ELEC-E5431- Osman Manzoor Ahmed - 721347 - Assignment 1

January 24, 2019

1 ELEC-E5431 – Large-Scale Data Analysis (LSD Analysis)

Home Work 1 Osman Manzoor Ahmed 721347

In every algorithm the output is as following: 1. X Matrix computed by the algorithm 2. X Optimal Matrix 3. Difference of the above mentioned matrices 4. Convergence rate plot 5. Plot of the X optimal and X created by the algorithm

Kindly scroll down to the ends of each algorithm output in order to see the plots.

```
In [13]: # All Libraries

         import pandas as pd
         from matplotlib import pyplot as plt
         from IPython.display import display, HTML
         import numpy as np
         from scipy import random, linalg
         from sklearn.datasets import make_spd_matrix
         import math
         from random import randint
```

```
In [33]: # ********************************** All Functions********************************

         # Gradient  Ax-b
         def gradient(A,b,x):
             #gradient = np.subtract(np.dot(np.transpose(A),x),b)
             gradient = np.subtract(np.dot(A,x),b)
             #gradient = np.subtract(b,np.dot(np.transpose(A),x))
             return gradient

         # Negative Gradient  b-Ax
         def neg_gradient(A,b,x):
             #gradient = np.subtract(np.dot(np.transpose(A),x),b)
             gradient = np.subtract(b,np.dot(np.transpose(A),x))
             return gradient

         #Error Function log || x_grad - x_opt ||
         def logerror(x_grad, x_opt):
```

```
    diff = x_grad - x_opt
    norm = np.linalg.norm(diff)
    log = np.log(norm)
    return log

# Gradient Descent Method
def gradient_descent(A, b, a, k,x_opt):
    #x = np.zeros((100,1))
    n = len(b)
    x = np.zeros((n,1))
    errors = []

    #for i in range(k):
    while True:
        # Calculate gradient
        grad = gradient(A,b,x)
        # Gradient descent formula
        x = x - a * grad
        #print("In")
        if(np.linalg.norm(grad) <= pow(10,-5)):
           break
        #empirical_errors.append(empirical_risk(X,Y,w))
        errors.append(logerror(x,x_opt))
    return errors,x

# Conjugate Gradient Descent Method
def conjugate_gradient_descent(A, b, a, k,x_opt):
    #x = np.zeros((100,1))
    n = len(b)
    x = np.zeros((n,1))
    p = b
    p_prev = np.zeros((n,1))
    errors = []
    checker = 1
    gamma = 0
    grad = np.ones((n,1))
    i = 0
    r = b
    r_new = b
    #for i in range(k):
    while True:
        # Calculate gradient
        i = i+1
        if(i == 1):
            p = r
            p_prev = p
        else:
            beta = np.divide(np.power(np.linalg.norm(r_new),2),np.power(np.linalg.norm
```

```
            p = np.add(r_new,np.dot(beta,p_prev))
            p_prev = p

        # conjugate Gradient formula
        x = x + a * p
        r = r_new
        r_new = neg_gradient(A,b,x)
        if(np.linalg.norm(r_new) <= pow(10,-5)):
            break
        # Calculate the gamma now
        #print("In")
        errors.append(logerror(x,x_opt))
    return errors,x

# Nesterovs Algorithm
def nesterov_algorithm(A, b, a, L, k,x_opt):
    #x = np.zeros((100,1))
    n = len(b)
    x = np.zeros((n,1))
    errors = []
    y = x
    alpha = a

    #for i in range(k):
    while True:
        # Calculate gradient
        x_prev = x
        grad = gradient(A,b,y)
        x = (np.subtract(y,(1/L)*grad))
        a_prev = alpha
        alpha = (1 + math.sqrt(1 + 4 * a_prev * a_prev))/2

        beta = (a_prev * (1-a_prev)) / (a_prev * a_prev + alpha)

        y = x + (beta * (np.subtract(x,x_prev)))

        if(np.linalg.norm(grad) <= pow(10,-5)):
            break
        # Calculate the gamma now
        #print("In")
        errors.append(logerror(x,x_opt))
    return errors,x

# Stochastic Coordinate Descent Method
def stochastic_coordinate_descent(A, b, a, k,x_opt):
    #x = np.zeros((100,1))
    n = len(b)
    x = np.zeros((n,1))
```

```
    errors = []

    for i in range(k):
    #while True:
        #for j in range(len(x)):
        random = randint(0, n-1)
        # Calculate gradient

        grad = gradient(A[random,random].reshape(1,1),b[random].reshape(1,1),x[random]
        #print(grad.shape)
        # Gradient descent formula
        x[random] = x[random] - a * grad
        #print("In")
        if(np.linalg.norm(grad) <= pow(10,-5)):
            break
        #empirical_errors.append(empirical_risk(X,Y,w))
        errors.append(logerror(x,x_opt))
    return errors
```

In [15]:
```
# Creation of Matrices A, b and X_opt(Optimal x)

# Generate a random symmetric, positive-definite matrix. Size 100*100
matrixSize = 100
A = make_spd_matrix(matrixSize, random_state=None)

# Check if the newly created matrix A is positive- definite. Check if all its Eigneva
if(np.all(np.linalg.eigvals(A) > 0)):
    print("Success, you have a positive definite matrix")

# Generate matrix b size 100*1 that should be in range of Matrix A
x = np.ptp(A,axis = 0)
b = np.reshape(x, (100,1))

# Generate X_Opt as X_opt = A(inverse)b
A_inverse = np.linalg.inv(A)
X_opt = np.dot(A_inverse,b)

#Calculate value of alpha as 1/trace(A)
alpha = 1/A.trace()
```

```
Success, you have a positive definite matrix
```

In [22]:
```
# **************************** Gradient Descent *******************

loggError, x_grad_hat = gradient_descent(A,b,alpha,1000,X_opt)
print("*******************************************************")
print("X Matrix computed by the gradient descent")
```

```
print(x_grad_hat)
print("*****************************************************")
print("X Optimal ")
print(X_opt)
print("*****************************************************")
diff_gradient_descent = np.subtract(x_grad_hat,X_opt)
print("Gradient Descent, Difference between x and x_optimal")
print(diff_gradient_descent)

plt.plot(loggError)
plt.xlabel('Number of Iterations')
plt.ylabel('Log Error')
plt.title('Plot of error according to Iterations(Gradient Descent)')
plt.show()

fig, axes = plt.subplots(1, 2,figsize=(12, 4))
axes[0].plot(X_opt)
axes[0].set_title("X optimal")
axes[1].plot(x_grad_hat,color="red")
axes[1].set_title("X Created by Gradient Descent")
```

```
*****************************************************
X Matrix computed by the gradient descent
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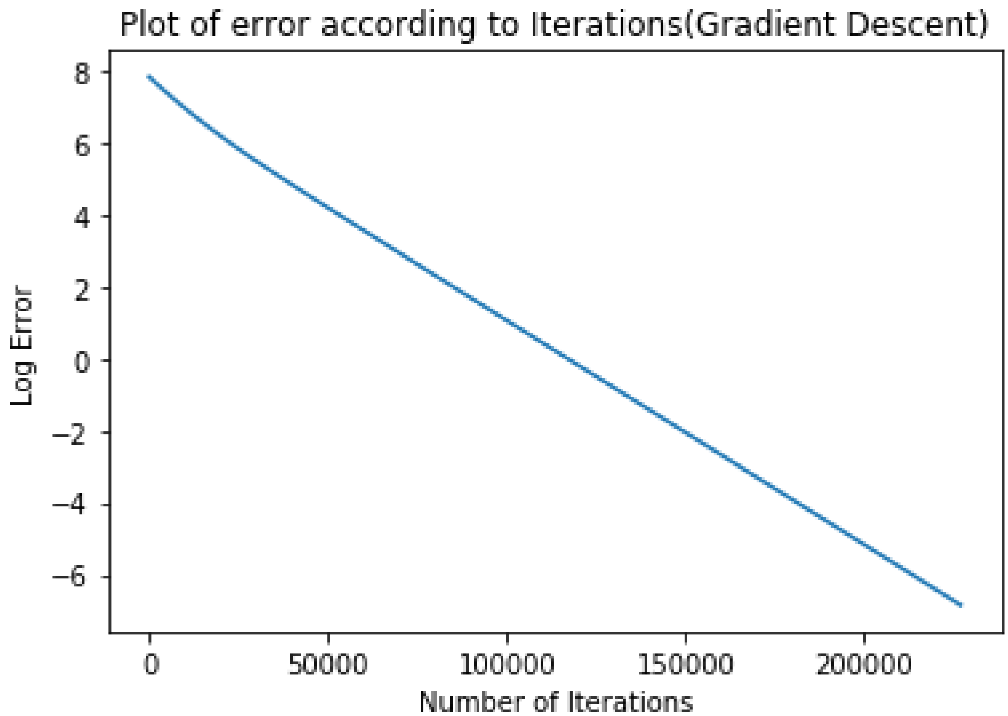
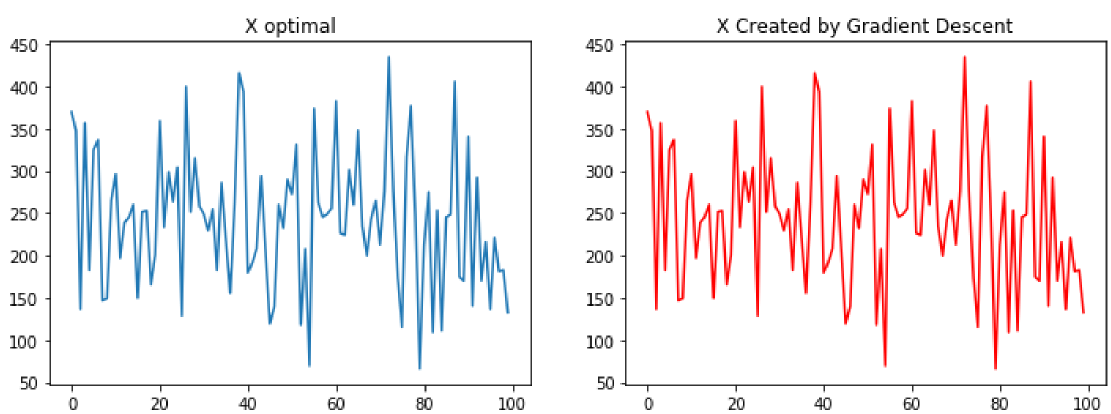


Out[22]: Text(0.5,1,'X Created by Gradient Descent')



In [21]:
```
#***************************** Conjugate Gradient Descent ************

logeError,x_conj_grad_hat = conjugate_gradient_descent(A,b,alpha,10000,X_opt)
print("*****************************************************")
print("X Matrix computed by the conjugate gradient descent")
print(x_conj_grad_hat)
```

```
print("****************************************************")
print("X Optimal ")
print(X_opt)
print("****************************************************")
diff_conj_gradient_descent = np.subtract(x_conj_grad_hat,X_opt)
print("Conjugate Gradient Descent, Difference between x and x_optimal")
print(diff_conj_gradient_descent)

plt.plot(logeError)
plt.xlabel('Number of Iterations')
plt.ylabel('Log Error')
plt.title('Plot of error according to Iterations(Conjugate Gradient Descent)')
plt.show()


fig, axes = plt.subplots(1, 2,figsize=(12, 4))
axes[0].plot(X_opt)
axes[0].set_title("X optimal")
axes[1].plot(x_conj_grad_hat,color="red")
axes[1].set_title("X Created by Conjugate Gradient Descent")
```

```
****************************************************
X Matrix computed by the conjugate gradient descent
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[ -6.37258567e-05]
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[  1.38664774e-04]
[  7.15708208e-05]
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[  2.28352164e-05]
[ -7.27580455e-05]
[ -3.81144012e-05]
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[ -1.12644087e-04]
[  9.90206041e-05]
[  7.42449759e-05]
[  7.17654278e-05]
[ -6.70692689e-05]
[ -7.45690407e-05]
[  7.15365498e-05]
[  3.52114274e-05]
[ -4.22733231e-06]
[ -1.16384926e-04]
```

```
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[  2.28640226e-06]
[  4.45992918e-05]
[  5.08304290e-05]
[ -1.77964949e-04]
[ -7.05692823e-05]
[  8.51842744e-05]
[  4.49760711e-06]
[  5.34011656e-05]
[  2.98732513e-06]
[ -8.54501853e-05]
[ -3.01391324e-05]
[  4.34371212e-05]
[  2.65443306e-05]
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[  2.02081708e-06]
[ -8.05740888e-05]
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[ -1.73036263e-05]
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[ -5.94362040e-05]
[  1.30704754e-04]
[ -2.88679851e-05]
[  7.41100615e-05]
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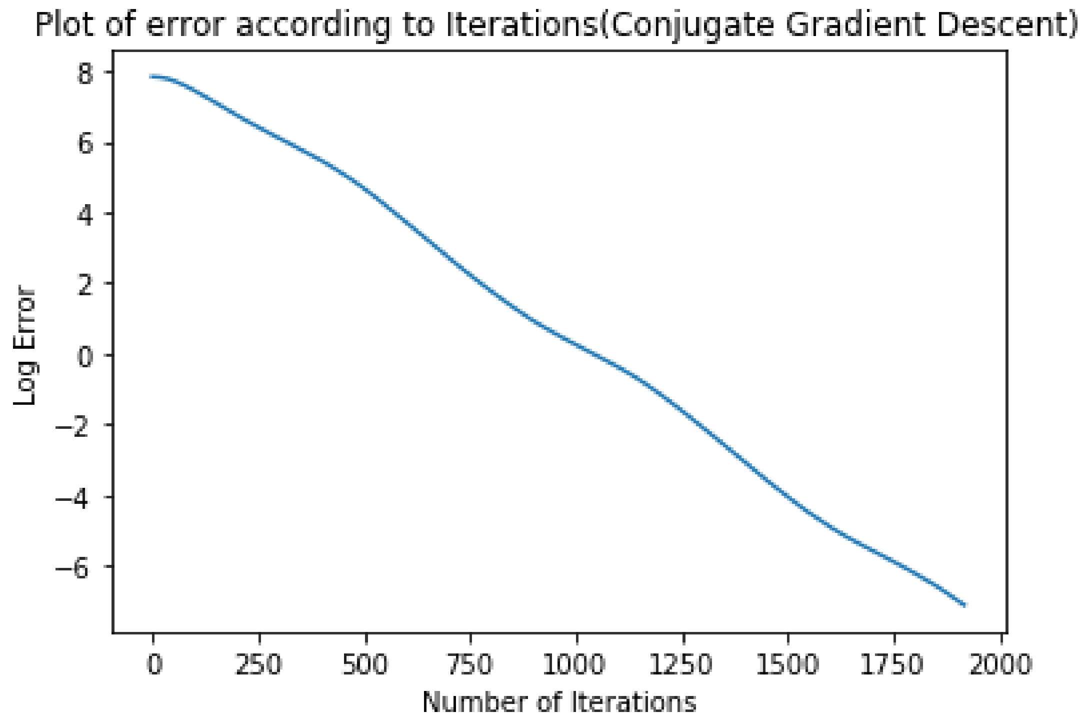
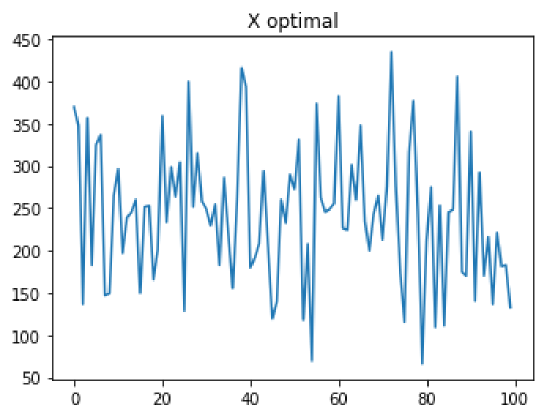
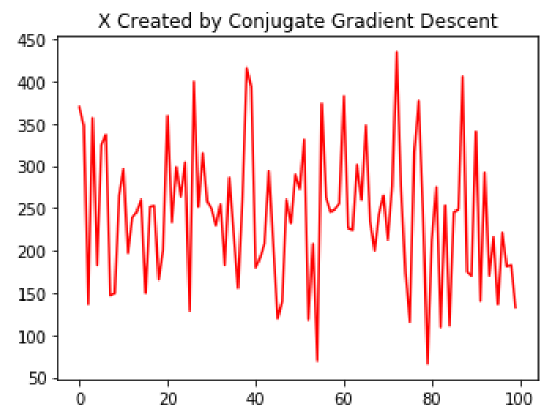


Out[21]: Text(0.5,1,'X Created by Conjugate Gradient Descent')





In [28]:
```
#****************************** Nesterovs Algorithm ************

logError,x_nest_algo = nesterov_algorithm(A,b,0,A.trace(),10000,X_opt)
print("*******************************************************")
print("X Matrix computed by the Nesterov Algorithm")
print(x_nest_algo)
```

```python
    print("*****************************************************")
    print("X Optimal ")
    print(X_opt)
    print("*****************************************************")
    diff_nest_algo = np.subtract(x_nest_algo,X_opt)
    print("Nesterov Algorithm, Difference between x and x_optimal")
    print(diff_nest_algo)

    plt.plot(logError)
    plt.xlabel('Number of Iterations')
    plt.ylabel('Log Error')
    plt.title('Plot of error according to Iterations(Nesterov Method)')
    plt.show()

    fig, axes = plt.subplots(1, 2,figsize=(12, 4))
    axes[0].plot(X_opt)
    axes[0].set_title("X optimal")
    axes[1].plot(x_nest_algo,color="red")
    axes[1].set_title("X Created by Nesterov Algorithm")
```

```
*****************************************************
X Matrix computed by the Nesterov Algorithm
[[ 369.85567513]
 [ 347.52930769]
 [ 136.57170756]
 [ 356.56884789]
 [ 182.62946928]
 [ 324.9442824 ]
 [ 336.91040735]
 [ 146.8814646 ]
 [ 149.54641457]
 [ 265.2008635 ]
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 [ 196.94285936]
 [ 238.76143669]
 [ 245.74076526]
 [ 260.33718727]
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 [ 298.68877416]
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 [ 304.4763547 ]
 [ 128.762478  ]
```

```
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[ 248.87602114]
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[ 272.84196638]
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[ 373.98026167]
[ 262.73466052]
[ 245.69804421]
[ 248.9715473 ]
[ 255.64950244]
[ 382.59159265]
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[ 224.49243135]
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[ 260.21281628]
[ 348.06374709]
[ 234.92285771]
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[ 212.87269534]
[ 275.81240103]
[ 434.99515081]
[ 276.08307149]
```

```
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```

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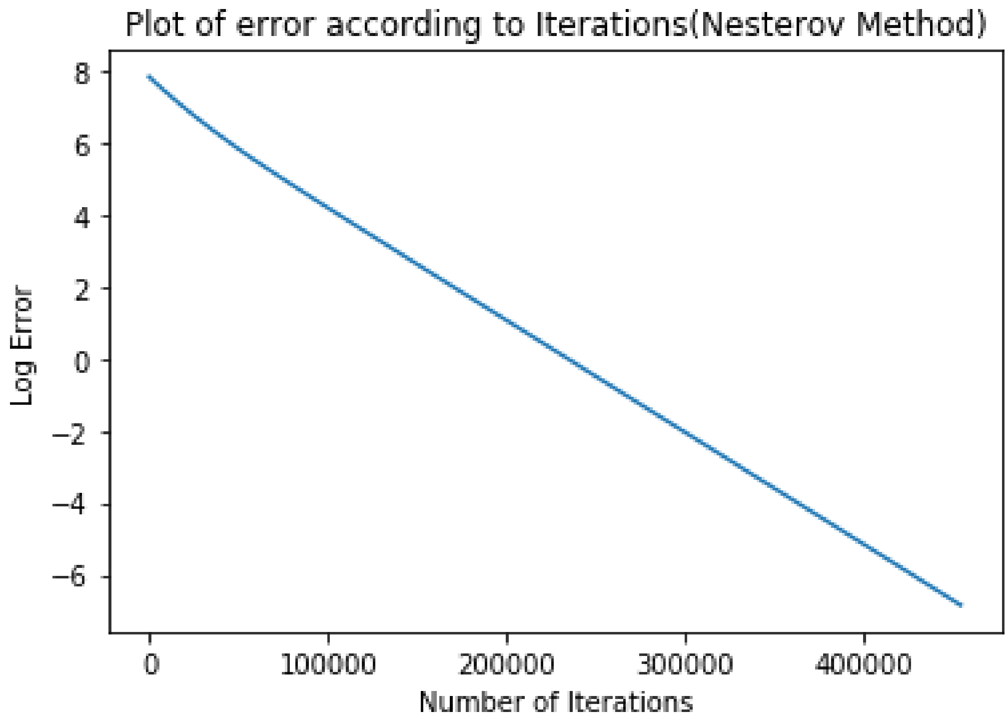
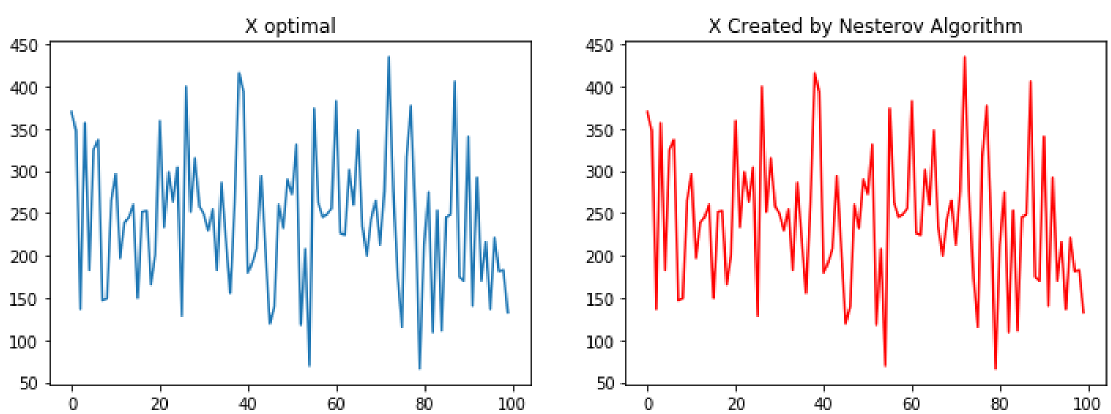


Out[28]: Text(0.5,1,'X Created by Nesterov Algorithm')




```
In [34]: #***************************** Stochastic Coordinate Descent Algorithm ***********

         logcoorError = stochastic_coordinate_descent(A,b,alpha,50000,X_opt)
         #print(len(logError))
         #print("Gradient X")
         #print(x_grad)
```

```
#print(x_grad.shape)

plt.plot(logcoorError)
plt.xlabel('Number of Iterations')
plt.ylabel('Log Error')
plt.title('Plot of error according to Iterations(Stochastic Coordinate Descent Method)
plt.show()
```