Adaptive Markets: Financial Evolution at the Speed of Thought

Andrew W. Lo, MIT

National Bank of Belgium and 11 Universities Finance Seminar

November 29, 2017

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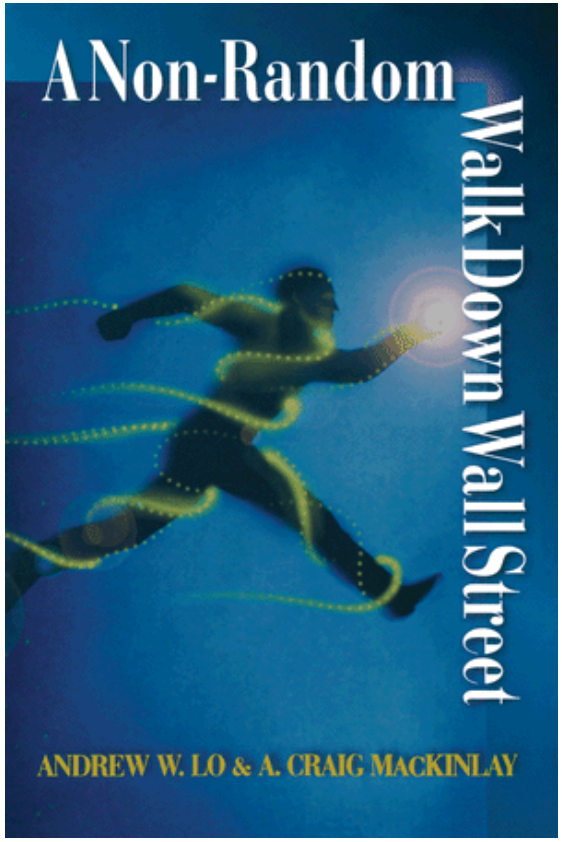


In a fashion analogous to Theorem 6.3.1, the behavior of Q_n under long-range dependent alternatives may now be derived in several steps using Lemmas A.2, A.3, and Theorem 6.3.2:

Theorem 6.3.3. *Let $\{\epsilon_t\}$ be a zero-mean stationary Gaussian stochastic process with autocovariance function γ_k such that*

$$\gamma_k \sim \begin{cases} k^{2H-2}L(k) & \text{for } H \in (\frac{1}{2}, 1) \text{ or,} \\ -k^{2H-2}L(k) & \text{for } H \in (0, \frac{1}{2}) \end{cases} \quad \text{as} \quad k \to \infty \qquad (6.3.14)$$

where $L(k)$ is a slowly varying function at infinity. Then as n and q increase without bound such that $(q/n) \to 0$, we have:

$$\text{(a)} \quad \operatorname{Max}_{1 \le k \le n} \frac{1}{\sigma_n} \sum_{j=1}^{k} (X_j - \overline{X}_n) \quad \Rightarrow \quad \operatorname{Max}_{0 \le \tau \le 1} W_H^o(\tau) \equiv M_H^o,$$

$$\text{(b)} \quad \operatorname{Min}_{1 \le k \le n} \frac{1}{\sigma_n} \sum_{j=1}^{k} (X_j - \overline{X}_n) \quad \Rightarrow \quad \operatorname{Min}_{0 \le \tau \le 1} W_H^o(\tau) \equiv m_H^o,$$

$$\text{(c)} \quad R_n \equiv \frac{\hat{\sigma}_n(q)\sqrt{n}}{\sigma_n} \cdot \frac{1}{\sqrt{n}} Q_n \quad \Rightarrow \quad M_H^o - m_H^o \equiv V_H,$$

$$\text{(d)} \quad a_n \equiv \frac{\sigma_n}{\hat{\sigma}_n(q)\sqrt{n}} \quad \xrightarrow{p} \quad \begin{cases} \infty & \text{for } H \in (\frac{1}{2}, 1), \\ 0 & \text{for } H \in (0, \frac{1}{2}), \end{cases}$$

$$\text{(e)} \quad \frac{1}{\sqrt{n}} Q_n = a_n R_n \quad \xrightarrow{p} \quad \begin{cases} \infty & \text{for } H \in (\frac{1}{2}, 1), \\ 0 & \text{for } H \in (0, \frac{1}{2}), \end{cases}$$

where $\hat{\sigma}_n(q)$ is defined in (6.3.6), σ_n is defined in Theorem 6.3.2, and $W_H^o(\tau) \equiv W_H(\tau) - \tau W_H(1)$.[20]

Theorem 6.3.3 shows that the modified rescaled range test is consistent against a class of long-range dependent stationary Gaussian alternatives. In the presence of positive strong dependence, the R/S statistic diverges in probability to infinity; in the presence of negative strong dependence, it converges in probability to zero. In either case, the probability of rejecting the null hypothesis approaches unity for all stationary Gaussian stochastic processes satisfying (6.3.14), a broad set of alternatives that includes all fractionally-differenced Gaussian ARIMA(p, d, q) models with $d \in (-\frac{1}{2}, \frac{1}{2})$.

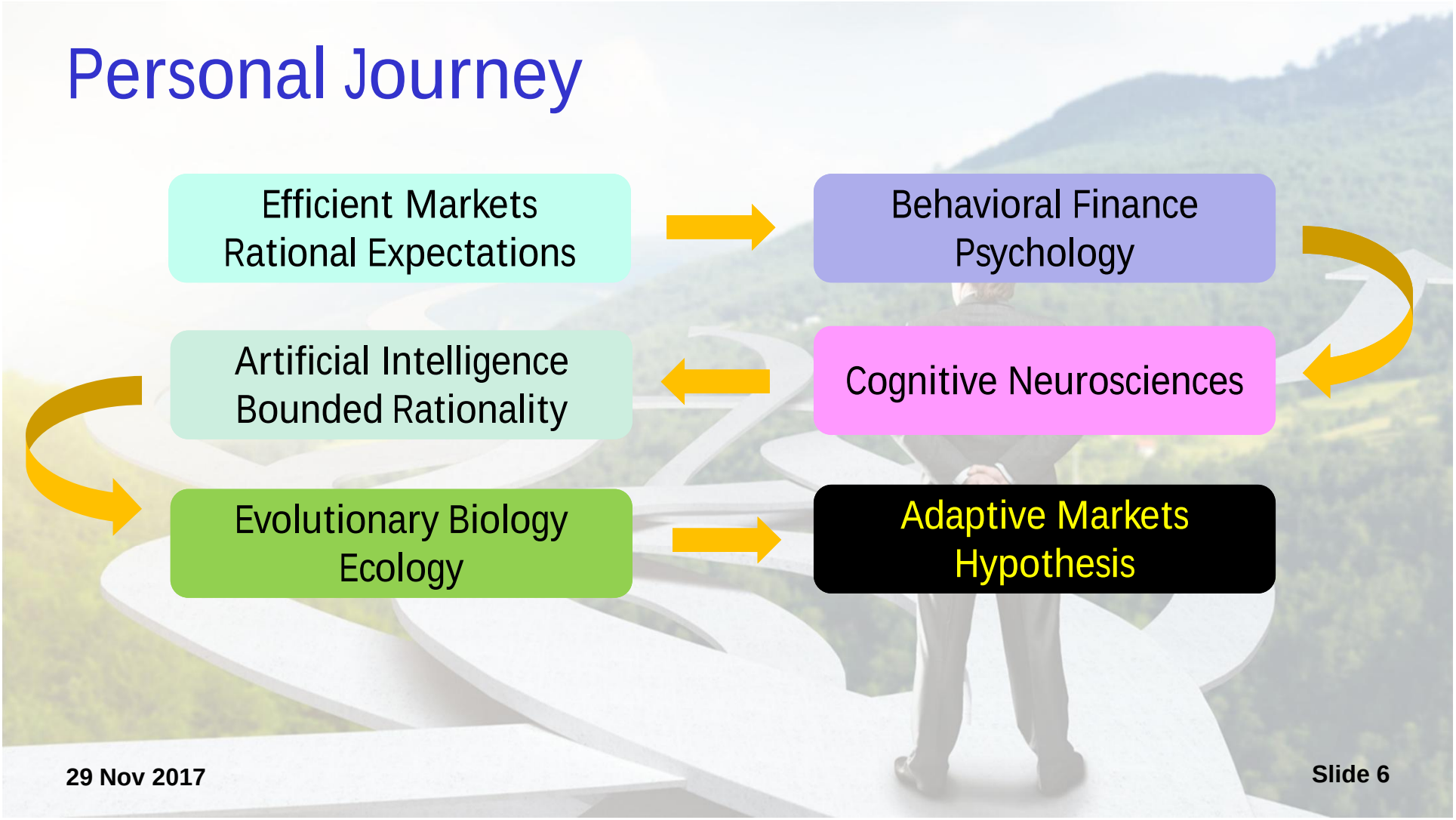
From (a) and (b) of Theorem 6.3.3 it is apparent that the normalized *population* rescaled, $R_n/\sqrt{n}$, converges to zero in probability. Therefore,

[20] Although it is tempting to call $W_H^o(\tau)$ a "fractional Brownian bridge," this is not the most natural definition despite the fact that it is "tied down." See Jonas (1983, Chapter 3.3) for a discussion.

Markets are efficient

People behave irrationally

Personal Journey



Summary

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- Traditional investment framework is flawed
- Not wrong, but incomplete (physics envy)
- Stable environment $\Rightarrow$ stable investment policies (EMH)
- Dynamic environment $\Rightarrow$ dynamic investment policies (AMH)
- The current environment is highly dynamic
- We must adapt to changing market conditions
- "it's ~~the economy~~ environment, stupid"
- The Adaptive Markets Hypothesis provides a framework for investing, risk management, financial regulation, and more

The Traditional Investment Paradigm

In the beginning...

$$R_{it} = \alpha_i + \beta_i F_t + \epsilon_{it}$$

Implications:

- Correlation matters; diversification
- Benchmarks, performance attribution
- Passive investing
- Indexation, hedging, portable alpha
- Risk budgeting
- Framework for fiduciary duties

The Traditional Investment Paradigm

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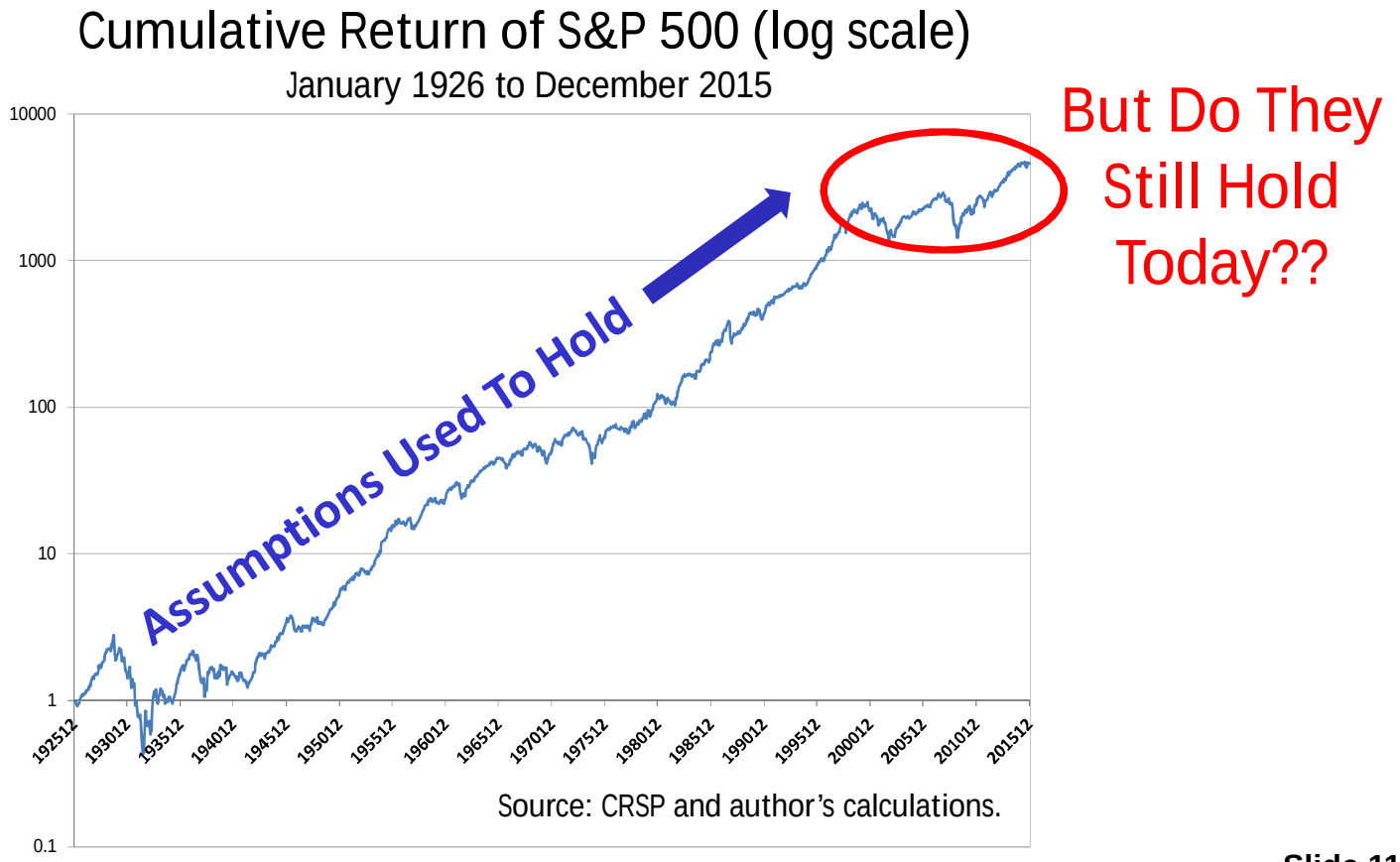
But This Framework Requires Several Key Assumptions:

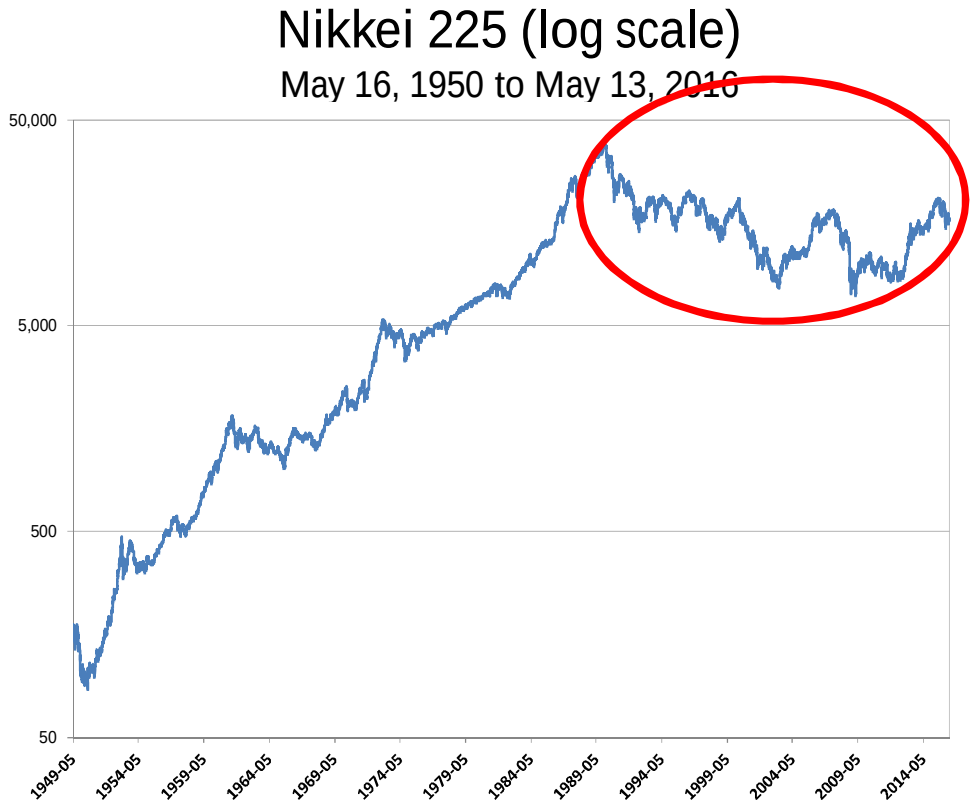
- Relationship is linear
- Relationship is static across time and circumstances
- Parameters can be accurately estimated
- Investors behave rationally
- Markets are stationary (static probability laws)
- Markets are efficient

What If Some of These Assumptions Don't Hold?

The Traditional Investment Paradigm



The Traditional Investment Paradigm



Have Alternatives Become Irrelevant?

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The Economist — Dec 22, 2012

Hedge funds

Going nowhere fast

Hedge funds have had another lousy year, to cap a disappointing decade

Dec 22nd 2012 | From the print edition

FINANCIAL TIMES — Mar 21, 2013

ft.com > companies >

March 21, 2013 7:07 pm

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'An industry in crisis': Hedge funds bleed $100 billion in 2016

Jeff Cox | @JeffCoxCNBCcom

Friday, 13 Jan 2017 | 2:02 PM ET

CNBC

SEARCH — NEW YORK POST — TIPS

Jun 13, 2017

BUSINESS

Hedge fund returns are still falling short of the S&P 500

By Carleton English — June 13, 2017 | 4:52pm

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Why are gas prices up and oil prices down?

Jonathon Trugman

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Hip new eateries coming to Upper West Side, Hell's Kitchen

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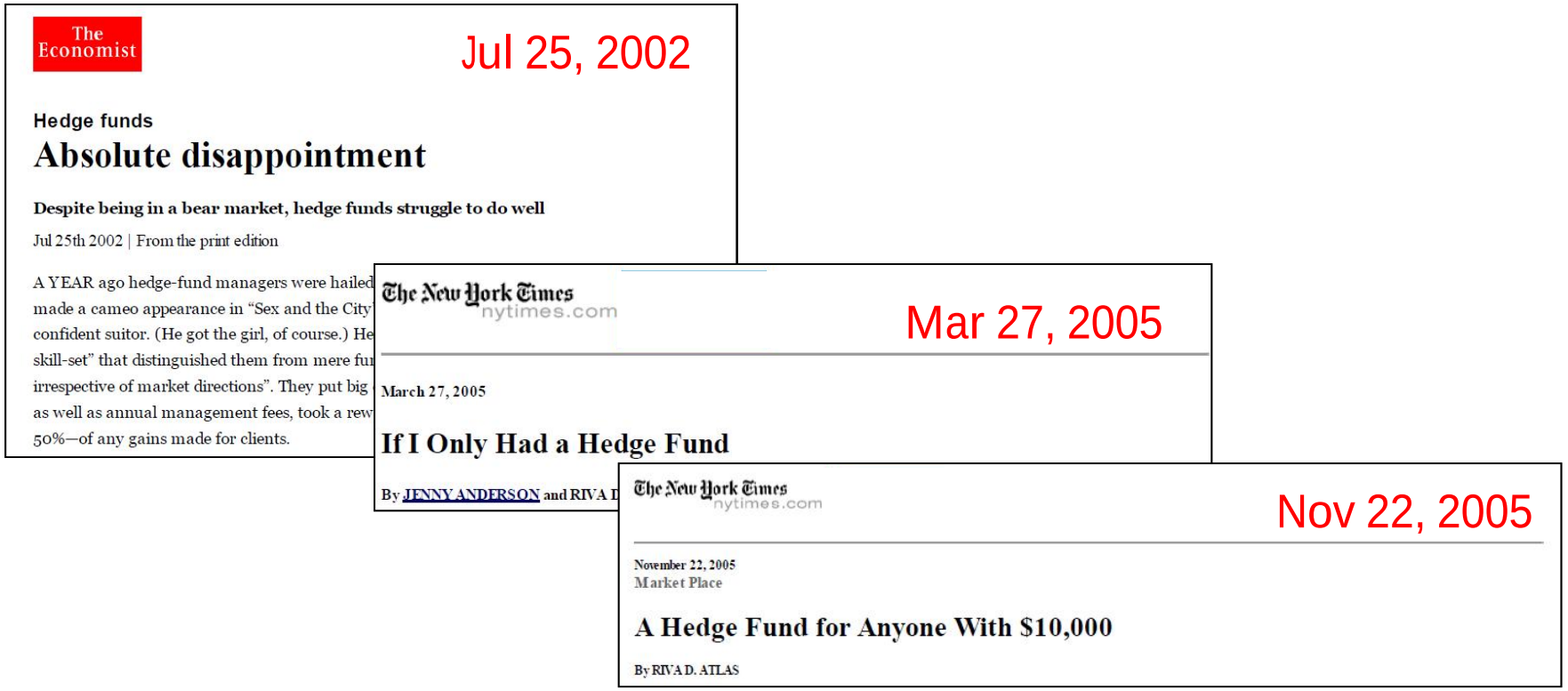
Have Alternatives Become Irrelevant?



The Economist

Jul 25, 2002

Hedge funds

Absolute disappointment

Despite being in a bear market, hedge funds struggle to do well

Jul 25th 2002 | From the print edition

A YEAR ago hedge-fund managers were hailed
made a cameo appearance in "Sex and the City
confident suitor. (He got the girl, of course.) He
skill-set" that distinguished them from mere fu
irrespective of market directions". They put big
as well as annual management fees, took a rew
50%—of any gains made for clients.

The New York Times nytimes.com

Mar 27, 2005

March 27, 2005

If I Only Had a Hedge Fund

By JENNY ANDERSON and RIVA D

The New York Times nytimes.com

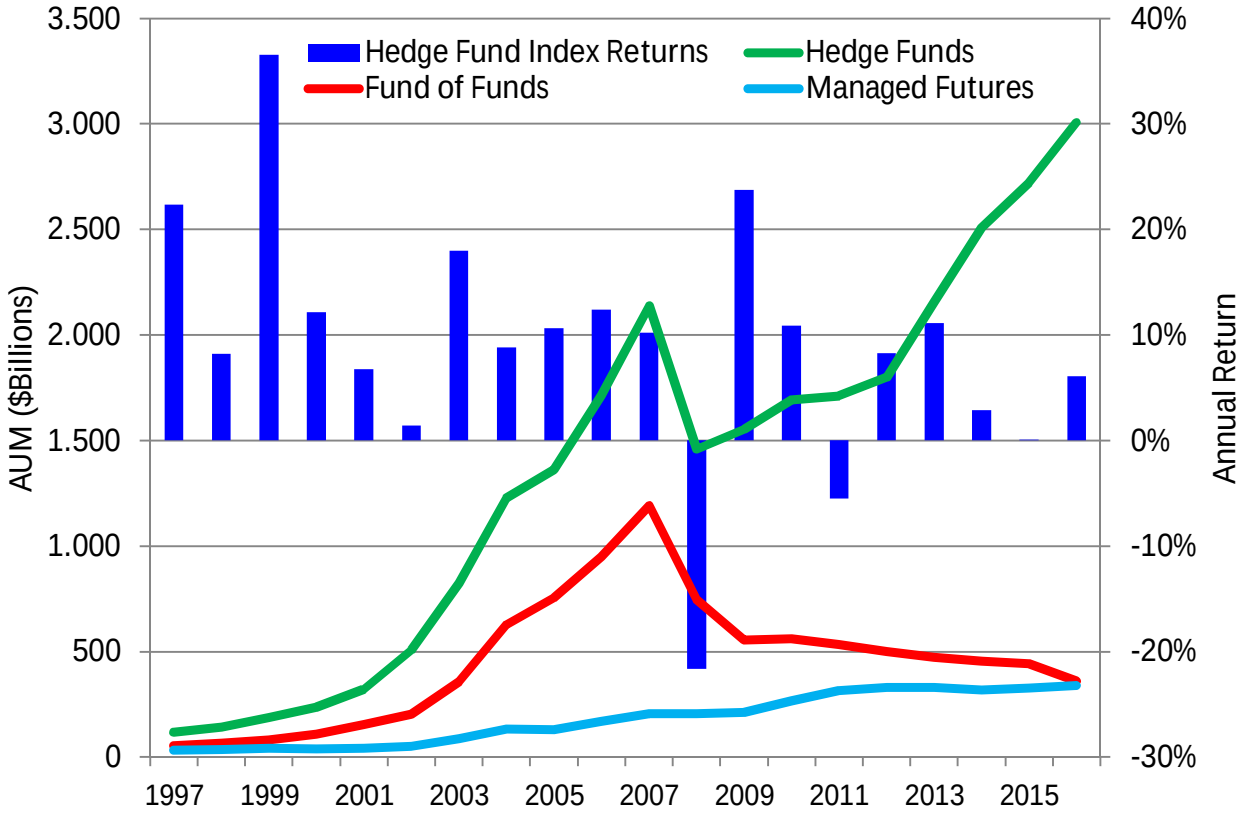
Nov 22, 2005

November 22, 2005
Market Place

A Hedge Fund for Anyone With $10,000

By RIVA D. ATLAS

“News of my death is greatly exaggerated…”



Source: www.barclayhedge.com

Pre- and Post-Crisis Hedge Fund Performance

Hedge Funds: A Dynamic Industry in Transition

Mila Getmansky,[1] Peter A. Lee,[2] and Andrew W. Lo[2,3]

Annu. Rev. Financ. Econ. 2015. 7:483–577

Keywords

hedge funds, alternative investments, investment management, long/short, illiquidity, financial crisis

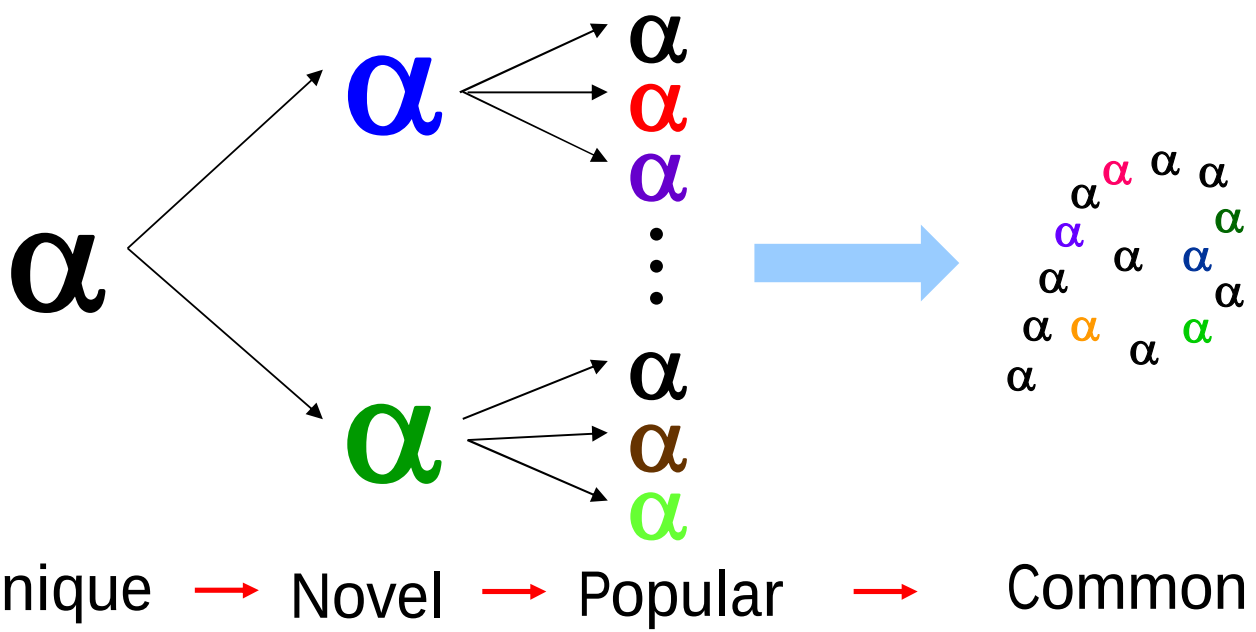
Abstract

The hedge-fund industry has grown rapidly over the past two decades, offering investors unique investment opportunities that often reflect more complex risk exposures than those of traditional investments. In this article, we present a selective review of the recent academic literature on hedge funds as well as updated empirical results for this industry. Our review is written from several distinct perspectives: the investor's, the portfolio manager's, the regulator's, and the academic's. Each of these perspectives offers a different set of insights into the financial system, and the combination provides surprisingly rich implications for the Efficient Markets Hypothesis, investment management, systemic risk, financial regulation, and other aspects of financial theory and practice.

Category	# Fund-Months	Ann. Mean (%)	Ann. SD (%)	Sharpe Ratio	Sortino Ratio	Skew.	Kurt.	MaxDD (%)	Corr. to S&P 500 (%)	ρ_1 (%)	Box-Q(3) p-value (%)
January 1996 to December 2006											
Convertible Arbitrage	7,827	8.1	4.3	0.95	1.53	-1.25	8.63	-8.70	42.9	45.9	0.0
Dedicated Short Bias	1,384	-2.3	18.8	-0.31	-0.58	0.59	4.17	-42.29	-76.8	8.9	19.1
Emerging Markets	12,673	11.6	15.7	0.47	0.69	-1.61	10.51	-49.26	58.5	28.0	0.8
Equity Market Neutral	11,537	6.5	3.0	0.82	1.82	2.05	16.06	-2.21	3.0	-11.7	22.4
Event Driven	18,565	9.4	5.2	1.02	1.55	-2.02	13.89	-12.56	54.7	32.3	0.2
Fixed Income Arbitrage	7,749	6.8	3.7	0.75	0.95	-3.56	24.53	-13.69	-1.0	42.7	0.0
Global Macro	8,948	4.7	6.1	0.14	0.26	0.46	4.17	-14.24	21.7	2.5	41.6
Long/Short Equity Hedge	69,160	11.1	9.8	0.71	1.33	0.15	5.31	-18.52	68.8	18.7	16.7
Managed Futures	13,761	5.0	9.8	0.11	0.20	0.14	2.97	-16.34	-8.7	0.0	72.9
Multi-Strategy	8,100	8.5	5.2	0.85	1.43	-0.73	5.16	-6.67	49.1	0.1	65.1
Fund of Funds	55,507	6.6	6.4	0.41	0.68	-0.33	6.51	-12.97	53.3	22.2	5.0
All Single Manager Funds	163,702	8.7	6.5	0.72	1.28	-0.26	5.46	-10.95	65.2	19.2	13.1
January 2010 to December 2014											
Convertible Arbitrage	3,940	3.0	5.7	0.52	0.96	-0.07	2.67	-10.20	50.4	10.6	63.6
Dedicated Short Bias	571	-1.5	7.3	-0.21	-0.33	-0.40	2.94	-22.56	-59.1	10.2	66.5
Emerging Markets	22,401	0.4	8.5	0.04	0.06	-0.67	3.84	-16.10	78.6	7.9	41.9
Equity Market Neutral	8,930	3.9	2.4	1.59	2.97	-0.69	4.00	-3.35	81.6	22.4	25.1
Event Driven	11,465	5.0	4.9	1.01	1.78	-0.70	3.04	-7.66	77.1	20.1	20.2
Fixed Income Arbitrage	7,202	5.0	1.7	2.90	6.29	-0.94	4.23	-1.03	54.1	-10.0	68.3
Global Macro	16,824	3.7	2.5	1.46	3.29	0.12	3.54	-2.03	63.2	9.7	68.7
Long/Short Equity Hedge	66,758	4.7	6.2	0.73	1.27	-0.54	3.42	-10.67	89.0	11.1	64.2
Managed Futures	23,471	2.8	7.5	0.36	0.71	0.11	2.28	-14.48	25.4	-12.4	78.4
Multi-Strategy	57,505	5.2	2.5	2.06	4.06	-0.87	5.52	-3.20	80.9	16.3	23.2
Fund of Funds	139,161	1.7	3.5	0.46	0.78	-0.55	2.73	-7.42	79.3	11.6	59.1
All Single Manager Funds	233,194	4.2	4.2	1.00	1.84	-0.39	3.63	-6.36	85.3	11.7	46.8

Source: Getmansky, Lee, Lo (2015, Table 14)

Hedge-Fund Strategy Life Cycle

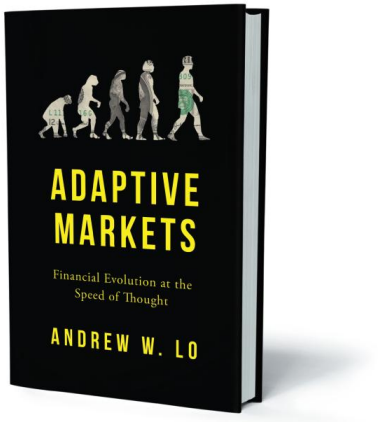


The Adaptive Markets Hypothesis

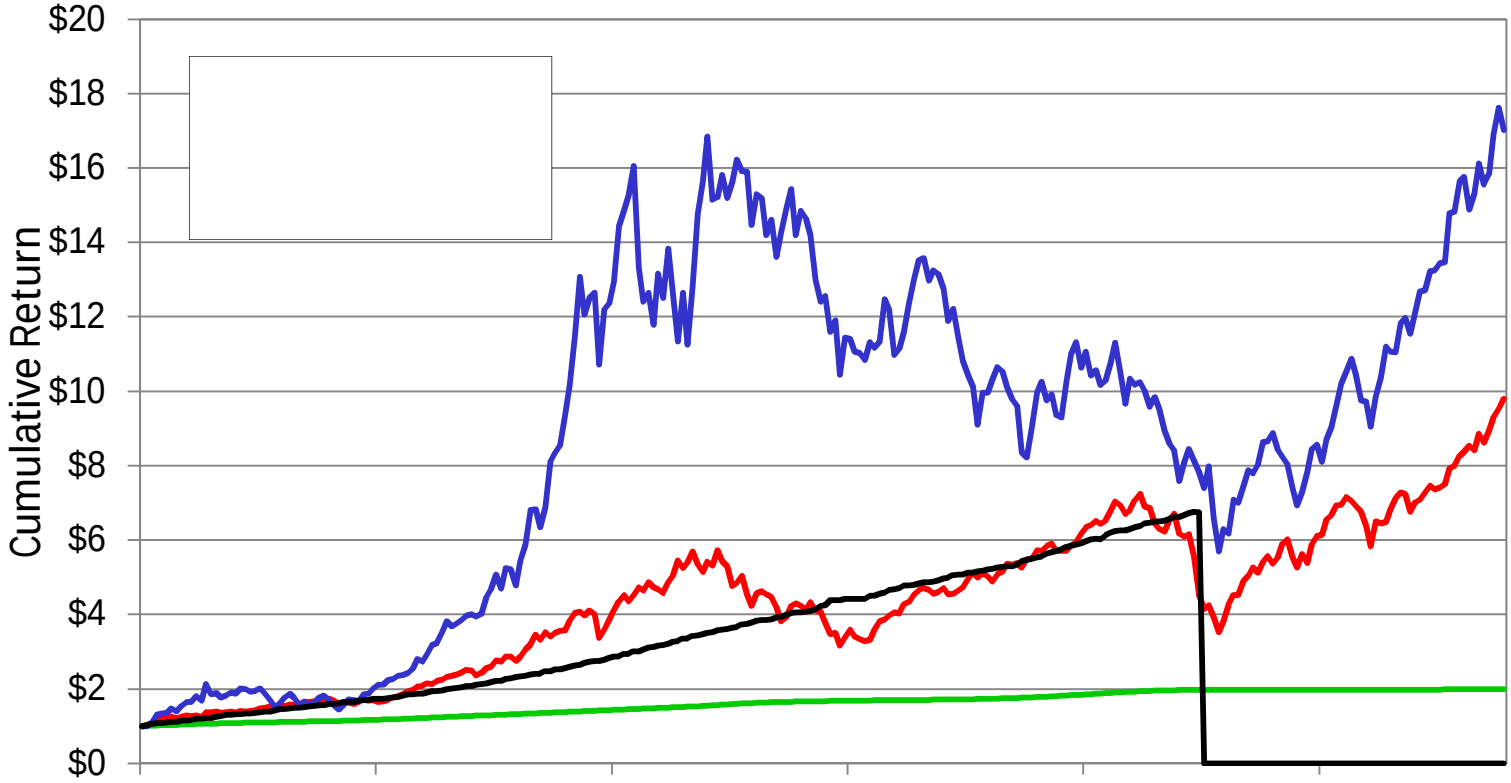
"Nothing makes sense in biology except in the light of evolution," Dobzhansky (1973)

"Nothing makes sense in the hedge fund industry except in the light of the Adaptive Markets Hypothesis," Lo (2017)

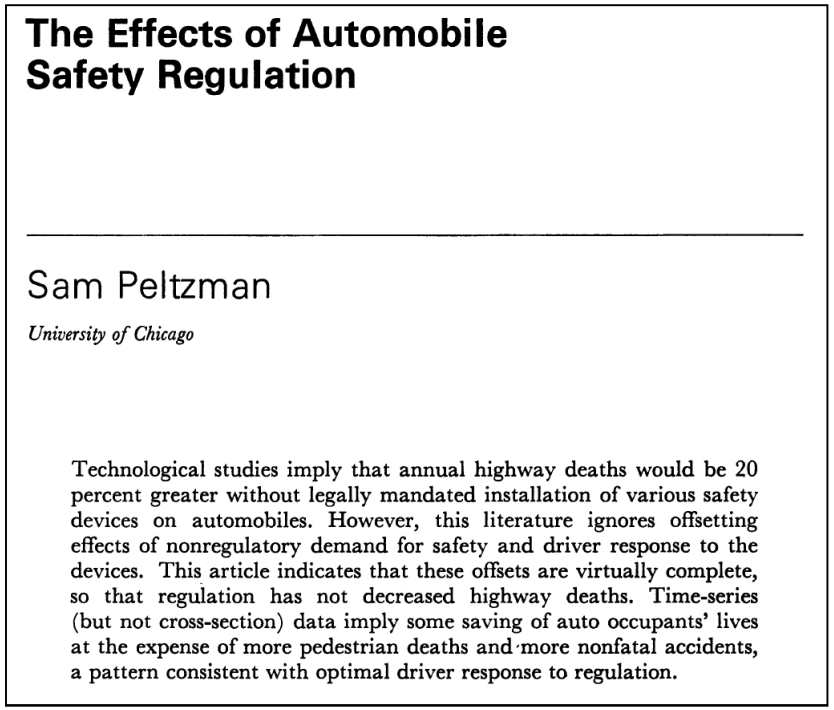
1. Individuals act in their own self-interest
2. Individuals make mistakes ("satisfice")
3. Individuals learn and adapt (heuristics)
4. Competition drives adaptation and innovation
5. Evolution determines market dynamics



What Do Investors Want?



Risk Perception and Adaptive Behavior



The Effects of Automobile Safety Regulation

Sam Peltzman

University of Chicago

Technological studies imply that annual highway deaths would be 20 percent greater without legally mandated installation of various safety devices on automobiles. However, this literature ignores offsetting effects of nonregulatory demand for safety and driver response to the devices. This article indicates that these offsets are virtually complete, so that regulation has not decreased highway deaths. Time-series (but not cross-section) data imply some saving of auto occupants' lives at the expense of more pedestrian deaths and more nonfatal accidents, a pattern consistent with optimal driver response to regulation.

Journal of Political Economy 83(1975), 677–726.

Risk Perception and Adaptive Behavior

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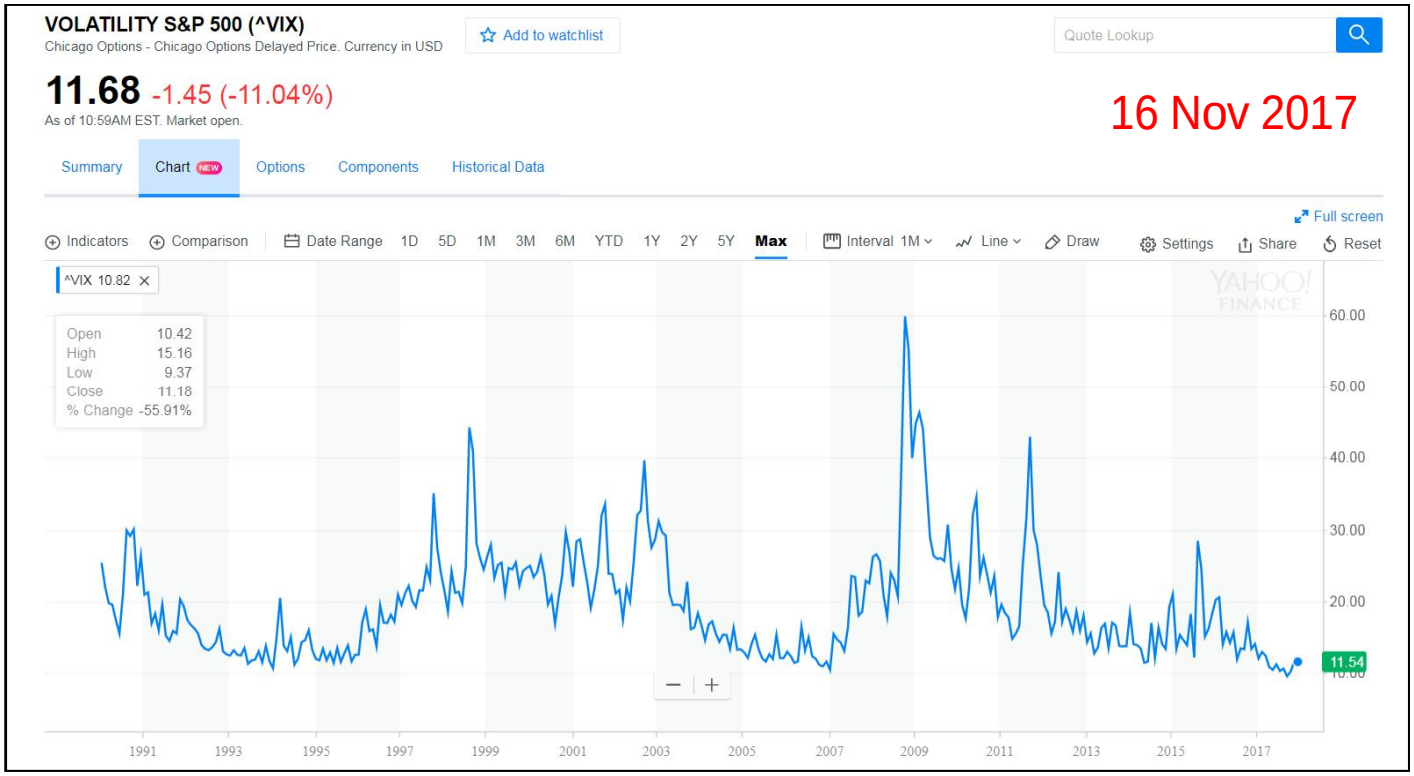
Automobile Safety Regulation and the Incentive to Drive Recklessly: Evidence from NASCAR

Russell S. Sobel* and Todd M. Nesbit†

When safety regulation makes automobiles safer, drivers may drive more recklessly, partially or completely offsetting effects on the overall level of safety. Evidence of these offsetting effects has been hard to find, however, primarily because of the aggregate nature of accident data. In this paper we explore how changes in the safety of automobiles used in the National Association for Stock Car Auto Racing (NASCAR) has altered the incentive of drivers to drive recklessly. This unique data set allows more accurate and objective measurement of the necessary variables to test for these effects at a microlevel. Our results strongly support the presence of these offsetting behavioral effects.

Southern Economic Journal 74(2007), 71–84.

Implications for the Current Ecosystem



A New Investment Paradigm Is Emerging

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Efficient Markets

- Long-only constraint
- Diversify across stocks and bonds
- Market-cap-weighted indexes
- Manage risk via asset allocation
- Alpha vs. market beta
- Markets are efficient
- Equities in the long run

Adaptive Markets

- Long/short strategies
- Diversify across more asset classes and strategies
- Passive transparent indexes
- Manage risk via active volatility scaling algorithms
- Alphas $\Rightarrow$ multiple betas
- Markets are adaptive
- "In the long run we're all dead," but make sure the short run doesn't kill you first

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What Is An Index??

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- Market-cap-weighted portfolio?

Jack Bogle (1997) on the Origins of the Vanguard Index Trust:

> The basic ideas go back a few years earlier. In 1969–1971, Wells Fargo Bank had worked from academic models to develop the principles and techniques leading to index investing. John A. McQuown and William L. Fouse pioneered the effort, which led to the construction of a $6 million index account for the pension fund of Samsonite Corporation. With a strategy based on an equal-weighted index of New York Stock Exchange equities, its execution was described as "a nightmare". The strategy was abandoned in 1976, replaced with a market-weighted strategy using the Standard & Poor's 500 Composite Stock Price Index. The first such models were accounts run by Wells Fargo for its own pension fund and for Illinois Bell.

What Is An Index??

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- Market-cap weighting requires little trading
- "Buy-and-hold" portfolio
- What if trading were cheaper, faster, and automatable?

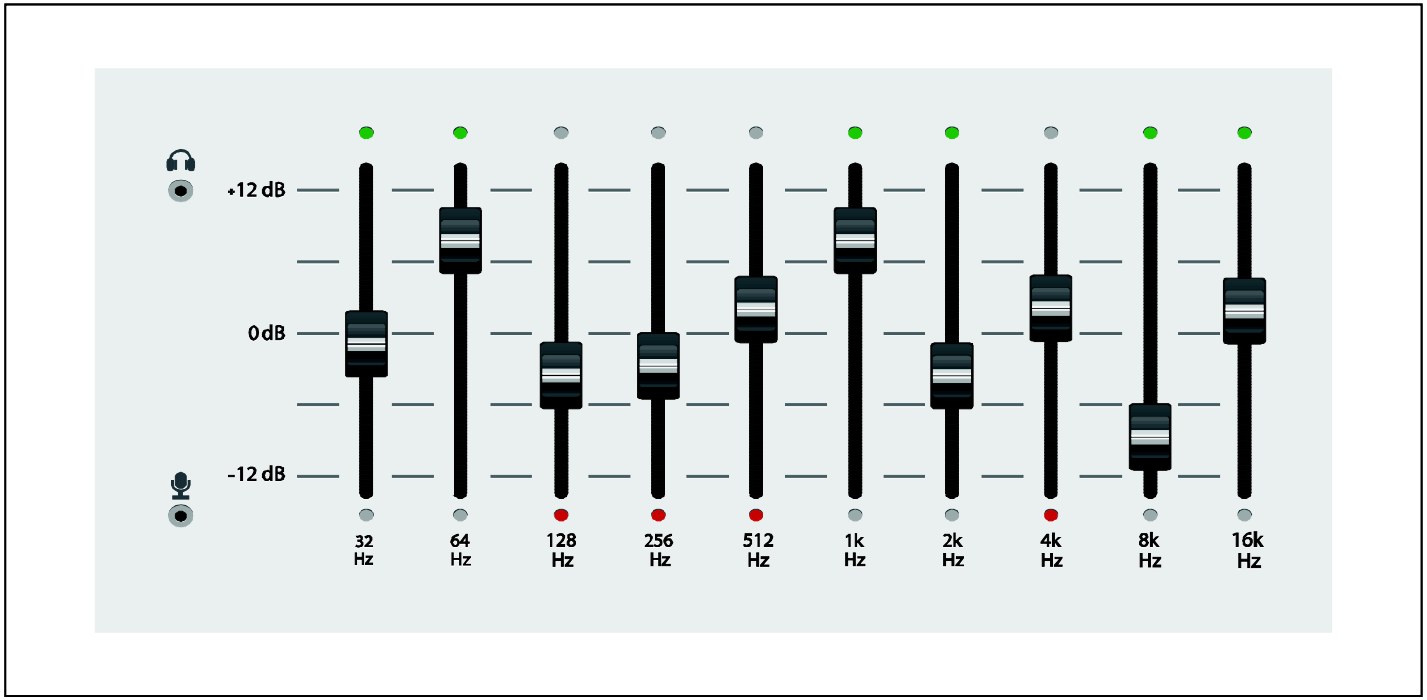
In·dex \ˈin-ˌdeks\ *noun*

An index is any portfolio strategy satisfying three properties: (1) it is completely transparent; (2) it is investable; and (3) it is totally systematic.

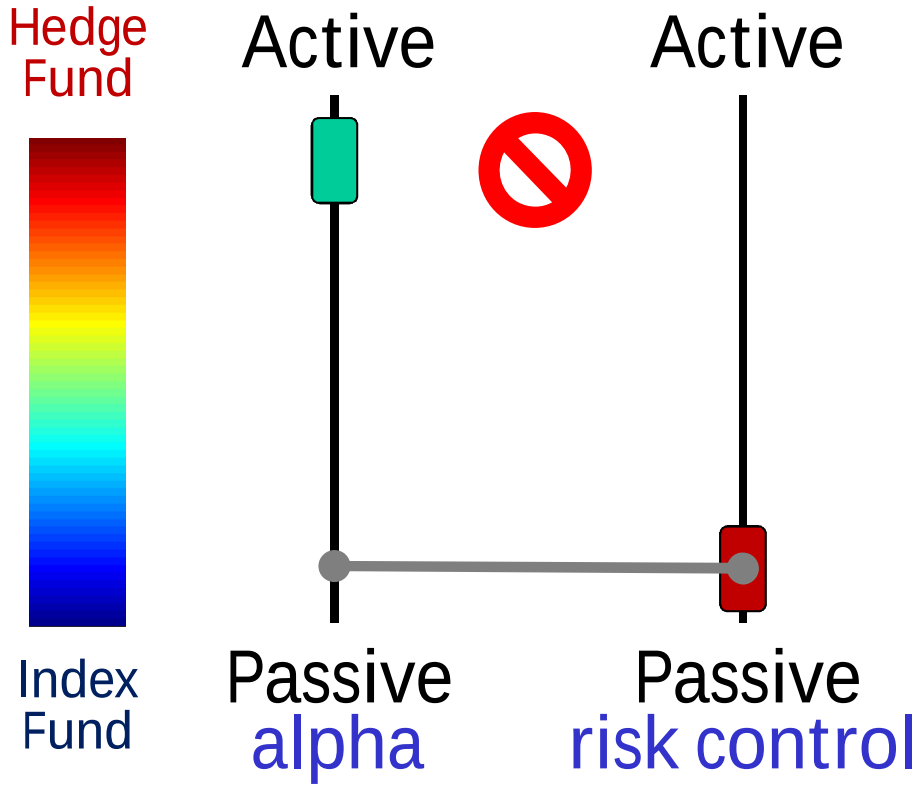
What Is An Index??

	Yes	No	Maybe
Value-weighted average?	✓		
Equal-weighted average?	✓		
Target-date fund?	✓		
FHFA House Price Index?		✓	
Hedge Fund Index?		✓	
Trend-following futures?			✓
Risk-managed large-cap core?			✓

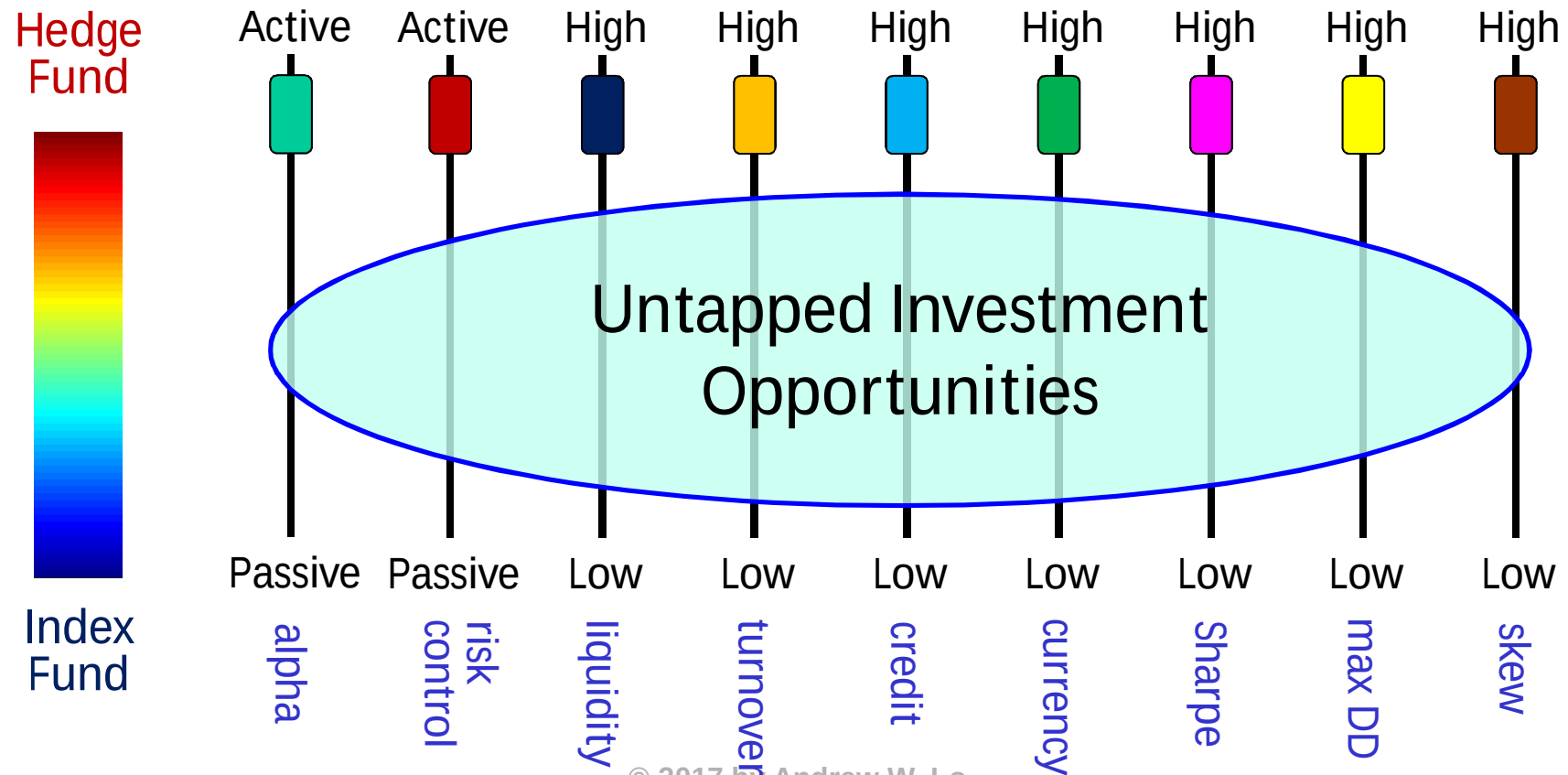
What Is An Index??



What Is An Index??



Full-Spectrum Investing



The Opportunity: Precision Indexes

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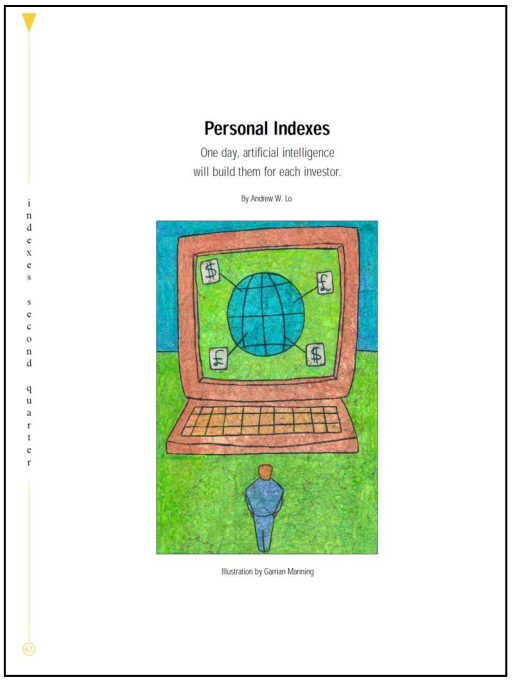
- Instead of the DowJones30®, FTSE100®, or S&P500®, imagine investing in the:
  - RichardZeckhauser30®, ArnieWood100®, or LarrySummers500®
- Imagine if such portfolios took into account income, expenses, age, health, taxes, and behavior
- Imagine if such portfolios were automated
- We have the hardware and software; we need the algorithms

This Idea Is Not New



"Artificial intelligence and active management are not at odds with indexation, but instead imply a more sophisticated set of indexes and portfolio management policies for the typical investor, something each of us can look forward to, perhaps within the next decade."

– Andrew W. Lo, *Journal of Indexes* Q2, 2001

So What's Missing?

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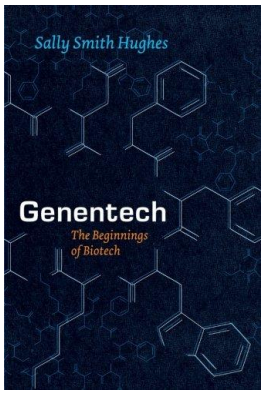
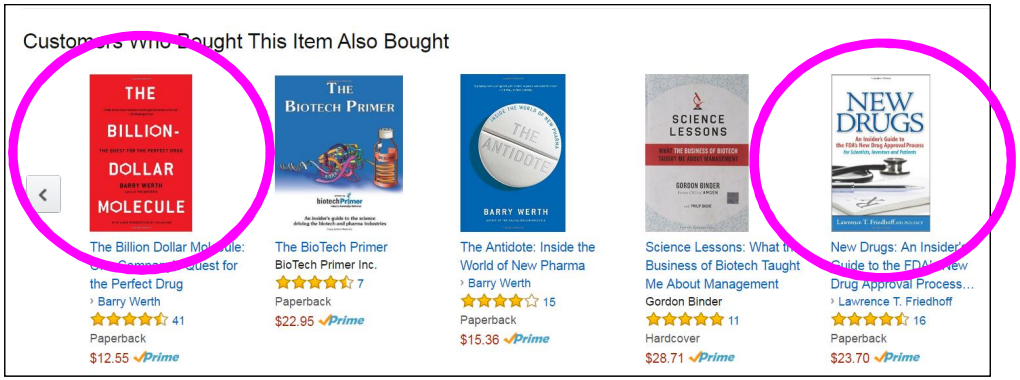
...Not Artificial Intelligence

Artificial Humanity

- We need an algorithm for investor behavior so we can counterbalance our least productive actions (e.g., loss aversion, overconfidence, overreaction, etc.)

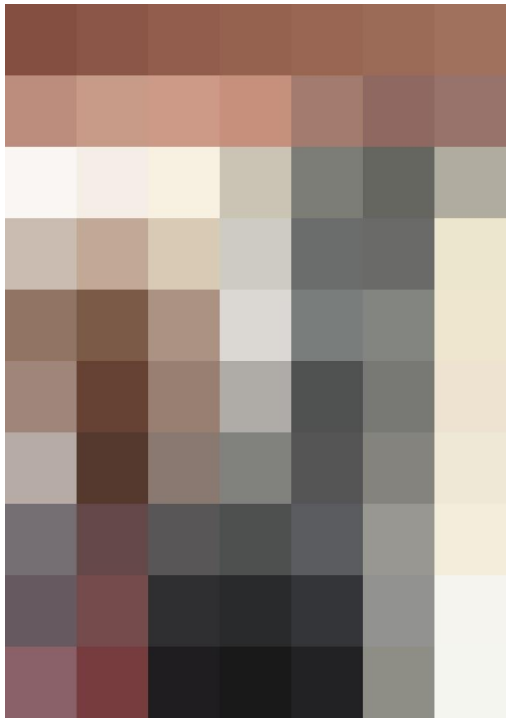
Artificial vs. Natural Intelligence

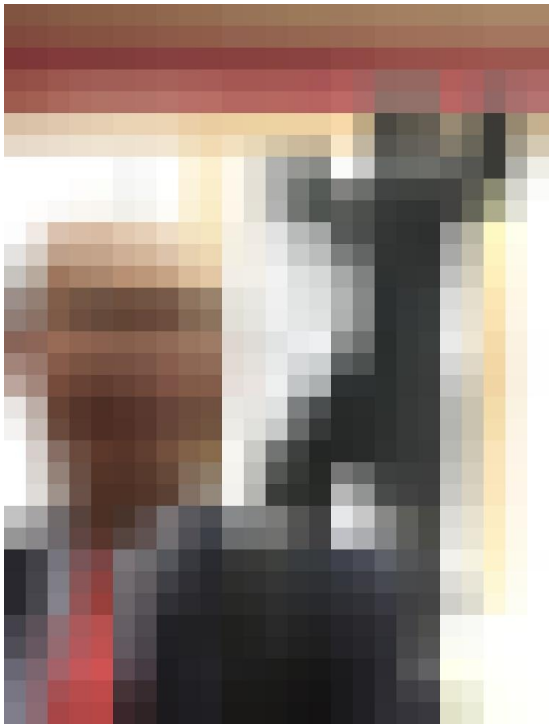




- Expert systems vs. machine-learning techniques
- Expensive storage $\Rightarrow$ small data, complex code
- Cheap storage $\Rightarrow$ big data, simple code
- This is closer to natural intelligence! Narrative vs. facts

Friend or Foe?





Friend or Foe?

José Susan

- Gender and sex orientation (4)
- Race/ethnicity (4)
- Age (4)
- Current home state (50)
- Religious affiliation (4)
- Political party (3)
- Economic status (3)
- Education (3)

345,600 Possible Types!

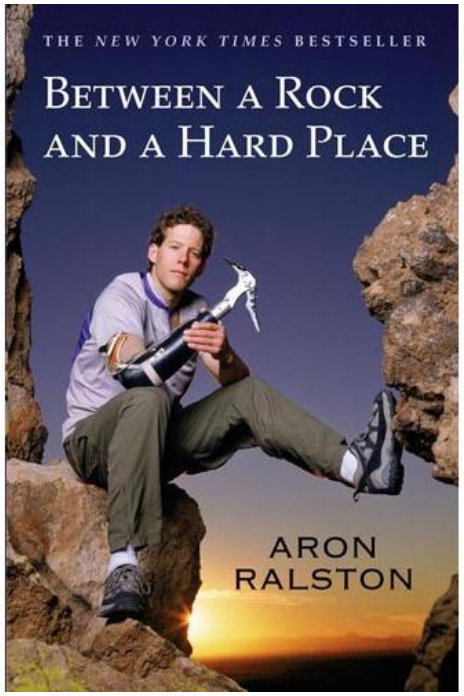
But Beware of Learning With Sparse Data


CAMBRIDGE SAVINGS BANK
Make The BETTER Mo


HARVARD APPAREL
GIFTS
USED BOOKS

Evolution at the Speed of Thought

Aron Lee Ralston, 4/26/03



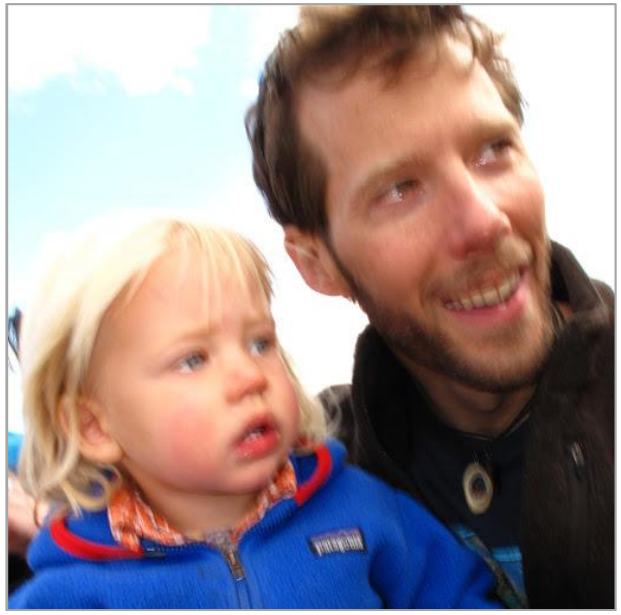
Evolution at the Speed of Thought

A blond three-year-old boy in a red polo shirt comes running across a sunlit hardwood floor in what I somehow know is my future home. By the same intuitive perception, I know the boy is my own. I bend to scoop him into my left arm, using my handless right arm to balance him, and we laugh together as I swing him up to my shoulder… Then, with a shock, the vision blinks out. I'm back in the canyon, echoes of his joyful sounds resonating in my mind, creating a subconscious reassurance that somehow I will survive this entrapment. Despite having already come to accept that I will die where I stand before help arrives, now I believe I will live.

That belief, that boy, changes everything for me.

– Aron Lee Ralston (2005)

Evolution at the Speed of Thought



We Need New Narratives In Finance!

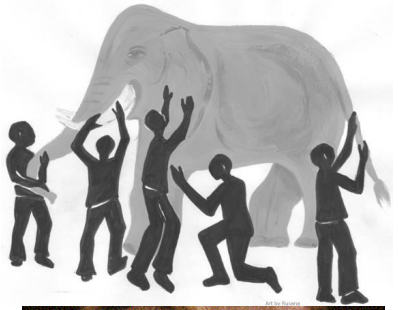
Conclusion

MIT LFE

"It Takes A Theory To Beat A Theory"

- Standard paradigm is not wrong, just incomplete
- Human behavior has been stable for 60,000 years
- Our environment has changed rapidly
- The mismatch can create challenges
- Evolution determines dynamics
- Competition, selection, innovation

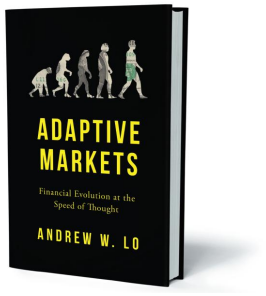
How Adaptive Are You?



Thank You!

For more on Adaptive Markets:

- http://bit.ly/2t3Sre6 (MIT Sloan Lecture)
- http://bit.ly/2ty6Rqp (Clarendon Lectures)
- http://alo.mit.edu (website)
- @AndrewWLo (Twitter)



Additional References

- Abbe, E., Khandani, A. and A. Lo, 2012, "Privacy-Preserving Methods for Sharing Financial Risk Exposures", *American Economic Review: Papers and Proceedings* 102, 65–70.
- Bisias, D., Flood, M., Lo, A., and S. Valavanis, 2012, "A Survey of Systemic Risk Analytics", *Annual Review of Financial Economics* 4, 255–296.
- Brennan, T. and A. Lo, 2012, "Do Labyrinthine Legal Limits on Leverage Lessen the Likelihood of Losses? An Analytical Framework", *Texas Law Review* 90, 1775–1810.
- Brennan, T. and A. Lo, 2014, "Dynamic Loss Probabilities and Implications for Financial Regulation", to appear in *Yale Journal on Regulation*.
- Butaru, F., Chen, Q., Clark, B., Das, S., Lo, A., and A. Siddique, 2015, "Risk and Risk Management in the Credit Card Industry" (June 14, 2015). Available at SSRN: http://ssrn.com/abstract=2618746 or http://dx.doi.org/10.2139/ssrn.2618746
- Campbell, J., Lo, A. and C. MacKinlay, 1997, *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press.
- Chan, N., Getmansky, M., Haas, S. and A. Lo, 2007, "Systemic Risk and Hedge Funds", in M. Carey and R. Stulz, eds., *The Risks of Financial Institutions and the Financial Sector*. Chicago, IL: University of Chicago Press.
- Getmansky, M., Lo, A. and I. Makarov, 2004, "An Econometric Analysis of Serial Correlation and Illiquidity in Hedge-Fund Returns", *Journal of Financial Economics* 74, 529–609.
- Getmansky, M., Lo, A. and S. Mei, 2004, "Sifting Through the Wreckage: Lessons from Recent Hedge-Fund Liquidations", *Journal of Investment Management* 2, 6–38.
- Khandani, A., Kim, A. and A. Lo, 2010, "Consumer Credit Risk Models via Machine-Learning Algorithms", *Journal of Banking & Finance* 34, 2767–2787.
- Khandani, A. and A. Lo, 2007, "What Happened to the Quants In August 2007?", *Journal of Investment Management* 5, 29–78.
- Khandani, A. and A. Lo, 2009, "What Happened to the Quants in August 2007?: Evidence from Factors and Transactions Data", to appear in Journal of Financial Markets.
- Khandani, A., Lo, A., and R. Merton, 2013, "Systemic Risk and the Refinancing Ratchet Effect", *Journal of Financial Economics* 108, 29–45.

Additional References

- Kirilenko, A., and A. Lo, 2013, "Moore's Law vs. Murphy's Law: Algorithmic Trading and Its Discontents", *Journal of Economic Perspectives* 27, 51–72.
- Li, W., Azar, Larochelle, D., Hill, P. and A. Lo, 2015, "Law Is Code: A Software Engineering Approach to Analyzing the United States Code", *Journal of Business & Technology Law* 10, 297–374.
- Lo, A., 2001, "Risk Management for Hedge Funds: Introduction and Overview", *Financial Analysts Journal* 57, 16–33 Lo, A., 2002, "The Statistics of Sharpe Ratios", *Financial Analysts Journal* 58, 36–50.
- Lo, A., 2002, "The Statistics of Sharpe Ratios", *Financial Analysts Journal* 58, 36–50.
- Lo, A., 2004, "The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective", *Journal of Portfolio Management* 30, 15–29.
- Lo, A., 2005, *The Dynamics of the Hedge Fund Industry*. Charlotte, NC: CFA Institute.
- Lo, A., 2005, "Reconciling Efficient Markets with Behavioral Finance: The Adaptive Markets Hypothesis", *Journal of Investment Consulting* 7, 21–44.
- Lo, A., 2007, "Where Do Alphas Come From?: Measuring the Value of Active Investment Management", to appear in *Journal of Investment Management*.
- Lo, A., 2012, "Reading About the Financial Crisis: A 21-Book Review", *Journal of Economic Literature* 50, 151–178.
- Lo, A., 2013, "Fear, Greed, and Financial Crises: A Cognitive Neurosciences Perspective", in J.P. Fouque and J. Langsam, eds., *Handbook of Systemic Risk*, Cambridge University Press.
- Lo, A. and C. MacKinlay, 1999, *A Non-Random Walk Down Wall Street*. Princeton, NJ: Princeton University Press.
- Lo, A., Petrov, C. and M. Wierzbicki, 2003, "It's 11pm—Do You Know Where Your Liquidity Is? The Mean-Variance-Liquidity Frontier", *Journal of Investment Management* 1, 55–93.