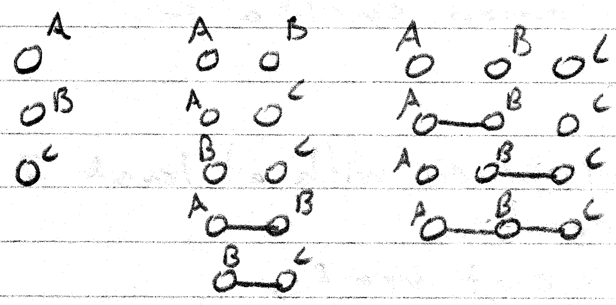
Assignment 3

1. How many Subgraphs does P_3 have.



12 subgraphs

2 Prove that a connected graph G with at least two edges is nonseperable if and only if any two incident edges of G lie on a comon cycle of G

Proof:

Let G be a connected graph with at least 2 edges
Suppose G is nonseperable
Therefore G has no cut vertex
Since G is nonseperable and has at least two edges, there must be some vertex V with at least 2 edges, e and f incident to V.
If we remove V we also remove e and f, and G remains connected.
Therefore some other path exists from the vertices adjacent to V throug e and f
Which implies any two incident edges must lie on some cycle in G
Suppose now, any two incident edges of G lie on a common cycle of G
Consider any vertex V of degree at least 2 and 2 of its neighbors U and W.
Since any two incident edges are on the same cycle we know U, V, and W must also be in the same cycle.
Thus if we removed V and the edges from V to U and W the graph remains connected since a path will still exists from U to W
Which implies G must be nonseperable.
Therefore, a connected graph G with at least 2 edges is nonseperable if and only if any two incident edges lie on a common cycle in G

QED

3 Prove that a 3 regular graph G has a cut vertex iF and only if G has a bridge.

Proof:

Let G be a 3-regular graph.
Suppose G has a cut vertex, V.
Then by removing V, G will be disconnected into 2 or 3 components.
IF 2 components are created 1 component will contain only one neighbor of V, and the edge from V to this neighbor would be a bridge
IF 3 components are created each component will contain only one neighbor of V and any edge from V to one of these neighbors would be a bridge
Suppose now G has a bridge, e.
By removing either endpoint of e we will also remove e thus making G disconnected.
Therefore both endpoints of e are cut vertices.
Therefore a 3-regular graph G has a cut vertex if and only if G has a bridge.

QED

4. Prove that for all $n \geq 5$ C_n^c is hamiltonian

Proof:

Base Case:

Let $G = C_5^c$. Note C_5^c is isomorphic to C_5

Thus $G = C_5$ which is hamiltonian

Inductive Hypothesis:

Assume C_k^c is hamiltonian, where $k \geq 5$

Inductive Step:

Let $V = \{V_1, V_2, \ldots V_{k-1}, V_k\}$ be the vertex set for C_k^c

If k is odd we can take the following hamiltonian cycle:

$V_1 V_3 \ldots V_k V_2 V_4 \ldots V_{k-1} V_1$

If k is even we can take the following hamiltonian cycle:

$V_1 V_3 \ldots V_{k-1} V_2 V_k V_{k-2} \ldots V_4 V_1$

Thus if we consider C_{k+1}^c then $k+1$ is either odd if k was even or even if k was odd, and we know the valid hamiltonian cycle for each of these cases

Therefore by induction C_k^c is hamiltonian for all $n \geq 5$.

QED

5. Prove that every tree has at most one perfect matching.

Proof:

Toward a contradiction, suppose a tree, T, has 2 distinct perfect matchings M and M'

Thus one of M and M' has an edge the other doesn't

Assume this edge e is in M

Let the endpoints of e be u and v

Since T is a tree there exists a single path from u to v for which e must be a part of.

However, if M' does not include e, then there must be some other edge ~~[illegible]~~ on the path from u to v which implies T contains a cycle $\Rightarrow\Leftarrow$

Therefore M must equal M'.

Therefore every tree has at most one perfect matching

QED

6. Meet 1 Meet 2 Meet 3 Meet 4

Meet 1:
9 1
8 2
7 3
6 5 4

Meet 2:
8 1 3
6 5
4
2 9 7

Meet 3:
9 6 1 4
3 7
8 5 2

Meet 4:
7 1
5 9
3
6 2 8 4

7. 5-regular no 1-Factor