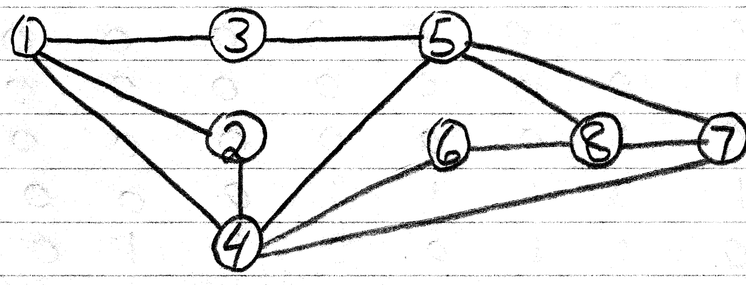
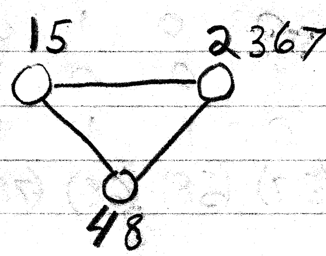
Assignment 1

1 a) 

b) 

$n = 3$

c) $\chi = 3$ since least $n = 3$

d) $H = \{ \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \},$
$\{ v_1 v_3, v_3 v_5, v_5 v_7, v_7 v_8, v_8 v_6, v_6 v_4, v_4 v_2, v_2 v_1 \} \}$

e) Adjacency

$$\begin{array}{c c}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} &
\begin{bmatrix}
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix} \\
 & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}
\end{array}$$

f) Incidence

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

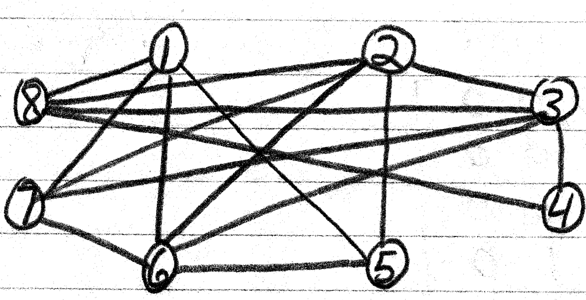
(12) (13) (14) (24) (35) (45) (46) (47) (57) (58) (68) (78)

g) G is not biportite because $\chi = 3$. To be biportite $\chi = 2$

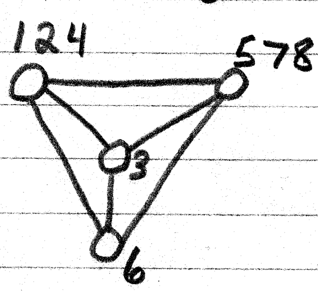
h) largest order $= 3$

1-2-4 } = 2 cliques of order 3
5-7-8 }

i)

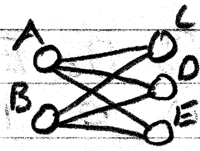


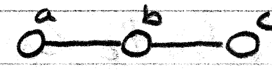
j)



$n = 4$

k) $\chi = 4$ since least $n = 4$

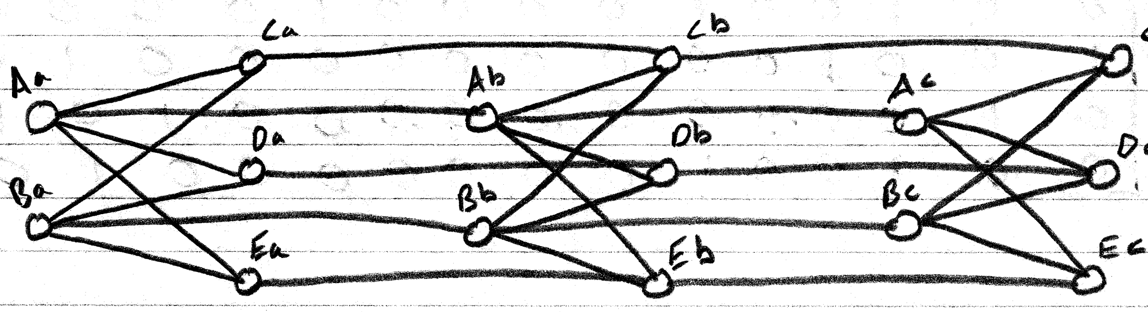
2 $K_{2,3}$: 

P_3: 

a) Adjacency

$$\begin{array}{c} A \\ B \\ C \\ D \\ E \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \end{array}$$

b) $K_{2,3} \square P_3$:



c)

$$
\begin{array}{c}
\\ Aa \\ Ab \\ Ac \\ Ba \\ Bb \\ Bc \\ Ca \\ Cb \\ Cc \\ Da \\ Db \\ Dc \\ Ea \\ Eb \\ Ec
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccccccc} Aa & Ab & Ac & Ba & Bb & Bc & Ca & Cb & Cc & Da & Db & Dc & Ea & Eb & Ec \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

3 Prove using mathematical induction that, forall $n \geq 3$, there is a homomorphism mapping P_n to P_2.

Proof:

Base Case:

Let $G = P_3 = \{\{v_1, v_2, v_3\}, \{v_1v_2, v_2v_3\}\}$

Then by the mapping $\varnothing(v_1) = v_1$ where $v_1v_2 \in E(P_2)$

$\varnothing(v_2) = v_2$

$\varnothing(v_3) = v_1$

Thus $\varnothing : P_3 \to P_2$

Induction Hypothesis:

Assume $\varnothing : P_k \to P_2$ where $k > 3$.

Inductive Step

Let $G = \{V(P_k) \cup \{v_{k+1}\}, E(P_k) \cup \{v_k v_{k+1}\}\} = P_{k+1}$

We know $\varnothing(P_k) = P_2$

Therefore $\varnothing(G) = \{V(P_2) \cup \{\varnothing(v_{k+1})\}, E(P_2) \cup \{\varnothing(v_k)\varnothing(v_{k+1})\}\}$

By the mapping of $\varnothing : P_k \to P_2$ we know:

$$\varnothing(v_k) = \begin{cases} v_1 & \text{if } k \equiv 1 \pmod 2 \\ v_2 & \text{if } k \equiv 0 \pmod 2 \end{cases}$$

Therefore if $k \equiv 1 \pmod 2$ then $k+1 \equiv 0 \pmod 2$

and if $k \equiv 0 \pmod 2$ then $k+1 \equiv 1 \pmod 2$

Therefore $\varnothing(G) = \{V(P_2) \cup \{v_1\}, E(P_2) \cup \{v_1v_2\}\}$

$= \{V(P_2), E(P_2)\}$

or $\varnothing(G) = \{V(P_2) \cup \{v_2\}, E(P_2) \cup \{v_1v_2\}\}$

$= \{V(P_2), E(P_2)\}$

Therefore by mathematical induction we have proven that for all $n \geq 3$, there is a homomorphism mapping P_n do P_2.

QED