MCG4340 Closed Loop Control System Lab

Lab Intro – Winter 2021

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Lab Location: D214

Prelab

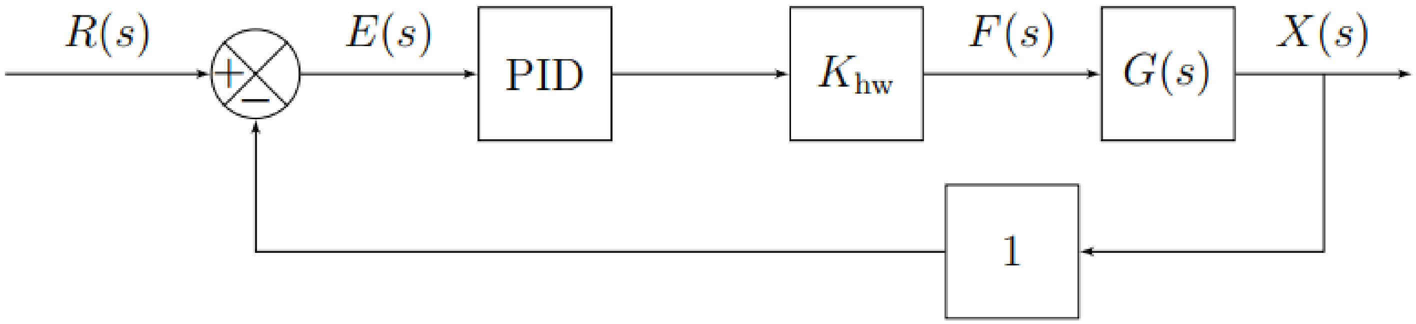
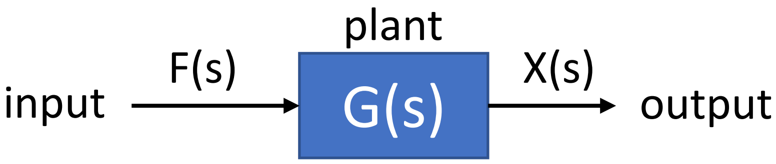


Figure 1: Block diagram of a closed loop system with unit feedback.

- Closed loop transfer function for PD and PID control
- Find characteristic polynomial for PD control
- Obtain expressions for natural frequency ω_n and damping ratio ζ
- Identify possible sources of error

Prelab

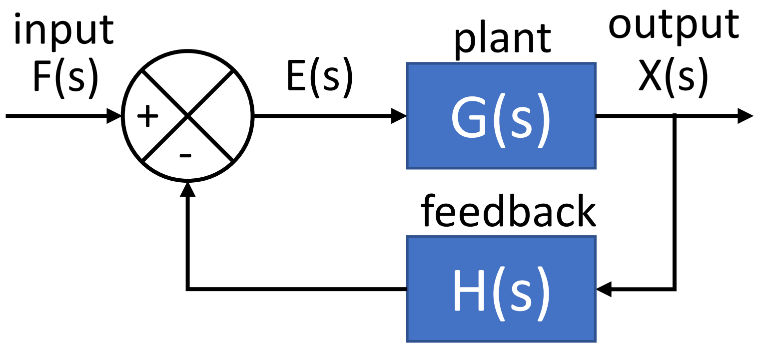
Open loop system review



$$X(s) = G(s)F(s)$$

$$OLTF = \frac{output}{input} = \frac{X(s)}{F(s)} = G(s)$$

Closed loop / feedback system review



$$FF = G(s)$$

$$FB = G(s)H(s)$$

$$CLTF = \frac{X(s)}{F(s)} = \frac{\sum FF}{1 + \sum FB}$$

Prelab

PID control system

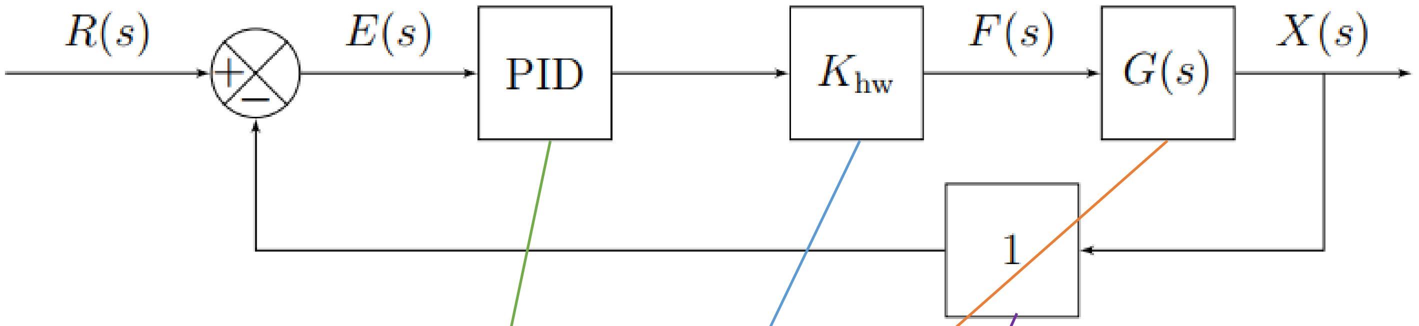


Figure 1: Block diagram of a closed loop system with unit feedback.

$$CLTF = \frac{FF}{1+FB} = \frac{\left(K_p + K_d s + \frac{K_i}{s}\right) K_{hw} \frac{1}{ms^2}}{1 + \left(K_p + K_d s + \frac{K_i}{s}\right) K_{hw} \frac{1}{ms^2}\, 1} = \boxed{\frac{K_{hw}\left(K_p + K_d s + \frac{K_i}{s}\right)}{ms^2 + K_{hw}\left(K_p + K_d s + \frac{K_i}{s}\right)}}$$

Prelab

Natural frequency and damping ratio (PD control)

$$CLTF = \frac{K_{hw}\left(K_p + K_d s\right)}{\boxed{ms^2 + K_{hw}K_d s + K_{hw}K_p}}$$

$$CE\colon\ ms^2 + K_{hw}K_d s + K_{hw}K_p$$

$$0 = s^2 + \boxed{\frac{K_{hw}K_d}{m}}s + \boxed{\frac{K_{hw}K_p}{m}} = s^2 + \boxed{2\zeta\omega_n}s + \boxed{\omega_n^2}$$

$$\omega_n = \sqrt{\frac{K_{hw}K_p}{m}}$$

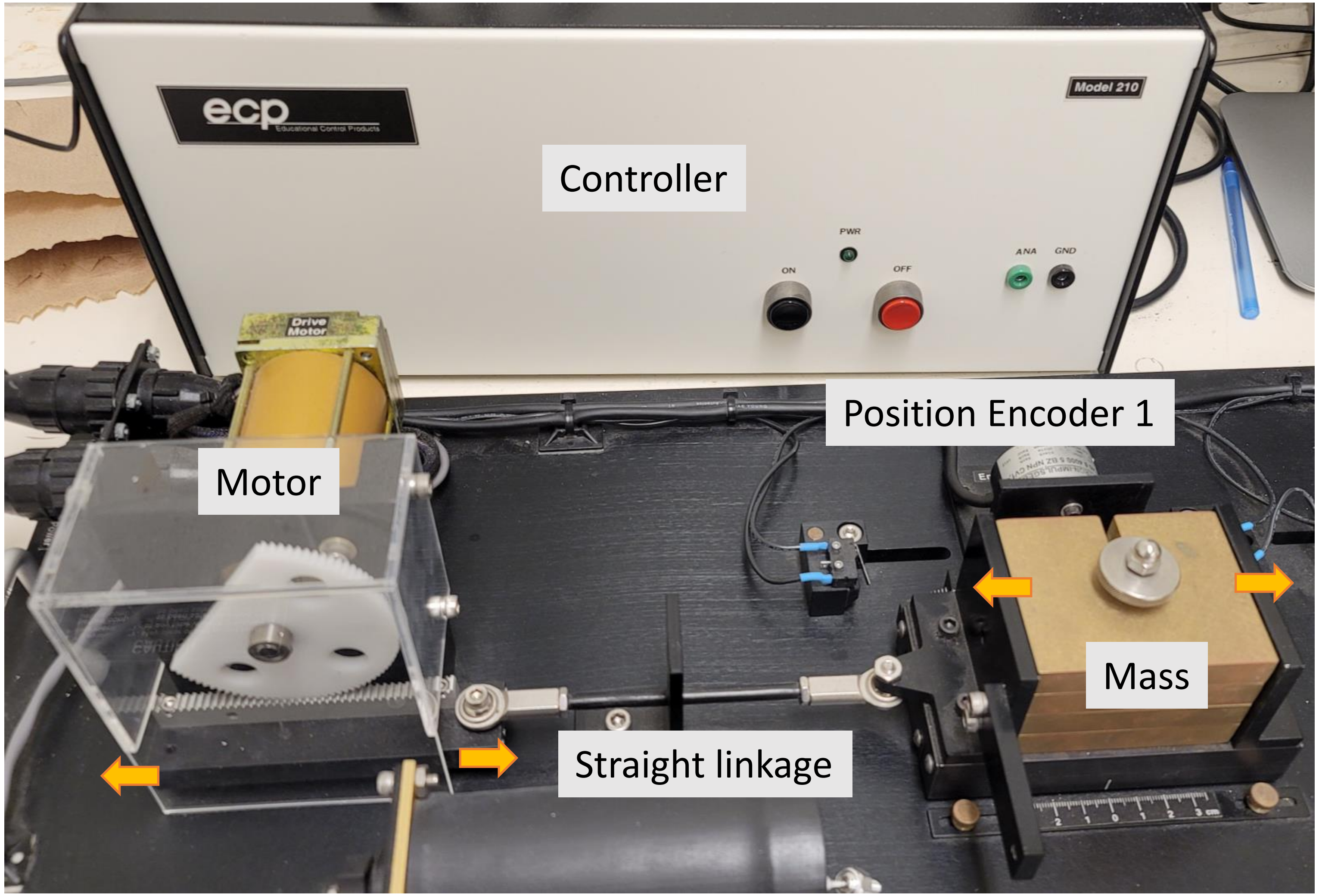
$$\zeta = \frac{K_{hw}K_d}{2m\omega_n} = \frac{K_d}{2}\sqrt{\frac{K_{hw}}{mK_p}}$$

Controller design:

$$K_p = \frac{\boxed{\omega_n^2}m}{K_{hw}}$$

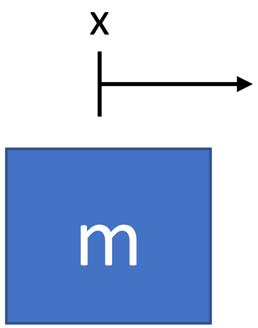
$$K_d = \frac{2\boxed{\zeta}\boxed{\omega_n}m}{K_{hw}}$$

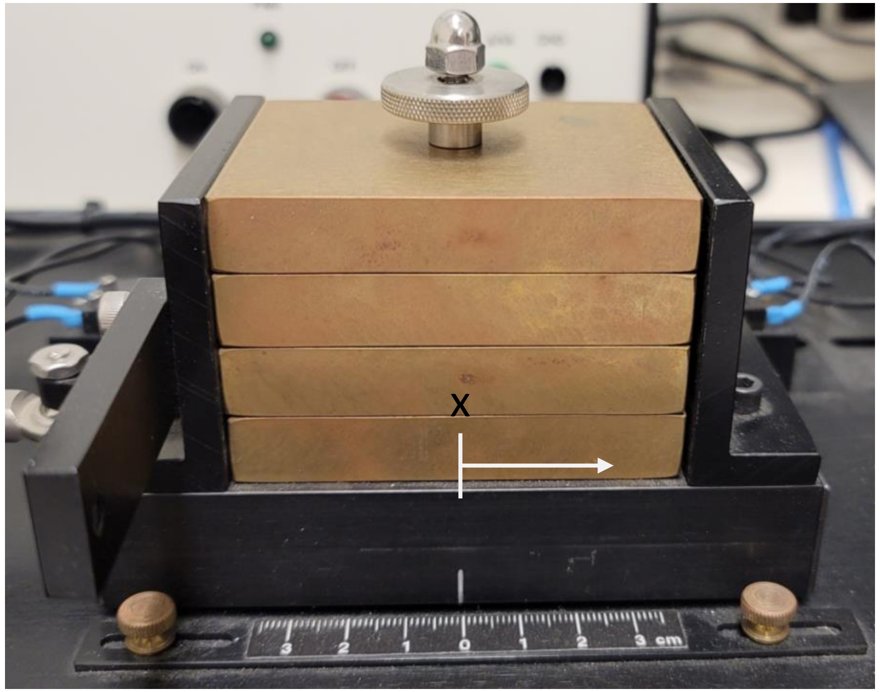
Experimental system



Experimental system

Purely inertial open system

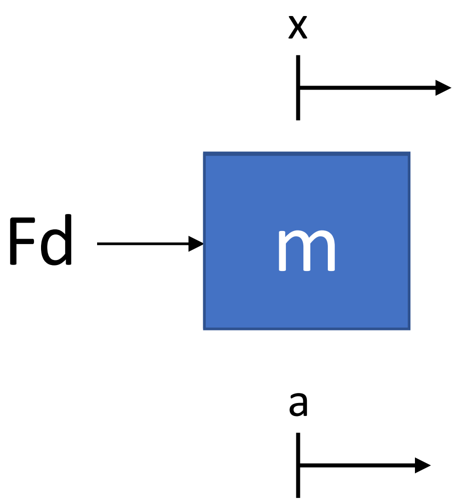




m = 4x500g + 0.77kg carriage

Experimental system

No constraints

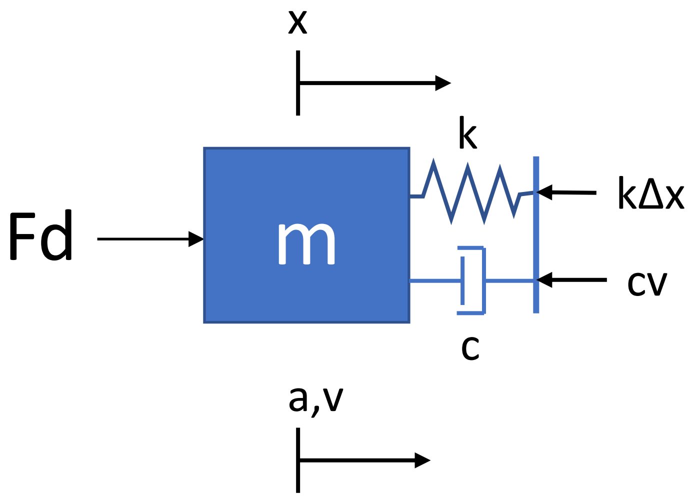


$\mathrm{F} = ma = m\ddot{x}$ $\longrightarrow$ Laplace transform: $F(s) = s^2 mX(s) - ms\cancelto{0}{x(0)} - m\cancelto{0}{\dot{x}(0)}$

$$OLTF = \frac{X(s)}{F(s)} = \frac{1}{ms^2}$$

Experimental system

With simple mechanical elements



$$\mathrm{F} = ma + cv + kx = m\ddot{x} + c\dot{x} + kx$$

$$F = s^2 mX(s) - \cancelto{0}{msx(0)} - \cancelto{0}{m\dot{x}(0)} + scX(s) - \cancelto{0}{cx(0)} + kX(s)$$

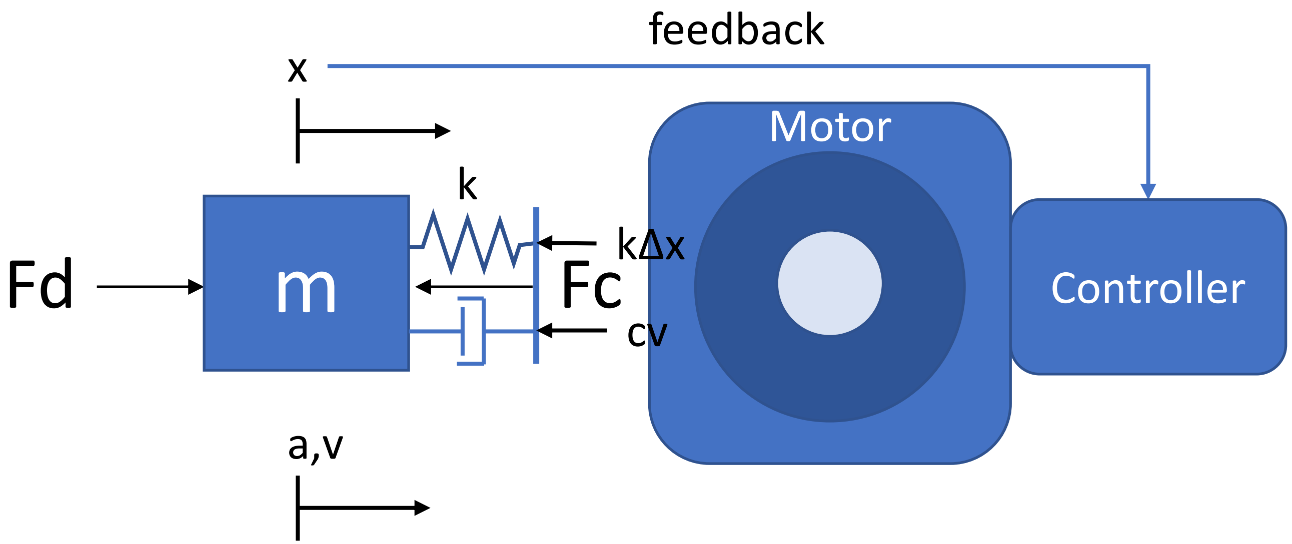
$$OLTF = \frac{X}{F} = \frac{1/m}{\boxed{s^2 + \frac{c}{m}s + \frac{k}{m}}}$$

$$CE: \; s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$2\zeta\omega_n = \frac{c}{m} \qquad \omega_n^2 = \frac{k}{m}$$

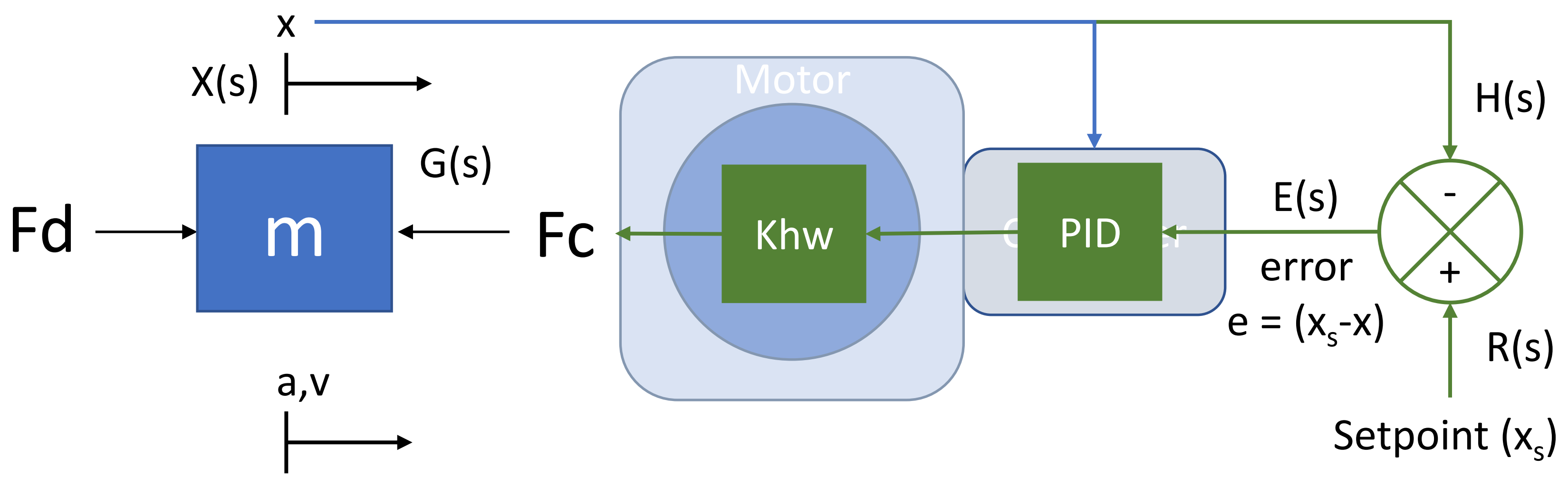
Experimental system

Replace simple mechanical elements with a control system



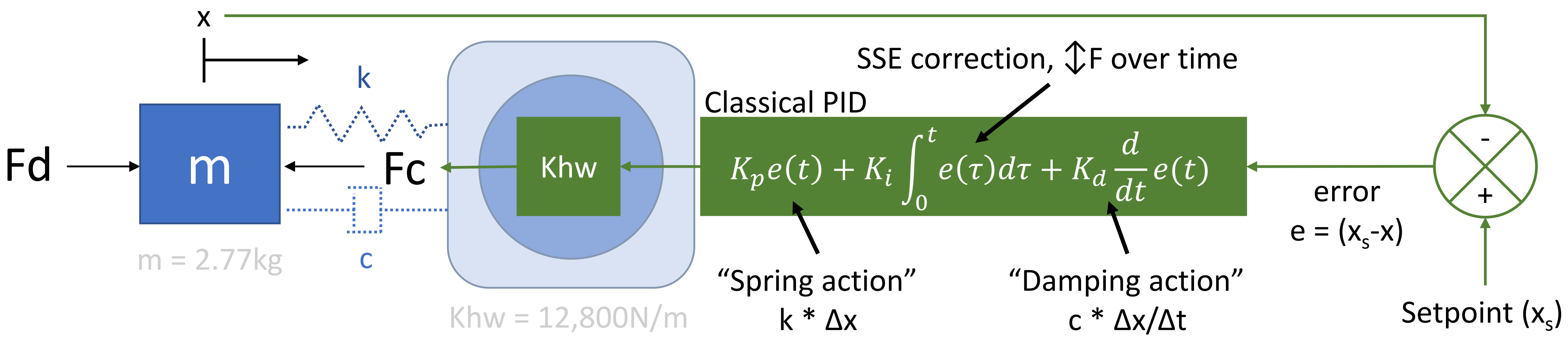
Experimental system

PID control system

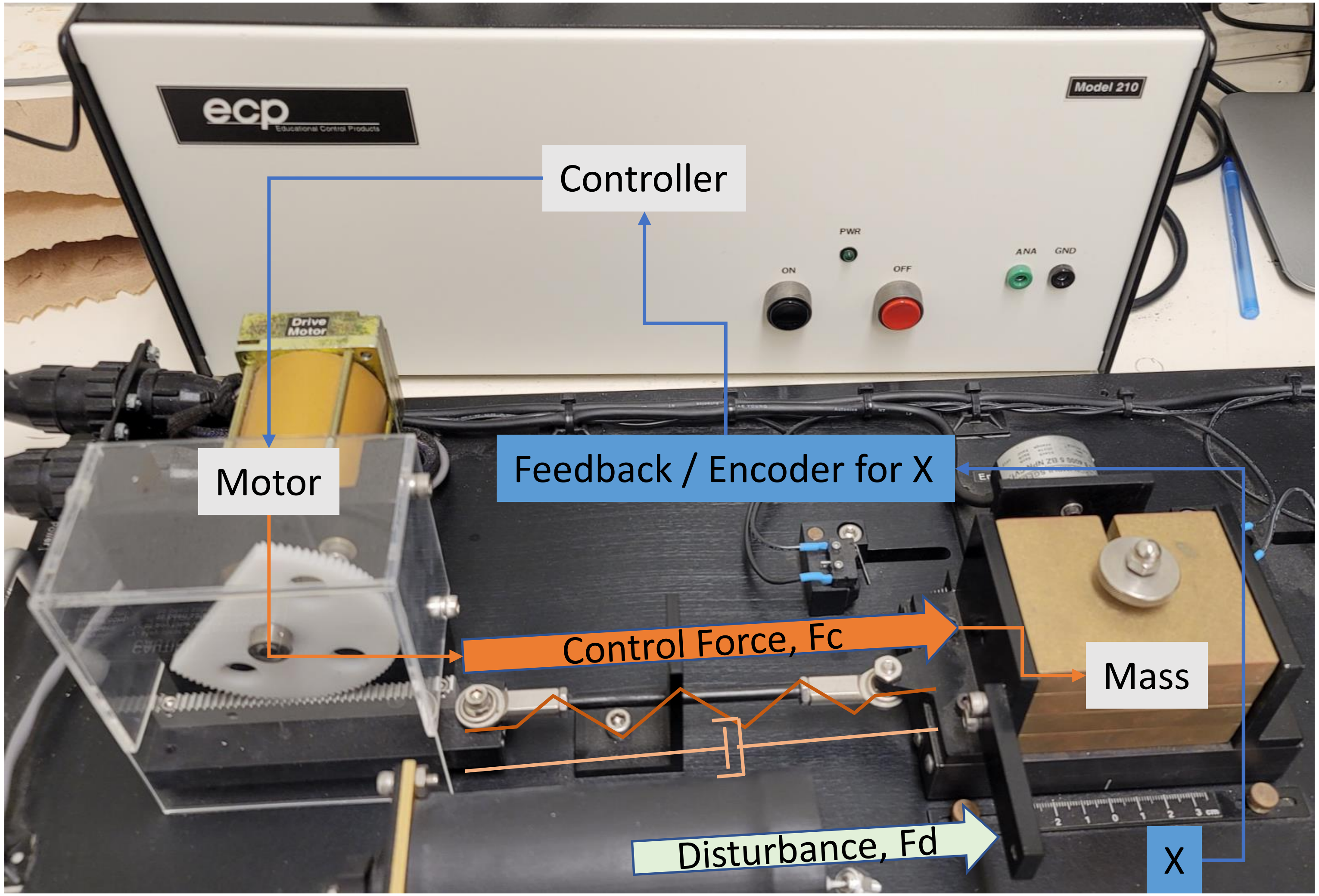


Experimental system

PID control system



Experimental system



Lab grading

Lab logbook = 10% of the grade

- Controls experiment = 2%

Formal lab report = 17% of the grade

- For lab assigned to the group

Lab logbook submission

Your lab group:

- **WED1**

Submit Controls Lab Logbook

- **March 5, 2021 by 14:30**

Lab logbook submission

Prelab logbook (20%)

- **Pass/fail**
- Submit at least 1 hr before the lab
- Attempted derivations:
    - transfer functions for PD, PID control of the system
    - natural frequency, damping ratio for PD controller in terms of Kp, Kd

Lab logbook submission

Lab logbook (80%)

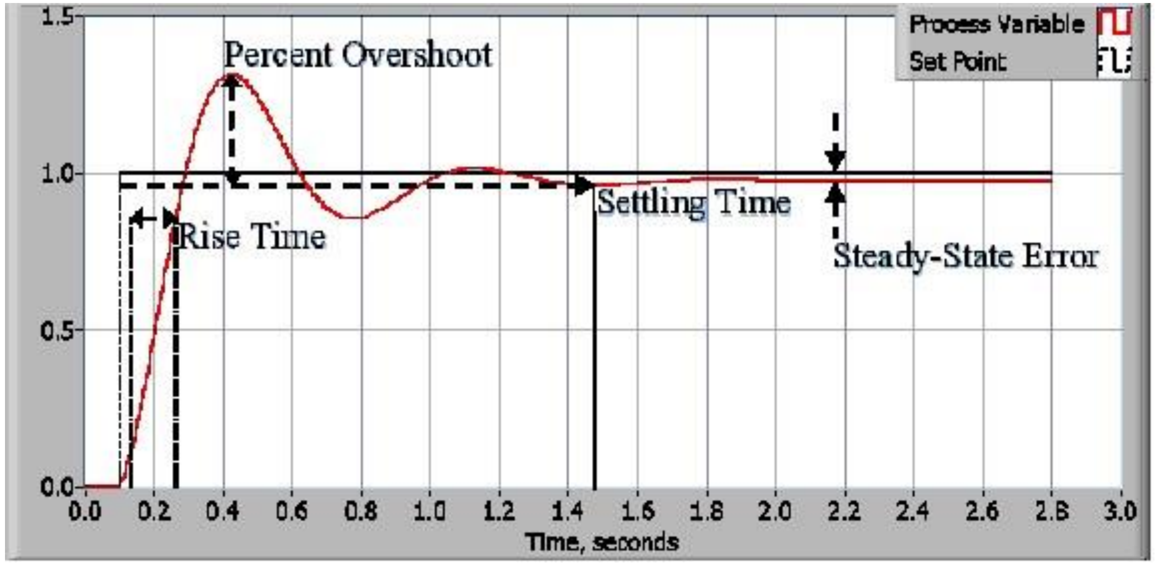
- **Content (56%)**
  - **Title page** – experiment title, date/time, group#, location (CBY D214), TA/Prof
  - **Objectives** – few sentences per lab manual, paraphrase
  - **Prelab** – derivations
  - **Methodology** – refer reader to the lab manual reference, but do note any changes to procedure (e.g. remote lab via Zoom, equipment issues, etc). Calculated values for Kp (3.3.3), Kd (3.3.8, 3.3.10), Kp & Kd (3.4a,b,c), Ki (4.1.18, 4.1.20)
  - **Observations** – observation notes during the experiment on the effects of Kp, Kd, Ki, steady state error, oscillations, frequency response, etc.
  - **Results and Analysis** – answer lab questions, can section by question# with appropriate plots/diagrams.
    - Derivations – *show* final value theorem derivation for steady state error of your PD and PID systems
    - Plots (*from raw data*):
      - Disturbance responses for proportional action (x2)
      - Step responses for under- critically- and over-damped PD system (x3)
      - Frequency response diagrams for under- critically- and over-damped PD system (x3)
      - Step response for integral action on the critically-damped case (x2)
  - **Conclusions** – main takeaway points from the lab

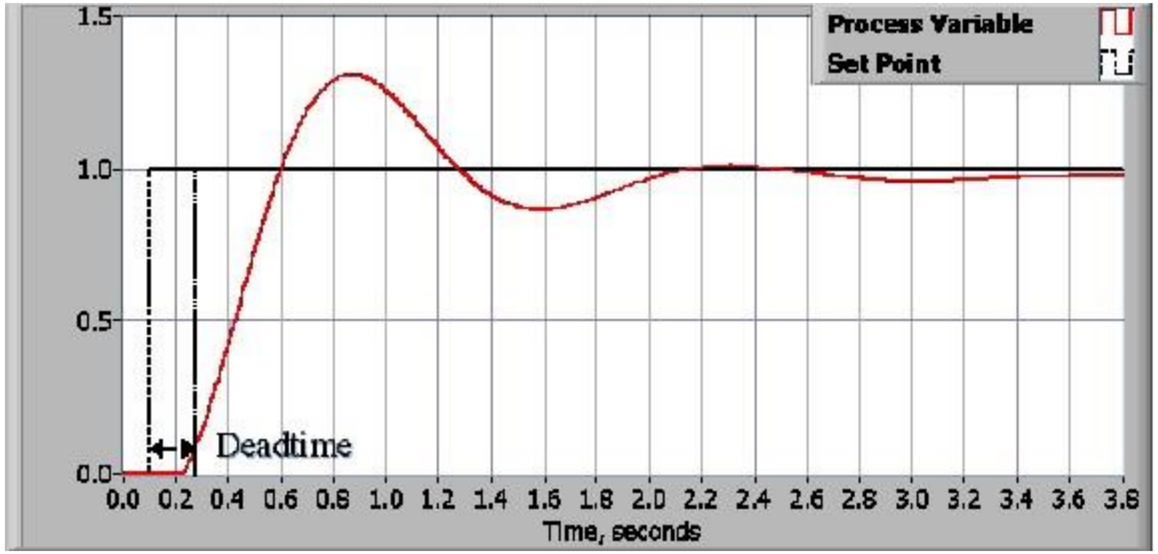
Lab logbook submission

- **Style (24%)**
  - Clear organization – see suggested subheadings from Content section
  - Readable plots – split into subplots or different plots if overlaying them looks too busy

Results

Control action features to pay attention to





https://www.ni.com/en-ca/innovations/white-papers/06/pid-theory-explained.html

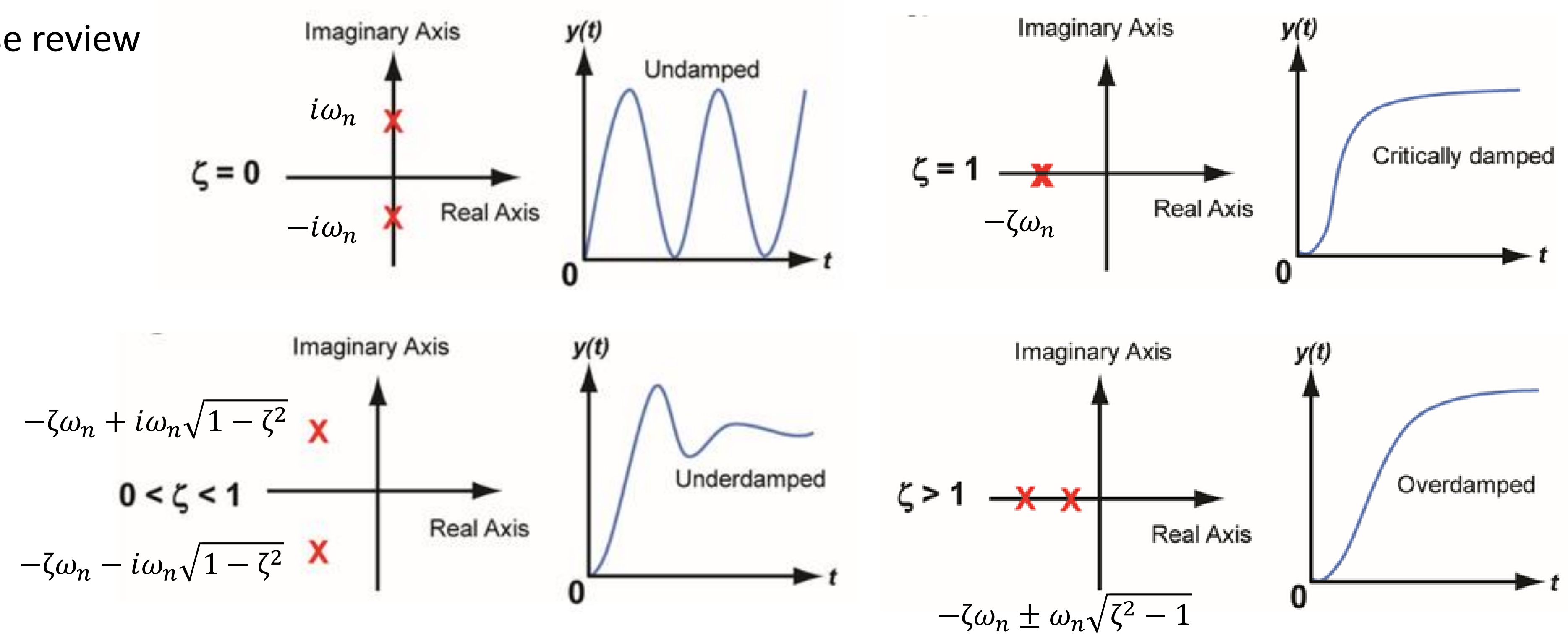
Results

2nd order system response review

$$\text{e.g. } \mathrm{F} = m\ddot{x} + c\dot{x} + kx$$

Characteristic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$



3rd order characteristic equation hint

$$(s + \alpha\omega_n)(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

Modified Fig 7 (Shin & Blers, 2010)

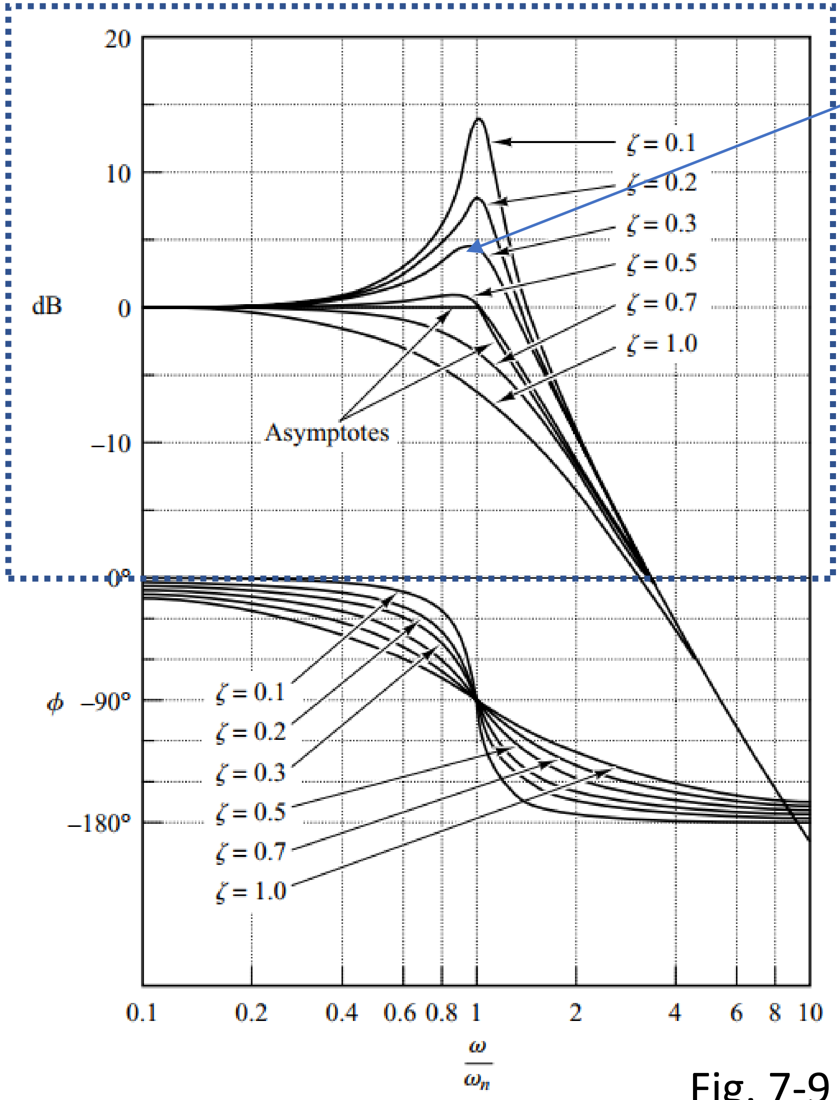
Shin, Yong-Jun & Bleris, Leonidas. (2010). Linear Control Theory for Gene Network Modeling. *PloS one.* 5. 10.1371/journal.pone.0012785.

Results

Bode plots

2nd order system response review

$$\mathrm{e.g.\,F} = m\ddot{x} + c\dot{x} + kx$$



$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$$

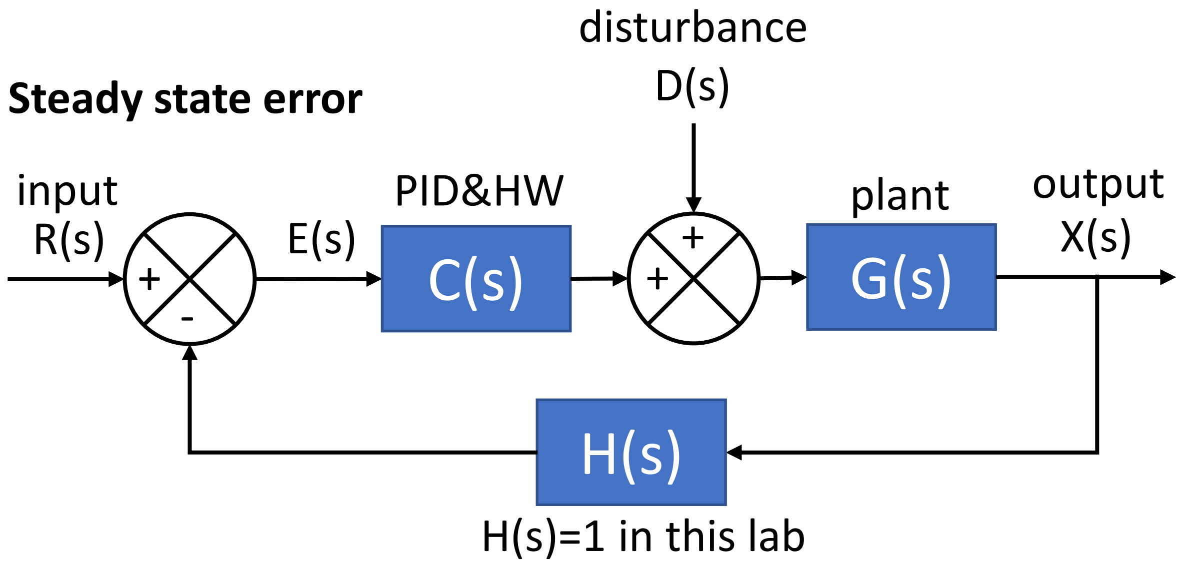
for $0 \le \zeta \le 0.707$

Fig. 7-9

Ogata. Modern Control Engineering (5th ed)

Results

Steady state error



By Final Value Theorem

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$E(s) = R(s) - X(s)H(s) = \frac{P(s)}{Q(s)}$$

FVT can be applied iff *E(s)* is:

- rational
- proper (degree P(s) < degree Q(s))

For inputs: $R(s), D(s)$

$$X(s) = \frac{[R(s)C(s) + D(s)]G(s)}{1 + C(s)G(s)H(s)}$$

$$E(s) = \frac{R(s) - D(s)G(s)H(s)}{1 + C(s)G(s)H(s)}$$

Expressions derived for the lab will be a bit simpler

$$Derivation\ hint:$$
$$(EC + D)G = X$$
$$E = R - XH$$

PID forms

<u>General/parallel form</u>

$$K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt} e(t)$$

$$K_p + \frac{K_i}{s} + K_d s$$

Bit easier mathematically
More physical meaning:

Kp – spring action
Ki – changes output over time
Kd – damping action

<u>Standard form</u>

$$K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{d}{dt} e(t) \right]$$

$$K_p + \frac{K_p}{T_i s} + K_p T_d s$$

More common in industry
More practical meaning:

Kp – scaling of error, integral and derivative components
Ti – integration time, error eliminated within Ti (s or samples)
Td – derivative time, error predicted at time Td (s or samples)