

Trigonometric Identities

Original available online at <https://bithole.dev/misc/trig-sheet.pdf>.

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \quad \csc u = \frac{1}{\sin u}$$

$$\cos u = \frac{1}{\sec u} \quad \sec u = \frac{1}{\cos u}$$

$$\tan u = \frac{1}{\cot u} \quad \cot u = \frac{1}{\tan u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$\tan^2 u + 1 = \sec^2 u$$

$$\cot^2 u + 1 = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Odd Identities

$$\sin(-u) = -\sin u \quad \cot(-u) = -\cot u$$

$$\tan(-u) = -\tan u \quad \csc(-u) = -\csc u$$

Even Identities

$$\cos(-u) = \cos u \quad \sec(-u) = \sec u$$

Sum/Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Identities

$$\sin(2u) = 2 \sin u \cos u$$

$$\begin{aligned}\cos(2u) &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u\end{aligned}$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Identities

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$

$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

Product-to-Sum Identities

$$\sin u \sin v = \frac{1}{2} [\cos(u - v) - \cos(u + v)]$$

$$\cos u \sin v = \frac{1}{2} [\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u + v) - \sin(u - v)]$$

Sum-to-Product Identities

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$