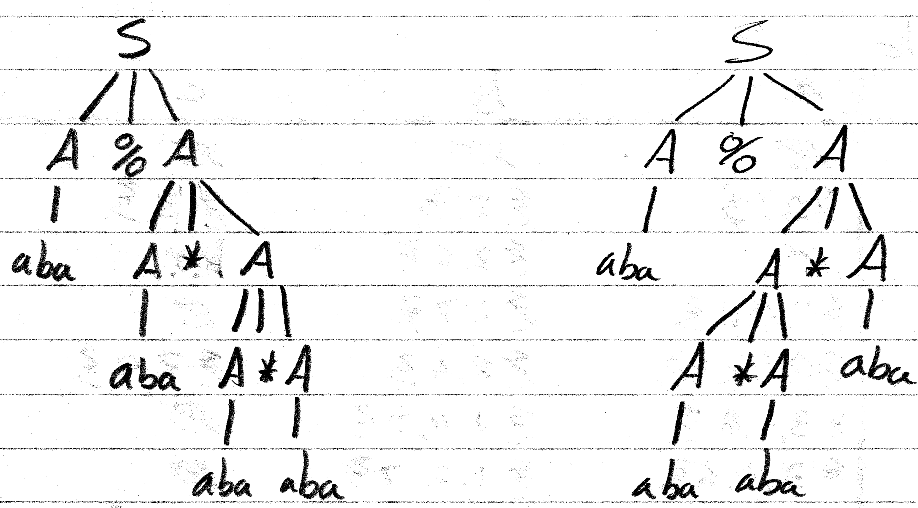
Theory of Computing Assignment 4

1. $L = \{ w \in \{a,b\}^* \mid |w|_a = |w|_b + 1 \}$

$S \rightarrow aA \mid bB \mid a$
$A \rightarrow abA \mid aAb \mid Aab \mid baA \mid bAa \mid Aba \mid ab \mid ba$
$B \rightarrow aaA \mid aAa \mid Aaa$

2. $S \rightarrow A \% A \mid A - B - A$
$A \rightarrow A * A \mid aba \mid b$
$B \rightarrow bab \mid a$



Both trees produce aba % aba * aba * aba.
There fore Grammar is ambigous

3. $S \rightarrow \varepsilon \mid b X_1 \mid a X_2 \mid b$
$X_0 \rightarrow b X_1 \mid a X_2 \mid b$
$X_1 \rightarrow a X_0 \mid b X_4 \mid a \mid b$
$X_2 \rightarrow a X_3 \mid b X_5$
$X_3 \rightarrow b X_6 \mid b$
$X_4 \rightarrow a X_3 \mid b X_2$
$X_5 \rightarrow a X_4 \mid b X_6 \mid b$
$X_6 \rightarrow a X_5$

4. NFA Table

	a	b	c
→0	{0,1}	{2}	{3}
1	{2,4}	{4}	{4}
2	∅	{3,5}	∅
3	{3,6,4}	{1,7}	∅
*4	∅	∅	∅
5	∅	{4}	∅
6	∅	{2}	∅
7	{4}	∅	∅

DFA Table

	a	b	c
∅	∅	∅	∅
→{0}	{0,1}	{2}	{3}
{2}	∅	{3,5}	∅
{3}	{3,4,6}	{1,7}	∅
{0,1}	{0,1,2,4}	{2,4}	{3,4}
{3,5}	{3,4,6}	{1,4,7}	∅
* {3,4,6}	{3,4,6}	{1,2,7}	∅
{1,7}	{2,4}	{4}	{4}
* {0,1,2,4}	{0,1,2,4}	{2,3,4,5}	{3,4}
* {2,4}	∅	{3,5}	∅
* {3,4}	{3,4,6}	{1,7}	∅
* {1,4,7}	{2,4}	{4}	{4}
{1,2,7}	{2,4}	{3,4,5}	{4}
* {4}	∅	∅	∅
* {2,3,4,5}	{3,4,6}	{1,3,4,5,7}	∅
* {3,4,5}	{3,4,6}	{1,4,7}	∅
* {1,3,4,5,7}	{2,3,4,6}	{1,4,7}	{4}
* {2,3,4,6}	{3,4,6}	{1,2,3,5,7}	∅
{1,2,3,5,7}	{2,3,4,6}	{1,3,4,5,7}	∅

5. a) $L = \{a^n b^{m+2} c^{n+1} \mid m,n \geq 0\}$

$S \to bbc \mid A \mid B$
$A \to abbBcc \mid aCcc$
$B \to bB \mid b$
$C \to aCc \mid abbBc \mid bb$

b) $L = \{a^{n-1} b^n c^m d^{m-2} \mid m,n \geq 3\}$

$S \to aabbbcccd \mid aaAbbbcccBd \mid aaAbbbcccd \mid aabbbcccBd$
$A \to aAb \mid ab$
$B \to cBd \mid cd$

c) $L = \{a^{2n} c^2 (bc)^{3n} \mid n \geq 4\}$

$S \to aaaaaaaacc\,bcbcbcbcbcbcbcbcbcbcbcbc$
$S \to aaaaaaaaA\,bcbcbcbcbcbcbcbcbcbcbcbc$
$A \to aaAbcbcbc$
$A \to aacc\,bcbcbc$

d) $L = \{uc^n v \mid 5|v|_a + 2|v|_b = 3|u|_a + 4|u|_b,\ n \geq 1\}$

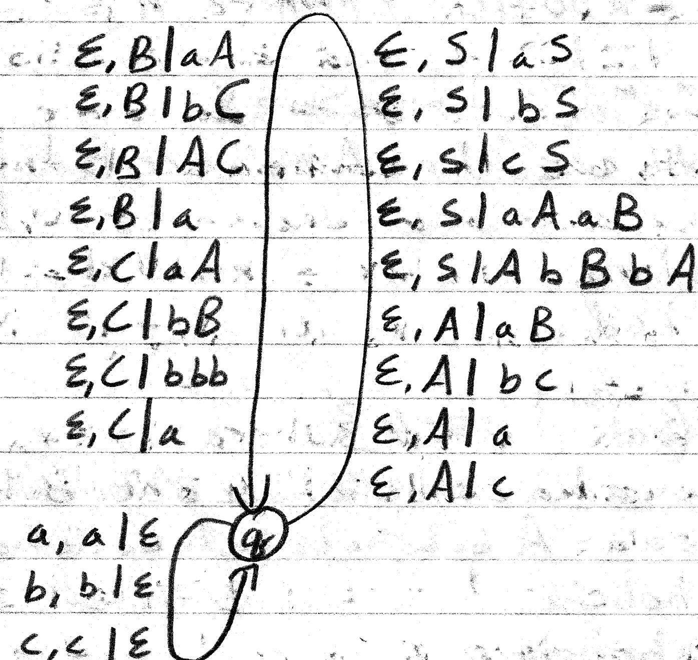
L is Context Free as we could construct a grammar such that uv is built such that $5|v|_a + 2|v|_b = 3|u|_a + 4|u|_b$. Then rename such that uCv where $C \to cC \mid c$. Thus since a grammar can be constructed L is context Free

6a) $S \to aS \mid bS \mid cS \mid aAaB \mid AbBbA$

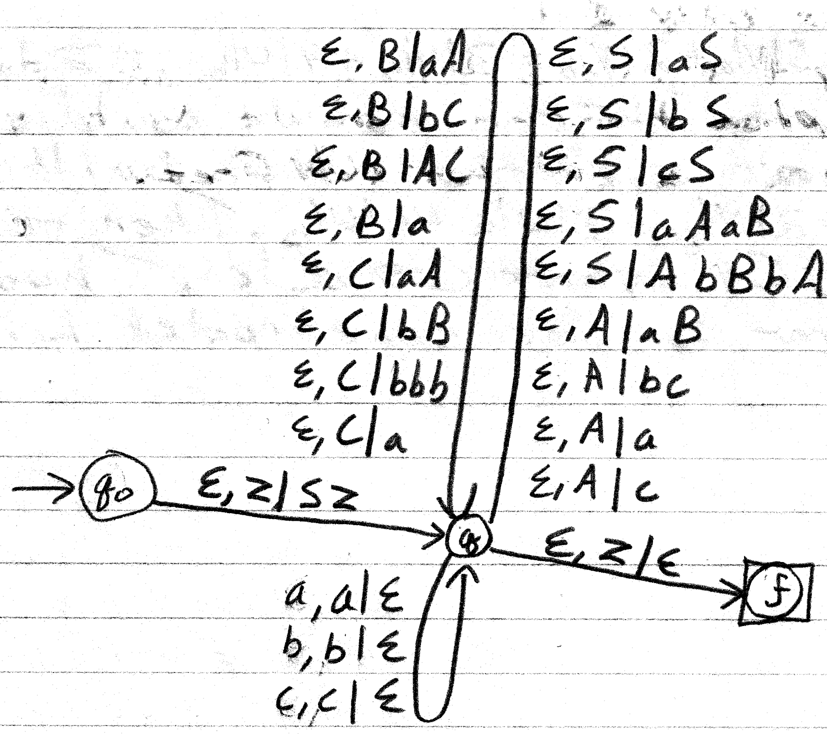
$A \to aB \mid bC \mid a \mid c$

$B \to aA \mid bC \mid AC \mid a$

$C \to aA \mid bB \mid bbb \mid a$



b)



7a) $L = \{ a^{p+3} \mid p \text{ is prime}, p > 7 \}$

Proof :

Assume L is context free.

Let n be the pumping constant.

Consider $z = a^{n+3}$ where n is prime and $n > 7$

Clearly $|z| \geq n$ and $z \in L$. So by the pumping lemma there must be some choice u, v, w, x, y satisfying the conditions of the pumping lemma.

But consider any choice of u, v, w, x, y for which $z = uvwxy$, $|vwx| \leq n$, $|vx| \geq 1$

Since $|vwx| \leq n$, $v = a^k$ $w = a^l$ $x = a^h$ where $k + h \geq 1$ and $k + l + h \leq n$

Thus $u = a^i$ $y = a^j$ where $i + k + l + h + j = n + 3$

Now consider uwy. By the pumping lemma $uwy \in L$.

But $uwy = a^i a^l a^j$, but $k + h \geq 1$ so $i + l + j < n + 3$

Which implies n is no longer prime.

Therefore $uwy \notin L$

Since $uwy \notin L$, we get a contradiction ; hence the assumption that L is context free is false.

Hence L is not context free.

QED

b) $L = \{ a^{2n} b^{2n} c^{3n} \mid n \geq 3 \}$

Proof:

Assume L is context free

Let n be the pumping constant

Consider $z = a^{2n} b^{2n} c^{3n}$ where $n \geq 3$

Clearly $|z| \geq n$ and $z \in L$. So by the pumping lemma there must be some choice u, v, w, x, y satisfying the conditions of the pumping lemma

But consider any choice of u, v, w, x, y for which $z = uvwxy$, $|vwx| \leq n$ and $|vx| \geq 1$

Since $|vwx| \leq n$ we have five cases:

<u>Case 1</u>

$v = a^k$ $w = a^l$ $x = a^h$ and $k+h \geq 1$, $k+l+h \leq n$

Thus $u = a^i$ $y = a^j b^{2n} c^{3n}$ and $i+k+l+h+j = 2n$

Conside uwy. By pumping lemma $uwy \in L$

$uwy = a^i a^l a^j b^{2n} c^{3n} = a^{i+l+j} b^{2n} c^{3n}$

Since $|vx| \geq 1$ then $i+l+j < 2n \Rightarrow i+l+j \neq 2n$

Thus $uwy \notin L$. We get a contradiction

<u>Case 2</u>

$v = b^k$ $w = b^l$ $x = b^h$ and $k+h \geq 1$ and $k+l+h \leq n$

Thus $u = a^{2n} b^i$ $y = b^j c^{3n}$ and $i+k+l+h+j = 2n$

Conside uwy. By pumping lemma $uwy \in L$

$uwy = a^{2n} b^i b^l b^j c^{3n} = a^{2n} b^{i+l+j} c^{3n}$

Since $|vx| \geq 1$ then $i+l+j < 2n \Rightarrow i+l+j \neq 2n$

Thus $uwy \notin L$. We get a contradiction

<u>Case 3</u>

$v = c^k$ $w = c^l$ $x = c^h$ and $k+h \geq 1$ and $k+l+h \leq n$

Thus $u = a^{2n} b^{2n} c^i$ $y = c^j$ and $i+k+l+h+j = 3n$

Consider uwy. By pumping lemma $uwy \in L$

$uwy = a^{2n} b^{2n} c^i c^l c^j = a^{2n} b^{2n} c^{i+l+j}$

Since $|vx| \geq 1$ then $i+l+j < 3n \Rightarrow i+l+j \neq 3n$

Thus $uwy \notin L$, we get a contradiction

b) continued.

Case 4

$V = a^k$ $W = a^l b^m$ $x = b^h$ and $k+h \geq 1$ and $k+l+m+h \leq$

Thus $U = a^i$ $y = b^j c^{3n}$ and $i+k+l+m+h+j = 4n$

Consider uwy. By the pumping lemma $uwy \in L$

$uwy = a^i a^l b^m b^j c^{3n} = a^{i+l} b^{m+j} c^{3n}$

Since $|vx| \geq 1$ then $i+l+m+j < 4n$

$\Rightarrow (i+l \neq m+j \neq 2n)$

OR $(i+l = m+j \Rightarrow i+l < 2n$ (or $m+j <$

$\Rightarrow i+l+n < 2n+n)$

Noting $3n = 2n+n$

Thus $uwy \notin L$, we get a contradiction.

Case 5

$V = b^k$ $w = b^l c^m$ $x = c^h$ and $k+h \geq 1$ and $k+l+m+h \leq n$

Thus $U = a^{2n} b^i$ $y = c^j$ and $i+k+l+m+h+j = 5n$

Consider uwy. By the pumping lemma $uwy \in L$

$uwy = a^{2n} b^i b^l c^m c^j = a^{2n} b^{i+l} c^{m+j}$

Since $|vx| \geq 1$ then $i+l+m+j < 5n$

$\Rightarrow (i+l = m+j \Rightarrow 2n = 3n)$

OR $(i+l \neq m+j \Rightarrow ((i+l > m+j \Rightarrow 2n > 3n$

or $(i+l < m+j \Rightarrow i+l+n < 2n+$

Thus $uwy \notin L$, we get a contradiction.

Since all cases are false, we get a contradiction; hence the assumption that L is Context free is False. Hence L is not context free.

QED

c) $L = \{a^{n^3+2n^2+5} \mid n \geq 4\}$

Proof:

Assume L is context free

Let n be the pumping constant

Consider $z = a^{n^3+2n^2+5}$ $n \geq 4$

Clearly $|z| \geq n$ and $z \in L$. So by the pumping lemma there must be some choice u, v, w, x, y satisfying the conditions of the pumping lemma

But consider any choice of u, v, w, x, y for which $z = uvwxy$, $|vwx| \leq n$, $|vx| \geq 1$

Since $|vwx| \leq n$, $v = a^k$ $w = a^l$ $x = a^h$ where $k+h \geq 1$ and $k+l+h \leq$

Thus $u = a^i$ $y = a^j$ and $i+k+l+h+j = n^3+2n^2+5$

Now consider uwy. By the pumping lemma $uwy \in L$

But $uwy = a^i a^l a^j = a^{i+l+j}$

Since $|vx| \geq 1$ then $i+l+j < n^3+2n^2+5$

Therefore $uwy \notin L$

Since $uwy \notin L$, we get a contradiction; hence the assumption that L is context free is false.

Hence L is not context free.

QED

d) $L = \{a^{2n} c^5 b^{3n} c^{4n} \mid n \geq 0\}$

Proof:

Assume L is context free

Let n be the pumping constant

Consider $z = a^{2n} c^5 b^{3n} c^{4n}$ $n \geq 0$

Clearly $|z| \geq n$ and $z \in L$. So by the pumping lemma there must be some choice u, v, w, x, y satisfying the conditions of the pumping lemma

But consider any choice of u, v, w, x, y for which $z = uvwxy$, $|vwx| \leq n$, $|vx| \geq 1$

Since $|vwx| \leq n$ we have 2 cases:

Case 1

vwx contains only one symbol

Therefore uwy contains less than $2n$ a's therefore the number of a's plus n will not equal $3n$,

OR uwy contains less than $3n$ b's therefore the number of b's plus n will not equal $4n$,

OR uwy contains less than $4n$ c's therefore the number of c's minus n will not equal $3n$

Thus $uwy \notin L$

Case 2

vwx straddles two symbols

Immediately we see if v or x contains any of c^5 uwy will not be in L

If w contains all of c^5 then taking uwy results that the number of a's is equal to ~~or greater~~ than the number of b's or the number of a's plus n is less than the number of b's,

OR uwy contains some b's and some c's therefore the number of b's is greater than or equal to the number of c's or the number of b's plus n is less than the number of c's. Thus $uwy \notin L$.

Since all cases are false. We know L is not Context Free. QED