Homework 7 solutions

Question 4

(a)

$$\begin{aligned}
\hat{B}_j^\dagger &= (i\hat{A}_{2j-1}\hat{A}_{2j})^\dagger && (1)\\
&= -i\hat{A}_{2j}^\dagger\hat{A}_{2j-1}^\dagger && (2)\\
&= -i\hat{A}_{2j}\hat{A}_{2j-1} && (3)\\
&= i\hat{A}_{2j-1}\hat{A}_{2j} = \hat{B}_j && (4)
\end{aligned}$$

(b)

$$\begin{aligned}
\hat{B}_j^2 &= (i\hat{A}_{2j-1}\hat{A}_{2j})^2 && (5)\\
&= -\hat{A}_{2j-1}\hat{A}_{2j}\hat{A}_{2j-1}\hat{A}_{2j} && (6)\\
&= \hat{A}_{2j-1}\hat{A}_{2j-1}\hat{A}_{2j}\hat{A}_{2j} && (7)\\
&= \hat{A}_{2j-1}^2\hat{A}_{2j}^2 && (8)\\
&= \mathbb{1}\mathbb{1} = \mathbb{1} && (9)
\end{aligned}$$

(c)

$$\begin{aligned}
[\hat{B}_j, \hat{H}] &= \hat{B}_j\hat{H} - \hat{H}\hat{B}_j && (10)\\
&= i\hat{A}_{2j-1}\hat{A}_{2j}\hat{H} - i\hat{H}\hat{A}_{2j-1}\hat{A}_{2j} && (11)\\
&= i\hat{A}_{2j-1}\hat{A}_{2j}\hat{H} - i\hat{A}_{2j-1}\hat{H}\hat{A}_{2j} && (12)\\
&= i\hat{A}_{2j-1}(\hat{A}_{2j}\hat{H} - \hat{H}\hat{A}_{2j}) && (13)\\
&= i\hat{A}_{2j-1}([\hat{A}_{2j}, \hat{H}]) = 0 && (14)
\end{aligned}$$

(d)

$$\hat{H}\hat{A}|\psi\rangle = \hat{A}\hat{H}|\psi\rangle = \hat{A}E|\psi\rangle = E\hat{A}|\psi\rangle \quad (15)$$

(d)