UNIVERSIDAD AUTÓNOMA DEL ESTADO DE MÉXICO
FACULTAD DE CIENCIAS
DEPARTAMENTO DE MATEMÁTICAS
Cálculo Diferencial Vectorial
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Tarea: Polinomio de Taylor

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En cada uno de los ejercicios, determinar la fórmula de Taylor de segundo orden para la función dada alrededor del punto x_0, y_0.

1. $f(x,y) = (x+y)^2$, donde $x_0 = 0$, $y_0 = 0$.

   Solución.

   $f(0,0) = (0+0)^2 = 0$

   $\dfrac{\partial f}{\partial x}(0,0) = 2(x+y)|_{(0,0)} = 0$

   $\dfrac{\partial f}{\partial y}(0,0) = 2(x+y)|_{(0,0)} = 0$

   $\dfrac{\partial^2 f}{\partial x^2}(0,0) = 2$

   $\dfrac{\partial^2 f}{\partial y \partial x}(0,0) = 2$

   $\dfrac{\partial^2 f}{\partial y^2}(0,0) = 2$

   Así, el Polinomio de Taylor es

$$\begin{aligned} f((x_0,y_0)+(h_1,h_2)) &= f(h_1,h_2) \\ &= f(0,0) + h_1 \frac{\partial f}{\partial x}(0,0) + h_2 \frac{\partial f}{\partial y}(0,0) + \frac{1}{2}\left(h_1^2 \frac{\partial^2 f}{\partial x^2}(0,0) + \right. \\ &\quad \left. h_1h_2 \frac{\partial^2 f}{\partial y \partial x}(0,0) + h_2h_1 \frac{\partial^2 f}{\partial x \partial y}(0,0) + h_2^2 \frac{\partial^2 f}{\partial y^2}(0,0)\right) \\ &= \frac{1}{2}\left(2h_1^2 + 2h_1h_2 + 2h_2h_1 + 2h_2^2\right) \\ &= h_1^2 + 2h_1h_2 + h_2^2 \end{aligned}$$

   Notemos que $f(x,y) = (x+y)^2 = x^2 + 2xy + y^2$. Por lo tanto, es igual a su Polinomio de Taylor.

2. $f(x,y) = \dfrac{1}{x^2+y^2+1}$, donde $x_0 = 0$, $y_0 = 0$.

   Solución.

   $f(0,0) = \dfrac{1}{0^2+0^2+1} = 1$

$$\frac{\partial f}{\partial x}(0,0) = -\frac{2x}{(x^2+y^2+1)^2}\bigg|_{(0,0)} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = -\frac{2y}{(x^2+y^2+1)^2}\bigg|_{(0,0)} = 0$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial x^2}(0,0) &= \frac{(x^2+y^2+1)^2(-2)+(2x)(2x)(2(x^2+y^2+1))}{(x^2+y^2+1)^4}\bigg|_{(0,0)} \\
&= \frac{-2(x^2+y^2+1)^2+8x^2(x^2+y^2+1)}{(x^2+y^2+1)^4}\bigg|_{(0,0)} \\
&= \frac{-2(x^2+y^2+1)+8x^2}{(x^2+y^2+1)^3}\bigg|_{(0,0)} \\
&= \frac{6x^2-2y^2-2}{(x^2+y^2+1)^3}\bigg|_{(0,0)} \\
&= -2
\end{aligned}$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \frac{8xy(x^2+y^2+1)}{(x^2+y^2+1)^4}\bigg|_{(0,0)} = 0$$

$$\begin{aligned}
\frac{\partial^2 f}{\partial y^2}(0,0) &= -\frac{(x^2+y^2+1)^2(-2)+(2y)(2y)(2(x^2+y^2+1))}{(x^2+y^2+1)^4}\bigg|_{(0,0)} \\
&= -\frac{-2(x^2+y^2+1)^2+8y^2(x^2+y^2+1)}{(x^2+y^2+1)^4}\bigg|_{(0,0)} \\
&= -\frac{-2(x^2+y^2+1)+8y^2}{(x^2+y^2+1)^3}\bigg|_{(0,0)} \\
&= -\frac{-2x^2+6y^2-2}{(x^2+y^2+1)^3}\bigg|_{(0,0)} \\
&= -2
\end{aligned}$$

Así, el Polinomio de Taylor es

$$\begin{aligned}
f((x_0,y_0)+(h_1,h_2)) &= f(h_1,h_2) \\
&= f(0,0) + h_1\frac{\partial f}{\partial x}(0,0) + h_2\frac{\partial f}{\partial y}(0,0) + \frac{1}{2}\left(h_1^2\frac{\partial^2 f}{\partial x^2}(0,0) + \right. \\
&\quad \left. h_1h_2\frac{\partial^2 f}{\partial y\partial x}(0,0) + h_2h_1\frac{\partial^2 f}{\partial x\partial y}(0,0) + h_2^2\frac{\partial^2 f}{\partial y^2}(0,0)\right) \\
&= 1 + \frac{1}{2}\left(-2h_1^2 - 2h_2^2\right) \\
&= 1 - h_1^2 - h_2^2
\end{aligned}$$

3. $f(x,y) = e^{x+y}$, donde $x_0 = 0$, $y_0 = 0$.

Solución.

$f(0,0) = e^{0+0} = 1$

$\dfrac{\partial f}{\partial x}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial f}{\partial y}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial^2 f}{\partial x^2}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial^2 f}{\partial y \partial x}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial^2 f}{\partial y^2}(0,0) = e^{x+y}|_{(0,0)} = 1$

Así, el Polinomio de Taylor es

$$\begin{aligned} f((x_0,y_0)+(h_1,h_2)) &= f(h_1,h_2) \\ &= f(0,0) + h_1 \frac{\partial f}{\partial x}(0,0) + h_2 \frac{\partial f}{\partial y}(0,0) + \frac{1}{2}\left(h_1^2 \frac{\partial^2 f}{\partial x^2}(0,0) + \right. \\ &\left. h_1 h_2 \frac{\partial^2 f}{\partial y \partial x}(0,0) + h_2 h_1 \frac{\partial^2 f}{\partial x \partial y}(0,0) + h_2^2 \frac{\partial^2 f}{\partial y^2}(0,0)\right) \\ &= 1 + h_1 + h_2 + \frac{1}{2}\left(h_1^2 + 2h_1h_2 + h_2^2\right) \\ &= 1 + h_1 + h_2 + h_1h_2 + \frac{h_1^2 + h_2^2}{2} \end{aligned}$$

4. $f(x,y) = e^{-x^2-y^2}\cos(xy)$, donde $x_0 = 0$, $y_0 = 0$.

Solución.

$f(0,0) = e^{-0^2-0^2}\cos(0 \cdot 0) = 1$

$\dfrac{\partial f}{\partial x}(0,0) = -2xe^{-x^2-y^2}\cos(xy) + e^{-x^2-y^2}\cos(xy)\Big|_{(0,0)} = 1$

$\dfrac{\partial f}{\partial y}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial^2 f}{\partial x^2}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial^2 f}{\partial y \partial x}(0,0) = e^{x+y}|_{(0,0)} = 1$

$\dfrac{\partial^2 f}{\partial y^2}(0,0) = e^{x+y}|_{(0,0)} = 1$

Así, el Polinomio de Taylor es

$$\begin{aligned}
f((x_0, y_0) + (h_1, h_2)) &= f(h_1, h_2) \\
&= f(0,0) + h_1 \frac{\partial f}{\partial x}(0,0) + h_2 \frac{\partial f}{\partial y}(0,0) + \frac{1}{2}\left(h_1^2 \frac{\partial^2 f}{\partial x^2}(0,0) + \right. \\
&\qquad \left. h_1 h_2 \frac{\partial^2 f}{\partial y \partial x}(0,0) + h_2 h_1 \frac{\partial^2 f}{\partial x \partial y}(0,0) + h_2^2 \frac{\partial^2 f}{\partial y^2}(0,0)\right) \\
&= 1 + h_1 + h_2 + \frac{1}{2}\left(h_1^2 + 2h_1h_2 + h_2^2\right) \\
&= 1 + h_1 + h_2 + h_1h_2 + \frac{h_1^2 + h_2^2}{2}
\end{aligned}$$

5. $f(x, y) = \operatorname{sen}(xy) + \cos(xy)$, donde $x_0 = 0$, $y_0 = 0$.

   Solución.

6. $f(x, y) = e^{(x-1)^2} \cos(y)$, donde $x_0 = 1$, $y_0 = 0$.

   Solución.