Automated Market Making

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Background

Electronic Exchanges

- What is an Exchange
  - Many different exchanges can take orders
  - Ex) NYSE, NASDAQ

- Sending an Order
  - Routes to any one of the exchanges to find the best price

Order Types

- Market Orders (MO’s) - Aggressive Orders
  - A trader indicates they want to buy or sell a quantity of shares at the best available price and usually results in immediate execution

- Limit Order (LO’s) - Passive Orders
  - A trader sending a LO indicates their desire to buy or sell at a given price and the trade will not result in immediate execution, but must wait until it is either matched with an incoming market order or it is withdrawn.

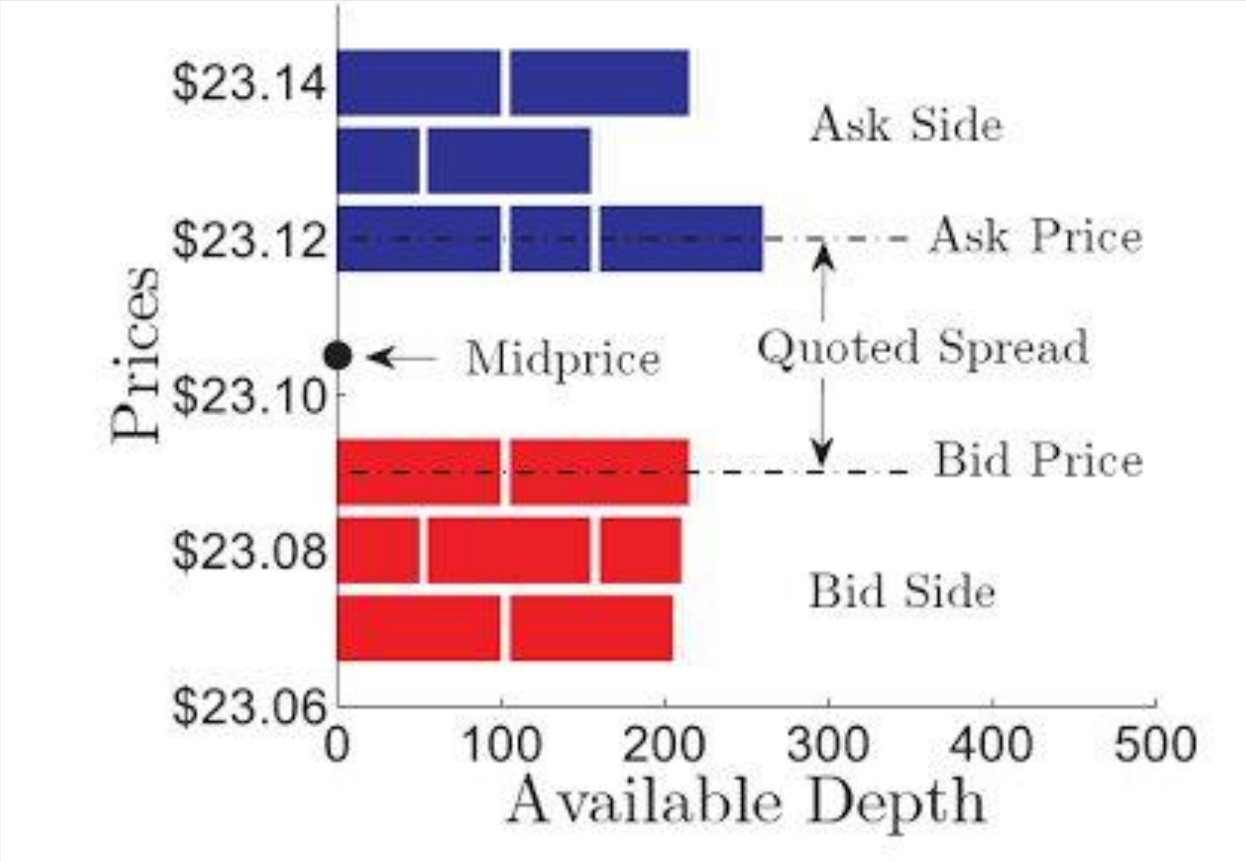
Limit Order Book

- The Limit Order Book
  - Keeps track of incoming and outgoing orders
  - A matching engine uses an algorithm to select orders that will be executed
  - Most markets prioritize MO’s over LO’s.

- MO comes in
  - A buy MO will be matched with current sell LO’s at the lowest price
  - A sell MO will be matched with current buy LO’s at the highest price
  - In either case, if the quantity of the LO at the best price is not enough to fill the MO, it must go deeper in the book and is known as “Walking the Book”

Limit Order Book

- Spread
  - Difference between the best ask and the bid price
- Midprice
  - Average of the bid and ask
- Depth of LO
  - Difference between a limit order and the midprice



Market Makers

- What is a Market Maker
  - Ex) Jane Street, Optiver
  - Providing liquidity in the market
- Goal
  - Choose the optimal depths from the midprice in which to post LO’s both for buy and sell.
- How do they make money
  - buying/selling around this spread
  - Intuitively: Market Makers have the luxury to wait and get good deals by posting LO’s

Simulating a Strategy

Value Function

- The Market Maker seeks a strategy (delta +-) that maximizes cash at the terminal date T.

- Cannot approach this in the traditional way of setting the derivative to zero due to the infinitely many values of delta

- Using the Hamilton Jacobi Bellman equations we find the optimal deltas (depths) for our Buy and Sell LO's at each time step t for every possible inventory, q.

- These optimal depths maximize cash at the terminal date

$$H^{\delta}(t,x,S,q) = \mathrm{E}_{t,x,q,S}[X_T + Q_T^{\delta}(S_T^{\delta} - \alpha Q_T^{\delta}) - \phi \int_t^T (Q_u)^2 du]$$

$$H(t,x,S,q) = \sup_{\delta^{\pm} \in A} H^{\delta}(t,x,S,q)$$

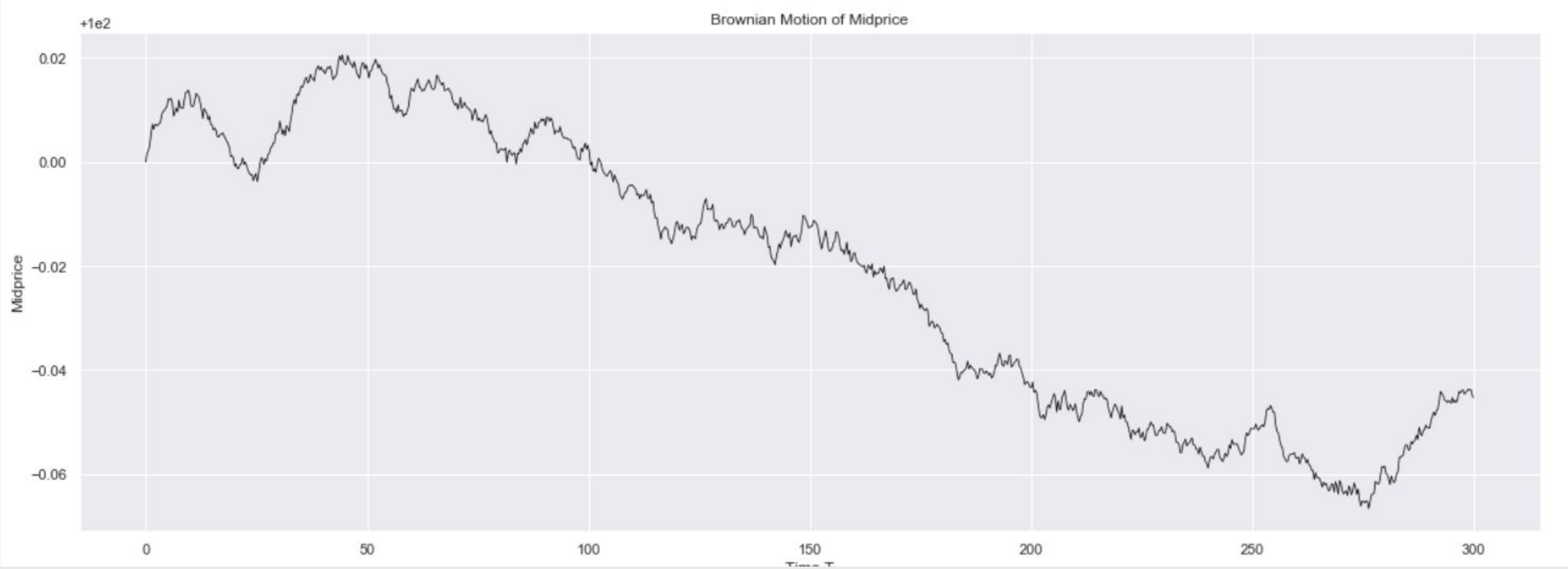
$$\partial H(t,x,S,q) + \sup_{u \in A}[\mathcal{L}_t^u H(t,x,S,q) + F(t,x,S,q)] = 0$$

Modeling the Midprice

$S = (S_t)_{0 \leq t \leq T}$, denotes the midprice

$S_t = S_0 = S_0 + \sigma W_t$ where $\sigma > 0$

$W = (W_t)_{0 \leq t \leq T}$ is standard Brownian Motion



Optimal Deltas (LO Depths) over Time

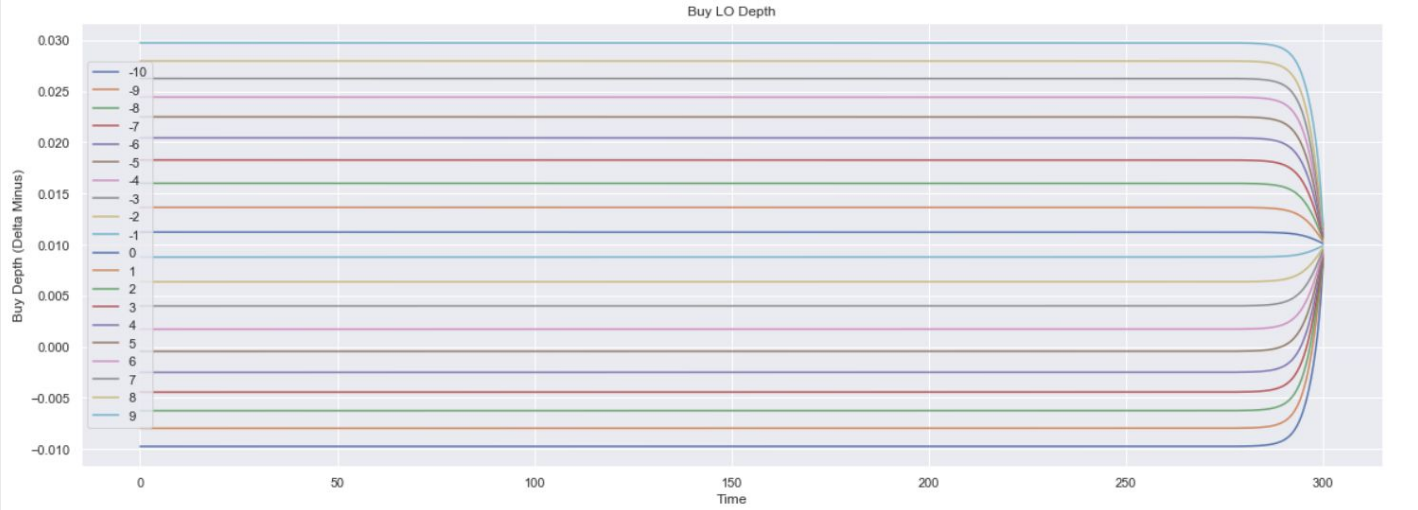
$\delta^{\pm} = (\delta^{\pm})_{0 \le t \le T}$ Represents the depth at which the agent posts limit orders

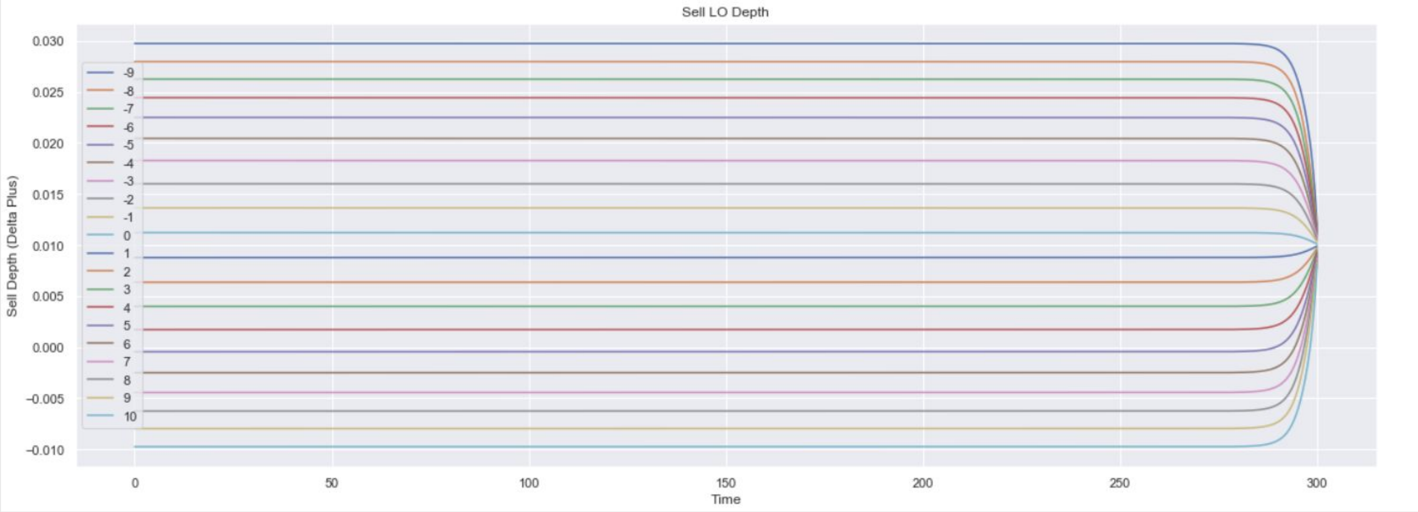
Sell LOs are posted at a price of $S_t + \delta_t^{+}$

Buy LOs are posted at a price of $S_t - \delta_t^{-}$

- Both the Sell and Buy Depths converge to zero at the terminal date due to the penalty parameter of selling the remaining inventory at time T.

- Intiutively: You want to end with as little inventory as possible because this model sells all remaining inventory at a cost with penalty at time T (terminal date).





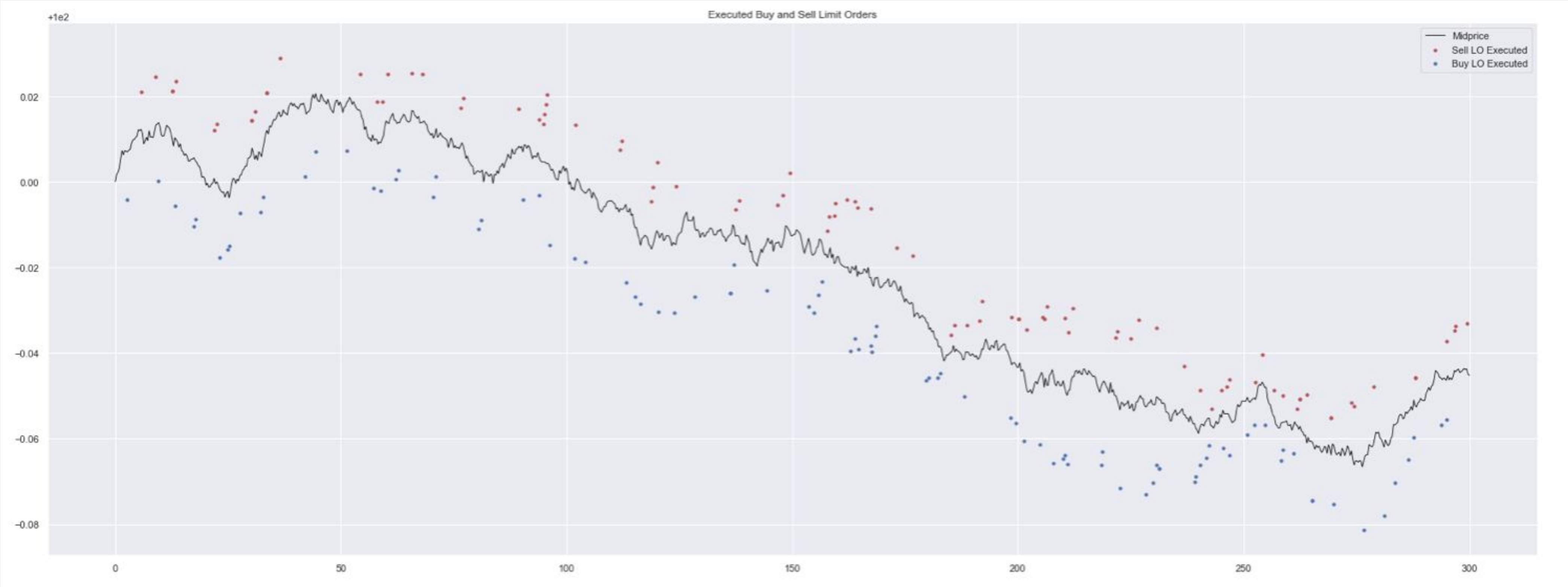
Optimal Depths and Midprice

Filled Market Orders

$M^{\pm} = (M^{\pm})_{0 \leq t \leq T}$ Denotes the counting processes corresponding to buy and sell market orders which arrive as Poisson Processes

Given that a Market Order has arrived, it is filled with probability $e^{-\kappa^{\pm}\delta_t^{\pm}}$

- Depending on current inventory levels at each point in time, the optimal depths are chosen and filled with the probability above
- The fill-probability, means the farther away from the midprice, the farther the MO must walk the book and results in a lower probability of being filled

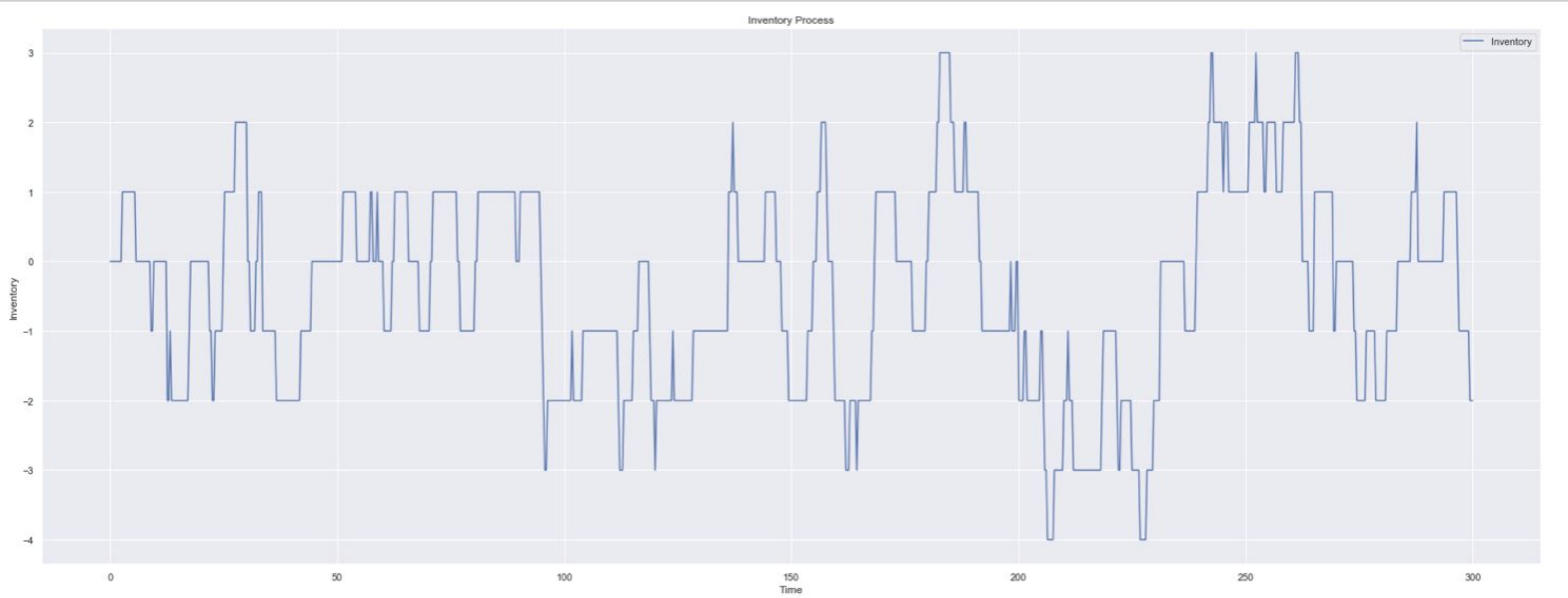


Inventory Process Over Time

$$Q^{\delta} = (Q_t^{\delta})_{0 \leq t \leq T} \text{ Denotes the Inventory Process}$$

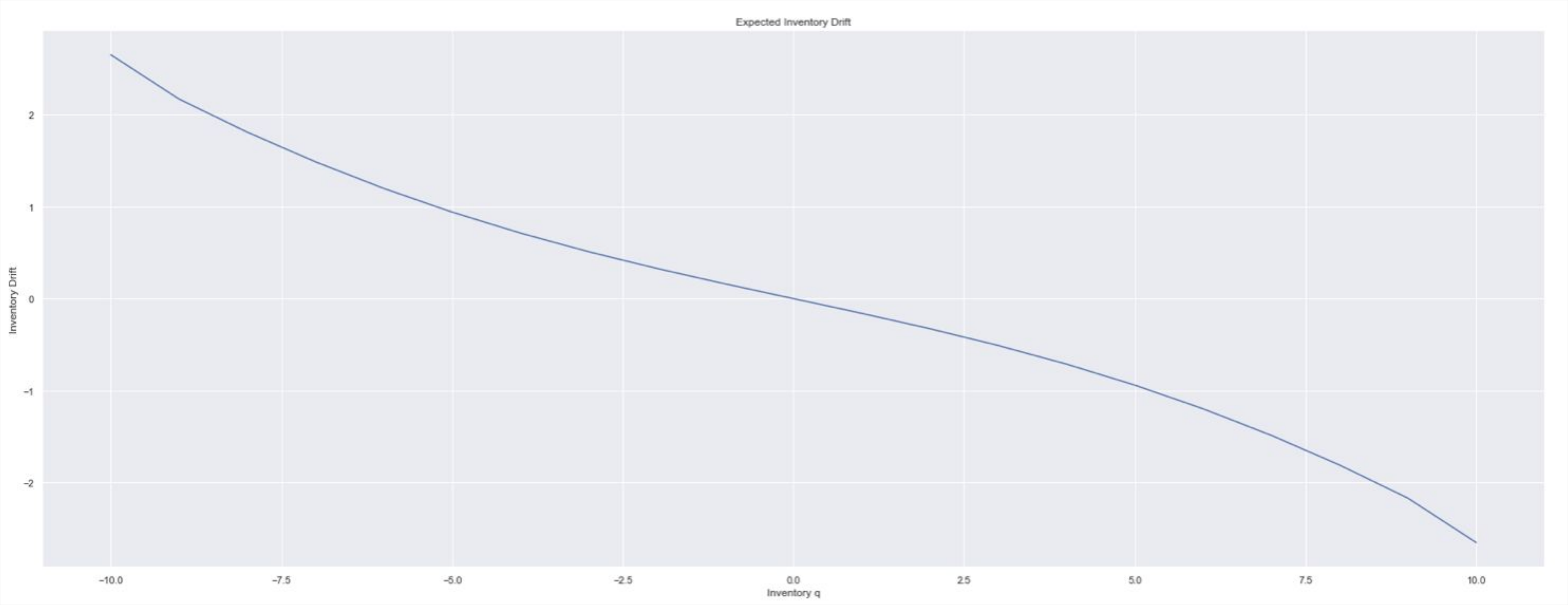
$$Q_t^{\delta} = N_t^{\delta,-} - N_t^{\delta,+}$$

Filled Buy - Filled Sell LO



Inventory Drift

- The optimal strategy induces mean reversion in inventories
- Caused by the asymmetry in optimal depths



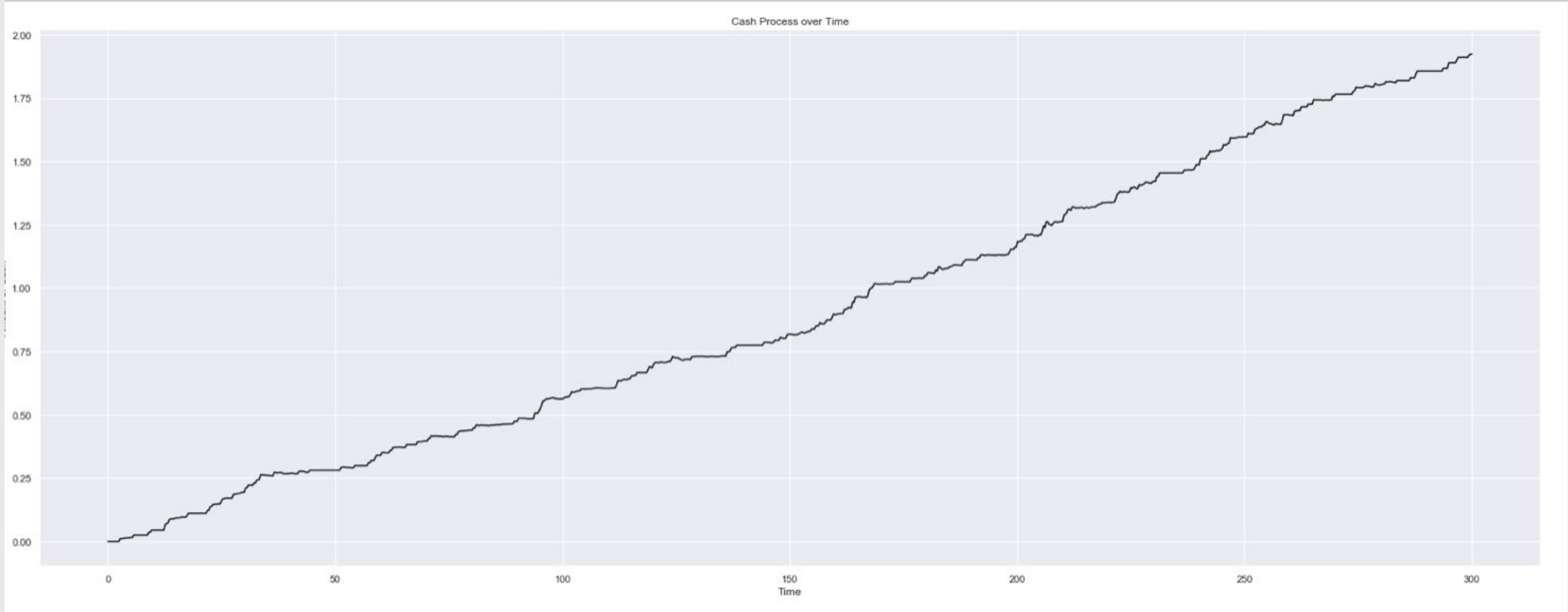
Cash Process Over Time

$X^\delta = (X_t^\delta)_{0 \leq t \leq T}$ Denotes the Market Makers cash process

$$dX_t^\delta = (S_t + \delta_t^+)dN_t^{\delta,+} - (S_t - \delta_t^-)dN_t^{\delta,-}$$

Increase in Cash from Sell LO

Outflow in Cash from Buy LO



Works Cited

- Cartea Álvaro. *Algorithmic and High-Frequency Trading*. Cambridge University Press, 2015.