31. $\dfrac{\mathrm{d}y}{\mathrm{d}t} - y = 1$; $y(0) = 0$

Solución.

$\mathscr{L}\left\{\dfrac{\mathrm{d}y}{\mathrm{d}t} - y\right\} = \mathscr{L}\{1\}$

$\Longrightarrow sY(s) - y(0) - Y(s) = \dfrac{1}{s}$

$\Longrightarrow Y(s)(s-1) = \dfrac{1}{s}$

$\Longrightarrow Y(s) = \dfrac{1}{s(s-1)}$

Sea $\dfrac{1}{s(s-1)} = \dfrac{A}{s} + \dfrac{B}{s-1}$

$\Longrightarrow 1 = A(s-1) + Bs = s(A+B) - A$

$\Longrightarrow A + B = 0$ y $-A = 1$

$\Longrightarrow A = -1$ y $B = 1$

Así, $Y(s) = -\dfrac{1}{s} + \dfrac{1}{s-1}$.

Aplicando Transformada Inversa de Laplace

$\mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{-\dfrac{1}{s} + \dfrac{1}{s-1}\right\}$

$\therefore y(t) = -1 + e^t$

32. $2\,\dfrac{\mathrm{d}y}{\mathrm{d}t} + y = 0$; $y(0) = -3$

Solución.

$\mathscr{L}\left\{2\,\dfrac{\mathrm{d}y}{\mathrm{d}t} + y\right\} = \mathscr{L}\{0\}$

$\Longrightarrow 2sY(s) - 2y(0) + Y(s) = 0$

$\Longrightarrow Y(s)(2s+1) = -6$

$\Longrightarrow Y(s) = \dfrac{-6}{2s+1}$

Aplicando Transformada Inversa de Laplace

$\mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{\dfrac{-6}{2s+1}\right\} = \mathscr{L}^{-1}\left\{\dfrac{-3}{s+\frac{1}{2}}\right\}$

$\therefore y(t) = -3e^{-\frac{t}{2}}$

33. $y' + 6y = e^{4t}$; $y(0) = 2$

Solución.

$\mathscr{L}\left\{y'+6y\right\}=\mathscr{L}\{e^{4t}\}$

$\Longrightarrow sY(s)-y(0)+6Y(s)=\dfrac{1}{s-4}$

$\Longrightarrow Y(s)(s+6)=\dfrac{1}{s-4}+2$

$\Longrightarrow Y(s)=\dfrac{2s-7}{(s-4)(s+6)}$

Sea $\dfrac{2s-7}{(s-4)(s+6)}=\dfrac{A}{s-4}+\dfrac{B}{s+6}$

$\Longrightarrow 2s-7=A(s+6)+B(s-4)=s(A+B)+6A-4B$

$$\Longrightarrow A+B=2 \tag{1}$$
$$6A-4B=-7 \tag{2}$$

De (1): $A=2-B$. Sustituyendo en (2) se tiene que $-7=6(2-B)-4B=12-6B-4B=12-10B\Longrightarrow B=\dfrac{19}{10}$. Así, $A=2-\dfrac{19}{10}=\dfrac{1}{10}$.

De esta manera, $Y(s)=\dfrac{\frac{1}{10}}{s-4}+\dfrac{\frac{19}{10}}{s+6}$.

Aplicando Transformada Inversa de Laplace

$\mathscr{L}^{-1}\{Y(s)\}=\mathscr{L}^{-1}\left\{\dfrac{\frac{1}{10}}{s-4}+\dfrac{\frac{19}{10}}{s+6}\right\}$

$\therefore y(t)=\dfrac{e^{4t}}{10}+\dfrac{19e^{-6t}}{10}$

34. $y'-y=2\cos(5t)$; $y(0)=0$

Solución.

$\mathscr{L}\left\{y'-y\right\}=\mathscr{L}\{2\cos(5t)\}$

$\Longrightarrow sY(s)-y(0)-Y(s)=\dfrac{2s}{s^2+25}$

$\Longrightarrow Y(s)(s-1)=\dfrac{2s}{s^2+25}$

$\Longrightarrow Y(s)=\dfrac{2s}{(s-1)(s^2+25)}$

Sea $\dfrac{2s}{(s^2+25)}=\dfrac{A}{s-1}+\dfrac{Bs+C}{s^2+25}$

$$\Longrightarrow 2s = A(s^2+25) + (Bs+C)(s-1) = s^2(A+B) + s(-B+C) + 25A - C$$

$$\begin{aligned}\Longrightarrow & A+B=0 && (3)\\ & -B+C=2 && (4)\\ & 25A-C=0 && (5)\end{aligned}$$

Sumando (4) y (5): 25A - B = 2. De (3): $A=-B$. Así, $-25B-B=-26B=2 \Longrightarrow B=-\dfrac{1}{13}$. Luego, $A=\dfrac{1}{13}$ y $C=\dfrac{25}{13}$.

Por lo que, $Y(s)=\dfrac{\frac{1}{13}}{s-1}-\dfrac{\frac{1}{13}s-\frac{25}{13}}{s^2+25}$

Aplicando Transformada Inversa de Laplace

$\mathscr{L}^{-1}\{Y(s)\}=\mathscr{L}^{-1}\left\{\dfrac{\frac{1}{13}}{s-1}-\dfrac{\frac{1}{13}s-\frac{25}{13}}{s^2+25}\right\}$

$\therefore y(t)=\dfrac{e^t}{13}-\dfrac{\cos(5t)}{13}+\dfrac{5\operatorname{sen}(5t)}{13}$