BIL 404/504
Homework 4
Due: The Day of the Final Exam

(To be Submitted on Paper in Your Final Exam Room)

Q1. (200pts) Consider an oligopolistic industry where n firms produce a single homogenous good subject to the industry (inverse) demand curve $P(Q) = \alpha - Q$. Above, Q is the industry total output, i.e., $Q = \sum_{i=1}^{n} q_i$ where q_i is the output of firm $i = 1, 2, \ldots, n$, $P(Q)$ is the price of the good, and $\alpha > 0$ is a fixed parameter. The cost function facing firm $i = 1, 2, \ldots, n$ is $C_i(q_i) = cq_i$, where $c \in (0, \alpha)$ is a fixed parameter.

(a) (10pts) For the oligopolistic industry described above, show that the Cournot (Nash) equilibrium output of each firm is equal to $(\alpha - c)/(n + 1)$.

(b) (190pts) You are asked to write a code in any computer programming language (or in any package like MATLAB) to explore whether the oligopolistic firms can learn the equilibrium asked in part (a) using a genetic algorithm (GA) that we studied in class referring to "*Arifovic, Jasmina (1994) Genetic algorithm learning and cobweb model. Journal of Economic Dynamics and Control, 18, 3–28*".

Your GA must contain a single population of n chromosomes at each iteration and use the four genetic operators, involving recombination, crossover, mutation, and election. To run your GA, you have to use the following specifications:

- $n = 20$
- $\alpha = 1023$
- $c = 0$
- $l = 10$ (the length of the binary strings)
- $p_{cross} = 0.500 + (YZ) \times 10^{-3}$ (the probability of crossover) (This is the benchmark level for your computations!)
- $p_{mut} = 0.00300 + (YZ) \times 10^{-5}$ (the probability of mutation) (This is the benchmark level for your computations!)

- YZ = the last two digits of your student ID number (Z is the last digit)
- $T = 1000$ (The number of iterations)

Outputs to Be Reported:

(110pts) (i) Your codes.

(40pts) (ii) Using the benchmark levels of p_{cross} and p_{mut} in your computations, prepare two graphs that respectively plot for each iteration $t = 1, 2, \ldots, 1000$ the industry output Q_t and the population variance of the individual outputs $\sigma_t^q = Var(q_{1,t}, q_{2,t}, \ldots, q_{n,t})$ associated with n chromosomes in the set A_t. Comment whether you observe convergence to the Cournot (Nash) equilibrium. If you do not, address possible reasons.

(20pts) (iii) Now, you are asked to make your computations after increasing the probability of crossover by 0.100 over the benchmark level (while keeping the probability of mutation p_{mut} at the benchmark level). Prepare two graphs that respectively plot for each iteration $t = 1, 2, \ldots, 1000$ the industry output Q_t and the population variance of the individual outputs $\sigma_t^q = Var(q_{1,t}, q_{2,t}, \ldots, q_{n,t})$ associated with n chromosomes in the set A_t. Comment on any differences (about the existence or speed of convergence) with respect to your findings in part (ii).

(20pts) (iv) Now, you are asked to make your computations after increasing the probability of mutation (p_{mut}) by 0.00100 over the benchmark level (while keeping the probability of crossover p_{cross} at the benchmark level). Prepare two graphs that respectively plot for each iteration $t = 1, 2, \ldots, 1000$ the industry output Q_t and the population variance of the individual outputs $\sigma_t^q = Var(q_{1,t}, q_{2,t}, \ldots, q_{n,t})$ associated with n chromosomes in the set A_t. Comment on any differences (about the existence or speed of convergence) with respect to your findings in part (ii).