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Course: *B.Sc. Mathematical Sciences*

Subject: *Numerical Methods Practical File*

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Practical 1(Bisection Method)

Ques 1. Find out the roots of the function f(x)=Cos(x) in the interval [0,2] using bisection method. Compute the approx value of the root after 14 iterations.
Solution:

```
f[x_] := Cos[x]

x0 = 0.0;
x1 = 2.0;
n = 14
14

If[f[x0] * f[x1] > 0, Print[
  "These Values do not fit in the given condition for IVT. Please change the values"],
 For[i = 1, i ≤ n, i++, a = (x0 + x1) / 2; Print[i, "th iteration value is: ", a];
   If[f[x0] * f[a] < 0, x1 = a, x0 = a];];]
1th iteration value is: 1.

2th iteration value is: 1.5

3th iteration value is: 1.75

4th iteration value is: 1.625

5th iteration value is: 1.5625

6th iteration value is: 1.59375

7th iteration value is: 1.57813

8th iteration value is: 1.57031

9th iteration value is: 1.57422

10th iteration value is: 1.57227

11th iteration value is: 1.57129

12th iteration value is: 1.5708

13th iteration value is: 1.57056

14th iteration value is: 1.57068
```

Ques 2. Find out the roots of the function f(x)=Cos(x) in the interval [0,2] using bisection method. Maximum iteration allowed are 20. Maximum error bound is 0.0001. Terminate the loop if any

condition is satisfied.
Solution:

```
f[x_] := Cos[x]

e = 0.0001;
x0 = 0.0;
x1 = 2.0;
n = 20;

If[f[x0] * f[x1] > 0, Print[
   "These values do not fit the given condition of IVT. Please change the values"],
  For[i = 1, i ≤ n, i++, a = (x0 + x1) / 2;
   If[Abs[(x1 - x0) / 2] < e, Return[a], Print[i "th iteration value is: ", a];
    Print["Estimated error in", i, " th iteration is: ", (x1 - x0) / 2];
    If[f[x0] * f[a] < 0, x1 = a, x0 = a]]];
  Print["Root  is:  ", a];
  Print["Estimated error in ", i, " the iteration is: ", (x1 - x0) / 2]]
```

```
th iteration value is: 1.
Estimated error in1 th iteration is: 1.
2 th iteration value is: 1.5
Estimated error in2 th iteration is: 0.5
3 th iteration value is: 1.75
Estimated error in3 th iteration is: 0.25
4 th iteration value is: 1.625
Estimated error in4 th iteration is: 0.125
5 th iteration value is: 1.5625
Estimated error in5 th iteration is: 0.0625
6 th iteration value is: 1.59375
Estimated error in6 th iteration is: 0.03125
7 th iteration value is: 1.57813
Estimated error in7 th iteration is: 0.015625
8 th iteration value is: 1.57031
Estimated error in8 th iteration is: 0.0078125
9 th iteration value is: 1.57422
Estimated error in9 th iteration is: 0.00390625
10 th iteration value is: 1.57227
Estimated error in10 th iteration is: 0.00195313
11 th iteration value is: 1.57129
Estimated error in11 th iteration is: 0.000976563
12 th iteration value is: 1.5708
Estimated error in12 th iteration is: 0.000488281
13 th iteration value is: 1.57056
Estimated error in13 th iteration is: 0.000244141
14 th iteration value is: 1.57068
Estimated error in14 th iteration is: 0.00012207
Return[1.57074]
```

Practical 2 (Regula-Falsi)

Q1. Find approximate root of Cos(x) in the interval (0,2) in 10 iterations.

Solution:

```
f[x_] := Cos[x]
x_0 = 0;
x_1 = 2;
e = 0.000001;
iterations = 10;
If · f[x_0] * f[x_1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For · n = 2, n ≤ iterations, n++,
   x_n = N · (x_{n-2} * f[x_{n-1}] - x_{n-1} * f[x_{n-2}]) / (f[x_{n-1}] - f[x_{n-2}]) · ;
   If[Abs[x_n - x_{n-1}] < e , Return[x_n]];
   Print[n - 1, "th iteration is: ", x_n];
   Print ["Estimated error: ", Abs[x_n - x_{n-1}]] · · ;
Print["Approximate Root ", x_n]
```

```
1th iteration is: 1.41228

Estimated error: 0.587717

2th iteration is: 1.57391

Estimated error: 0.161623

3th iteration is: 1.57078

Estimated error: 0.0031228

4th iteration is: 1.5708

Estimated error: 0.0000128049

Return[1.5708]

Approximate Root 1.5708
```

Q2. Find approximate root of following equations using the secant method:

i) $^{3} - 3x^2 + 2x + 5 = 0$

ii) Cos(x) - xe^{x} = 0

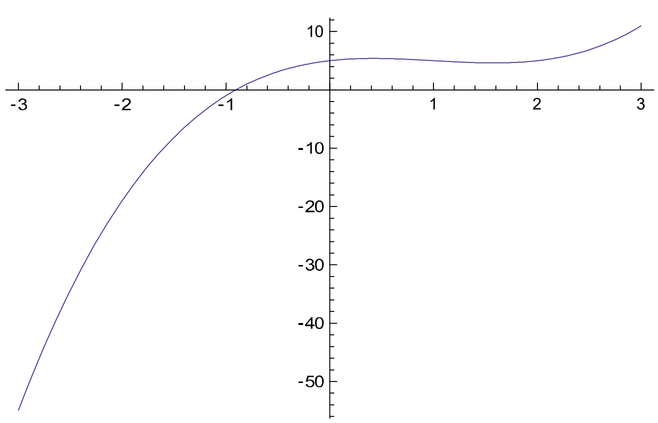
iii) $e^{-x} = x$

Solution:

i) $^{3} - 3x^2 + 2x + 5 = 0$

```
f[x_] := x^3 - 3 x^2 + 2 x + 5
x0  = -1;
x1 = 0;
e = 0.000001;
iterations = 10;
If · f[x0] * f[x1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For · n = 2, n ≤ iterations, n++,
   xn = N · (xn-2 * f[xn-1] - xn-1 * f[xn-2]) / (f[xn-1] - f[xn-2]) · ;
   If[Abs[xn - xn-1] < e , Return[xn]];
   Print[n - 1, "th iteration is: ", xn];
   Print ["Estimated error: ", Abs[xn - xn-1]] · · ;
Print["Approximate Root ", xn]
Plot[f[x], {x, -3, 3}]
```

```
1th iteration is: -0.833333

Estimated error: 0.833333

2th iteration is: -0.962567

Estimated error: 0.129234

3th iteration is: -0.901757

Estimated error: 0.0608094

4th iteration is: -0.904081

Estimated error: 0.00232406

5th iteration is: -0.904161

Estimated error: 0.0000794928

Return[-0.904161]

Approximate Root -0.904161
```



ii) Cos(x) - xe^{x} = 0

```
f[x_] := Cos[x] - x ⋅ˣ
x₀ = -2;
x₁ = -1;
e = 0.000001;
iterations = 10;
If ⋅ f[x₀] * f[x₁] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For ⋅ n = 2, n ≤ iterations, n++,
                xₙ₋₂ * f[xₙ₋₁] - xₙ₋₁ * f[xₙ₋₂]
   xₙ  =  N ⋅  ──────────────────────────────── ⋅;
                      f[xₙ₋₁] - f[xₙ₋₂]
   If[Abs[xₙ - xₙ₋₁] < e , Return[xₙ]];
   Print[n - 1, "th iteration is: ", xₙ];
   Print ["Estimated error: ", Abs[xₙ - xₙ₋₁]] ⋅ ⋅ ;
Print["Approximate Root ", xₙ]
Plot[f[x], {x, -3, 3}]
```

1th iteration is: -1.86193

Estimated error: 0.861932

2th iteration is: -1.86407

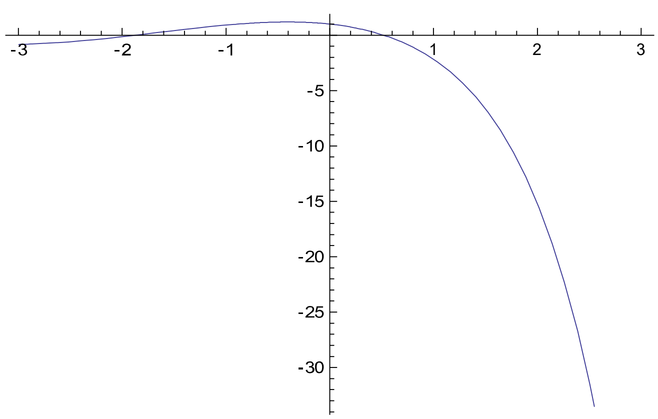
Estimated error: 0.00214255

3th iteration is: -1.864

Estimated error: 0.0000795107

Return[-1.864]

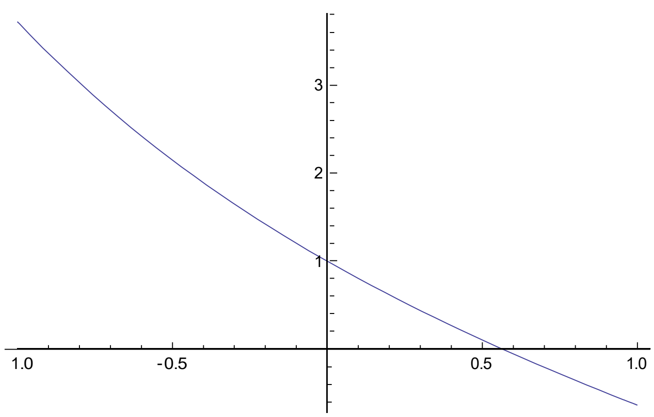
Approximate Root -1.864



iii) $e^{-x} = x$

```
f[x_] := · -x - x
x0 = 0;
x1 = 1;
e = 0.000001;
iterations = 10;
If · f[x0] * f[x1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For · n = 2, n ≤ iterations, n++,
   xn = N · (x(n-2) * f[x(n-1)] - x(n-1) * f[x(n-2)]) / (f[x(n-1)] - f[x(n-2)]) · ;
   If[Abs[xn - x(n-1)] < e , Return[xn]];
   Print[n - 1, "th iteration is: ", xn];
   Print ["Estimated error: ", Abs[xn - x(n-1)]] · · ;
Print["Approximate Root ", xn]
Plot[f[x], {x, -1, 1}]
```

```
1th iteration is: 0.6127

Estimated error: 0.3873

2th iteration is: 0.563838

Estimated error: 0.0488614

3th iteration is: 0.56717

Estimated error: 0.00333197

4th iteration is: 0.567143

Estimated error: 0.0000270518

Return[0.567143]

Approximate Root 0.567143
```



Practical 2 (Secant Method)

Ques 1. Solve x^3 - 2x - 5 = 0
Solution:

```
f[x_] := x^3 - 2 x - 5
x0 = 2;
x1 = 3;
e = 0.000001;
iterations = 10;
If · f[x0] * f[x1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For · n = 2, n ≤ iterations, n++,
   xn = N · xn-1 - f[xn-1] * (xn-1 - xn-2)/(f[xn-1] - f[xn-2]) ·;
   If[Abs[xn - xn-1] < e , Return[xn]];
   Print[n - 1, "th iteration is: ", xn];
   Print ["Estimated error: ", Abs[xn - xn-1]] · ·;
Print["Approximate Root ", xn]
```

```
1th iteration is: 2.05882
Estimated error: 0.941176
2th iteration is: 2.08126
Estimated error: 0.0224401
3th iteration is: 2.09482
Estimated error: 0.0135605
4th iteration is: 2.09455
Estimated error: 0.000274715
5th iteration is: 2.09455
Estimated error: 2.05019×10^-6
Return[2.09455]
Approximate Root 2.09455
```

Q2. Find approximate root of following equations using the secant method:

i) $^3 - 3x^2 + 2x + 5 = 0$

ii) $\text{Cos}(x) - xe^x = 0$

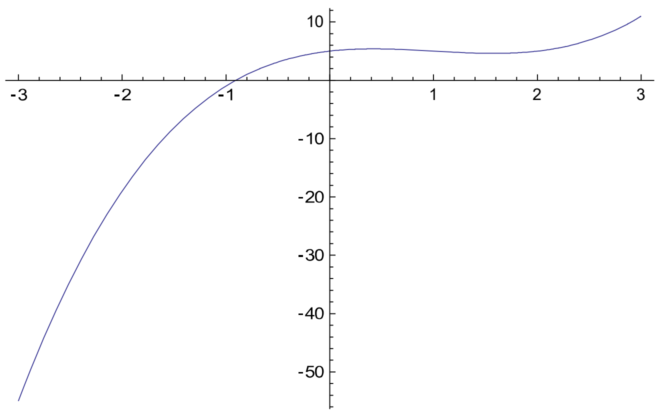
iii) $e^{-x} = x$

Solution:

i) $^{3} - 3x^{2} + 2x + 5 = 0$

```
g[x_] := x^3 - 3 x^2 + 2 x + 5
x0 = -1;
x1 = 0;
e = 0.000001;
iterations = 10;
If · g[x0] * g[x1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For · n = 2, n ≤ iterations, n++,
   xn = N · xn-1 - g[xn-1] * (xn-1 - xn-2)/(g[xn-1] - g[xn-2]) · ;
   If[Abs[xn - xn-1] < e , Return[xn]];
   Print[n - 1, "th iteration is: ", xn];
   Print ["Estimated error: ", Abs[xn - xn-1]] · · ;
Print["Approximate Root ", xn]

Plot[g[x], {x, -3, 3}]
```

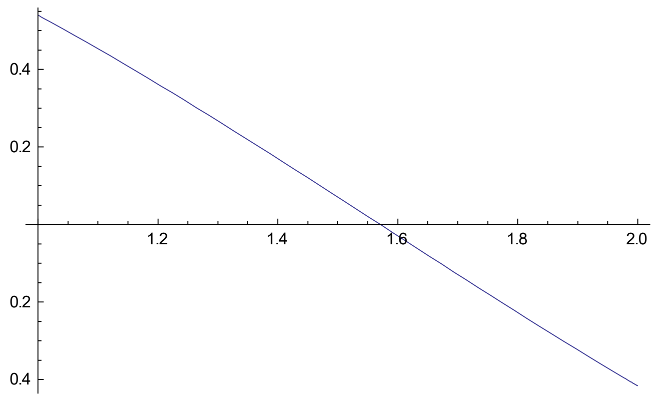
```
1th iteration is: -0.833333

Estimated error: 0.833333

2th iteration is: -0.962567

Estimated error: 0.129234

3th iteration is: -0.901757

Estimated error: 0.0608094

4th iteration is: -0.904081

Estimated error: 0.00232406

5th iteration is: -0.904161

Estimated error: 0.0000794928

Return[-0.904161]

Approximate Root -0.904161
```



ii) Cos(x) - xe^x = 0

```
k[x_] := Cos[x] - x e^(x)
x0 = 1;
x1 = 2;
e = 0.000001;
iterations = 10;
If[k[x0] * k[x1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For[n = 2, n ≤ iterations, n++,
   xn = N[xn-1 - k[xn-1] * (xn-1 - xn-2)/(k[xn-1] - k[xn-2])];
   If[Abs[xn - xn-1] < e , Return[xn]];
   Print[n-1, "th iteration is: ", xn];
   Print ["Estimated error: ", Abs[xn - xn-1]]]];
Print["Approximate Root ", xn]

Plot[k[x], {x, 1, 2}]
```

```
1th iteration is: 1.5649

Estimated error: 0.435096

2th iteration is: 1.57098

Estimated error: 0.00607467

3th iteration is: 1.5708

Estimated error: 0.000182263

Return[1.5708]

Approximate Root 1.5708
```



iii) $e^{-x} = x$

```
k[x_] := · -x - x
x0 = 0;
x1 = 1;
e = 0.000001;
iterations = 10;
If · k[x0] * k[x1] > 0,
  Print["These values do not satisfy the IVP so change the initial value."],
  For · n = 2, n ≤ iterations, n++,
                                  x(n-1) - x(n-2)
   xn = N · x(n-1) - k[x(n-1)] * ─────────────────── · ;
                                k[x(n-1)] - k[x(n-2)]
   If[Abs[xn - x(n-1)] < e , Return[xn]];
   Print[n - 1, "th iteration is: ", xn];
   Print ["Estimated error: ", Abs[xn - x(n-1)]] · · ;
Print["Approximate Root ", xn]

Plot[k[x], {x, -6, 2}]
```

1th iteration is: 0.6127

Estimated error: 0.3873

2th iteration is: 0.563838

Estimated error: 0.0488614

3th iteration is: 0.56717

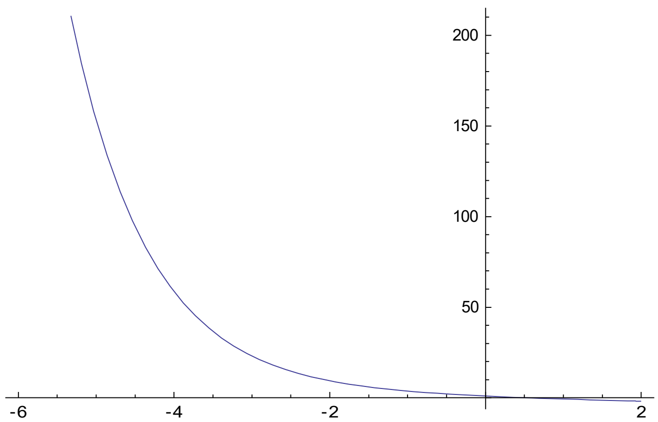
Estimated error: 0.00333197

4th iteration is: 0.567143

Estimated error: 0.0000270518

Return[0.567143]

Approximate Root 0.567143

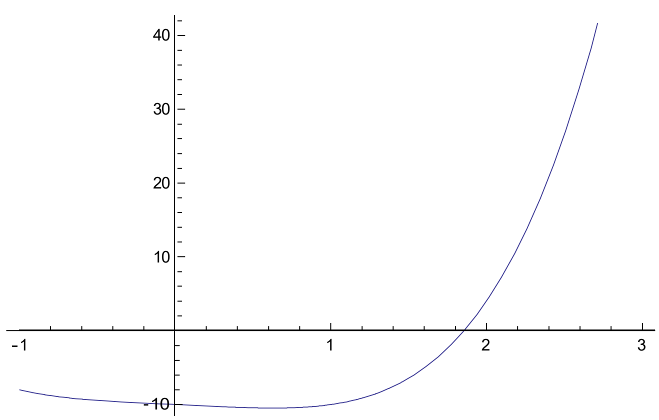


Practical 3 (Newton-Raphson Method)

Q1. Find approximate root of the equation ${}^4 - x - 10 = 0$
Solution:

```
f[x_] := x^4 - x - 10
x_0 = 2;
e = 5 * 10^-5;
iterations = 5;
For · n = 1, n ≤ iterations, n++,
  x_n = N · x_(n-1) - f[x_(n-1)] / f'[x_(n-1)] · ;
  If[Abs[x_n - x_(n-1)] < e, Return[x_n]];
  Print[n, "th iteration's value is: ", x_n];
  Print["Estimated error is: ", Abs[x_n - x_(n-1)]] · ;
Print["Final approximate root is: ", x_n]
Plot[f[x], {x, -1, 3}]
```

```
1th iteration's value is: 1.87097

Estimated error is: 0.129032

2th iteration's value is: 1.85578

Estimated error is: 0.015187

3th iteration's value is: 1.85558

Estimated error is: 0.000196141

Return[1.85558]

Final approximate root is: 1.85558
```

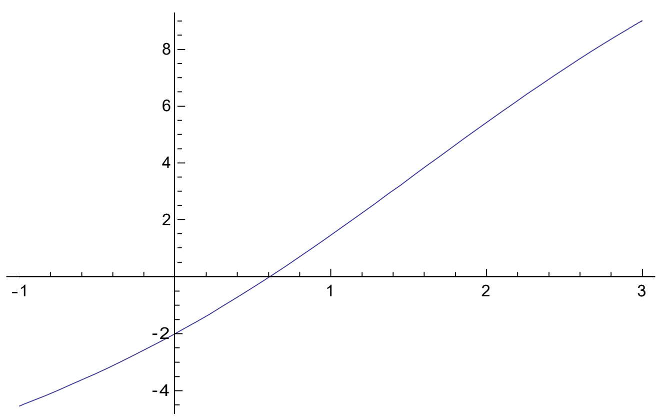


Q2. Find the approximate root of the following:
i) 3x - Cos(x) - 1
ii) Cosx - xe^x=0
iii) $e^{-x} = x$

Solution:
i)3x - Cos(x) - 1

```
f[x_] := 3 x - Cos[x] - 1
x0 = 1;
e = 5 * 10^-5;
iterations = 5;
For · n = 1, n ≤ iterations, n++,
  xn = N · xn-1 - f[xn-1]/f'[xn-1] · ;
  If[Abs[xn - xn-1] < e, Return[xn]];
  Print[n, "th iteration's value is: ", xn];
  Print["Estimated error is: ", Abs[xn - xn-1]] · ;
Print["Final approximate root is: ", xn]
Plot[f[x], {x, -1, 3}]
```

```
1th iteration's value is: 0.620016

Estimated error is: 0.379984

2th iteration's value is: 0.607121

Estimated error is: 0.0128953

Return[0.607102]

Final approximate root is: 0.607102
```

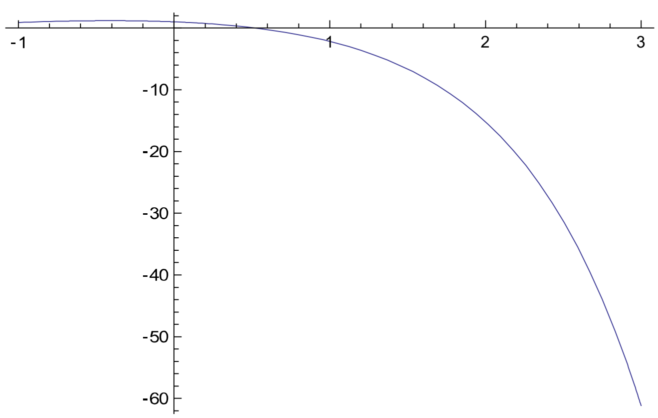


Solution:

ii) Cosx - xe^x=0

```
g[x_] := Cos[x] - x * ⅇ^x
x₀ = 1;
err = 5 * 10^-5;
iterations = 5;
For[n = 1, n ≤ iterations, n++,
  xₙ = N[xₙ₋₁ - g[xₙ₋₁] / g'[xₙ₋₁]];
  If[Abs[xₙ - xₙ₋₁] < err, Return[xₙ]];
  Print[n, "th iteration's value is: ", xₙ];

  Print["Estimated error is: ", Abs[xₙ - xₙ₋₁]]];
Print["Final approximate root is: ", xₙ]
Plot[g[x], {x, -1, 3}]
```

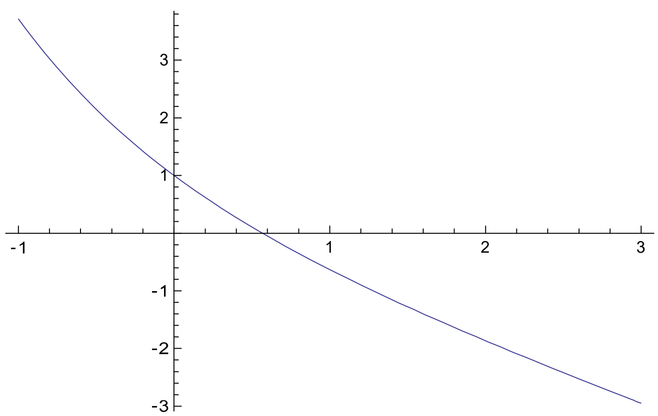
```
1th iteration's value is: 0.653079

Estimated error is: 0.346921

2th iteration's value is: 0.531343

Estimated error is: 0.121736

3th iteration's value is: 0.51791

Estimated error is: 0.0134335

4th iteration's value is: 0.517757

Estimated error is: 0.00015253

Return[0.517757]

Final approximate root is: 0.517757
```



iii) $e^{-x} =$
Solution:

```
g[x_] := · -x - x
x0 = 1;
err = 5 * 10^-5;
iterations = 5;
For · n = 1, n ≤ iterations, n++,
  xn = N · xn-1 - g[xn-1] / g'[xn-1] · ;
  If[Abs[xn - xn-1] < err, Return[xn]];
  Print[n, "th iteration's value is: ", xn];
  Print["Estimated error is: ", Abs[xn - xn-1]] · ;
Print["Final approximate root is: ", xn]
Plot[g[x], {x, -1, 3}]
```

```
1th iteration's value is: 0.537883

Estimated error is: 0.462117

2th iteration's value is: 0.566987

Estimated error is: 0.0291041

3th iteration's value is: 0.567143

Estimated error is: 0.000156295

Return[0.567143]

Final approximate root is: 0.567143
```



Practical 4

Gauss Jordan Method

Q1. Solve the given system of equations using Gauss Jordan Method:

y + z = 2
2x + 3z = 5
x + y + z = 3

Solution:

```
A = {{0, 1, 1, 2}, {2, 0, 3, 5}, {1, 1, 1, 3}}
{{0, 1, 1, 2}, {2, 0, 3, 5}, {1, 1, 1, 3}}

A // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 3 & 5 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

```
RowReduce[A] // MatrixForm
```

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

```
Solve[{x · 1, y · 1, z · 1}, {x, y, z}]
{{x · 1, y · 1, z · 1}}
```

Gauss Elimination Method

Q1. Solve the given system of equations using Gauss Jordan Method:

2x + y + z = 4
3x + 5y + 2z = 15

$2x + y + 4z = 18$

Solution:

```
A = {{2, 1, 1}, {3, 5, 2}, {2, 1, 4}};
A // MatrixForm
B = {4, 15, 18};
B // MatrixForm
m1 = Length[A];
m2 = Length[B];

x = Table[0, {m1}];

If[m1 ≠ m2, Print["This system of equation can not be solved"],
 Table[AppendTo[A ⟦i⟧ , B ⟦i⟧ ], {i, m1}];
 Print["[A|B]=", A // MatrixForm];
 For[i = 1, i ≤ m1 - 1, i++, s = Abs[A ⟦i, i⟧ ];
  c = i;
  For[j = i + 1, j ≤ m1, j++, If[Abs[A ⟦j, i⟧ ] > s, s = A ⟦j, i⟧ ;
    c = j;]];
  For[k = 1, k ≤ m1 + 1, k++, d[k] = A ⟦i, k⟧ ;
   A ⟦i, k⟧  = A ⟦c, k⟧ ;
   A ⟦c, k⟧  = d[k]];
  Print["step", i, A // MatrixForm];
  For[j = i + 1, j ≤ m1, j++, m = A ⟦j, i⟧ / A ⟦i, i⟧ ;
   For[k = 1, k ≤ m1 + 1, k++, A ⟦j, k⟧  = A ⟦j, k⟧ - (m * A ⟦i, k⟧ )];];
  Print[A // MatrixForm];];
 For[i = 0, i ≤ m1 - 1, i++,
  x ⟦m1 - i⟧  = (A ⟦m1 - i, m1 + 1⟧ - Sum[A ⟦m1 - i, j⟧  * x ⟦j⟧ , {j, m1 - i + 1, m1}]) /
     A ⟦m1 - i, m1 - i⟧ ;];
 Print["x =", x // MatrixForm];]
```

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 15 \\ 18 \end{pmatrix}$$

$$[A|B]=\begin{pmatrix} 2 & 1 & 1 & 4 \\ 3 & 5 & 2 & 15 \\ 2 & 1 & 4 & 18 \end{pmatrix}$$

$$\text{step1}\begin{pmatrix} 3 & 5 & 2 & 15 \\ 2 & 1 & 1 & 4 \\ 2 & 1 & 4 & 18 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 5 & 2 & 15 \\ 0 & -\frac{7}{3} & -\frac{1}{3} & -6 \\ 0 & -\frac{7}{3} & \frac{8}{3} & 8 \end{pmatrix}$$

$$\text{step2}\begin{pmatrix} 3 & 5 & 2 & 15 \\ 0 & -\frac{7}{3} & -\frac{1}{3} & -6 \\ 0 & -\frac{7}{3} & \frac{8}{3} & 8 \end{pmatrix}$$

$$\begin{Vmatrix} 3 & 5 & 2 & 15 \\ 0 & -\frac{7}{3} & -\frac{1}{3} & -6 \\ 0 & 0 & 3 & 14 \end{Vmatrix}$$

$$x = \begin{pmatrix} -\frac{9}{7} \\ \frac{40}{21} \\ \frac{14}{3} \end{pmatrix}$$

Practical 5

Gauss Jacobi Method

Q1. Find the given system of given system of equations.

i) $27x_1 + 6x_2 - x_3 = 85, \quad 6x_1 + 15x_2 + 2x_3 = 72, \quad x_1 + x_2 + 54x_3 = 110$
ii) $5x_1 + 2x_2 + x_3 = 10, \quad 3x_1 + 7x_2 + 4x_3 = 21, \quad x_1 + x_2 + 9x_3 = 12$
iii) $10x_1 - 2x_2 + x_3 = 12, \quad x_1 + 9x_2 - x_3 = 10, \quad 2x_1 - x_2 + 11x_3 = 20$

Solution:
i)

```
n = 3; (* number of variables *)
a = {{27, 6, -1}, {6, 15, 2}, {1, 1, 54}};
MatrixForm[a]
x = {0, 0, 0} (* initial value of x⁰1,x⁰2,x⁰3 *)

b = {85, 72, 110}

For[k = 1, k ≤ 5, k++,
 For[i = 1, i ≤ n, i++,
  x [[i]] = (b [[i]] - Sum[a [[i, j]] * x [[j]] , {j, 1, i - 1}] - Sum[a [[i, j]] * x [[j]] , {j, i + 1, n}]) /
    a [[i, i]] ];
 For[m = 1, m ≤ n, m++, x [[m]] = N[x [[m]] ]]]
For[p = 1, p ≤ n, p++, Print["x[", p, "]=", x [[p]] ]]
```

$$\begin{pmatrix} 27 & 6 & -1 \\ 6 & 15 & 2 \\ 1 & 1 & 54 \end{pmatrix}$$

```
{0, 0, 0}

{85, 72, 110}

x[1]=2.42548

x[2]=3.57302

x[3]=1.92595
```

Solution.

ii)

```
n = 3; (* number of variables *)
a = {{5, 2, 1}, {3, 7, 4}, {1, 1, 9}};
MatrixForm[a]
x = {0, 0, 0} (* initial value of x⁰1,x⁰2,x⁰3 *)

b = {10, 21, 12}

For[k = 1, k ≤ 5, k++,
 For[i = 1, i ≤ n, i++,
  x ⟦i⟧ = (b ⟦i⟧ - Sum[a ⟦i, j⟧ * x ⟦j⟧ , {j, 1, i - 1}] - Sum[a ⟦i, j⟧ * x ⟦j⟧ , {j, i + 1, n}]) /
    a ⟦i, i⟧ ];
 For[m = 1, m ≤ n, m++, x ⟦m⟧ = N[x ⟦m⟧ ]]]
For[p = 1, p ≤ n, p++, Print["x[", p, "]=", x ⟦p⟧ ]]
```

$$\begin{pmatrix} 5 & 2 & 1 \\ 3 & 7 & 4 \\ 1 & 1 & 9 \end{pmatrix}$$

```
{0, 0, 0}

{10, 21, 12}

x[1]=0.999005

x[2]=2.00009

x[3]=1.0001
```

Solution:

iii)

```
n = 3; (* number of variables *)
a = {{10, -2, 1}, {1, 9, -1}, {2, -1, 11}};
MatrixForm[a]
x = {0, 0, 0} (* initial value of x⁰1,x⁰2,x⁰3 *)

b = {12, 10, 20}

For[k = 1, k ≤ 5, k++,
 For[i = 1, i ≤ n, i++,
  x ⟦i⟧ = (b ⟦i⟧ - Sum[a ⟦i, j⟧ * x ⟦j⟧ , {j, 1, i - 1}] - Sum[a ⟦i, j⟧ * x ⟦j⟧ , {j, i + 1, n}]) /
    a ⟦i, i⟧ ];
 For[m = 1, m ≤ n, m++, x ⟦m⟧ = N[x ⟦m⟧ ]]]
For[p = 1, p ≤ n, p++, Print["x[", p, "]=", x ⟦p⟧ ]]
```

$$\begin{pmatrix} 10 & -2 & 1 \\ 1 & 9 & -1 \\ 2 & -1 & 11 \end{pmatrix}$$

```
{0, 0, 0}

{12, 10, 20}
```

```
x[1]=1.26241
x[2]=1.15906
x[3]=1.69402
```

Gauss Seidel Method

Q1. Solve the following equations using Gauss Seidel Method:

i) $27 x_1 + 6 x_2 - x_3 = 85, \quad 6 x_1 + 15 x_2 + 2 x_3 = 72, \quad x_1 + x_2 + 54 x_3 = 110$

ii) $5 x_1 + 2 x_2 + x_3 = 10, \quad 3 x_1 + 7 x_2 + 4 x_3 = 21, \quad x_1 + x_2 + 9 x_3 = 12$

iii) $10 x_1 - 2 x_2 + x_3 = 12, \quad x_1 + 9 x_2 - x_3 = 10, \quad 2 x_1 - x_2 + 11 x_3 = 20$

Solution:

i)

```
n = 3; (* number of variables *)
a = {{27, 6, -1}, {6, 15, 2}, {1, 1, 54}};
MatrixForm[a]
x = {0, 0, 0} (* initial value of x⁰1,x⁰2,x⁰3 *)

b = {85, 72, 110}

For[k = 1, k ≤ 5, k++,
 For[i = 1, i ≤ n, i++,
  y [[i]] = (b [[i]] - Sum[a [[i, j]] * y [[j]] , {j, 1, i - 1}] - Sum[a [[i, j]] * x [[j]] , {j, i + 1, n}]) /
    a [[i, i]];
 For[m = 1, m ≤ n, m++, x [[m]] = N[y [[m]] ]]]
For[p = 1, p ≤ n, p++, Print["x[", p, "]=", x [[p]] ]]
```

$$\begin{pmatrix} 27 & 6 & -1 \\ 6 & 15 & 2 \\ 1 & 1 & 54 \end{pmatrix}$$

```
{0, 0, 0}

{85, 72, 110}

x[1]=2.42548
x[2]=3.57302
x[3]=1.92595
```

Solution.
ii)

```
n = 3; (* number of variables *)
a = {{5, 2, 1}, {3, 7, 4}, {1, 1, 9}};
MatrixForm[a]
x = {0, 0, 0} (* initial value of x01,x02,x03 *)

b = {10, 21, 12}

For[k = 1, k ≤ 5, k++,
 For[i = 1, i ≤ n, i++,
  y ⟦i⟧ = (b ⟦i⟧ - Sum[a ⟦i, j⟧ * y ⟦j⟧ , {j, 1, i - 1}] - Sum[a ⟦i, j⟧ * x ⟦j⟧ , {j, i + 1, n}]) /
    a ⟦i, i⟧ ];
 For[m = 1, m ≤ n, m++, x ⟦m⟧ = N[y ⟦m⟧ ]]]
For[p = 1, p ≤ n, p++, Print["x[", p, "]=", x ⟦p⟧ ]]
```

$$\begin{pmatrix} 5 & 2 & 1 \\ 3 & 7 & 4 \\ 1 & 1 & 9 \end{pmatrix}$$

```
{0, 0, 0}

{10, 21, 12}

x[1]=0.999005

x[2]=2.00009

x[3]=1.0001
```

Solution:
iii)

```
n = 3; (* number of variables *)
a = {{10, -2, 1}, {1, 9, -1}, {2, -1, 11}};
MatrixForm[a]
x = {0, 0, 0} (* initial value of x⁰1,x⁰2,x⁰3 *)

b = {12, 10, 20}

For[k = 1, k ≤ 5, k++,
 For[i = 1, i ≤ n, i++,
  y [[i]] = (b [[i]] - Sum[a [[i, j]] * y [[j]] , {j, 1, i - 1}] - Sum[a [[i, j]] * x [[j]] , {j, i + 1, n}]) /
    a [[i, i]] ];
 For[m = 1, m ≤ n, m++, x [[m]] = N[y [[m]] ]]]
For[p = 1, p ≤ n, p++, Print["x[", p, "]=", x [[p]] ]]
```

$$\begin{pmatrix} 10 & -2 & 1 \\ 1 & 9 & -1 \\ 2 & -1 & 11 \end{pmatrix}$$

```
{0, 0, 0}

{12, 10, 20}

x[1]=1.26241

x[2]=1.15906

x[3]=1.69402
```

Practical 6

Newton Interpolation

Q1. For the data (3,293), (5,508) , (6,585), (9,764). Using Newton interpolation, find interpolation polynomial p(x) and also find p(5.6).

```
points = {{3, 293}, {5, 508}, {6, 585}, {9, 764}}
n  =  Length[points]
y = points [[All, 1]]
f = points [[All, 2]]
dd[k_] := Sum[(f [[i]]  / Product[If[Equal[j, i], 1, (y [[i]] - y [[j]] )], {j, 1, k}]), {i, 1, k}];
p[x_] = Sum[(dd[i] * Product[If[i ≤ j, 1, x - y [[j]] ], {j, 1, i - 1}]), {i, 1, n}]
Simplify[p[x]]
Evaluate[p[5.6]]
```

```
{{3, 293}, {5, 508}, {6, 585}, {9, 764}}

4

{3, 5, 6, 9}

{293, 508, 585, 764}
```

$$293 + \frac{215}{2}(-3 + x) - \frac{61}{6}(-5 + x)(-3 + x) + \frac{35}{36}(-6 + x)(-5 + x)(-3 + x)$$

$$\frac{1}{36}(-9702 + 9003\, x - 856\, x^2 + 35\, x^3)$$

```
556.033
```

Lagrange Interpolation

Q1. For the data (-1, 5), (0, 1), (1, 1), (2,11) find the interpolation polynomial P(x) and find P(1.5) using langrange interpolation.
Solution:

```
xi = {-1, 0, 1, 2}
fi = {5, 1, 1, 11}
n = Length[xi]
For · k = 1, k ≤ n, k++,
```

$$L_k[x\_] = \left(\prod_{j=1}^{k-1} (x - xi[[j]]) / (xi[[k]] - xi[[j]])\right) * \left(\prod_{j=k+1}^{n} (x - xi[[j]]) / (xi[[k]] - xi[[j]])\right) \cdot$$

$$p[x\_] = \sum_{k=1}^{n} L_k[x] * fi[[k]] ;$$

```
Print["Lagrange Interpolation Polynomial P[x] = ", p[x]]
Print["Simplify Lagrange Interpolation Polynomial P[x] = ", Simplify[p[x]]]
Print["Value of P[x] at x = 1.5 is: ", p[1.5]]
```

```
{-1, 0, 1, 2}

{5, 1, 1, 11}

4

Lagrange Interpolation Polynomial P[x] =
```

$$-\frac{5}{6}(1-x)(2-x)x + \frac{1}{2}(1-x)(2-x)(1+x) + \frac{1}{2}(2-x)x(1+x) + \frac{11}{6}(-1+x)x(1+x)$$

Simplify Lagrange Interpolation Polynomial P[x] = $1 - 3x + 2x^2 + x^3$

Value of P[x] at x = 1.5 is: 4.375

Practical 7

Trapezoidal Rule

Q1. Use trapezoidal rule to integrate $f(x) = e^{x^2}$ from 0 to 2 for n = 10.
Solution:

```
f[x_] = Exp[x^2];
a = 0;
b = 2;
n = 10;
h = (b - a) / n;
app = N[(h / 2) * (f[a] + 2 * Sum[f[i], {i, a + h, b - h, h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 2}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

```
The exact value of f[x] is: 16.4526

The approximate value of f[x] is: 17.1702

The error is: 0.717582
```

Q2. Using the trapezoidal rule, evaluate the following:

i) $\int_0^1 \frac{x^2}{1+x^3} \cdot$ for n = 5.

ii) $\int_0^6 \frac{1}{1+x^2} \cdot x$ for n = 6.

iii) $\int_0^{0.6} \cdot^{-x^2} \cdot x$ for n = 6.

Solution:

i) $\int_0^1 \frac{x^2}{1+x^3} \cdot$ for n = 5.

```
f[x_] = x^2 / (1 + x^3);
a = 0;
b = 1;
n = 5;
h = (b - a) / n;
app = N[(h / 2) * (f[a] + 2 * Sum[f[i], {i, a + h, b - h, h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 1}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

```
The exact value of f[x] is: 0.231049

The approximate value of f[x] is: 0.231878

The error is: 0.000829247
```

ii) $\int_0^6 \frac{1}{1+x^2} \cdot x$ for n = 6.

```
f[x_] = 1 / (1 + x^2);
a = 0;
b = 6;
n = 6;
h = (b - a) / n;
app = N[(h / 2) * (f[a] + 2 * Sum[f[i], {i, a + h, b - h, h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 6}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

```
The exact value of f[x] is: 1.40565

The approximate value of f[x] is: 1.4108

The error is: 0.00515093
```

iii) $\int_0^{0.6} e^{-x^2} \, dx$ for n = 6.

```
f[x_] = Exp[(-x)^2];
a = 0;
b = 0.6;
n = 6;
h = (b - a) / n;
app = N[(h / 2) * (f[a] + 2 * Sum[f[i], {i, a + h, b - h, h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 0.6}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

```
The exact value of f[x] is: 0.680492

The approximate value of f[x] is: 0.681924

The error is: 0.00143156
```

Simpson’s 1/3 Rule

Q1. Using Simpson’s 1/3 Rule integrate $f(x) = \frac{x^2}{1+x^3}$ from 0 to 1 for n = 4.

Solution:

```
f[x_] = x^2 / (1 + x^3);
a = 0;
b = 1;
n = 4;
h = (b - a) / n;
app = N[(h / 3) * (f[a] + 4 * Sum[f[i], {i, a + h, b - h, 2 h}] +
      2 * Sum[f[i], {i, a + 2 h, b - 2 h, 2 h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 1}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

```
The exact value of f[x] is: 0.231049

The approximate value of f[x] is: 0.231085

The error is: 0.0000355959
```

Q2. Using simpson’s 1/3 rule, evaluate the following:

i) $\int_0^1 \frac{1}{x^2+6x+10} \, dx$ for n = 10.

ii) $\int_0^6 \frac{1}{1+x^2} \, dx$ for n = 6.

iii) $\int_{0}^{0.6} e^{-x^2} \, dx$ for n = 6.

Solution:

i) $\int_{0}^{1} \frac{1}{x^2+6x+10} \, dx$ for n = 10.

```
f[x_] = 1 / (x^2 + 6 x + 10);
a = 0;
b = 1;
n = 10;
h = (b - a) / n;
app = N[(h / 3) * (f[a] + 4 * Sum[f[i], {i, a + h, b - h, 2 h}] +
       2 * Sum[f[i], {i, a + 2 h, b - 2 h, 2 h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 1}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

The exact value of f[x] is: 0.0767719

The approximate value of f[x] is: 0.0767719

The error is: 2.23635×10^{-8}

ii) $\int_{0}^{6} \frac{1}{1+x^2} \, dx$ for n = 6.

```
f[x_] = 1 / (1 + x^2);
a = 0;
b = 6;
n = 6;
h = (b - a) / n;
app = N[(h / 3) * (f[a] + 4 * Sum[f[i], {i, a + h, b - h, 2 h}] +
       2 * Sum[f[i], {i, a + 2 h, b - 2 h, 2 h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 6}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

The exact value of f[x] is: 1.40565

The approximate value of f[x] is: 1.36617

The error is: 0.0394742

iii) $\int_0^{0.6} e^{-x^2}\, dx$ for n = 6.

```
f[x_] = Exp[(-x)^2];
a = 0;
b = 0.6;
n = 6;
h = (b - a) / n;
app = N[(h / 3) * (f[a] + 4 * Sum[f[i], {i, a + h, b - h, 2 h}] +
      2 * Sum[f[i], {i, a + 2 h, b - 2 h, 2 h}] + f[b])];
Ex = N[Integrate[f[x], {x, 0, 0.6}]];
Print["The exact value of f[x] is: ", Ex]
Print["The approximate value of f[x] is: ", app]
Print["The error is: ", Abs[Ex - app]]
```

```
The exact value of f[x] is: 0.680492

The approximate value of f[x] is: 0.680499

The error is: 7.00825×10^-6
```

Practical 8 (Euler's Method)

Q1. Using Euler's method find an approximate value of y corresponding x = 1, for the first order ODE f(x,y) = x + y and y = 1 at x = 0.

Solution:

```
f[x_, y_] := x + y;
a = 0;
b = 1;
n = 10;
y[0] = 1;
h = (b - a) / n;
For[i = 0, i ≤ n, i++, x[i] = a + h * i;
 y[i + 1] = y[i] + h * f[x[i], y[i]];
 Print["Value at x[", i, "]=", x[i], " is ", N[y[i]]]]
```

Value at x[0]=0 is 1.

Value at x[1]= $\frac{1}{10}$ is 1.1

Value at x[2]= $\frac{1}{5}$ is 1.22

Value at x[3]= $\frac{3}{10}$ is 1.362

Value at x[4]= $\frac{2}{5}$ is 1.5282

Value at x[5]= $\frac{1}{2}$ is 1.72102

Value at x[6]= $\frac{3}{5}$ is 1.94312

Value at x[7]= $\frac{7}{10}$ is 2.19743

Value at x[8]= $\frac{4}{5}$ is 2.48718

Value at x[9]= $\frac{9}{10}$ is 2.8159

Value at x[10]=1 is 3.18748

Q2.

i) Using Euler’s method find an approximate value of y corresponding x = 0.1, for the first order ODE f(x,y) = (y-x)/(x+y) and y = 1 at x = 0.
ii) Using Euler’s method find an approximate value of y corresponding x = 0.4, for the first order ODE f(x,y) = y + e^x and y = 0 at x = 0.
iii) Using Euler’s method find an approximate value of y corresponding x = 1.2, for the first order ODE f(x,y) =log(x+y) and y = 2 at x = 0.

Solution:
i) Using Euler’s method find an approximate value of y corresponding x = 0.1, for the first order ODE f(x,y) = (y-x)/(x+y) and y = 1 at x = 0.

```
f[x_, y_] := (y - x) / (x + y);
a = 0;
b = 0.1;
n = 10;
y[0] = 1;
h = (b - a) / n;
For[i = 0, i ≤ n, i++, x[i] = a + h * i;
 y[i + 1] = y[i] + h * f[x[i], y[i]];
 Print["Value at x[", i, "]=", x[i], " is ", N[y[i]]]]
```

```
Value at x[0]=0. is 1.
Value at x[1]=0.01 is 1.01
Value at x[2]=0.02 is 1.0198
Value at x[3]=0.03 is 1.02942
Value at x[4]=0.04 is 1.03885
Value at x[5]=0.05 is 1.04811
Value at x[6]=0.06 is 1.0572
Value at x[7]=0.07 is 1.06613
Value at x[8]=0.08 is 1.07489
Value at x[9]=0.09 is 1.08351
Value at x[10]=0.1 is 1.09198
```

ii) Using Euler’s method find an approximate value of y corresponding x = 0.4, for the first order ODE f(x,y) = y + e^x and y = 0 at x = 0.

```
f[x_, y_] := y + ·^x;
a = 0;
b = 0.4;
n = 10;
y[0] = 0;
h = (b - a) / n;
For[i = 0, i ≤ n, i++, x[i] = a + h * i;
 y[i + 1] = y[i] + h * f[x[i], y[i]];
 Print["Value at x[", i, "]=", x[i], " is ", N[y[i]]]]
```

```
Value at x[0]=0. is 0.
Value at x[1]=0.04 is 0.04
Value at x[2]=0.08 is 0.0832324
Value at x[3]=0.12 is 0.129893
Value at x[4]=0.16 is 0.180189
Value at x[5]=0.2 is 0.234337
Value at x[6]=0.24 is 0.292566
Value at x[7]=0.28 is 0.355119
Value at x[8]=0.32 is 0.422249
Value at x[9]=0.36 is 0.494224
Value at x[10]=0.4 is 0.571326
```

iii) Using Euler’s method find an approximate value of y corresponding x = 1.2, for the first order ODE f(x,y) =log(x+y) and y = 2 at x = 0.

```
f[x_, y_] := Log[x + y];
a = 0;
b = 1.2;
n = 10;
y[0] = 2;
h = (b - a) / n;
For[i = 0, i ≤ n, i++, x[i] = a + h * i;
 y[i + 1] = y[i] + h * f[x[i], y[i]];
 Print["Value at x[", i, "]=", x[i], " is ", N[y[i]]]]
```

```
Value at x[0]=0. is 2.
Value at x[1]=0.12 is 2.08318
Value at x[2]=0.24 is 2.17797
Value at x[3]=0.36 is 2.28392
Value at x[4]=0.48 is 2.40059
Value at x[5]=0.6 is 2.52755
Value at x[6]=0.72 is 2.66438
Value at x[7]=0.84 is 2.81068
Value at x[8]=0.96 is 2.96607
Value at x[9]=1.08 is 3.13018
Value at x[10]=1.2 is 3.30268
```