13. $y'' - 2y' + 2y = 0$; $y(0) = 0$, $y'(0) = 1$

Solución.

$\mathscr{L}\{y'' - 2y' + 2y\} = \mathscr{L}\{0\}$

$\Longrightarrow s^2Y(s) - sy(0) - y'(0) - 2sY(s) - 2y(0) + 2Y(s) = 0$

$\Longrightarrow (s^2 - 2s + 2)Y(s) - 1 = 0$

$\Longrightarrow Y(s) = \dfrac{1}{s^2 - 2s + 2} = \dfrac{1}{(s-1)^2 + 1}$

Aplicando Transformada Inversa de Laplace

$\mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{\dfrac{1}{(s-1)^2 + 1}\right\}$

$\therefore y(t) = e^t \operatorname{sen}(t)$.

21. $y'' - 2y' + 2y = \cos(t)$; $y(0) = 1, y'(0) = 0$

Solución.

$\mathscr{L}\{y'' - 2y' + 2y\} = \mathscr{L}\{\cos(t)\}$

$\Longrightarrow s^2Y(s) - sy(0) - y'(0) - 2sY(s) + 2y(0) + 2Y(s) = \dfrac{s}{s^2+1}$

$\Longrightarrow (s^2 - 2s + 2)Y(s) - s + 2 = \dfrac{s}{s^2+1}$

$\Longrightarrow (s^2 - 2s + 2)Y(s) = \dfrac{s + (s-2)(s^2+1)}{s^2+1} = \dfrac{s^3 - 2s^2 + 2s - 2}{s^2+1}$

$\Longrightarrow Y(s) = \dfrac{s^3 - 2s^2 + 2s - 2}{(s^2+1)(s^2 - 2s + 2)}$

Sea $\dfrac{s^3 - 2s^2 + 2s - 2}{(s^2+1)(s^2 - 2s + 2)} = \dfrac{As + B}{s^2+1} + \dfrac{Cs + D}{s^2 - 2s + 2}$

$\Longrightarrow s^3 - 2s^2 + 2s - 2 = (As + B)(s^2 - 2s + 2) + (Cs + D)(s^2 + 1)$

$\Longrightarrow s^3 - 2s^2 + 2s - 2 = s^3(A + C) + s^2(-2A + B + D) + s(2A - 2B + C) + 2B + D$

$$\begin{aligned}
\Longrightarrow & A + C = 1 && (1)\\
& -2A + B + D = -2 && (2)\\
& 2A - 2B + C = 2 && (3)\\
& 2B + D = -2 && (4)
\end{aligned}$$

De (1) se tiene que $A = 1 - C$ y de (4) que $D = -2 - 2B$. Sustituyendo en (2) y en (3):

$$\begin{aligned}
& -2(1 - C) + B - 2 - 2B = -B + 2C - 4 = -2 \Longrightarrow -B + 2C = 2 && (5)\\
& 2(1 - C) - 2B + C = -2B - C + 2 = 2 \Longrightarrow -2B - C = 0 && (6)
\end{aligned}$$

Sumando (5) y 2·(6): $-5B = 2 \Longrightarrow B = -\dfrac{2}{5}$. Así, $D = -2 - 2\left(-\dfrac{2}{5}\right) = -\dfrac{6}{5}$.
Sustituyendo B en (6) se da que $-2\left(-\dfrac{2}{5}\right) - C = 0 \Longrightarrow C = \dfrac{4}{5}$. Sustituyendo esto en (1): $A + \dfrac{4}{5} = 1 \Longrightarrow A = \dfrac{1}{5}$.

De esta forma, $Y(s) = \dfrac{\frac{1}{5}s - \frac{2}{5}}{s^2+1} + \dfrac{\frac{4}{5}s - \frac{6}{5}}{s^2 - 2s + 2}$

Aplicando Transformada Inversa de Laplace

$$\mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{\frac{\frac{1}{5}s - \frac{2}{5}}{s^2+1} + \frac{\frac{4}{5}s - \frac{6}{5}}{s^2 - 2s + 2}\right\}$$

$$\Longrightarrow y(t) = \mathscr{L}^{-1}\left\{\frac{1}{5}\left(\frac{s}{s^2+1}\right) - \frac{2}{5}\left(\frac{1}{s^2+1}\right) + \frac{4}{5}\left(\frac{s-1}{(s-1)^2+1}\right) - \frac{2}{5}\left(\frac{1}{(s-1)^2+1}\right)\right\}$$

$$\Longrightarrow y(t) = \frac{1}{5}\cos(t) - \frac{2}{5}\operatorname{sen}(t) + \frac{4}{5}e^t\cos(t) - \frac{2}{5}e^t\operatorname{sen}(t)$$

$$\therefore y(t) = \cos(t)\left(\frac{1+4e^t}{5}\right) - 2\operatorname{sen}(t)\left(\frac{1+e^t}{5}\right)$$