MTH 4300, Lecture 8

Towers of Hanoi;
Pointers;
Pointers and Arrays
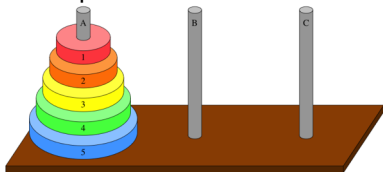
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March 2, 2025

This is one of the most famous recursive problems. There are n disks on the left pole, stacked in order of decreasing size. The goal is to move this stack to the right pole.

The rules: you can only move one disk at a time, and you can never place a disk on to of a smaller disk.

https://www.mathsisfun.com/games/towerofhanoi.html

How do we solve this? Here's a strategy:

- Move the smallest $n - 1$ disks temporarily to the pole we're **not** targeting, leaving the biggest disk uncovered.
- Move the biggest disk from the original pole to the pole we **are** targeting.
- Move the smallest $n - 1$ disks from the pole they're on to the target pole.

But how do we move the smallest $n - 1$ disks? That's what recursion is for. (What would the base case be? The instructions only make sense when $n > 1$, so $n = 1$ ought to be the base case.)

Let's write a program that gives explicit instructions for the disk moves. Every instruction will be of the form

   Move disk d from pole x to pole y.

L8x1_hanoi.cpp

Towers of Hanoi

One of the trickiest parts of this problem is understanding what the signature line of the function should be. I have written it as:

`void towers(int num_d, char f, char t, char aux)`

where `num_d` represents the number of disks we're trying to move; `f` represents the "from" pole (A, B or C) which we're moving the disks from, and `t` represents the pole which we're moving disks to. And `aux` represents the third pole, which can be used for temporary storage.
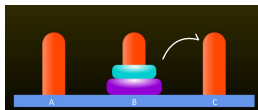
For example, for this picture, I would call

`towers(2, 'B', 'C', 'A')`

and it would print

```
Move disk 1 from pole B to pole A.
Move disk 2 from pole B to pole C.
Move disk 1 from pole A to pole C.
```

2. Pointers

One of the reasons for using C++ is that it allows you to work with memory in a fairly direct manner, so that you can design data structures that are complex, but can still be manipulated efficiently. To do this, we need to be able to work with addresses.

A pointer is a variable that is meant to hold the *address* of a variable of a particular type. For example, suppose you wanted a variable p that contains the address of an int. Then you would declare

        int* p;

The * means you're not declaring an int variable, but rather a variable which holds the address of an int (a "pointer to an int"). You could then store the address of int x to that variable by writing

        p = &x;

Note that the declaration of the pointer variable can also be written as
        int *p;
with the asterisk attached to the variable rather than the data type. This is more common, in fact, because it allows declarations like
        int *p, q, r, *s;
where q and r are regular int variables, and p and s are pointers to ints.

Pointers

In addition to playing a role in declaration, the ∗ symbol also is an operator: the <u>dereference</u> operator.

> *If you have a pointer variable p whose value is an address, then ∗p is the value stored at that address.*

In other words, ∗p means: look at the address in p; go to that address; and read the value that is there.

L8x2_pointer.cpp

When we declare int x = 937357 and int ∗p, here's what's getting created in the stack:

Then, when we set p = &x, here's how things get updated:

Finally, when we cout << ∗p, we first read the content of p – which is an address, 0x044 – then go to that address and read the contents *there*: 937357.

Pointer variables can of course be used as the left side or as the right side of an assignment. E.g., if p and q are both pointers of the same type, then

       p = q;

would cause the address stored in q to be copied into p.

Likewise, *p and *q can be used on both sides of an assignment:

       *p = *q;

would take the value stored at address *q, and assign it to the memory space whose address is *p.

L8x3_value.cpp

(We sometimes say that *p is an *l-value* since it can be used on the left side of an assignment.)

3. Pointers and Arrays

Whenever the name of an array is referenced without referring to a particular element, it is treated as a pointer. This phenomenon is known as the array *decaying* to a pointer.

For example: suppose I declare int arr[5]. As we discussed,

cout << arr

WON'T print out the entire array – instead it will print out the address of the first element. (char arrays are an exception to this behavior.)

Recall that arrays are stored in memory in consecutive addresses:

Then, arr would be 0x00. Also, if you add an integer n to an array, you obtain the address of the entry n positions down the array. So, for example, arr + 1 would be the address of arr[1], which would be 0x04; arr + 2 would be 0x08, etc. In fact, arr[i] is just a shorthand way of writing *(arr + i). This is where 0-based indexing comes from!

L8x4_array.cpp

Note that attempting to access out-of-bounds memory is undefined behavior. One possibility is a *segmentation fault*, where a program attempts to access a memory location outside of that reserved for the program.

Pointers and Arrays

The following example illustrates a couple of points.

- If a formal parameter of a function is declared as type T * for any data type T, then that function can be called with arrays of type T as parameters.
- Pointers are frequently used to traverse an array. For example, within the code

```
int arr[3] = {14, 25, 36};
int *end = arr + 3;
for(int *item = arr; item != end; ++item){ ...
```

the variable item would hold the address of each element of arr in succession.

L8x5_function.cpp