Assignment 4

1) How many Subgraphs does a labelled P_5 have?

		Vertizes				
		1	2	3	4	5
Edges	0	5	10	10	5	1
	1	x	4	12	12	4
	2	x	x	3	9	6
	3	x	x	x	2	4
	4	x	x	x	x	1
		5	14	25	28	16

88 Subgraphs

2) Prove that every 3-regular bridgeless graph contains a 2-factor

Proof:

Suppose G is a 3-regular graph that has no bridges
By Petersen's theorem G must have a perfect matching and thus a 1-factor, F.
Let $F' = G - F$
Since G is 3-regular and F is a perfect matching removing the edges in F from G will cause all vertices in G to decrease in degree by one since F spans all vertices of G.
Therefore the degree of all vertices in F' is 2 and F' spans all the vertices of G.
Thus F' is a 2-Factor of G.
Therefore every 3-regular bridgeless graph contains a 2-factor.

QED

3/ Prove that $C_n \square K_2$ is 1-factorable for $n \geq 4$.

Proof:

Base Case:

Let $n=4$

Thus $G = C_4 \square K_2 = Q_3$

Q_3 has 3 1-factors, one for each dimension, and they cover all edges

Thus $C_4 \square K_2$ is 1-factorable.

Inductive Hypothesis:

Assume $C_k \square K_2$ is 1-factorable for $k \geq 4$

Inductive Step

Suppose k is even.

Then C_k will have 2 1-factors and be 1-factorable.

Thus $C_k \square K_2$ will have 2 1-factors in each of the C_k and another 1-factor from the k edges connecting each C_k.

Therefore $C_k \square K_2$ is 1-factorable when k is even.

Suppose instead k is odd.

Then C_k will not have a 1-factor as 1 vertex cannot be matched

Thus in $C_k \square K_2$ there will be one or more unmatched vertex in each C_k that can be matched by one or more of the k edges connecting each C_k.

Therefore $C_k \square K_2$ is 1-factorable when k is odd.

Thus for $C_{k+1} \square K_2$, $k+1$ is odd if k is even and vice versa, and we know if the cycle is odd or even the product is 1 factorable.

Therefore by mathematical induction $C_n \square K_2$ is 1-factorable for $n \geq 4$.

QED

4/ Find a P_4 decomposition of K_7