.239 -8.406 -3.636 -3.017  1.694 -1.032  0.000
```

```
# Fitted values of the covariance matrix and the mean vector
fitted(model3.fit)
```

```
## $cov
##    O1     O2     O3     A1     A2     A3     EP     SP     HP     DP
## O1  0.235
## O2  0.150  0.219
## O3  0.184  0.172  0.263
## A1  0.049  0.045  0.056  0.254
## A2  0.044  0.041  0.051  0.165  0.222
## A3  0.050  0.047  0.058  0.187  0.170  0.231
## EP -0.122 -0.114 -0.140 -0.110 -0.100 -0.113  0.505
## SP -0.108 -0.101 -0.124 -0.097 -0.088 -0.100  0.249  0.468
## HP -0.125 -0.117 -0.144 -0.113 -0.103 -0.116  0.289  0.256  2.434
## DP -0.091 -0.085 -0.104 -0.082 -0.074 -0.084  0.209  0.185  0.216  0.285
```

```
# List all parameter values
parameterEstimates(model3.fit)
```

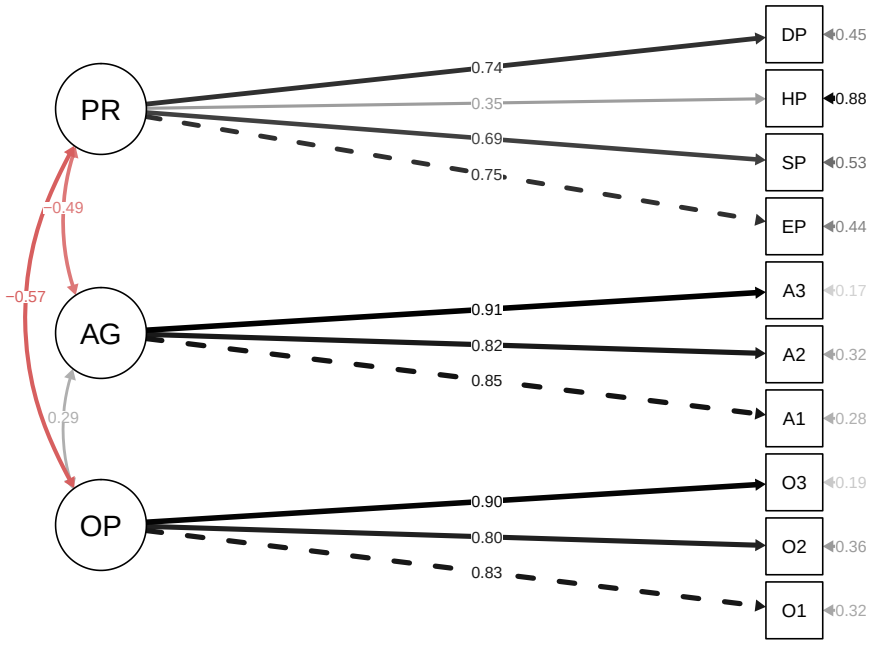
```
##    lhs op rhs    est    se       z pvalue ci.lower ci.upper
## 1   OP =~  O1  1.000 0.000      NA     NA    1.000    1.000
## 2   OP =~  O2  0.934 0.036  26.185      0    0.864    1.004
## 3   OP =~  O3  1.149 0.040  28.900      0    1.071    1.227
## 4   AG =~  A1  1.000 0.000      NA     NA    1.000    1.000
## 5   AG =~  A2  0.910 0.032  28.812      0    0.848    0.972
## 6   AG =~  A3  1.030 0.032  31.899      0    0.967    1.094
## 7   PR =~  EP  1.000 0.000      NA     NA    1.000    1.000
## 8   PR =~  SP  0.886 0.051  17.348      0    0.786    0.986
## 9   PR =~  HP  1.030 0.112   9.160      0    0.809    1.250
## 10  PR =~  DP  0.746 0.041  18.308      0    0.666    0.826
## 11  OP ~~  OP  0.160 0.011  14.156      0    0.138    0.182
## 12  OP ~~  AG  0.049 0.007   7.105      0    0.035    0.062
## 13  OP ~~  PR -0.122 0.011 -11.371      0   -0.143   -0.101
## 14  AG ~~  AG  0.182 0.012  14.822      0    0.158    0.206
## 15  AG ~~  PR -0.110 0.011 -10.241      0   -0.131   -0.089
## 16  PR ~~  PR  0.281 0.025  11.385      0    0.232    0.329
## 17  O1 ~~  O1  0.074 0.005  14.555      0    0.064    0.084
```

```
## 18  O2 ~~  O2  0.079 0.005  15.837      0    0.069    0.089
## 19  O3 ~~  O3  0.051 0.005   9.630      0    0.041    0.061
## 20  A1 ~~  A1  0.072 0.005  14.461      0    0.062    0.082
## 21  A2 ~~  A2  0.072 0.005  15.697      0    0.063    0.081
## 22  A3 ~~  A3  0.038 0.004   9.152      0    0.030    0.046
## 23  EP ~~  EP  0.224 0.016  14.198      0    0.193    0.255
## 24  SP ~~  SP  0.248 0.015  16.146      0    0.218    0.278
## 25  HP ~~  HP  2.137 0.107  20.052      0    1.928    2.346
## 26  DP ~~  DP  0.128 0.009  14.376      0    0.111    0.146
```

```
# Check the model setup
inspect(model3.fit)
```

```
## $lambda
##    OP AG PR
## O1  0  0  0
## O2  1  0  0
## O3  2  0  0
## A1  0  0  0
## A2  0  3  0
## A3  0  4  0
## EP  0  0  0
## SP  0  0  5
## HP  0  0  6
## DP  0  0  7
##
## $theta
##    O1 O2 O3 A1 A2 A3 EP SP HP DP
## O1 14
## O2  0 15
## O3  0  0 16
## A1  0  0  0 17
## A2  0  0  0  0 18
## A3  0  0  0  0  0 19
## EP  0  0  0  0  0  0 20
## SP  0  0  0  0  0  0  0 21
## HP  0  0  0  0  0  0  0  0 22
## DP  0  0  0  0  0  0  0  0  0 23
##
## $psi
##    OP AG PR
## OP  8
## AG  9 11
## PR 10 12 13
```

```
# Visualize the path model
semPaths(model3.fit,
         rotation = 2,
         layout = "tree2",
         what = "std",
         posCol = "black",
         edge.width = 0.5,
         style = "Lisrel",
         fade = T,
         edge.label.position = 0.55)
```



Model 3: Refined CFA of personality (T)

```
# Step 1: Model specification
model3b <- '
            # Measurement models
            OP =~ O1 + O2 + O3
            AG =~ A1 + A2 + A3
            PR =~ EP + SP + HP + DP

            # Covariance structure
            OP ~~ OP + AG + PR
            AG ~~ AG + PR
            PR ~~ PR

            # Residual covariance
            A1 ~~ A3

'

# Step 2: Model estimation
model3b.fit <- sem(model3b,
                   data = Bergh,
                   meanstructure = FALSE,
                   estimator = "ML")

# Step 3: Evaluate the model
# Summary
summary(model3b.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

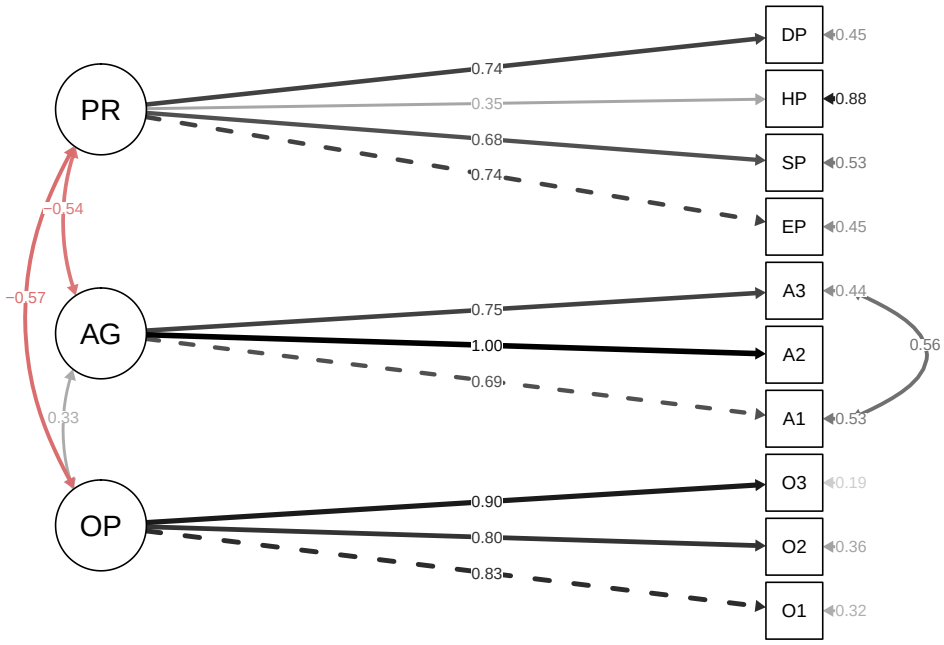
```
## lavaan 0.6-5 ended normally after 62 iterations
##
```

```
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         24
##
##   Number of observations                           861
##
## Model Test User Model:
##
##   Test statistic                               118.256
##   Degrees of freedom                                31
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              4270.205
##   Degrees of freedom                                45
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.979
##   Tucker-Lewis Index (TLI)                       0.970
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -5638.625
##   Loglikelihood unrestricted model (H1)      -5579.497
##
##   Akaike (AIC)                               11325.249
##   Bayesian (BIC)                             11439.444
##   Sample-size adjusted Bayesian (BIC)        11363.226
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.057
##   90 Percent confidence interval - lower         0.046
##   90 Percent confidence interval - upper         0.068
##   P-value RMSEA <= 0.05                          0.131
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.043
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP =~
##     O1                1.000                               0.400    0.827
##     O2                0.934    0.036   26.188    0.000    0.374    0.799
```

```
##     O3                1.149    0.040   28.921    0.000    0.460    0.898
##   AG =~
##     A1                1.000                              0.346    0.687
##     A2                1.361    0.086   15.756    0.000    0.471    0.999
##     A3                1.036    0.033   31.662    0.000    0.358    0.746
##   PR =~
##     EP                1.000                              0.529    0.744
##     SP                0.887    0.051   17.460    0.000    0.469    0.685
##     HP                1.031    0.112    9.177    0.000    0.545    0.349
##     DP                0.750    0.040   18.535    0.000    0.397    0.744
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP ~~
##     AG                0.046    0.006    7.516    0.000    0.330    0.330
##     PR               -0.121    0.011  -11.372    0.000   -0.573   -0.573
##   AG ~~
##     PR               -0.098    0.010   -9.409    0.000   -0.536   -0.536
##  .A1 ~~
##    .A3                0.066    0.008    8.266    0.000    0.066    0.560
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OP                0.160    0.011   14.158    0.000    1.000    1.000
##     AG                0.120    0.012    9.879    0.000    1.000    1.000
##     PR                0.279    0.024   11.413    0.000    1.000    1.000
##    .O1                0.074    0.005   14.570    0.000    0.074    0.317
##    .O2                0.079    0.005   15.846    0.000    0.079    0.361
##    .O3                0.051    0.005    9.643    0.000    0.051    0.194
##    .A1                0.134    0.009   14.890    0.000    0.134    0.528
##    .A2                0.000    0.012    0.027    0.979    0.000    0.001
##    .A3                0.102    0.008   12.293    0.000    0.102    0.444
##    .EP                0.225    0.016   14.456    0.000    0.225    0.447
##    .SP                0.249    0.015   16.300    0.000    0.249    0.531
##    .HP                2.138    0.106   20.073    0.000    2.138    0.878
##    .DP                0.127    0.009   14.467    0.000    0.127    0.447
##
## R-Square:
##                    Estimate
##     O1                0.683
##     O2                0.639
##     O3                0.806
##     A1                0.472
##     A2                0.999
##     A3                0.556
##     EP                0.553
##     SP                0.469
##     HP                0.122
##     DP                0.553
```

```
# Visualize the path model
semPaths(model3b.fit,
         rotation = 2,
         layout = "tree2",
```

```
         what = "std",
         posCol = "black",
         edge.width = 0.5,
         style = "Lisrel",
         fade = T,
         edge.label.position = 0.55)
```




```
## Model comparison: Model 3 vs. refined Model 3
anova(model3.fit, model3b.fit)
```

```
## Chi-Squared Difference Test
##
##             Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model3b.fit 31 11325 11439 118.26
## model3.fit  32 11392 11501 186.62     68.364       1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model 4: Structural equation model

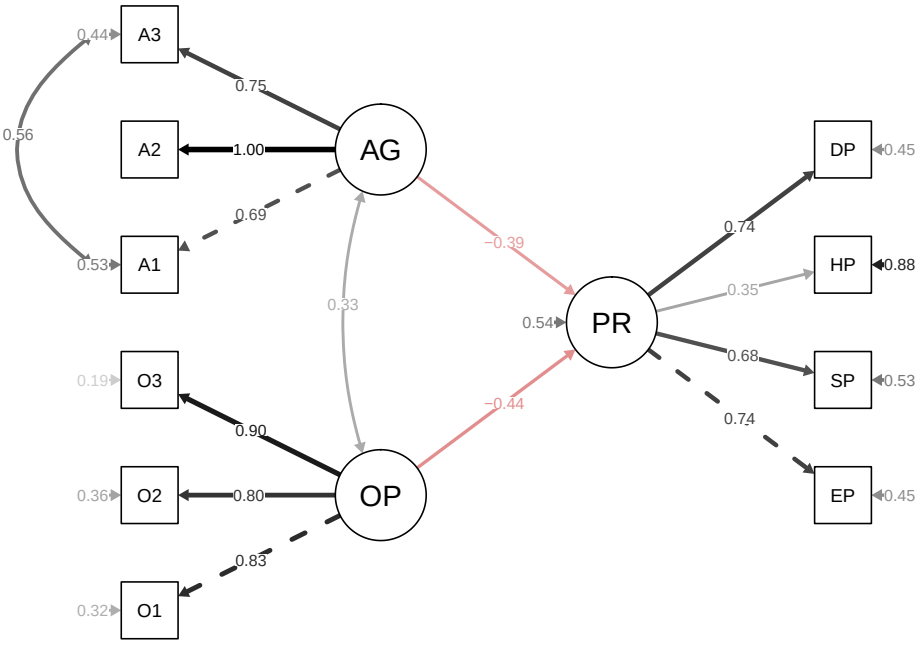
```
# Step 1: Model specification
model4 <- '
            # Measurement models
            OP =~ O1 + O2 + O3
            AG =~ A1 + A2 + A3
            PR =~ EP + SP + HP + DP

            # Residual covariance
            A1 ~~ A3

            # Structural model
            PR ~ b1*OP + b2*AG

'

# Step 2: Model estimation
model4.fit <- sem(model4,
                  data = Bergh,
                  meanstructure = FALSE,
                  estimator = "ML")

# Step 3: Evaluate the model
# Summary
summary(model4.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-5 ended normally after 55 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         24
##
##   Number of observations                           861
##
## Model Test User Model:
##
##   Test statistic                               118.256
##   Degrees of freedom                                31
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              4270.205
##   Degrees of freedom                                45
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.979
##   Tucker-Lewis Index (TLI)                       0.970
##
## Loglikelihood and Information Criteria:
```

```
##
##   Loglikelihood user model (H0)              -5638.625
##   Loglikelihood unrestricted model (H1)      -5579.497
##
##   Akaike (AIC)                               11325.249
##   Bayesian (BIC)                             11439.444
##   Sample-size adjusted Bayesian (BIC)        11363.226
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.057
##   90 Percent confidence interval - lower         0.046
##   90 Percent confidence interval - upper         0.068
##   P-value RMSEA <= 0.05                          0.131
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.043
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP =~
##     O1                1.000                               0.400    0.827
##     O2                0.934    0.036   26.188    0.000    0.374    0.799
##     O3                1.149    0.040   28.921    0.000    0.460    0.898
##   AG =~
##     A1                1.000                               0.346    0.687
##     A2                1.361    0.086   15.756    0.000    0.471    0.999
##     A3                1.036    0.033   31.662    0.000    0.358    0.746
##   PR =~
##     EP                1.000                               0.529    0.744
##     SP                0.887    0.051   17.460    0.000    0.469    0.685
##     HP                1.031    0.112    9.177    0.000    0.545    0.349
##     DP                0.750    0.040   18.535    0.000    0.397    0.744
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   PR ~
##     OP        (b1)   -0.587    0.053  -11.106    0.000   -0.444   -0.444
##     AG        (b2)   -0.595    0.058  -10.172    0.000   -0.390   -0.390
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .A1 ~~
##    .A3                0.066    0.008    8.266    0.000    0.066    0.560
##   OP ~~
##     AG                0.046    0.006    7.516    0.000    0.330    0.330
##
```

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .O1                0.074    0.005   14.570    0.000    0.074    0.317
##    .O2                0.079    0.005   15.846    0.000    0.079    0.361
##    .O3                0.051    0.005    9.643    0.000    0.051    0.194
##    .A1                0.134    0.009   14.890    0.000    0.134    0.528
##    .A2                0.000    0.012    0.027    0.979    0.000    0.001
##    .A3                0.102    0.008   12.293    0.000    0.102    0.444
##    .EP                0.225    0.016   14.456    0.000    0.225    0.447
##    .SP                0.249    0.015   16.300    0.000    0.249    0.531
##    .HP                2.138    0.106   20.073    0.000    2.138    0.878
##    .DP                0.127    0.009   14.467    0.000    0.127    0.447
##     OP                0.160    0.011   14.158    0.000    1.000    1.000
##     AG                0.120    0.012    9.879    0.000    1.000    1.000
##    .PR                0.150    0.015    9.937    0.000    0.536    0.536
##
## R-Square:
##                    Estimate
##     O1                0.683
##     O2                0.639
##     O3                0.806
##     A1                0.472
##     A2                0.999
##     A3                0.556
##     EP                0.553
##     SP                0.469
##     HP                0.122
##     DP                0.553
##     PR                0.464
```

```r
# Visualize the path model
semPaths(model4.fit,
         rotation = 2,
         layout = "tree2",
         what = "std",
         posCol = "black",
         edge.width = 0.5,
         style = "Lisrel",
         fade = T,
         edge.label.position = 0.55)
```




```
## Hypothesis testing
lavTestWald(model4.fit, constraints = "b1 == b2")
```

```
## $stat
## [1] 0.009016331
##
## $df
## [1] 1
##
## $p.value
## [1] 0.9243511
##
## $se
## [1] "standard"
```

Model 5: Multi-group SEM (Gender differences in the structural parameters)

```
# Step 1: Model specification
model5 <- '
            # Measurement models
            OP =~ O1 + O2 + O3
            AG =~ A1 + A2 + A3
            PR =~ EP + SP + HP + DP

            # Covariance structure
            OP ~~ OP + AG
            AG ~~ AG

            # Residual covariance
            A1 ~~ A3

            # Structural model
            PR ~ c(a1,b1)*OP + c(a2,b2)*AG

'

# Step 2: Model estimation
# Only allow for differences in the structural parameters
# Keep all other parameters equal (measurement invariance)
model5.fit <- sem(model5,
                  data = Bergh,
                  meanstructure = FALSE,
                  estimator = "ML",
                  group = "gender",
                  group.equal = c("loadings", "residuals"))

# Step 3: Evaluate the model
# Summary
summary(model5.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-5 ended normally after 60 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         48
##   Number of equality constraints                    17
##   Row rank of the constraints matrix                17
##
##   Number of observations per group:
##     male                                           249
##     female                                         612
##
## Model Test User Model:
##
##   Test statistic                               208.998
##   Degrees of freedom                                79
##   P-value (Chi-square)                           0.000
```

```
##   Test statistic for each group:
##     male                                       83.323
##     female                                    125.675
##
## Model Test Baseline Model:
##
##   Test statistic                             4207.254
##   Degrees of freedom                              90
##   P-value                                      0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                  0.968
##   Tucker-Lewis Index (TLI)                     0.964
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)             -5575.804
##   Loglikelihood unrestricted model (H1)     -5471.305
##
##   Akaike (AIC)                              11213.608
##   Bayesian (BIC)                            11361.109
##   Sample-size adjusted Bayesian (BIC)       11262.661
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                        0.062
##   90 Percent confidence interval - lower       0.052
##   90 Percent confidence interval - upper       0.072
##   P-value RMSEA <= 0.05                        0.028
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                         0.067
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
##
## Group 1 [male]:
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP =~
##     O1                1.000                               0.410    0.833
##     O2      (.p2.)    0.932    0.036   26.006    0.000    0.382    0.806
##     O3      (.p3.)    1.148    0.040   28.727    0.000    0.471    0.902
##   AG =~
##     A1                1.000                               0.346    0.689
##     A2      (.p5.)    1.327    0.089   14.919    0.000    0.459    0.993
##     A3      (.p6.)    1.021    0.033   31.242    0.000    0.353    0.744
```

```
##   PR =~
##     EP                1.000                               0.552    0.761
##     SP      (.p8.)    0.822    0.049   16.706    0.000    0.454    0.685
##     HP      (.p9.)    1.029    0.114    9.020    0.000    0.568    0.363
##     DP      (.10.)    0.733    0.041   17.857    0.000    0.405    0.748
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   PR ~
##     OP        (a1)   -0.620    0.097   -6.403    0.000   -0.461   -0.461
##     AG        (a2)   -0.510    0.108   -4.719    0.000   -0.320   -0.320
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP ~~
##     AG                0.059    0.011    5.315    0.000    0.414    0.414
##  .A1 ~~
##    .A3                0.058    0.009    6.338    0.000    0.058    0.502
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OP                0.168    0.019    9.083    0.000    1.000    1.000
##     AG                0.120    0.015    7.753    0.000    1.000    1.000
##    .O1      (.17.)    0.074    0.005   14.561    0.000    0.074    0.306
##    .O2      (.18.)    0.079    0.005   15.870    0.000    0.079    0.351
##    .O3      (.19.)    0.051    0.005    9.612    0.000    0.051    0.186
##    .A1      (.20.)    0.132    0.009   14.304    0.000    0.132    0.525
##    .A2      (.21.)    0.003    0.012    0.244    0.807    0.003    0.013
##    .A3      (.22.)    0.101    0.008   11.859    0.000    0.101    0.447
##    .EP      (.23.)    0.221    0.016   14.053    0.000    0.221    0.421
##    .SP      (.24.)    0.233    0.014   16.504    0.000    0.233    0.531
##    .HP      (.25.)    2.133    0.106   20.044    0.000    2.133    0.869
##    .DP      (.26.)    0.129    0.009   14.552    0.000    0.129    0.440
##    .PR                0.172    0.026    6.590    0.000    0.564    0.564
##
## R-Square:
##                    Estimate
##     O1                0.694
##     O2                0.649
##     O3                0.814
##     A1                0.475
##     A2                0.987
##     A3                0.553
##     EP                0.579
##     SP                0.469
##     HP                0.131
##     DP                0.560
##     PR                0.436
##
##
## Group 2 [female]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

```
##   OP =~
##     O1                1.000                               0.394    0.822
##     O2      (.p2.)    0.932    0.036   26.006    0.000    0.367    0.794
##     O3      (.p3.)    1.148    0.040   28.727    0.000    0.452    0.895
##   AG =~
##     A1                1.000                               0.343    0.686
##     A2      (.p5.)    1.327    0.089   14.919    0.000    0.455    0.993
##     A3      (.p6.)    1.021    0.033   31.242    0.000    0.351    0.741
##   PR =~
##     EP                1.000                               0.511    0.736
##     SP      (.p8.)    0.822    0.049   16.706    0.000    0.420    0.656
##     HP      (.p9.)    1.029    0.114    9.020    0.000    0.526    0.339
##     DP      (.10.)    0.733    0.041   17.857    0.000    0.375    0.722
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   PR ~
##     OP        (b1)   -0.586    0.060   -9.722    0.000   -0.451   -0.451
##     AG        (b2)   -0.568    0.066   -8.614    0.000   -0.381   -0.381
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP ~~
##     AG                0.037    0.007    5.683    0.000    0.276    0.276
##  .A1 ~~
##    .A3                0.067    0.008    8.011    0.000    0.067    0.576
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OP                0.155    0.012   12.660    0.000    1.000    1.000
##     AG                0.118    0.013    9.258    0.000    1.000    1.000
##    .O1      (.17.)    0.074    0.005   14.561    0.000    0.074    0.324
##    .O2      (.18.)    0.079    0.005   15.870    0.000    0.079    0.370
##    .O3      (.19.)    0.051    0.005    9.612    0.000    0.051    0.199
##    .A1      (.20.)    0.132    0.009   14.304    0.000    0.132    0.529
##    .A2      (.21.)    0.003    0.012    0.244    0.807    0.003    0.014
##    .A3      (.22.)    0.101    0.008   11.859    0.000    0.101    0.450
##    .EP      (.23.)    0.221    0.016   14.053    0.000    0.221    0.459
##    .SP      (.24.)    0.233    0.014   16.504    0.000    0.233    0.569
##    .HP      (.25.)    2.133    0.106   20.044    0.000    2.133    0.885
##    .DP      (.26.)    0.129    0.009   14.552    0.000    0.129    0.478
##    .PR                0.145    0.017    8.743    0.000    0.556    0.556
##
## R-Square:
##                    Estimate
##     O1                0.676
##     O2                0.630
##     O3                0.801
##     A1                0.471
##     A2                0.986
##     A3                0.550
##     EP                0.541
##     SP                0.431
##     HP                0.115
```

```
##     DP                0.522
##     PR                0.444
```

```
# Hypothesis testing
lavTestWald(model5.fit, constraints = "a1==b1")
```

```
## $stat
## [1] 0.0955577
##
## $df
## [1] 1
##
## $p.value
## [1] 0.7572271
##
## $se
## [1] "standard"
```

```
lavTestWald(model5.fit, constraints = "a2==b2")
```

```
## $stat
## [1] 0.2275054
##
## $df
## [1] 1
##
## $p.value
## [1] 0.6333798
##
## $se
## [1] "standard"
```

Model 6: Multi-group SEM with equal structural parameters

```
# Step 1: Model specification
model6 <- '
            # Measurement models
            OP =~ O1 + O2 + O3
            AG =~ A1 + A2 + A3
            PR =~ EP + SP + HP + DP

            # Covariance structure
            OP ~~ OP + AG
            AG ~~ AG

            # Residual covariance
            A1 ~~ A3

            # Structural model
            PR ~ OP + AG

'

# Step 2: Model estimation
```

```r
model6.fit <- sem(model6,
                  data = Bergh,
                  meanstructure = FALSE,
                  estimator = "ML",
                  group = "gender",
                  group.equal = c("loadings",
                                  "residuals",
                                  "regressions"))

# Summary
summary(model6.fit, rsquare = TRUE, fit.measures = TRUE, standardized = TRUE)
```

```
## lavaan 0.6-5 ended normally after 59 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of free parameters                         48
##   Number of equality constraints                    19
##   Row rank of the constraints matrix                19
##
##   Number of observations per group:
##     male                                           249
##     female                                         612
##
## Model Test User Model:
##
##   Test statistic                               209.237
##   Degrees of freedom                                81
##   P-value (Chi-square)                           0.000
##   Test statistic for each group:
##     male                                        83.388
##     female                                     125.849
##
## Model Test Baseline Model:
##
##   Test statistic                              4207.254
##   Degrees of freedom                                90
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.969
##   Tucker-Lewis Index (TLI)                       0.965
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -5575.924
##   Loglikelihood unrestricted model (H1)      -5471.305
##
##   Akaike (AIC)                               11209.847
##   Bayesian (BIC)                             11347.832
##   Sample-size adjusted Bayesian (BIC)        11255.736
##
## Root Mean Square Error of Approximation:
```

```
##
##   RMSEA                                          0.061
##   90 Percent confidence interval - lower         0.051
##   90 Percent confidence interval - upper         0.071
##   P-value RMSEA <= 0.05                          0.041
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.067
##
## Parameter Estimates:
##
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##   Standard errors                             Standard
##
##
## Group 1 [male]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP =~
##     O1                1.000                               0.411    0.833
##     O2      (.p2.)    0.932    0.036   26.007    0.000    0.383    0.806
##     O3      (.p3.)    1.148    0.040   28.723    0.000    0.471    0.902
##   AG =~
##     A1                1.000                               0.346    0.690
##     A2      (.p5.)    1.322    0.088   14.987    0.000    0.458    0.991
##     A3      (.p6.)    1.022    0.033   31.242    0.000    0.354    0.745
##   PR =~
##     EP                1.000                               0.554    0.762
##     SP      (.p8.)    0.822    0.049   16.716    0.000    0.456    0.686
##     HP      (.p9.)    1.029    0.114    9.019    0.000    0.570    0.364
##     DP      (.10.)    0.733    0.041   17.855    0.000    0.406    0.749
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   PR ~
##     OP      (.15.)   -0.593    0.053  -11.116    0.000   -0.439   -0.439
##     AG      (.16.)   -0.554    0.058   -9.488    0.000   -0.346   -0.346
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP ~~
##     AG                0.059    0.011    5.315    0.000    0.414    0.414
##  .A1 ~~
##    .A3                0.058    0.009    6.303    0.000    0.058    0.500
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OP                0.169    0.019    9.099    0.000    1.000    1.000
##     AG                0.120    0.015    7.760    0.000    1.000    1.000
##    .O1      (.17.)    0.074    0.005   14.560    0.000    0.074    0.306
##    .O2      (.18.)    0.079    0.005   15.867    0.000    0.079    0.351
```

```
##    .O3      (.19.)    0.051    0.005    9.610    0.000    0.051    0.186
##    .A1      (.20.)    0.132    0.009   14.287    0.000    0.132    0.524
##    .A2      (.21.)    0.004    0.012    0.312    0.755    0.004    0.017
##    .A3      (.22.)    0.100    0.008   11.834    0.000    0.100    0.445
##    .EP      (.23.)    0.221    0.016   14.050    0.000    0.221    0.419
##    .SP      (.24.)    0.233    0.014   16.496    0.000    0.233    0.529
##    .HP      (.25.)    2.133    0.106   20.044    0.000    2.133    0.868
##    .DP      (.26.)    0.129    0.009   14.562    0.000    0.129    0.439
##    .PR                0.172    0.026    6.611    0.000    0.561    0.561
##
## R-Square:
##                    Estimate
##     O1                0.694
##     O2                0.649
##     O3                0.814
##     A1                0.476
##     A2                0.983
##     A3                0.555
##     EP                0.581
##     SP                0.471
##     HP                0.132
##     DP                0.561
##     PR                0.439
##
##
## Group 2 [female]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP =~
##     O1                1.000                               0.393    0.822
##     O2      (.p2.)    0.932    0.036   26.007    0.000    0.367    0.794
##     O3      (.p3.)    1.148    0.040   28.723    0.000    0.452    0.895
##   AG =~
##     A1                1.000                               0.344    0.688
##     A2      (.p5.)    1.322    0.088   14.987    0.000    0.455    0.991
##     A3      (.p6.)    1.022    0.033   31.242    0.000    0.351    0.743
##   PR =~
##     EP                1.000                               0.511    0.735
##     SP      (.p8.)    0.822    0.049   16.716    0.000    0.420    0.656
##     HP      (.p9.)    1.029    0.114    9.019    0.000    0.525    0.338
##     DP      (.10.)    0.733    0.041   17.855    0.000    0.374    0.722
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   PR ~
##     OP      (.15.)   -0.593    0.053  -11.116    0.000   -0.457   -0.457
##     AG      (.16.)   -0.554    0.058   -9.488    0.000   -0.374   -0.374
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   OP ~~
##     AG                0.038    0.007    5.698    0.000    0.277    0.277
##  .A1 ~~
```

```
##    .A3               0.066    0.008    7.978    0.000    0.066    0.575
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##     OP                0.155    0.012   12.667    0.000    1.000    1.000
##     AG                0.118    0.013    9.279    0.000    1.000    1.000
##    .O1       (.17.)   0.074    0.005   14.560    0.000    0.074    0.324
##    .O2       (.18.)   0.079    0.005   15.867    0.000    0.079    0.370
##    .O3       (.19.)   0.051    0.005    9.610    0.000    0.051    0.199
##    .A1       (.20.)   0.132    0.009   14.287    0.000    0.132    0.527
##    .A2       (.21.)   0.004    0.012    0.312    0.755    0.004    0.017
##    .A3       (.22.)   0.100    0.008   11.834    0.000    0.100    0.448
##    .EP       (.23.)   0.221    0.016   14.050    0.000    0.221    0.459
##    .SP       (.24.)   0.233    0.014   16.496    0.000    0.233    0.569
##    .HP       (.25.)   2.133    0.106   20.044    0.000    2.133    0.885
##    .DP       (.26.)   0.129    0.009   14.562    0.000    0.129    0.479
##    .PR                0.145    0.017    8.747    0.000    0.557    0.557
##
## R-Square:
##                    Estimate
##     O1                0.676
##     O2                0.630
##     O3                0.801
##     A1                0.473
##     A2                0.983
##     A3                0.552
##     EP                0.541
##     SP                0.431
##     HP                0.115
##     DP                0.521
##     PR                0.443
```

```
# Model comparison
anova(model5.fit, model6.fit)
```

```
## Chi-Squared Difference Test
##
##             Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## model5.fit 79 11214 11361 209.00
## model6.fit 81 11210 11348 209.24     0.2392       2     0.8873
```

R session info

```
sessionInfo()
```

```
## R version 3.6.3 (2020-02-29)
## Platform: x86_64-apple-darwin15.6.0 (64-bit)
## Running under: macOS Sierra 10.12.6
##
## Matrix products: default
## BLAS:   /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] corrplot_0.84   MPsychoR_0.10-7 semPlot_1.1.2   lavaan_0.6-5
##
## loaded via a namespace (and not attached):
##   [1] nlme_3.1-144        RColorBrewer_1.1-2  mi_1.0
##   [4] tools_3.6.3         backports_1.1.6     R6_2.4.1
##   [7] d3Network_0.5.2.1   rpart_4.1-15        Hmisc_4.3-1
##  [10] colorspace_1.4-1    nnet_7.3-12         tidyselect_1.0.0
##  [13] gridExtra_2.3       mnormt_1.5-7        compiler_3.6.3
##  [16] qgraph_1.6.5        fdrtool_1.2.15      htmlTable_1.13.3
##  [19] regsem_1.5.2        scales_1.1.0        checkmate_2.0.0
##  [22] psych_1.9.12.31     pbapply_1.4-2       sem_3.1-9
##  [25] stringr_1.4.0       digest_0.6.25       pbivnorm_0.6.0
##  [28] foreign_0.8-75      minqa_1.2.4         rmarkdown_2.1
##  [31] base64enc_0.1-3     jpeg_0.1-8.1        pkgconfig_2.0.3
##  [34] htmltools_0.4.0     lme4_1.1-23         lisrelToR_0.1.4
##  [37] htmlwidgets_1.5.1   rlang_0.4.6         huge_1.3.4
##  [40] rstudioapi_0.11     gtools_3.8.1        acepack_1.4.1
##  [43] dplyr_0.8.5         zip_2.0.4           magrittr_1.5
##  [46] OpenMx_2.17.3       Formula_1.2-3       Matrix_1.2-18
##  [49] Rcpp_1.0.4.6        munsell_0.5.0       abind_1.4-5
##  [52] rockchalk_1.8.144   lifecycle_0.2.0     whisker_0.4
##  [55] stringi_1.4.6       yaml_2.2.1          carData_3.0-3
##  [58] MASS_7.3-51.5       plyr_1.8.6          matrixcalc_1.0-3
##  [61] grid_3.6.3          parallel_3.6.3      crayon_1.3.4
##  [64] lattice_0.20-40     kutils_1.69         splines_3.6.3
##  [67] knitr_1.28          pillar_1.4.3        igraph_1.2.5
##  [70] rjson_0.2.20        boot_1.3-24         corpcor_1.6.9
##  [73] BDgraph_2.62        codetools_0.2-16    reshape2_1.4.4
##  [76] stats4_3.6.3        XML_3.99-0.3        glue_1.4.0
##  [79] evaluate_0.14       latticeExtra_0.6-29 data.table_1.12.8
##  [82] png_0.1-7           vctrs_0.2.4         nloptr_1.2.2.1
##  [85] gtable_0.3.0        purrr_0.3.4         assertthat_0.2.1
##  [88] ggplot2_3.3.0       xfun_0.13           openxlsx_4.1.4
##  [91] xtable_1.8-4        coda_0.19-3         Rsolnp_1.16
```

```
##  [94] survival_3.1-8      glasso_1.11         truncnorm_1.0-8
##  [97] tibble_3.0.1        arm_1.10-1          cluster_2.1.0
## [100] statmod_1.4.34      ellipsis_0.3.0
```