Theory of Computing Assignment 3

1. a) $bb^*(a^*+b+a)^*b^*b$

Simplify

$b(a+b)^*b$

The set of all strings over the alphabet $\{a,b\}$ that begin and end with b

b) $(a^*+b)^*a(a^*)^*a$

Simplify

$(a+b)^*aa^*a$

$(a+b)^*aa$

The set of all strings over the alphabet $\{a,b\}$ that end with aa

c) $(ab)^*ab(\varepsilon+a) + ba(ba)^*(b+\varepsilon)$

Simplify

$(ab)^+(\varepsilon+a) + (ba)^+(b+\varepsilon)$

$(ab)^+ + (ab)^+a + (ba)^+b + (ba)^+$

The set of all strings consisting of alternating a and (a and b alternates at least once)

d) $b^*ab^*(b^*ab^*b^*b^*ab^*ab^*ab^*)^*$

Simplify

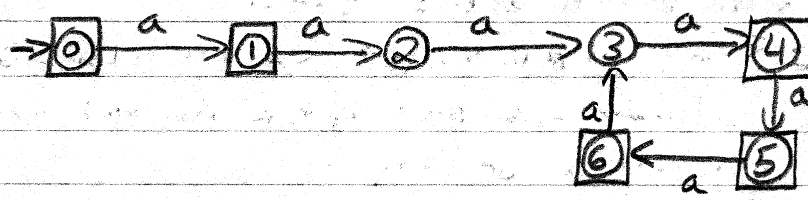
$b^*ab^*(b^*ab^*ab^*ab^*ab^*)^*$

The set of all strings over the alphabet $\{a,b\}$ having the number of a's a multiple of 4 plus 1

2 a) $L = \{ a^n b^m a^k \mid n \geq 2, m \geq 2, k \geq 3 \}$

$aaa^* bbb^* aaaa^*$

b) $L = \{ a^n \mid n \neq 2 \wedge n \not\equiv 3 \pmod 4 \}$



c) $L = \{ a^n b \mid n \geq 1 \} \cup \{ a b^m \mid m \geq 3 \}$

$aa^* b + abbbb^*$

d) L_1 is regular
$L = \{ wcv \in \{a,b,c\}^* \mid (2|w|_a + 3|v|_b) \equiv 3 \pmod 5)\ w, v \in L_1$
Since L_1 is regular L_1 has some regex R thus L will have the regex $R'cR''$ where R' and R'' are modified versions of R such that $2|w|_a + 3|w|_b \equiv 3 \pmod 5$ and we know we can modify R since L_1 is regular and therefore closed under regular operations.

3a) $L = \{a^n b^l a^m \mid |n-m| \leq 1, l > 3\}$

Proof:

Suppose L is regular

Let p be the pumping length.

Consider $W_p = a^p b^\alpha a^{p+1}$ where $\alpha > 3$

Clearly $|W_p| \geq p$ and $W_p \in L$. So by the pumping lemma, there must be some choice x, y, z satisfying the conditions of the pumping lemma.

But, consider any choice of x, y, z for which $W_p = xyz$, $|xy| \leq p$ and $|y| \geq 1$

Since $|xy| \leq p$, $x = a^k$ $y = a^h$ where $h \neq 0$ and $k + h \leq p$

Thus $z = a^j b^\alpha a^{p+1}$ where $k + h + j = p$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = a^k a^j b^\alpha a^{p+1} = a^{k+j} b^\alpha a^{p+1}$

Since $|y| \geq 1$ we know $h \neq 0$

So we know $k + j < p$

Thus $|k + j - (p+1)| \leq 1 \Rightarrow k + j = p$

Therefore xz is not in L

Since xz is not in L, we get a contradiction; hence the assumption that L is regular is false. Hence L is not regular.

QED

3b) $L_1 \subseteq \{a,b,c\}^*$

$L = \{wcv \in \{a,b,c\}^* \mid 2|w|_a = |w|_b \quad w,v \in L_1 \cap \{a,b\}^*\}$

It is not possible to determine if L is regular or not as we don't know if L_1 is regular or not regular. We don't know if the conditions imposed on L_1 are capable of satisfying the conditions imposed on L.

4 a) $L = \{c^m b^n a^n \mid n > 0, m \geq 2\}$

Proof:

Suppose L is regular.

Let p be the pumping length.

Consider $w_p = c^{\alpha} b^{p-\alpha} a^{p-\alpha}$ where $\alpha \geq 2$

Clearly $|w_p| \geq p$ and $w_p \in L$. So by the pumping lemma there must be some choice x, y, z satisfying the conditions of the pumping lemma

But, consider any choice of x, y, z for which $w_p = xyz$, $|xy| \leq p$ and $|y| \geq 1$

Since $|xy| \leq p$, we have two cases:

<u>Case 1</u>

$x = c^{\alpha-k}$ $y = c^k b^h$ where $h \neq 0$ and $\alpha + h \leq p$

Thus $z = b^l a^{p-\alpha}$ where $\alpha + h + l = p$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = c^{\alpha-k} b^l a^{p-\alpha}$

Since $|y| \geq 1$ we know $h \neq 0$

So we know $\alpha + l < p \Rightarrow l < p - \alpha$

Thus xz is not in L, we get a contradiction.

<u>Case 2</u>

$x = c^{\alpha} b^k$ $y = b^h$ where $h \neq 0$ and $\alpha + k + h \leq p$

Thus $z = b^l a^{p-\alpha}$ where $\alpha + k + h + l = p$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = c^{\alpha} b^k b^l a^{p-\alpha} = c^{\alpha} b^{k+l} a^{p-\alpha}$

Since $|y| \geq 1$ we know $h \neq 0$

So we know $\alpha + k + l < p \Rightarrow k + l < p - \alpha$

Thus xz is not in L, we get a contradiction

Since all cases are false, we get a contradiction; hence the assumption that L is regular is false. Hence L is not regular.

QED

4b) $L = \{ c a^n b^{n+k+2} a^k \mid n > 0,\ 2 < k < n \}$

Proof:

Suppose L is regular

Let p be the pumping length

Consider $W_p = c\,a^{p-1} b^{p-1+\alpha+2} a^{\alpha}$ where $p-1 > 3$ and $2 < \alpha < p-1$

Clearly $|W_p| \geq p$ and $W_p \in L$. So by the pumping lemma there must be some choice x, y, z satisfying the conditions of the pumping lemma

But, consider any choice of x, y, z for which $W_p = xyz$ $|xy| \leq p$ and $|y| \geq 1$

Since $|xy| \leq p$, we have two cases :

Case 1

$x = \varepsilon$ $y = c\,a^m$ where $m \leq p-1$

Thus $z = a^l b^{p-1+\alpha+2} a^{\alpha}$ where $m + l = p-1$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = \varepsilon a^l b^{p-1+\alpha+2} a^{\alpha} = a^l b^{p-1+\alpha+2} a^{\alpha}$

Clearly xz is not in L, we get a contradiction

Case 2

$x = c\,a^m$ $y = a^h$ where $h \neq 0$ and $m + h \leq p-1$

Thus $z = a^l b^{p-1+\alpha+2} a^{\alpha}$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = c a^m a^l b^{p-1+\alpha+2} a^{\alpha} = c\,a^{m+l} b^{p-1+\alpha+2} a^{\alpha}$

Since $|y| \geq 1$ we know $h \neq 0$

So we know $m + l < p-1 \Rightarrow p-1 < 3$

$\Rightarrow 2 < \alpha < 3$

Thus xz is not in L, we get a contradiction.

Since all cases are false, we get a contradiction; hence the assumption that L is regular is False. Hence L is not regular.

QED

4c) $L = \{c^m a^n b^l \mid n \neq l,\ m > 2\}$

Proof:

Suppose L is regular

Let p be the pumping length

Consider $w_p = c^\alpha a^{p-\alpha} b^{p-\beta}$ $\quad \alpha > 2 \quad \beta > \alpha$

Clearly $|w_p| \geq p$ and $w_p \in L$. So by the pumping lemma there must be some choice x, y, z satisfying the conditions of the pumping lemma

But consider any choice of x, y, z for which $w_p = xyz$ $|xy| \leq p$ and $|y| \geq 1$

Since $|xy| \leq p$, we have two cases:

<u>Case 1</u>

$x = c^{\alpha - k}$ $\quad y = c^k a^h$ where $h \neq 0$ and $\alpha + h \leq p$

Thus $z = a^j b^{p-\beta}$ where $\alpha + h + j = p$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = c^{\alpha - k} a^j b^{p-\beta}$

Since $|y| \geq 1$ we know $h \neq 0$

So we know $\alpha + j < p \Rightarrow j < p - \alpha \Rightarrow j \leq p - \beta$

Thus xz is not in L, we get a contradiction

<u>Case 2</u>

$x = c^\alpha a^k$ $\quad y = a^h$ where $h \neq 0$ and $\alpha + k + h \leq p$

Thus $z = a^j b^{p-\beta}$ where $\alpha + k + h + j = p$

Now consider xz. By the pumping lemma $xz \in L$

But $xz = c^\alpha a^k a^j b^{p-\beta} = c^\alpha a^{k+j} b^{p-\beta}$

Since $|y| \geq 1$ we know $h \neq 0$

So we know $\alpha + k + j < p \Rightarrow k + j < p - \alpha \Rightarrow k + j \leq p - \beta$

Thus xz is not in L, we get a contradiction

Since all cases are false, we get a contradiction; hence the assumption that L is regular is false. Hence L is not regular.

QED