13 – Recursion

COMP 125 Programming with Python

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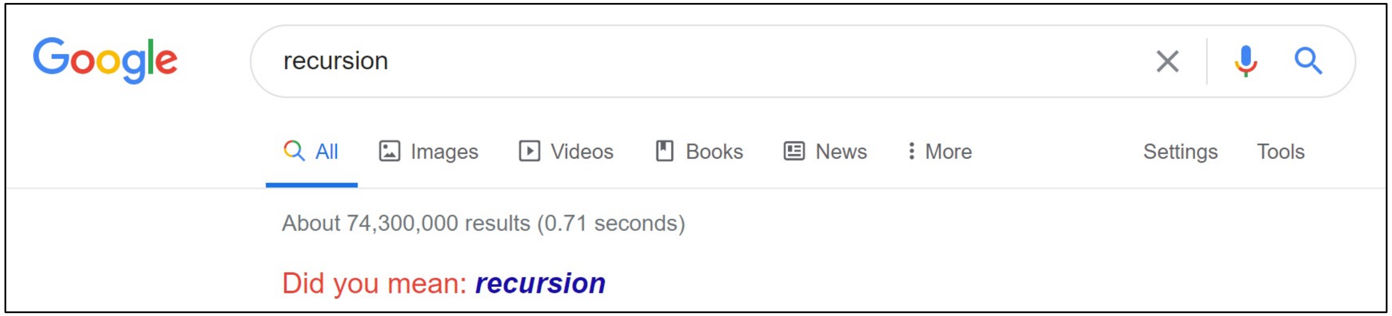
Recursion



- Recursion is an extremely powerful problem-solving technique
  - It breaks a problem into smaller identical problems and uses **the same function** to solve these smaller problems
  - It is an alternative to iterative solutions, which use loops

"In order to understand recursion, you must first understand recursion."

- Facts about recursive solutions
  - A recursive function calls itself
  - Each recursive call solves an identical but a smaller problem
  - Base case must be defined (it enables to stop the recursive calls)
  - Eventually, one of the smaller problems must be the base case

Factorial function (*iterative solution*)

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$$

$$f(n) = \begin{cases} 1 & \text{if n} = 0 \\ n \cdot f(n-1) & \text{otherwise} \end{cases}$$

```python
# assumes that n is non-negative
def iterative_factorial(n):
    if n == 0:
        return 1
    else:
        result = 1
        for i in range(1, n + 1):
            result *= i
        return result
```

Factorial function (*recursive solution*)

```python
# assumes that n is non-negative
def recursive_factorial(n):
    if n == 0:
        return 1
    else:
        return n * recursive_factorial(n - 1)
```

This is the base case, it stops recursive function calls

The function calls itself but a smaller argument

- Facts about recursive solutions
  - A recursive function calls itself
  - Each recursive call solves an identical but a smaller problem
  - Base case must be defined (it enables to stop the recursive calls)
  - Eventually, one of the smaller problems must be the base case

What happens if n is a negative integer?

Factorial function (*recursive solution*)

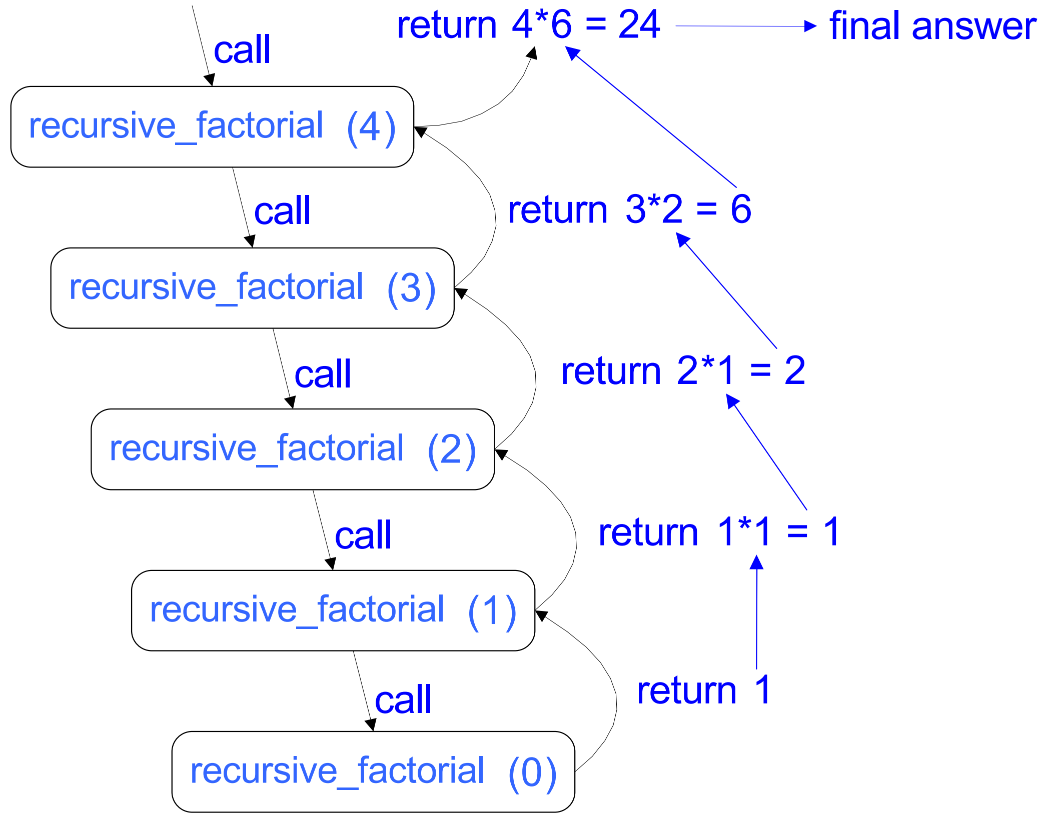
```
In [4]: recursive_factorial(-5)


  File "/Users/cigdem/Desktop/recursion.py", line 17, in recursive_factorial
    return n * recursive_factorial(n - 1)

  File "/Users/cigdem/Desktop/recursion.py", line 17, in recursive_factorial
    return n * recursive_factorial(n - 1)

  File "/Users/cigdem/Desktop/recursion.py", line 17, in recursive_factorial
    return n * recursive_factorial(n - 1)

  File "/Users/cigdem/Desktop/recursion.py", line 14, in recursive_factorial
    if n == 0:

RecursionError: maximum recursion depth exceeded in comparison
```

You have to be sure that the function eventually reaches the base case

The number of times that a function calls itself is known as the depth of recursion

Visualizing recursion

- Recursion trace
  - A box for each recursive call
  - An arrow from each caller to callee
  - An arrow from each callee to caller showing return value



Summing numbers from 1 to N

```
def iterative_sum(N):
    result = 0
    for i in range(N + 1):
        result += i
    return result
```

```
def recursive_sum(N):
    if N <= 0:
        return 0
    if N == 1:
        return 1
    return N + recursive_sum(N - 1)
```

It is possible to have more than one base case!

Fibonacci numbers

Recursive definition

$$F(N) = F(N - 1) + F(N - 2), \text{ for all } N > 2$$

Base cases

$$F(1) = 1$$

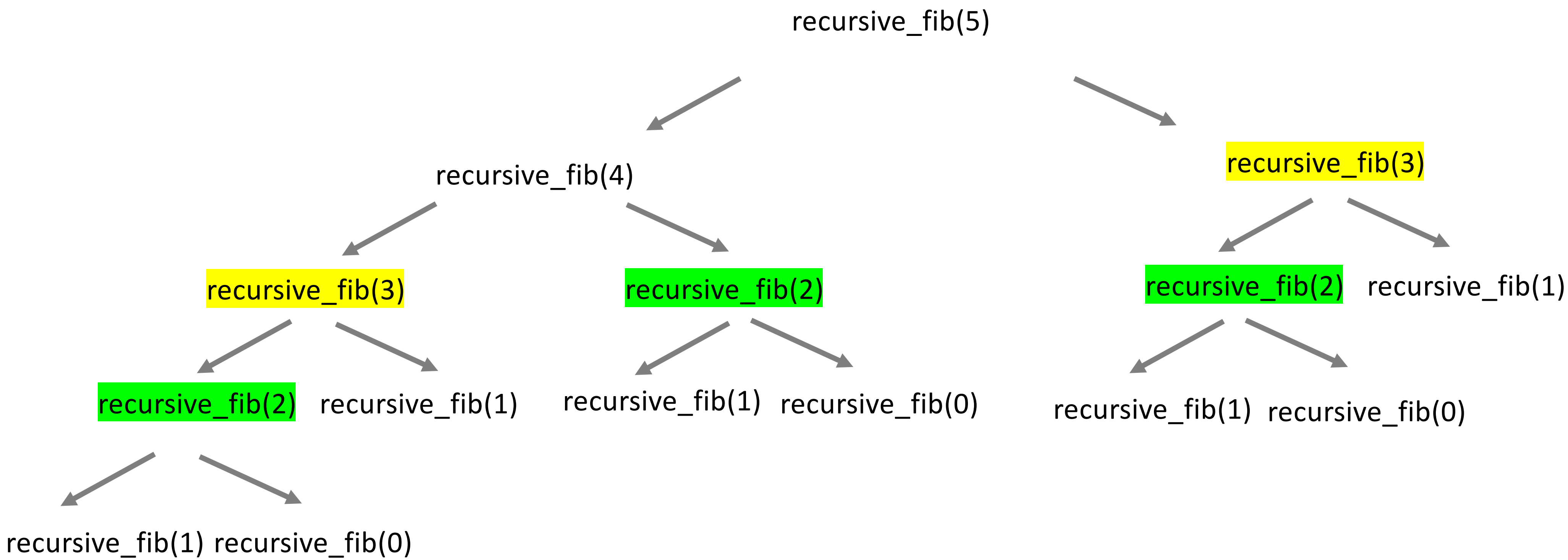
$$F(2) = 1$$

This is a bad example of recursion. It is too inefficient!!!

```
def iterative_fib(N):
    if N <= 0:
        return 0
    if N == 1 or N == 2:
        return 1
    first = 1
    second = 1
    for i in range(3, N + 1):
        result = first + second
        first = second
        second = result
    return result
```

```
def recursive_fib(N):
    if N <= 0:
        return 0
    if N == 1 or N == 2:
        return 1
    return recursive_fib(N - 1) + \
           recursive_fib(N - 2)
```

Visualizing recursion: fibonacci



Note the repeated calls!

What does it mean?

- Run the iterative and recursive functions for different values of N and measure the computational time

```python
import time

N = int(input('Enter N: '))

start = time.time()
res = iterative_fib(N)
end = time.time()
print(f"Iterative: {end - start:.6f} sec")

start = time.time()
res = recursive_fib(N)
end = time.time()
print(f"Recursive: {end - start:.6f} sec")
```

```
In [1]: runfile('/Users/cigdem/Des

Enter N: 5
Iterative: 0.000004 sec
Recursive: 0.000005 sec

In [2]: runfile('/Users/cigdem/Des

Enter N: 10
Iterative: 0.000006 sec
Recursive: 0.000045 sec

In [3]: runfile('/Users/cigdem/Des

Enter N: 20
Iterative: 0.000010 sec
Recursive: 0.005093 sec

In [4]: runfile('/Users/cigdem/Des

Enter N: 30
Iterative: 0.000011 sec
Recursive: 0.239757 sec

In [5]: runfile('/Users/cigdem/Des

Enter N: 40
Iterative: 0.000008 sec
Recursive: 29.496105 sec
```

What does it mean?

- Run the iterative and recursive functions for different values of N and measure the computational time

```
import time

N = int(input('Enter N: '))

start = time.time()
res = iterative_fib(N)
end = time.time()
print(f"Iterative: {end - start:.6f} sec")

start = time.time()
res = recursive_fib(N)
end = time.time()
print(f"Recursive: {end - start:.6f} sec")
```

```
In [5]: runfile('/Users/cigdem/Des

Enter N: 40
Iterative: 0.000008 sec
Recursive: 29.496105 sec

In [6]: runfile('/Users/cigdem/Des

Enter N: 41
Iterative: 0.000012 sec
Recursive: 53.246312 sec

In [7]: runfile('/Users/cigdem/Des

Enter N: 42
Iterative: 0.000013 sec
Recursive: 81.497787 sec

In [8]: runfile('/Users/cigdem/Des

Enter N: 43
Iterative: 0.000012 sec
Recursive: 131.133424 sec

In [9]: runfile('/Users/cigdem/Des

Enter N: 44
Iterative: 0.000012 sec
Recursive: 217.246881 sec
```

Recursion and efficiency

- Some recursive functions are so inefficient that they should not be used

- Factors contributing to this inefficiency
  - Inherent inefficiency of some recursive functions, such as the recursive_fib(…) function
  - Overhead associated with function calls

- Do not use a recursive function if it is inefficient and there is a clear and efficient implementation of the same algorithm using loops

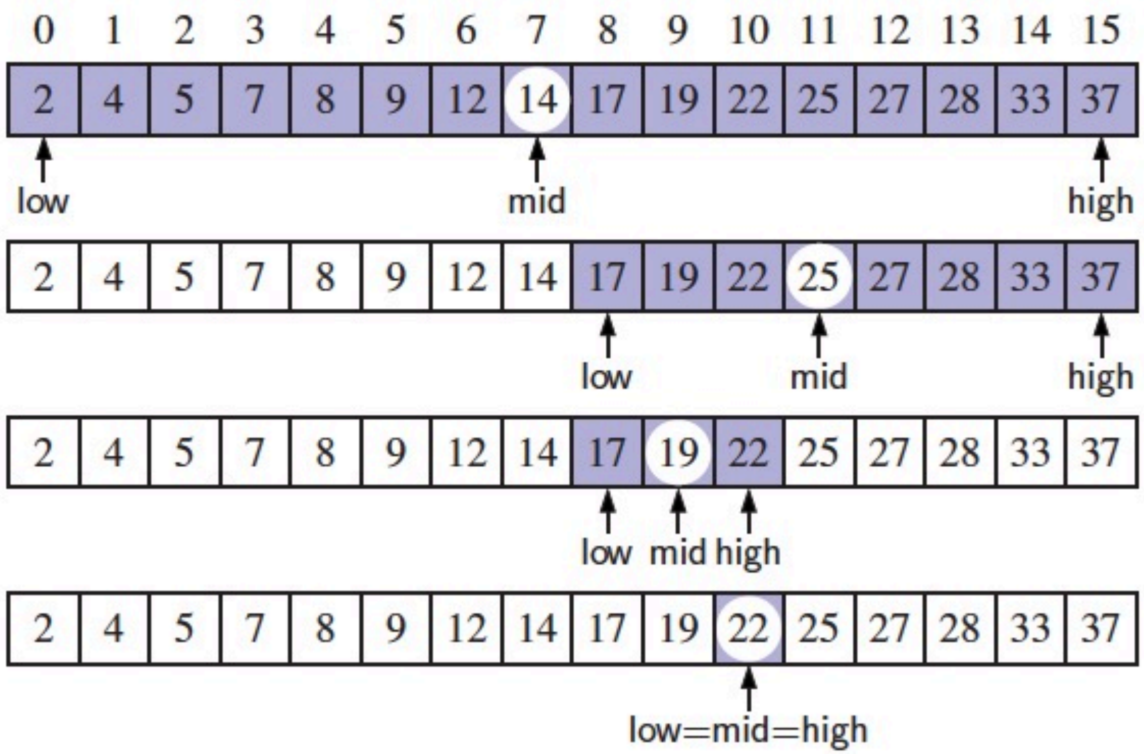
- **Efficiency (time complexity) analyses are discussed in more advanced CS courses**

Binary search

- This search algorithm works only on ***sorted sequences***
- It is a **very efficient search** algorithm

Find 22

- If the key equals data[mid], then we have found the target
- If key < data[mid], then we recur on the first half of the sequence
- If key > data[mid], then we recur on the second half of the sequence



```python
def binary_search(L, key, low, high):
    if low > high:
        return -1

    mid = (low + high) // 2 #integer division
    if L[mid] == key:
        return mid
    elif L[mid] > key:
        return binary_search(L, key, low, mid - 1)
    else:
        return binary_search(L, key, mid + 1, high)


def search(L, key):
    return binary_search(L, key, 0, len(L) - 1)

def main():
    D = [  3,  7, 10, 12, 14, 16, 19, 20, 23, 26, \
          32, 34, 37, 41, 43, 48, 52, 56, 59, 62, \
          67, 70, 72, 79, 81, 84, 89, 92, 95, 98 ]
    print(search(D, 16))
    print(search(D, 48))
    print(search(D, 77))

main()
```

```
In [1]: runfile('/Users/
5
15
-1
```

This is a very efficient search algorithm

It works very fast on even large sequences (e.g., containing millions of items)

It is much much faster than the linear search algorithm

Indirect recursion

- **Direct recursion**: function calls itself
  - e.g., binary_search(…) has a function call to binary_search(…)

- **Indirect recursion**: one function calls another function, which in turn calls the first function
  - e.g., function foo(…) calls function bar(…), and function bar(…) calls function foo(…)
  - Also called mutual recursion
  - Not restricted to two functions, can have a longer chain

Example

- Write two recursive functions that take an integer and return whether the integer is even or odd, respectively.

```
def isEven(N):
    if N == 0:
        return True
    return isOdd(N - 1)

def isOdd(N):
    if N == 0:
        return False
    return isEven(N - 1)
```

This is just to give you an indirect recursion example.

You know better/faster ways of implementing these functions

(N % 2 == 0 or N % 2 == 1)