KOÇ ÜNİVERSİTESİ

Lab06

COMP 125 Programming with Python

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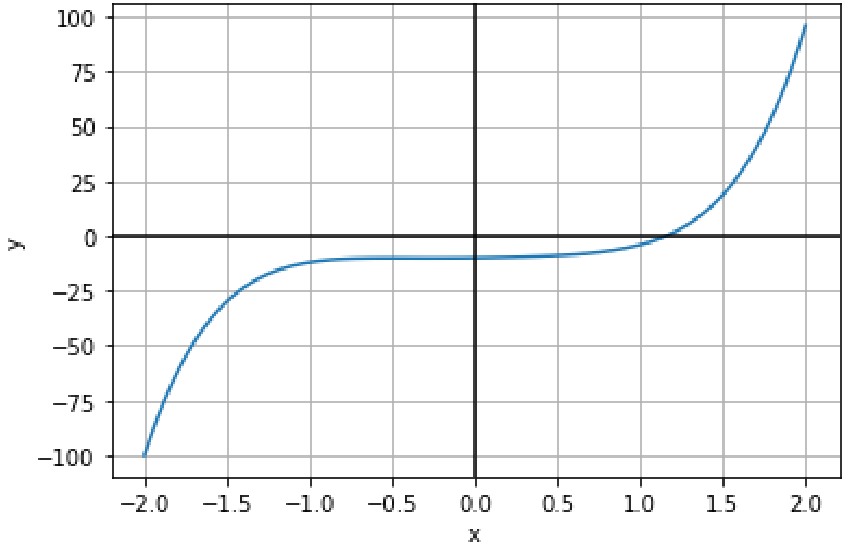
Finding the root(s) of a function

The root of a function f(x) is defined as the value (x), where the function evaluates to zero: f(x)=0

For example $f(x) = 3x^5 + 2x^2 + x - 10$ has one root in the interval [-2,2].

If we plot this function, we get:



How can we find the value of this root?

Bisection Method

Bisection method is a robust, but inefficient, method for finding the root(s) of a function numerically.

Let's suppose that we have two x-values **xpos** and **xneg** where the function has positive and negative values:

f(xpos) > 0 and **f(xneg)<0**.

Since the function changes sign between **xneg** and **xpos**,

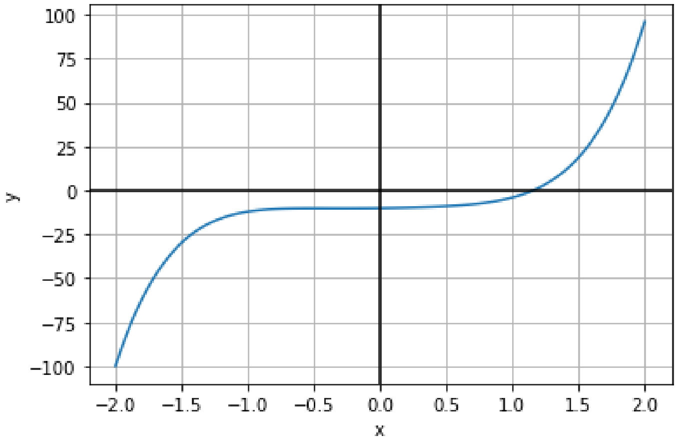
there must be at least one root in the interval **[xneg, xpos]**

For example for the function

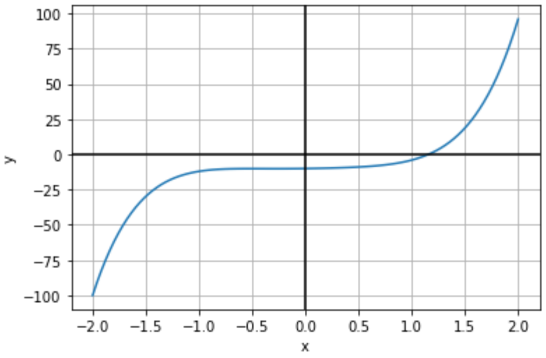
$$f(x) = 3x^5 + 2x^2 + x - 10$$

we can use the interval **[-2,2]**, since

$f(-2) = -100$ and $f(2) = 96$.



Bisection Method: Algorithm



Bisection method is an iterative method. The basic steps are:

1) Find the mid point of xneg and xpos.

$$xnew = \frac{xneg + xpos}{2}$$

2) Evaluate the function at this point

$$f(xnew)$$

3) If $f(xnew) > 0$, set

xpos = xnew,

otherwise set

xneg = xnew.

4) Now we have a new (narrower) interval that includes the root.

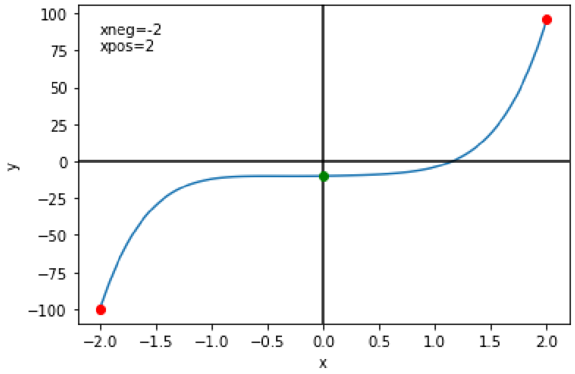
5) Repeat until the interval width

$$\delta = |xpos - xneg|$$

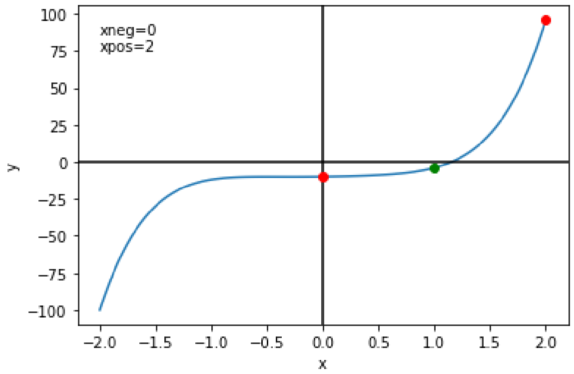
$\text{is less than a certain threshold value (e.g. } 0.000001)$

Let’s try

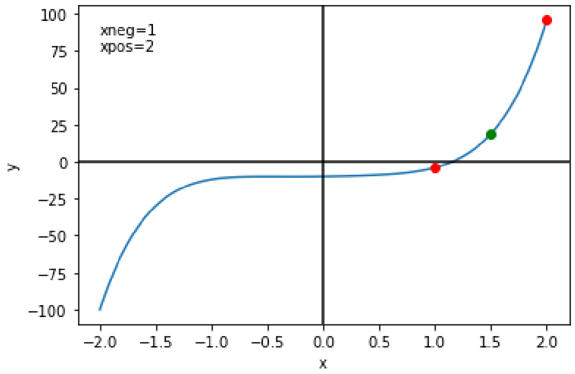
1)



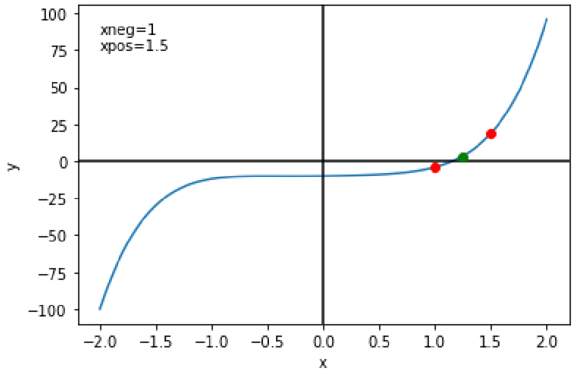
2)



3)



4)



Let’s implement the Bisection method

- Download the file lab06.zip and unzip it to the Desktop.
- Start Spyder and navigate to Lab06 folder

```
In [1]: pwd
Out[1]: 'C:\\Users\\msayar'

In [2]: cd Desktop
C:\Users\msayar\Desktop

In [3]: cd Lab06
C:\Users\msayar\Desktop\Lab06

In [4]: pwd
Out[4]: 'C:\\Users\\msayar\\Desktop\\Lab06'

In [5]:
```

Import plot module

- In the folder Lab06 we have a file “plot.py”

```python
import matplotlib.pyplot as plt
import numpy as np

def plot(func, xmin, xmax):
    """ Requires three arguments:
    func is the name of the function to be plotted.
    xmin and xmax define the x-limits of the plot.
    """
    n_points=100
    x=np.arange(xmin,xmax+(xmax-xmin)/(n_points-1),(xmax-xmin)/(n_points-1))
    f_x=[]
    for i in range(len(x)):
        f_x +=[func(x[i])]

    plt.plot(x,f_x)
    plt.axhline(y=0, color='k')
    plt.axvline(x=0, color='k')
    plt.xlabel("x")
    plt.ylabel("y")
    plt.show(block=False)
    return
```

- Import this module

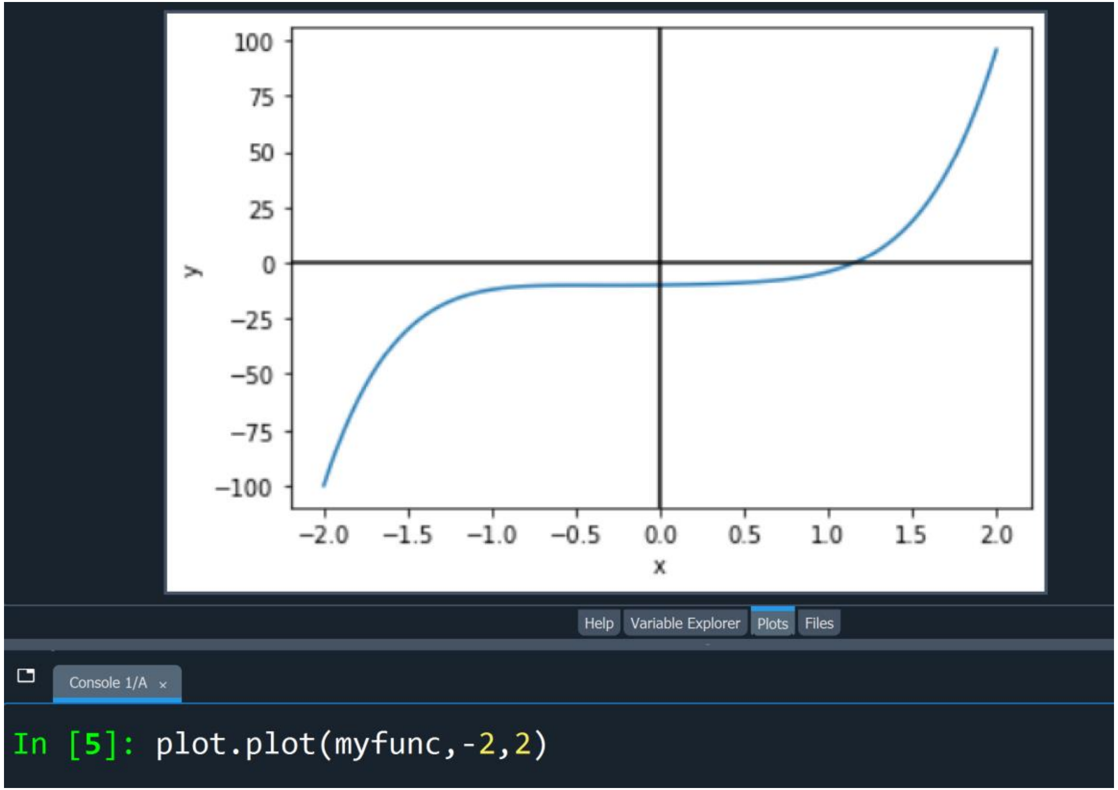
```
In [2]: import plot

In [3]: help(plot.plot)
Help on function plot in module plot:

plot(func, xmin, xmax)
    Requires three arguments:
    func is the name of the function to be plotted.
    xmin and xmax define the x-limits of the plot.
```

Implement “myfunc” function

- Function myfunc
- Should take one argument: x
- Should return the value of $f(x) = 3x^5 + 2x^2 + x - 10$

```python
def myfunc(x):
    """Evaluates the value of the function at x"""

    return
```

Let’s test it



100
75
50
25
0
−25
−50
−75
−100
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x
Help
Variable Explorer
Plots
Files
Console 1/A
In [5]: plot.plot(myfunc,-2,2)

Implement the “bisection_update” function

```
def bisection_update(func, xneg, xpos):
    """
    Performs one iteration of the Bisection algorithm
    Takes three arguments
    1) name of the function f(x))
    2) xneg, such that f(xneg)<0.
    3) xpos, such that f(xpos)>0.

    Returns the updated xneg and xpos values
    """


    "Write your code here"


    return xneg, xpos
```

Implement the “bisection” method

- Should check the given xneg and xpos to make sure that f(xneg)<0 and f(xpos)>0
- If not, return after printing the message: "Please provide two x-values where the sign of the function differs.“
- Use $\delta = 0.00001$as convergence criteria
- Use the “bisection_update” function to iterate until the convergence criteria is satisfied: $\delta = |xpos - xneg|$
- At every iteration print out the current xneg, xpos, delta, f(xneg) and f(pos)
- If found, return the root value

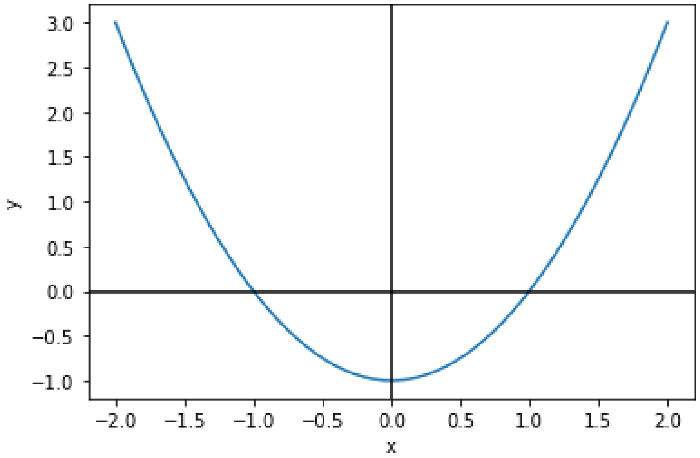
```
def bisection(func, xneg, xpos):
    """ Takes three arguments
    1) name of the function f(x))
    2) xneg, such that f(xneg)<0.
    3) xpos, such that f(xpos)>0.

    Returns the root if found
    """


    "Write your code here"

    return root
```

Sample output

```
xneg=-2.00000e+00, xpos=2.00000e+00, delta=4.00000e+00, f(xneg)=-1.00000e+02, f(xpos)=9.60000e+01
xneg=0.00000e+00, xpos=2.00000e+00, delta=2.00000e+00, f(xneg)=-1.00000e+01, f(xpos)=9.60000e+01
xneg=1.00000e+00, xpos=2.00000e+00, delta=1.00000e+00, f(xneg)=-4.00000e+00, f(xpos)=9.60000e+01
xneg=1.00000e+00, xpos=1.50000e+00, delta=5.00000e-01, f(xneg)=-4.00000e+00, f(xpos)=1.87812e+01
xneg=1.00000e+00, xpos=1.25000e+00, delta=2.50000e-01, f(xneg)=-4.00000e+00, f(xpos)=3.53027e+00
xneg=1.12500e+00, xpos=1.25000e+00, delta=1.25000e-01, f(xneg)=-9.37653e-01, f(xpos)=3.53027e+00
xneg=1.12500e+00, xpos=1.18750e+00, delta=6.25000e-02, f(xneg)=-9.37653e-01, f(xpos)=1.09199e+00
xneg=1.12500e+00, xpos=1.15625e+00, delta=3.12500e-02, f(xneg)=-9.37653e-01, f(xpos)=2.99110e-02
xneg=1.14062e+00, xpos=1.15625e+00, delta=1.56250e-02, f(xneg)=-4.65229e-01, f(xpos)=2.99110e-02
xneg=1.14844e+00, xpos=1.15625e+00, delta=7.81250e-03, f(xneg)=-2.20555e-01, f(xpos)=2.99110e-02
xneg=1.15234e+00, xpos=1.15625e+00, delta=3.90625e-03, f(xneg)=-9.60528e-02, f(xpos)=2.99110e-02
xneg=1.15430e+00, xpos=1.15625e+00, delta=1.95312e-03, f(xneg)=-3.32545e-02, f(xpos)=2.99110e-02
xneg=1.15527e+00, xpos=1.15625e+00, delta=9.76562e-04, f(xneg)=-1.71778e-03, f(xpos)=2.99110e-02
xneg=1.15527e+00, xpos=1.15576e+00, delta=4.88281e-04, f(xneg)=-1.71778e-03, f(xpos)=1.40851e-02
xneg=1.15527e+00, xpos=1.15552e+00, delta=2.44141e-04, f(xneg)=-1.71778e-03, f(xpos)=6.18078e-03
xneg=1.15527e+00, xpos=1.15540e+00, delta=1.22070e-04, f(xneg)=-1.71778e-03, f(xpos)=2.23078e-03
xneg=1.15527e+00, xpos=1.15533e+00, delta=6.10352e-05, f(xneg)=-1.71778e-03, f(xpos)=2.56317e-04
xneg=1.15530e+00, xpos=1.15533e+00, delta=3.05176e-05, f(xneg)=-7.30778e-04, f(xpos)=2.56317e-04
xneg=1.15532e+00, xpos=1.15533e+00, delta=1.52588e-05, f(xneg)=-2.37242e-04, f(xpos)=2.56317e-04
Root of the function myfunc is:  1.155323028564453l
```

Test with the following functions and initial values

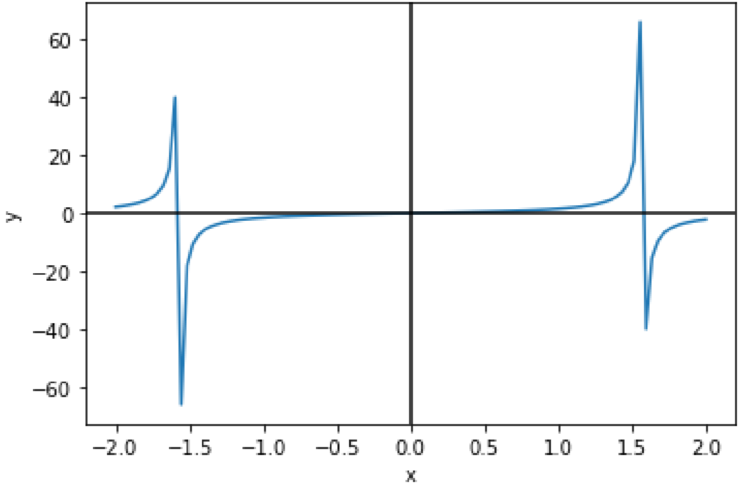
- $f(x) = x^2 - 1; xneg = 0; xpos = 1.5$
- Hint: implement these new functions with new names. You can simply call bisection function by changing the argument name.




```
In [35]: plot.plot(myfunc2,-2,2)

In [36]: bisection(myfunc2,0,1.5)
```

Test with the following functions and initial values

- $f(x) = \tan(x)\,; xneg = -0.5; xpos = 1.$
- Hint: implement these new functions with new names. You can simply call bisection function by changing the argument name.




```
In [42]: plot.plot(myfunc3,-2,2)

In [43]: bisection(myfunc3,-0.5,1)
```

Warning: What happens if you have a singularity.
Test the bisection method with xpos=-2.0 and xneg=-1.0.
Bisection method Works when the function is continuous.