ELEC-E5431- Osman Manzoor Ahmed - 721347 - Assignment 2

February 1, 2019

1 ELEC-E5431 – Large-Scale Data Analysis (LSD Analysis)

Home Work 2 Osman Manzoor Ahmed 721347

In every algorithm the output is as following: 1. Convergance plot according to the difference between the norm value of the function w.r.t the optimal value of X and algo computed value to X 2. In the last convergance plot of all the three algos.

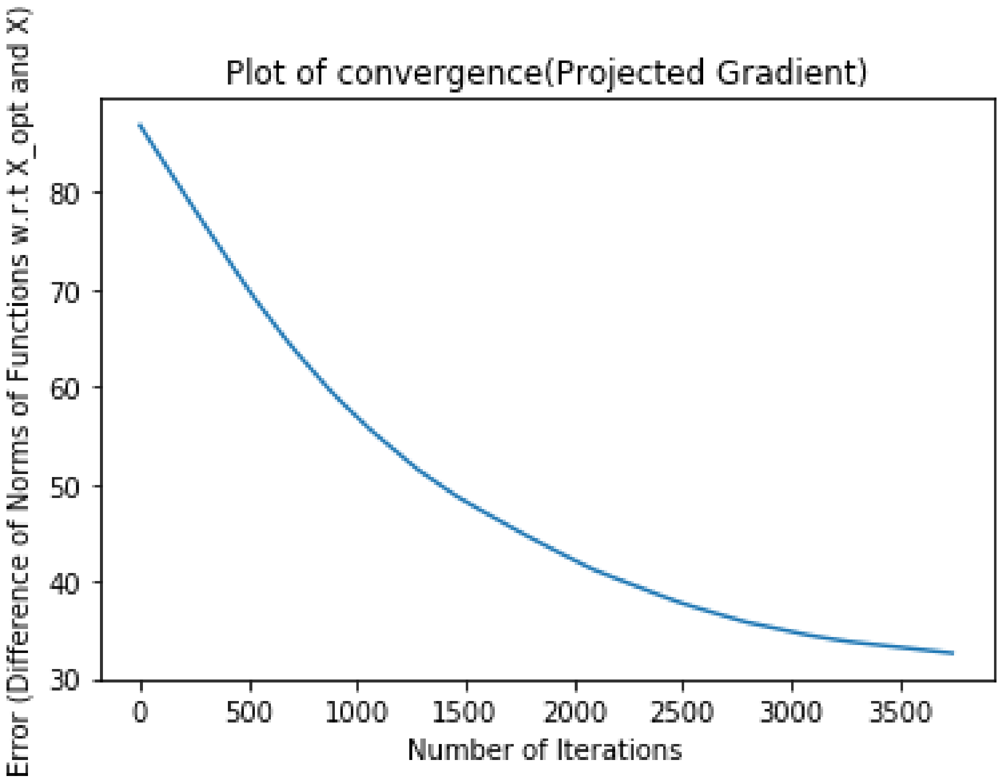
In [38]:
```
# All Libraries

import pandas as pd
from matplotlib import pyplot as plt
from IPython.display import display, HTML
import numpy as np
from scipy import random, linalg
from sklearn.datasets import make_spd_matrix
import math
from random import randint
from scipy.linalg import qr
from scipy.stats import ortho_group
```

In [39]:
```
#Matrix A of size 12*100

A = np.random.randn(20, 100)

#matrix x that is sparse with 3 non zero values
x_opt = np.zeros((100,1))
x_opt[8] = 0.88
x_opt[55] = 5.5
x_opt[74] = 1.7


# Creation of Matrix b
b = np.dot(A,x_opt)

# random value of ta (used in Proximal Gradient)
ta = 10
```

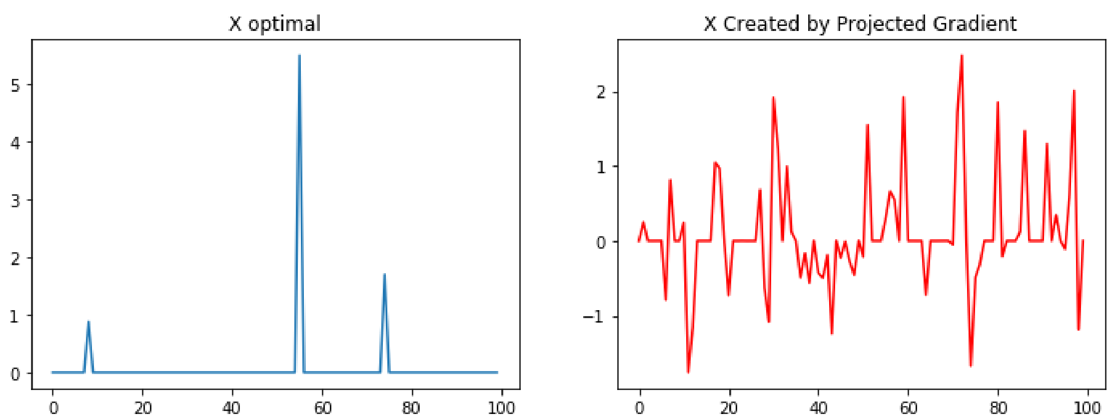
```
        # random value of P (used in ADMM)
        P  = 0.1

        # value of Alpha

        #alpha = 1.0 / np.linalg.norm(np.dot(A.T,A), 2)
        Aprod = np.dot(A.T,A)
        alpha = 1.0/ Aprod.trace()
        #alpha = 1.0/ A.trace()
```

```
In [51]: # Projected Subgradient Method for LASSO

        # Gradient
        def proj_gradient(A,b,x):

            Ident = np.identity(A.shape[1])
            Prod = np.dot(np.dot(A.T,np.linalg.inv(np.dot(A,A.T))),A)
            inv = np.subtract(Ident,Prod)
            inver = np.dot(inv,np.sign(x))

            return inver

        def sign(xx):
            def proj_elementwise(x):
                    if x > 0:
                        return 1
                    elif x < 0:
                        return -1
                    else:
                        return 0.0
            return np.vectorize(proj_elementwise)(xx)

        def calculate_lasso(x):
            x = x.reshape(-1)
            return np.sum(np.abs(x))

        def projected_Gradient(A,b,alpha,k,ta,norm_xopt):
            n = A.shape[1]
            x = np.random.normal(0, 1, (n,1))
            #x = np.ones((n,1))
            #x = np.zeros((n,1))
            values_of_Lasso = []
            norm_values = []
            norm_values.append(calculate_lasso(x))
            #for i in range(k):
            while(True):
                grad = proj_gradient(A,b,x)
                x = x - alpha * grad
```

```python
        norm_x = calculate_lasso(x)
        norm_values.append(norm_x)
        values_of_Lasso.append(norm_x - norm_xopt)

        error = abs(norm_values[-1] - norm_values[-2])
        if(error <= pow(10,-5)):
            break
    return values_of_Lasso,x


# Running of Projected Gradient
projected_norm_xopt = np.sum(np.abs(x_opt))
projected_error,projected_val_x = projected_Gradient(A,b,alpha,100000,ta,projected_no

plt.plot(projected_error)
plt.xlabel('Number of Iterations')
plt.ylabel('Error (Difference of Norms of Functions w.r.t X_opt and X)')
plt.title('Plot of convergence(Projected Gradient)')
plt.show()

fig, axes = plt.subplots(1, 2,figsize=(12, 4))
axes[0].plot(x_opt)
axes[0].set_title("X optimal")
axes[1].plot(projected_val_x,color="red")
axes[1].set_title("X Created by Projected Gradient")
```



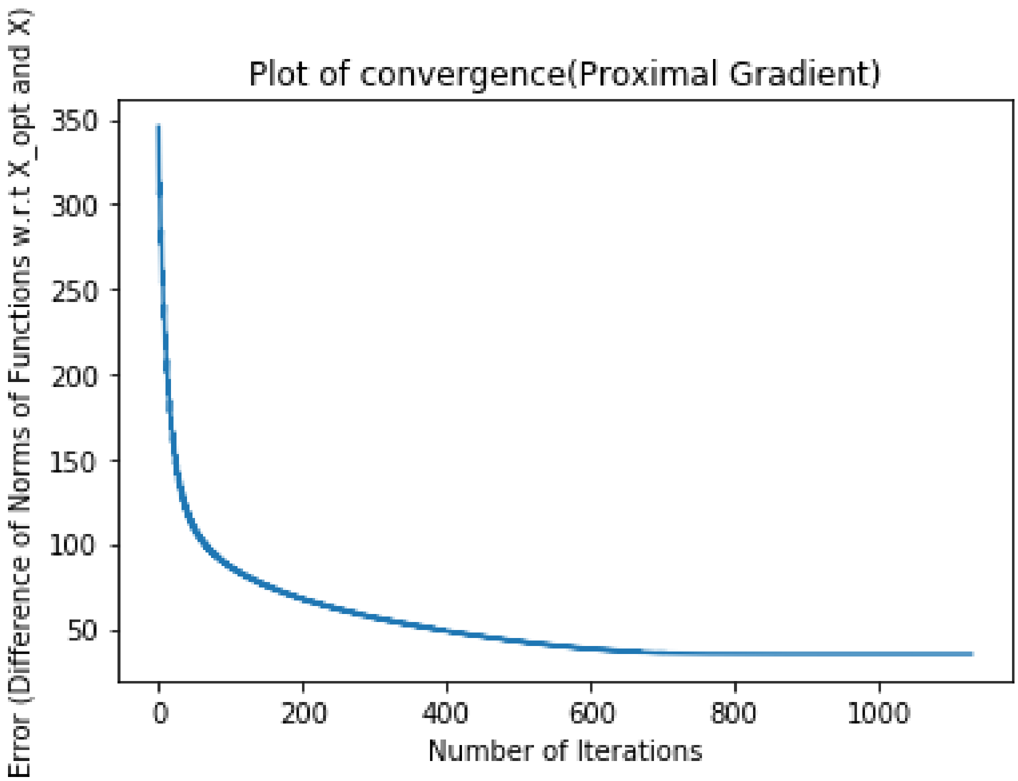

```
Out[51]: Text(0.5,1,'X Created by Projected Gradient')
```



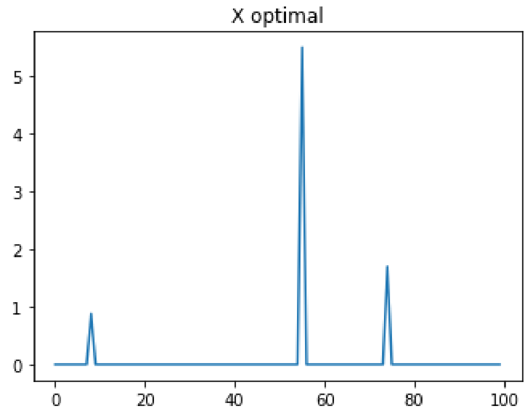

```
In [57]: # Proximal Gradient Algorithm for Lasso

         # Gradient  A.T(AX-b)
         def prox_gradient(A,b,x):
```

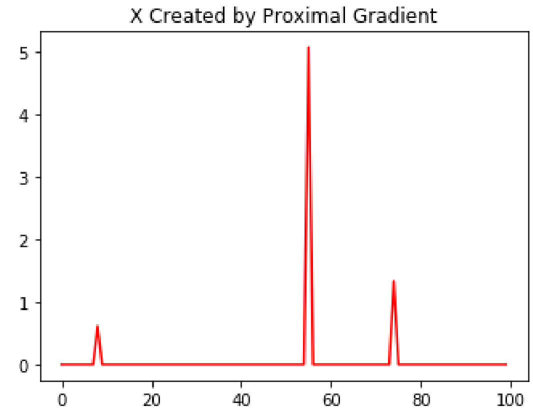
```
    gradient = np.dot(np.transpose(A),np.subtract(np.dot(A,x),b))
    return gradient

def soft(y,ta):
    def prox_elementwise(y, t):
            if y > t:
                return y - t
            elif y < -t:
                return t - y
            else:
                return 0.0
    return np.vectorize(prox_elementwise)(y, ta)

def calculate_lasso(A,b,x,ta):
    grad = np.subtract(np.dot(A,x),b)
    norm_of_grad = ((np.linalg.norm(grad)) ** 2)*0.5
    x_l1_norm = np.linalg.norm(x, ord=1) * ta
    return norm_of_grad + x_l1_norm

def proximal_Gradient(A,b,alpha,k,ta,norm_xopt):
    n = A.shape[1]
    #x = np.random.normal(0, 1, (n,1))
    x = np.zeros((n,1))
    values_of_Lasso = []
    norm_values = []
    norm_values.append(calculate_lasso(A,b,x,ta))
    #for i in range(k):
    while(True):
        grad = prox_gradient(A,b,x)
        x = soft(x - (alpha * grad),alpha * ta)
        norm_x = calculate_lasso(A,b,x,ta)
        norm_values.append(norm_x)
        values_of_Lasso.append(norm_x - norm_xopt)

        error = abs(norm_values[-1] - norm_values[-2])
        if(error <= pow(10,-5)):
           break

    return values_of_Lasso,x


# Running of Proximal Gradient
proximal_norm_xopt = np.sum(np.square(np.dot(A, x_opt) - b)) + ta * np.sum(np.abs(x_op
proximal_error,proximal_val_x = proximal_Gradient(A,b,alpha,100000,ta,proximal_norm_xo

plt.plot(proximal_error)
plt.xlabel('Number of Iterations')
plt.ylabel('Error (Difference of Norms of Functions w.r.t X_opt and X)')
```

```
plt.title('Plot of convergence(Proximal Gradient)')
plt.show()


fig, axes = plt.subplots(1, 2,figsize=(12, 4))
axes[0].plot(x_opt)
axes[0].set_title("X optimal")
axes[1].plot(proximal_val_x,color="red")
axes[1].set_title("X Created by Proximal Gradient")
```



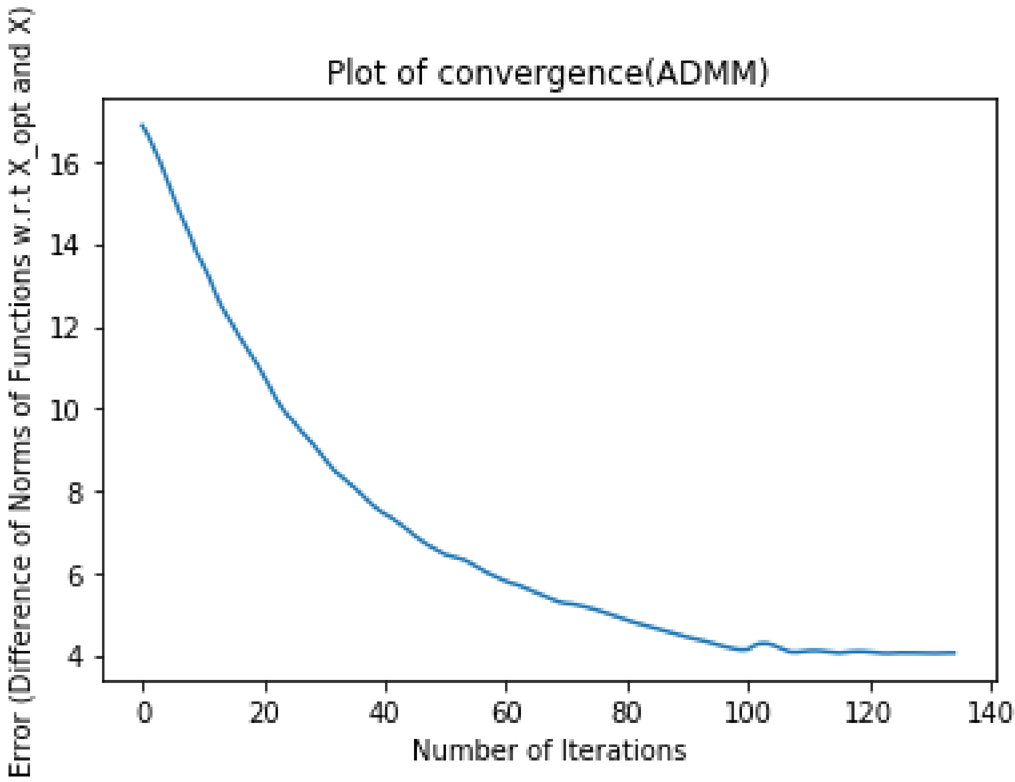

```
Out[57]: Text(0.5,1,'X Created by Proximal Gradient')
```



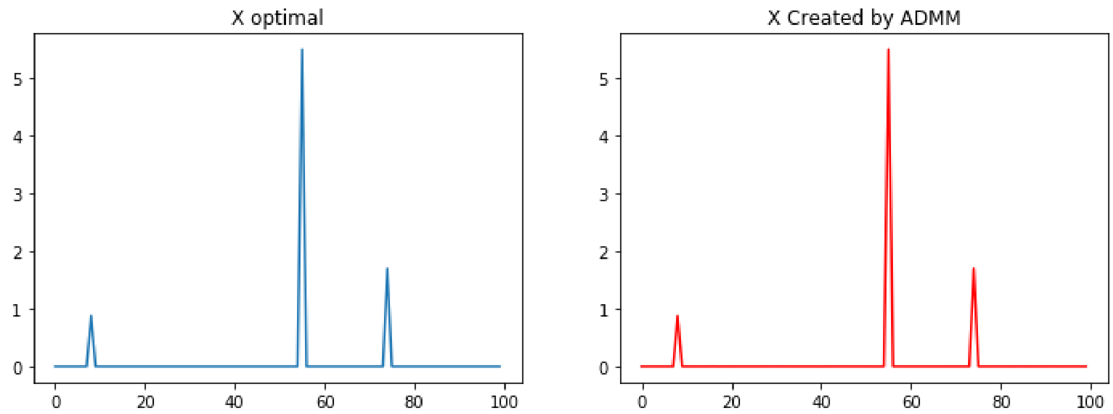


In [53]:
```
# ADMM Algorithm for Lasso

# Gradient
def ADMM_gradient(A,b,z,y,P):

    Ident = np.identity(A.shape[1])
    part_a = np.add(np.dot(A.T,A),P * Ident)
    part_b = np.subtract(np.add(np.dot(A.T,b),P * z),y)

    final_part = np.dot(np.linalg.inv(part_a),part_b)

    return final_part

def soft_admm(y,ta):
    def admm_elementwise(y, t):
            if y >= t:
                return y - t
            elif y <= -t:
                return t + y
            else:
                return 0.0
    return np.vectorize(admm_elementwise)(y, ta)

def calculate_lasso(A,b,x,ta):
    grad = np.subtract(np.dot(A,x),b)
    norm_of_grad = ((np.linalg.norm(grad)) ** 2)*0.5
    x_l1_norm = np.linalg.norm(x, ord=1)
    return norm_of_grad + x_l1_norm

def admm_lasso(A,b,alpha,k,P,norm_xopt):
    n = A.shape[1]
    #x = np.random.normal(0, 1, (n,1))
```

```
    #x = np.ones((n,1))
    x = np.zeros((n,1))
    z = np.zeros((n,1))
    u = np.zeros((n,1))
    values_of_Lasso = []
    #x = np.dot(np.dot(A.T,np.linalg.inv(np.dot(A,A.T))),b)
    norm_values = []
    norm_values.append(calculate_lasso(A,b,x,ta))
    #for i in range(k):
    while(True):
        grad = ADMM_gradient(A,b,z,u,P)
        x = grad
        z = soft_admm(np.add(x,u),alpha/P)
        u = np.add(u,np.subtract(x,z) * P)

        norm_x = calculate_lasso(A,b,x,ta)
        norm_values.append(norm_x)
        values_of_Lasso.append(norm_x - norm_xopt)

        error = abs(norm_values[-1] - norm_values[-2])
        if(error <= pow(10,-5)):
           break
    return values_of_Lasso,x


# Running of Proximal Gradient
admm_norm_xopt = np.sum(np.square(np.dot(A, x_opt) - b)) +  np.sum(np.abs(x_opt)) * 0
admm_error,admm_val_x = admm_lasso(A,b,alpha,10000,P,admm_norm_xopt)

plt.plot(admm_error)
plt.xlabel('Number of Iterations')
plt.ylabel('Error (Difference of Norms of Functions w.r.t X_opt and X)')
plt.title('Plot of convergence(ADMM)')
plt.show()


fig, axes = plt.subplots(1, 2,figsize=(12, 4))
axes[0].plot(x_opt)
axes[0].set_title("X optimal")
axes[1].plot(admm_val_x,color="red")
axes[1].set_title("X Created by ADMM")
```



Out[53]: Text(0.5,1,'X Created by ADMM')



In [54]: # Convergance Plot of all

```
plt.plot(projected_error)
plt.plot(proximal_error)
```

```python
plt.plot(admm_error)

plt.legend(['Projected Gradient', 'Proximal Gradient', 'ADMM'], loc='upper right')

plt.show()
```