Chapter 11, $E = mc^2$ and all that.

Learning Notes, Physics for Poets

1 The Meaning of $E = mc^2$

Ever since Hiroshima, this formula has been associated in the public mind with nuclear energy. However, we must emphasize (at the risk of repetition) that this formula applies equally well to all forms of energy. It is a universal description of nature; as valid for a bonfire as for a nuclear weapon.

$e = mc^2$ is a statement saying that in fact for all practical purposes mass and energy are identical; mass is energy and all energy has mass. c is just a conversion factor from units of mass to units of energy, much like converting from miles to km.

2 The Mass Increase

So just as energy can change, mass is no longer a constant. What we used to call mass, we must now define as the rest mass, i.e. the mass of the object at rest. Because as we speed up an object to relativistic speeds, its mass changes; from our point of view (i.e. our frame of reference) the object starts to get more and more massive.

So we introduce the symbol m_0 to designate the mass of an object at rest (the "constant" mass we are familiar with) and the changing or relativistic mass then becomes:

$$m = \gamma m_0$$

From this equation, we can see that at rest our Lorentz factor is 0 and our mass is equal to our rest mass, however as our object speeds up, our Lorentz factor increases and our object starts becoming more massive. As our object's speed approaches that of light we see that our mass approaches

infinity. Thus, this *relativistic mass increase* enforces the speed of light as our upper boundary; our maximum speed.

but what does this mean? It means that as we do more work to increase the object's speed near that of light, we are still increasing it's momentum, but our energy investment is going towards increasing the object's mass while the increases in speed are very small and harder and harder to achieve.

3 Summary

Despite relativistic reinterpretations of space and time, momentum conservation survives if we simply allow mass to vary with velocity according to the Lorentz factor.