Homework 6 solutions

Question 3

(a) Time-independent Schroedinger equation

$$E\psi = \hat{H}\psi \tag{1}$$

with Hamilton operator given by

$$\hat{H} = v\frac{\hbar}{i}\frac{d}{dx} + \frac{1}{2}m\omega^2 x^2 \tag{2}$$

therefore

$$\begin{aligned}
E\psi &= v\frac{\hbar}{i}\frac{d}{dx}\psi + \frac{1}{2}m\omega^2 x^2\psi && (3)\\
\frac{E}{\hbar v}\psi &= -i\frac{d}{dx}\psi + \frac{m\omega^2}{2\hbar v}x^2\psi && (4)\\
\frac{1}{\lambda}\psi &= -i\frac{d}{dx}\psi + \frac{1}{x_0^3}x^2\psi && (5)\\
\left(\frac{1}{\lambda} - \frac{1}{x_0^3}x^2\right)\psi + i\frac{d}{dx}\psi &= 0 && (6)
\end{aligned}$$

(b)

$$\begin{aligned}
\left(\frac{1}{\lambda} - \frac{1}{x_0^3}x^2\right)\psi + i\frac{d}{dx}\psi &= 0 && (7)\\
id\psi\frac{1}{\psi} &= \left(\frac{1}{x_0^3}x^2 - \frac{1}{\lambda}\right)dx && (8)\\
\ln(\psi) &= -i\left(\frac{1}{3x_0^3}x^3 - \frac{x}{\lambda} + C\right) && (9)\\
\psi &= C'\exp\left[-i\left(\frac{1}{3x_0^3}x^3 - \frac{x}{\lambda}\right)\right] && (10)
\end{aligned}$$

(c)

Question 4

(a)

(b)

(c)