Theory of Computing Assignment 1

1. a) $L_{20} = L_1 \cap L_2$

$L_1 = \{1010w \mid w \in \{0,1,2\}^*\}$

$L_2 = \{w01011 \mid w \in \{0,1,2\}^*\}$

$L_{20} = \{1w11 \mid w = 010 \vee w = 01010 \vee w = 010x010,\ x \in \{0,1,2\}^*\}$

b) $L_{21} = 01011\Sigma^* \cap \Sigma^* 1010$

$= \{w \mid w = 01011010 \vee 01011x1010,\ x \in \Sigma^*\}$

c) $L_{22} = L_{13}$

$= \{w \mid w = (0212)^n \vee w = (1202)^n,\ n \in \mathbb{Z},\ n > 0\}$

d) $L_{23} = L_6$

$= \{x1y \mid |y| = 4,\ x, y \in \{0,1,2\}^*\}$

e) $L_{24} = L_7 \cap L_8$

$L_7 = \{22021w \mid w \in \{0,1,2\}^*\}$

$L_8 = \{w22021 \mid w \in \{0,1,2\}^*\}$

$L_{24} = \{w \mid w = 22021 \vee 22021w22021,\ w \in \{0,1,2\}^*\}$

f) $L_{25} = L_{11} \setminus L_{12}$

$L_{11} = \{w \mid |w|_a \equiv 0 \pmod 7\}$

$L_{12} = \{w \mid |w|_b \equiv 0 \pmod 5\}$

$L_{25} = \{w \mid |w|_a \equiv 0 \pmod 7 \wedge |w|_b \not\equiv 0 \pmod 5,\ w \in \{a,b\}^*\}$

g) $L_{26} = h^{-1}(L_4)$

$L_4 = \{w \mid |w|_0 \equiv 1 \pmod 2,\ w \in \{0,1,2\}^*\}$

$L_{26} = \{w \mid |w|_a \equiv 1 \pmod 2,\ w \in \{a,b\}^*\}$

h) $L_{27} = h^{-1}(L_1^R) \cap h^{-1}(L_5)$

$L_1^R = \{ w0101 \mid w \in \{0,1,2\}^* \}$

$h^{-1}(L_1^R) = \{ waa \mid w \in \{a,b\}^* \}$

$L_5 = \{ w \mid |w|_1 \equiv 0 \pmod 2, w \in \{0,1,2\}^* \}$

$h^{-1}(L_5) = \{ w \mid |w|_a + |w|_b \equiv 0 \pmod 2, w \in \{a,b\}^* \}$

$L_{27} = \{ waa \mid |w|_a + |w|_b \equiv 0 \pmod 2, w \in \{a,b\}^* \}$

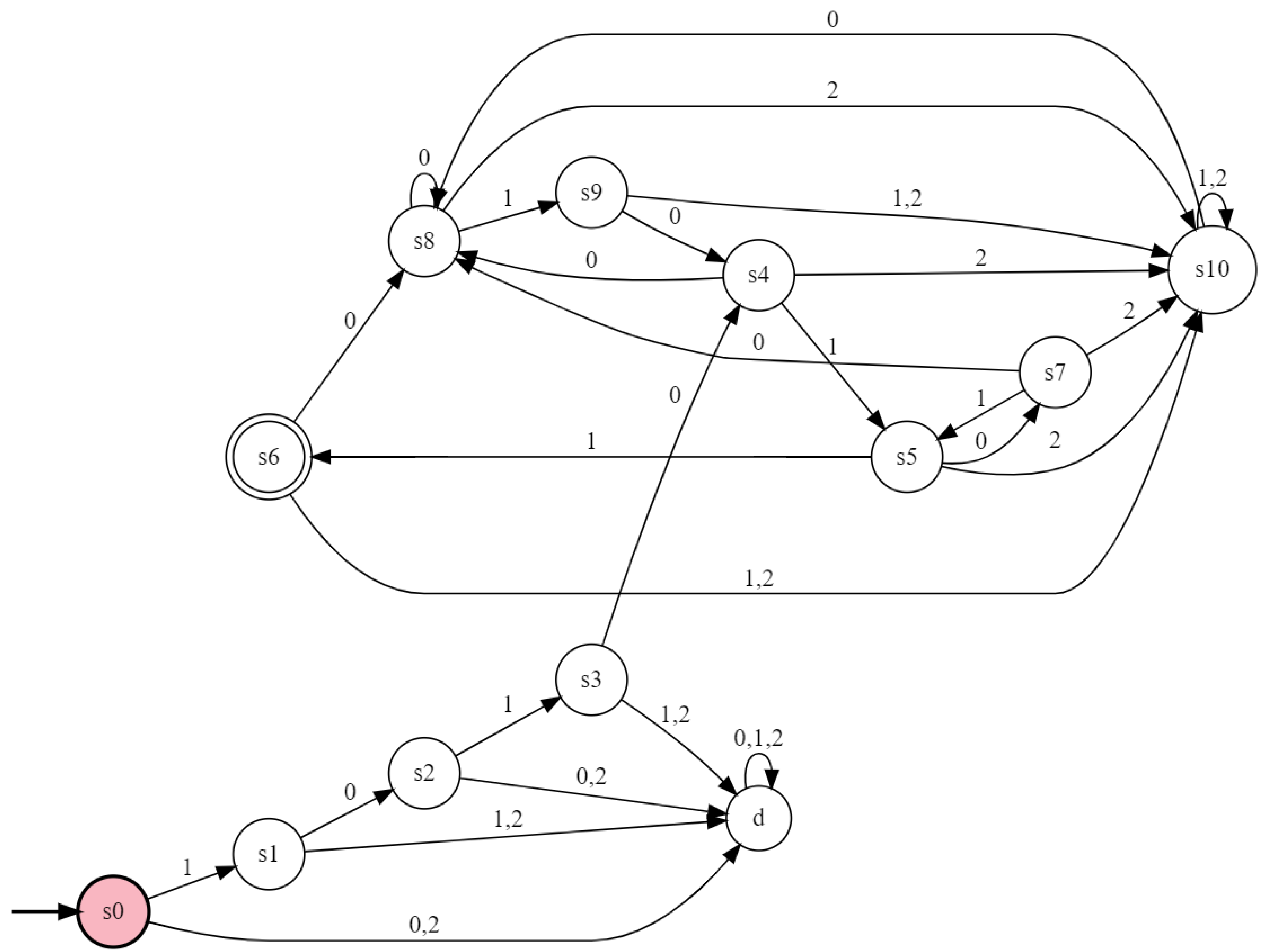
i) $L_{28} = g(L_1^R)$

$L_1^R = \{ w0101 \mid w \in \{0,1,2\}^* \}$

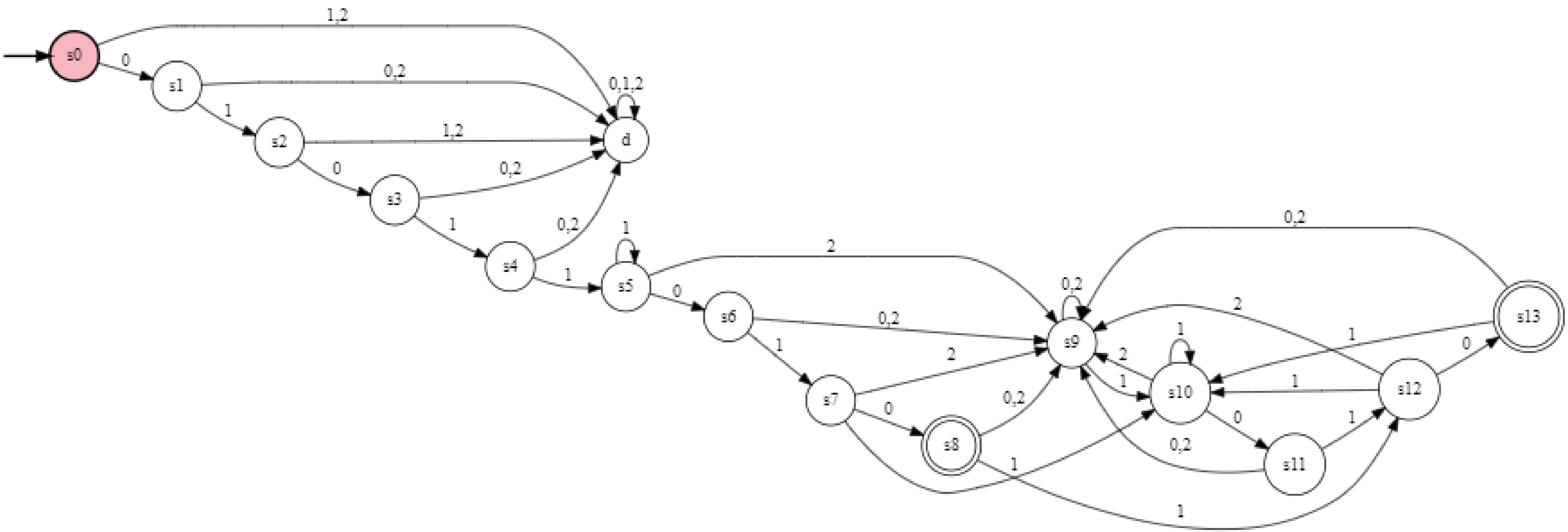
$L_{28} = \{ wabaaba \mid w = \varepsilon \vee w = aw \vee w = baw \}$

Question 2

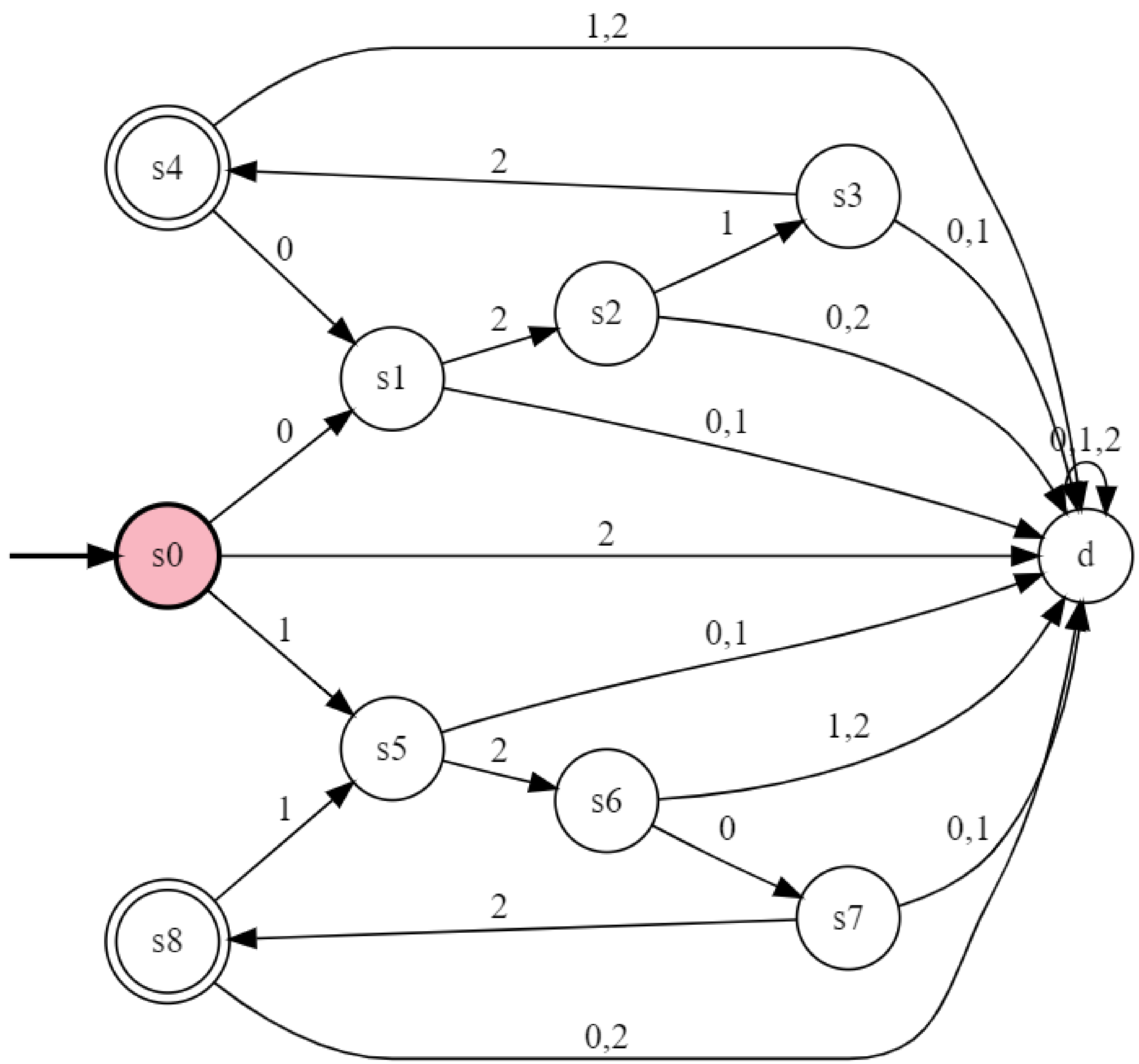
a) L_{20}



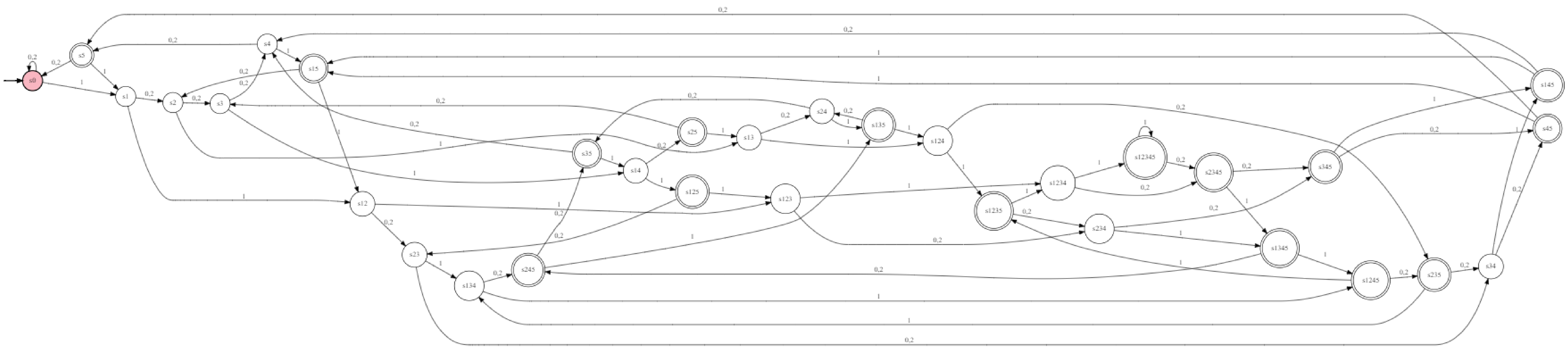
b) L_{21}



c) L_{22}

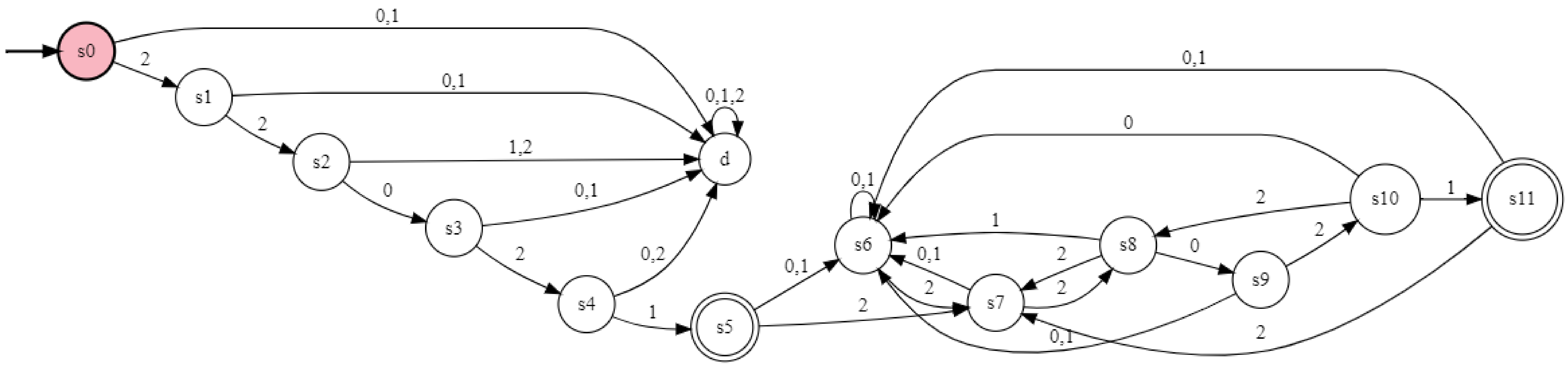


d) L_{23}

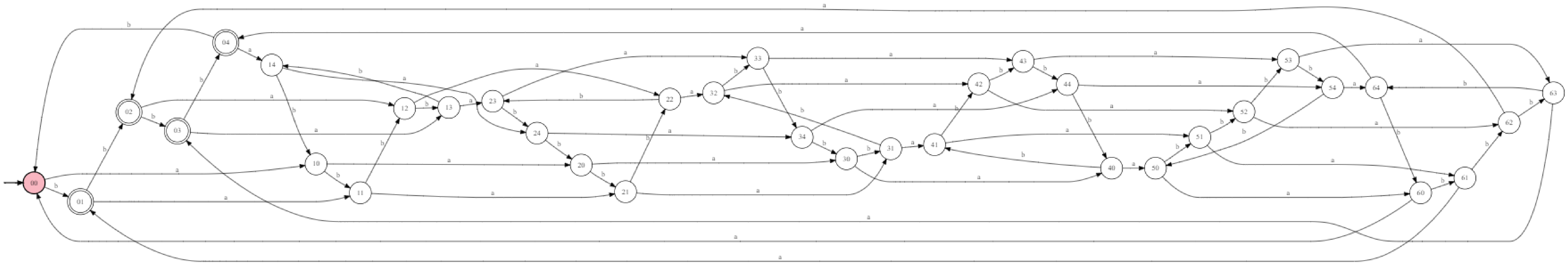


NOTE: if it is difficult to see the transitions you can refer to the grail file ‘Question 2/Part D/DFA’

e) L_{24}

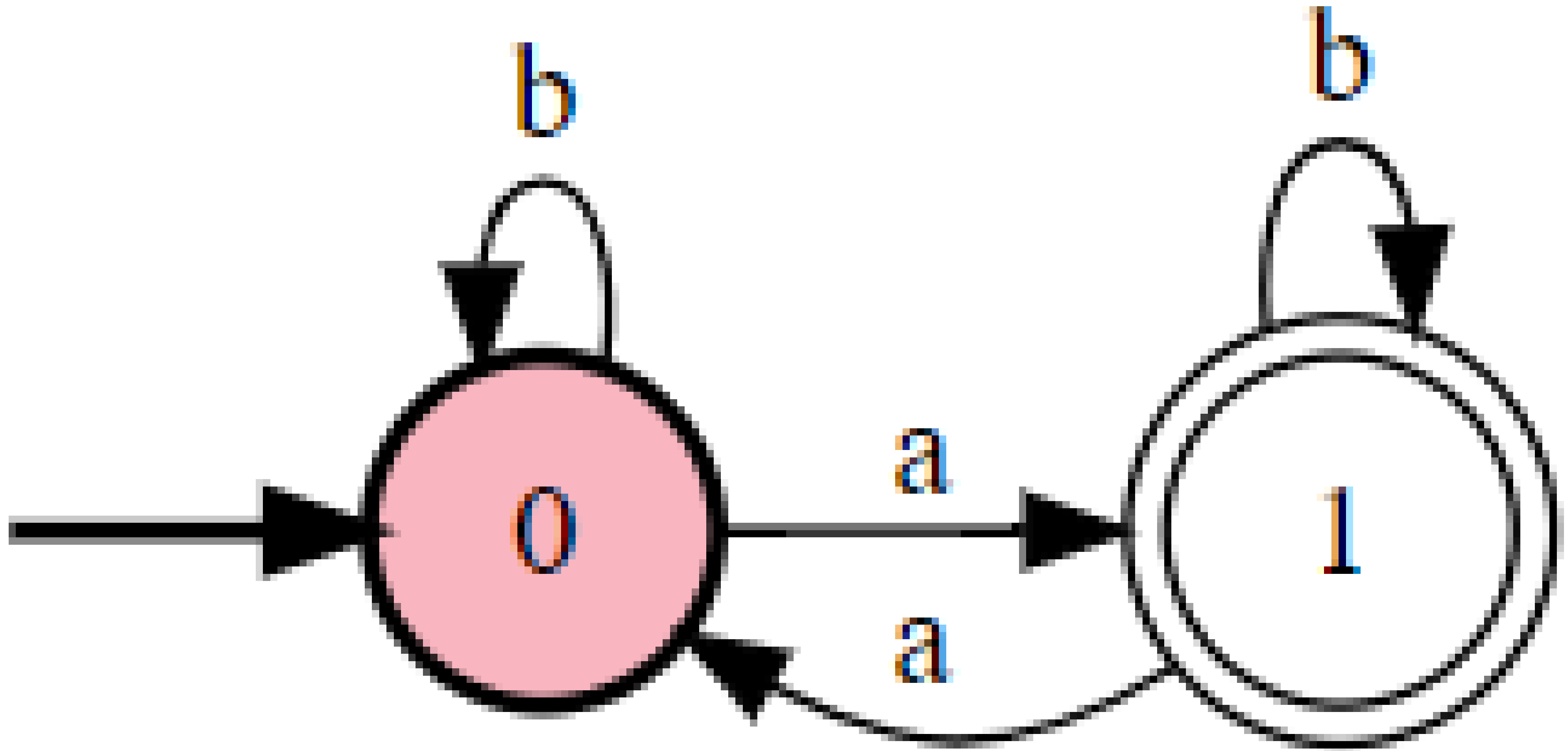


f) L_{25}

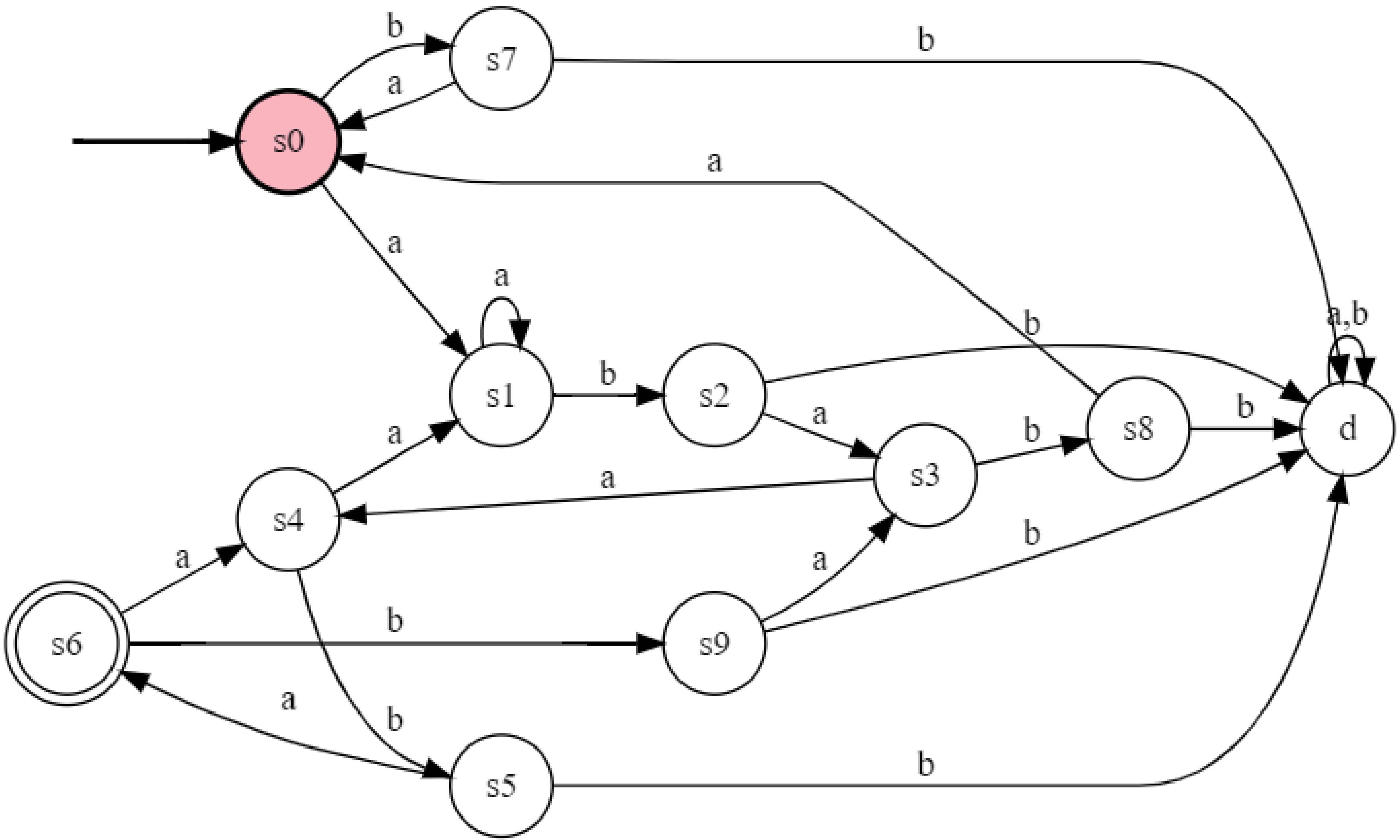


NOTE: if it is difficult to see the transitions you can refer to the grail file 'Question 2/Part F/DFA'

g) L_{26}

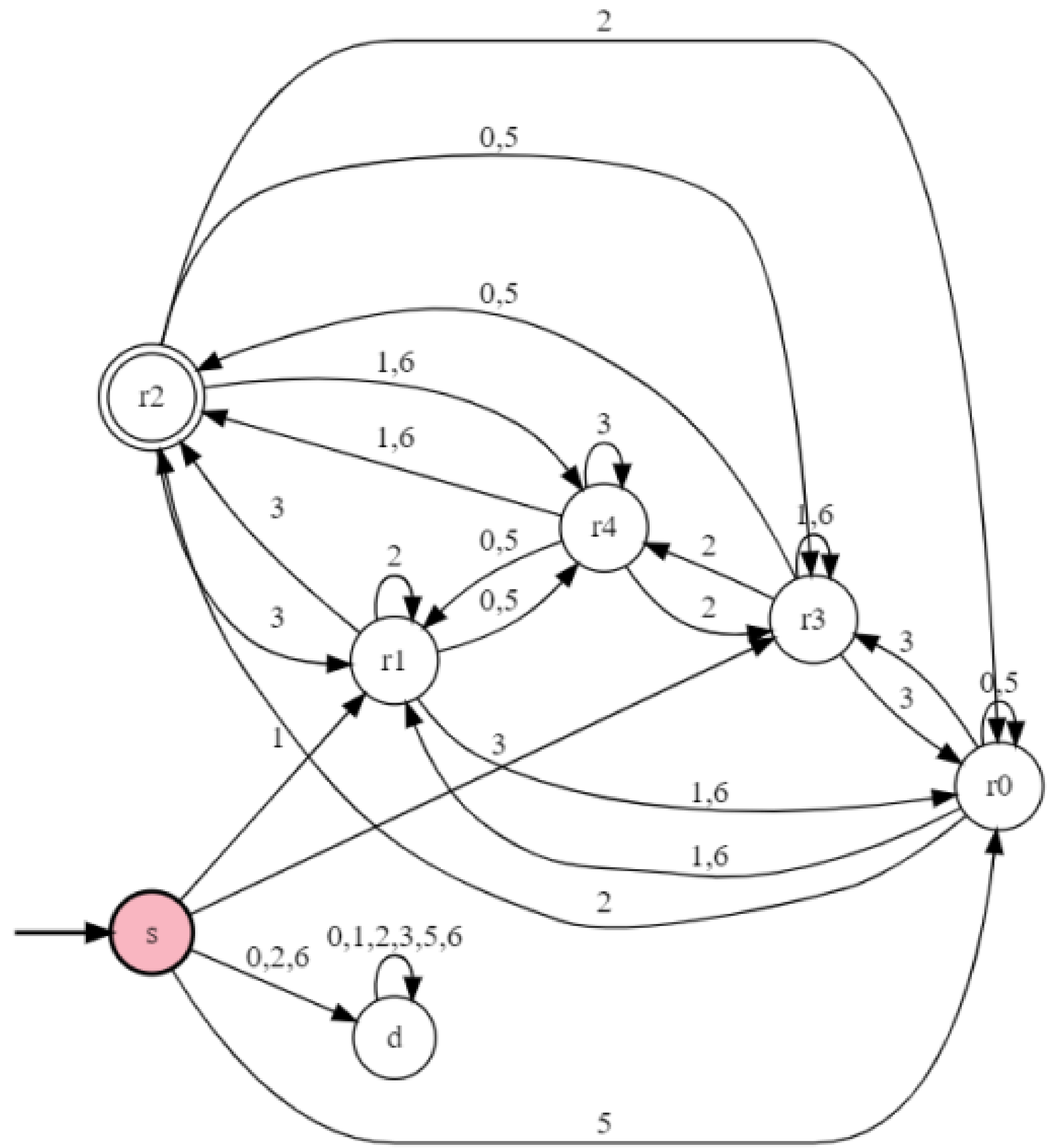


h) L_{28}

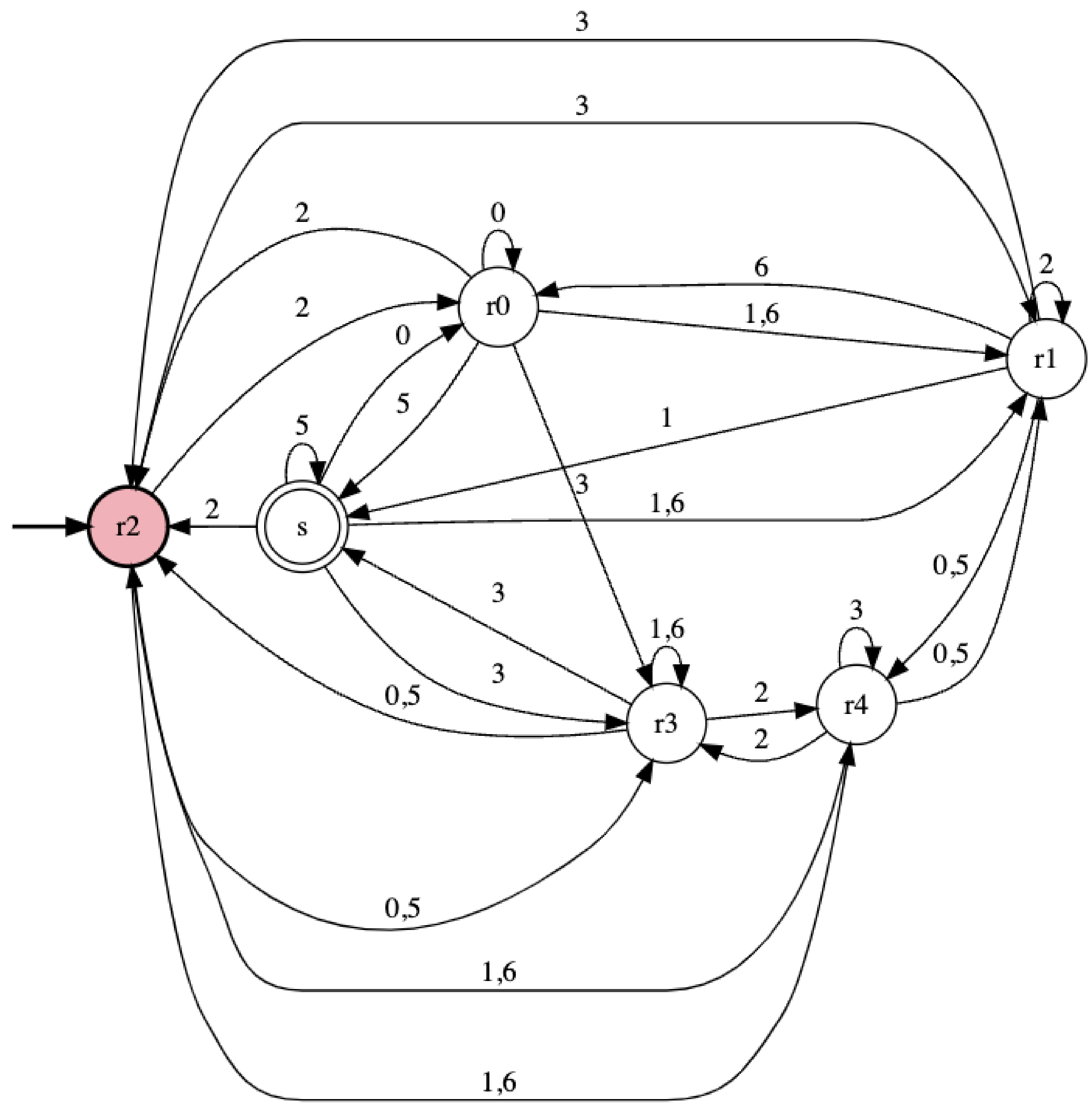


Question 3

a) the set of all strings beginning with a 1, 3 or 5, that, when the string is interpreted as an integer in base 9, is a multiple of 5 plus 2

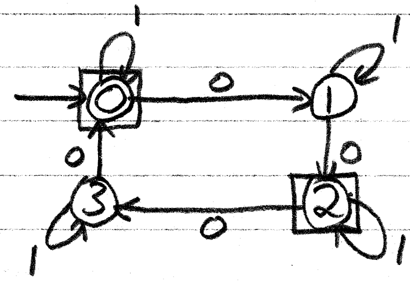


b) The set of all strings that ends with an 1, 3, or 5 and when the string is interpreted in reverse as an integer in base 9, is a multiple of 5 plus 2



4.

	0	1
$\rightarrow *0$	1	0
1	2	1
$*2$	3	2
3	0	3



$L = \{ w \mid |w|_0 \equiv 0 \pmod 2),\ w \in \{0,1\}^* \}$

Proof

Base case

Assume $w = \varepsilon$. Therefore w contains no 0's and $|w|_0 \equiv 0 \pmod 2$ holds true, and $|w| = 0$

Inductive hypothesis

w has an even number of 0's

Inductive Step

Consider the two possibilities $w = 0u$ and $w = 1u$ where $u = \{0,1\}^*$

If $w = 1u$ then $w \in L$ iff $|u|_0 \equiv 0 \pmod 2$ thus $|w| \geq 1$.

If $w = 0u$ then $w \in L$ iff $|u|_0 \equiv 1 \pmod 2$ thus $|w| \geq 2$.

Therefore, by induction we have proven that the $|w|$ of a word in L is $|w| \geq 0$.

QED