

# Digital Design

## Week 6: Finite State Machines



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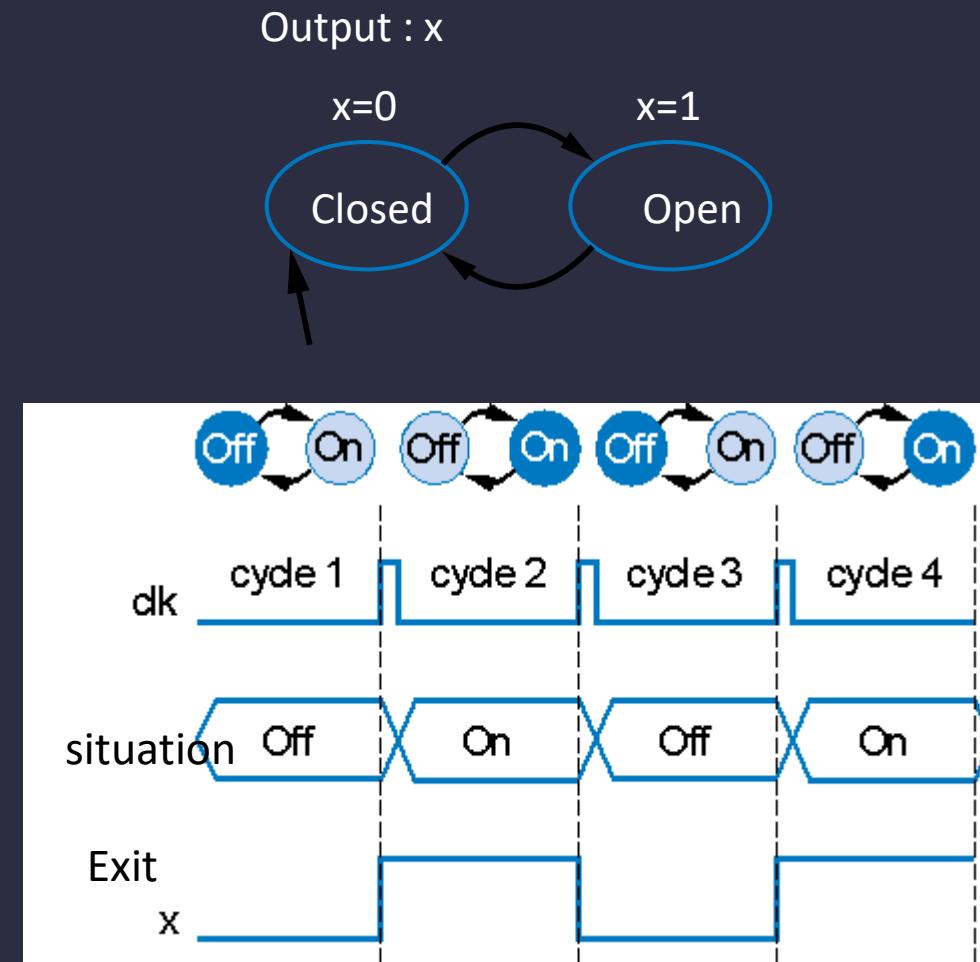
# Course

- Finite State Machines



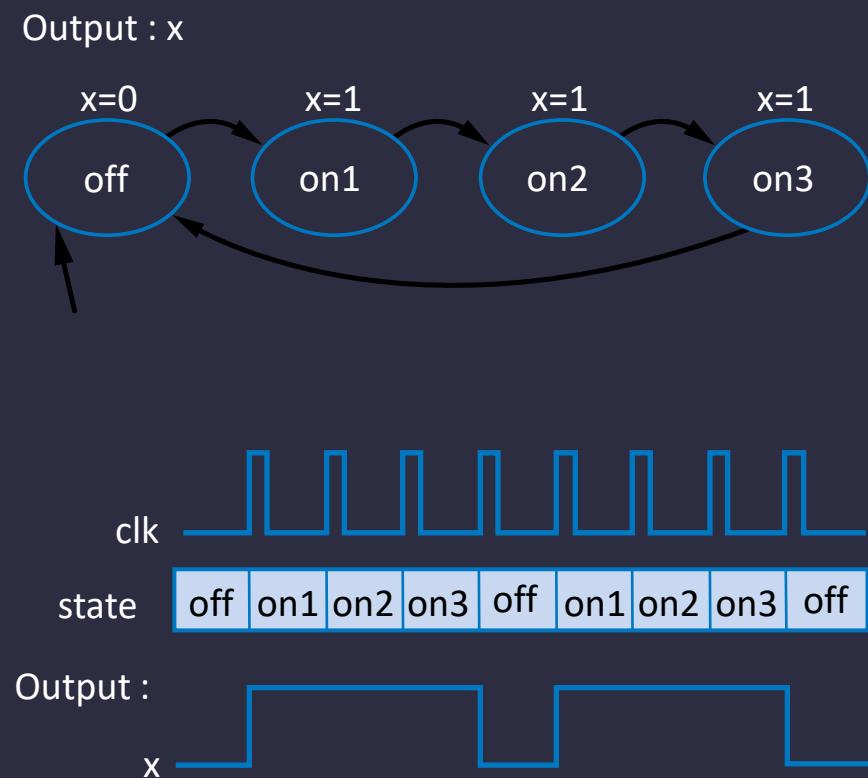
# Determining the Behavior of the Sequential Circuit FSM

- Finite-State Machine (FSM)
  - It is a representation of the behavior of the sequential circuit according to the states.
  - Lists states and transitions between states
    - Example : Let there be an output signal named X and each clock change value in cycle
    - Two states : “Off” ( $x=0$ ), and “On” ( $x=1$ )
    - There are transitions from the Off state to the On state and from the On state to the Off state.
    - The initial state is indicated by an arrow.



# FSM Example : 0,1,1,1, repetition

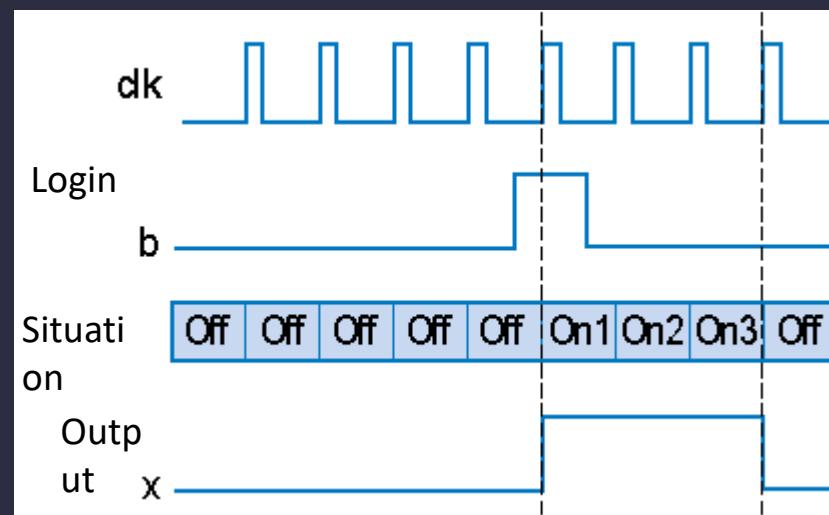
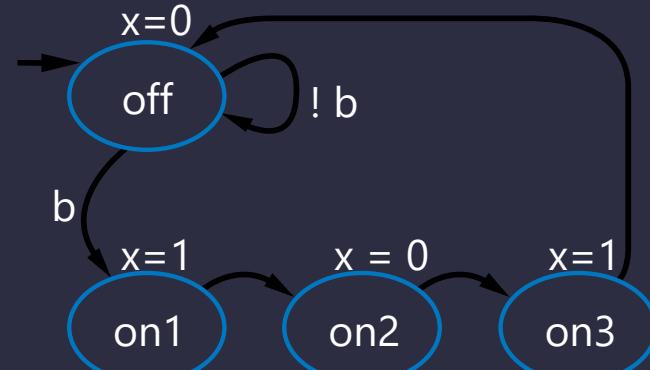
- In order  $0, 1, 1, 1, 0, 1, 1, 1, \dots$  A circuit that gives its outputs will be designed.
  - Each value is a clock It appears in the cycle .
- Can be designed with FSM
  - There are 4 states



# FSM Example

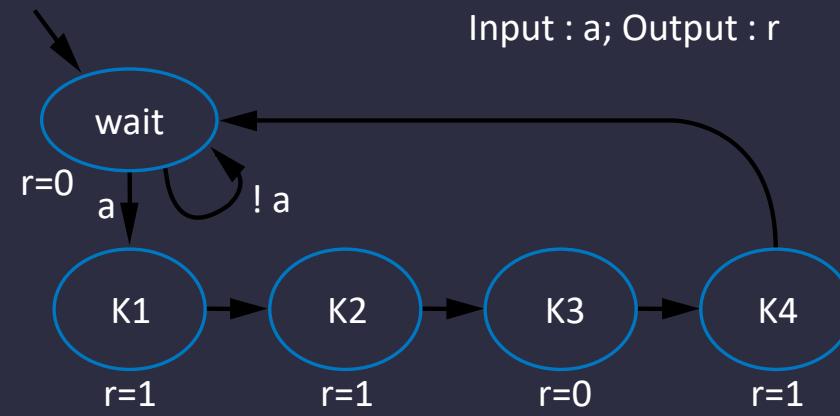
take a one-bit input named b,  
 Output pattern 101 and waits for  
 input b again.

Inputs : b; Outputs : x



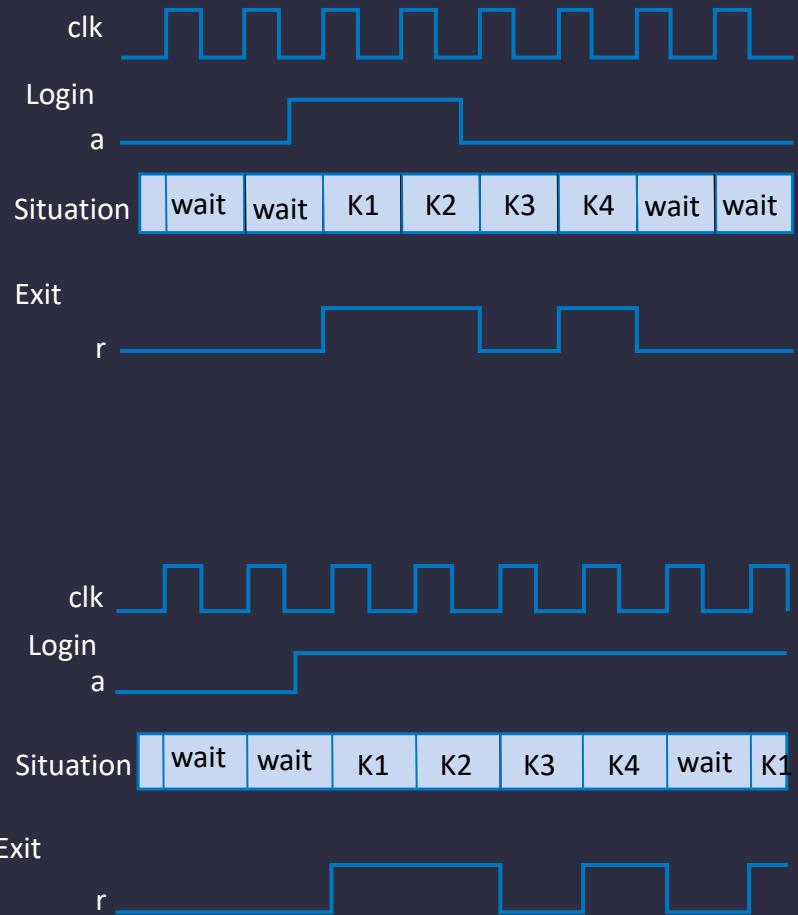
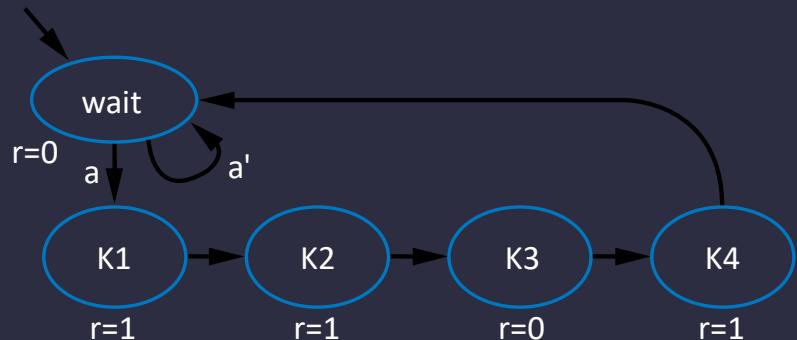
# FSM Example: Car Key

- Car keys contain a chip that holds the key's identity.
  - When the key is inserted in the car, it asks the key for identification.
  - Sends key credential, if not correct the tool won't start
- FSM
  - Wait for request ( $a=1$ )
  - Send ID ( For example , 1101 )



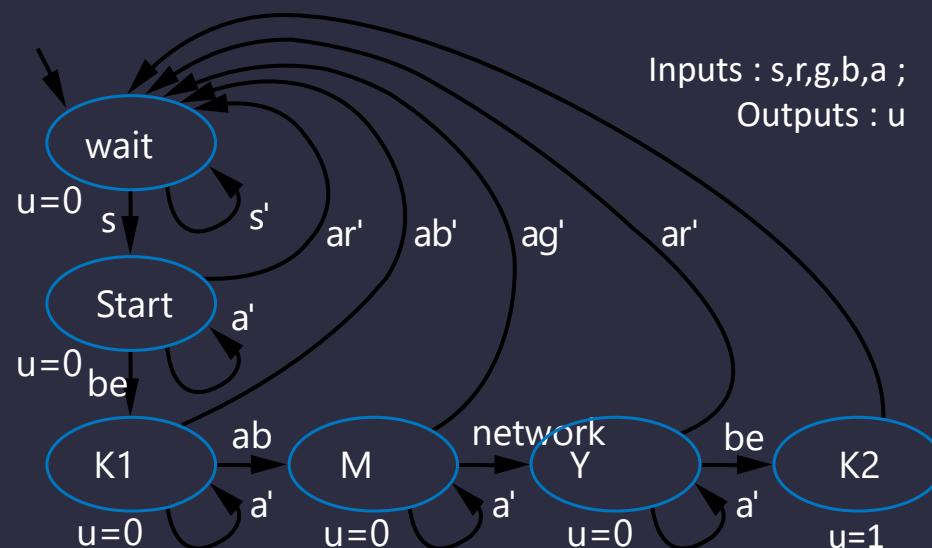
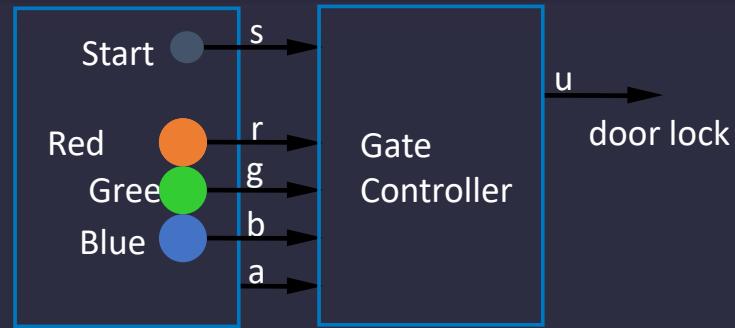
# FSM Example: Car Key

- FSM Timings

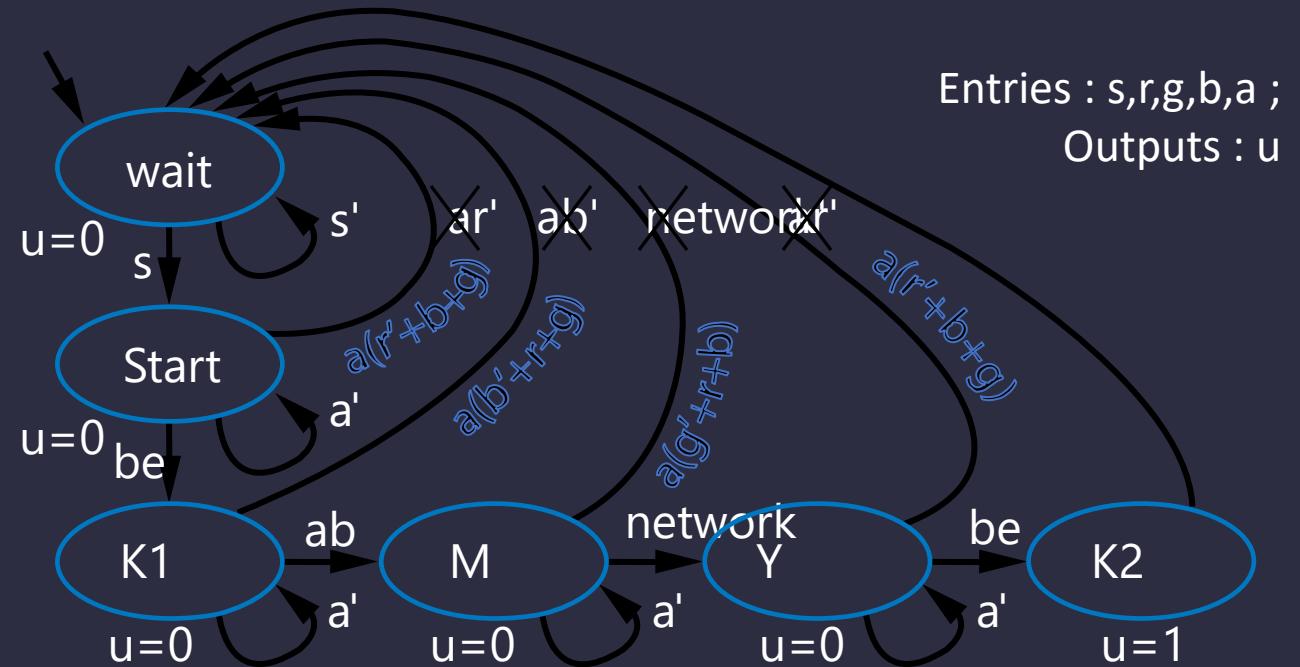


# FSM Example: Code Detector

- A door is only opened when the following button sequence is pressed ( $u=1$ )
  - start , red , blue , green , red
- $s, r, g, b$ 
  - If input a, buttons of other color
- FSM
  - While in wait state, start when start
  - When start is pressed
    - If it's red, go to K1
    - Then, if it's blue, M
    - Then, if green, Y
    - Then, if it's red, K2
      - In this case, the signal to open the door is generated ( $u=1$ )
  - Any wrong input will revert to wait state



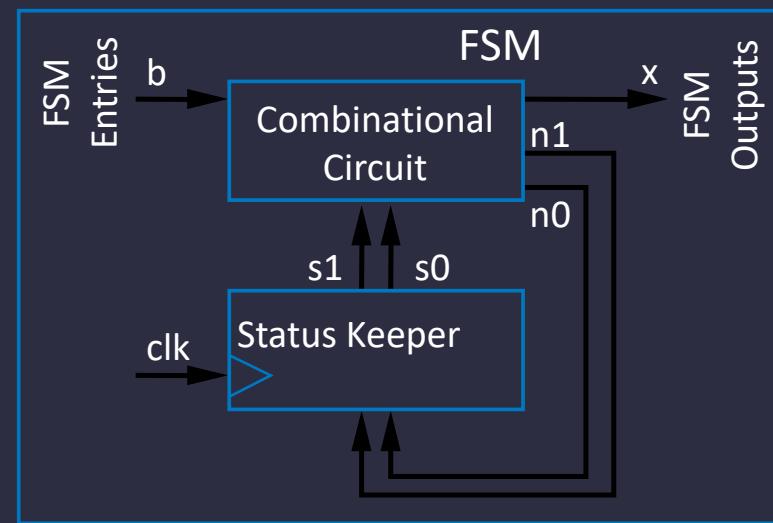
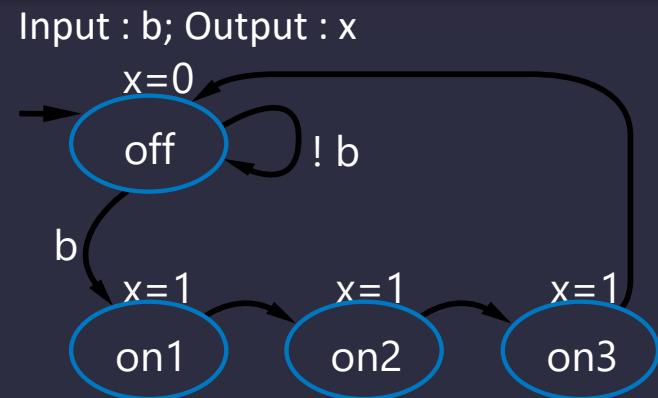
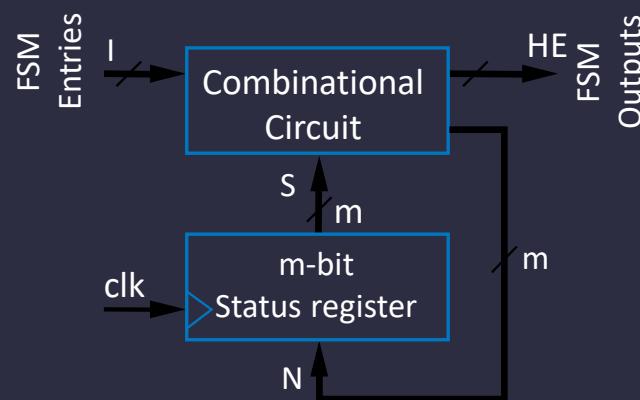
# FSM Example: Code Detector, Improvements



In the new state machine, pressing the wrong button returns to the "Wait" state.

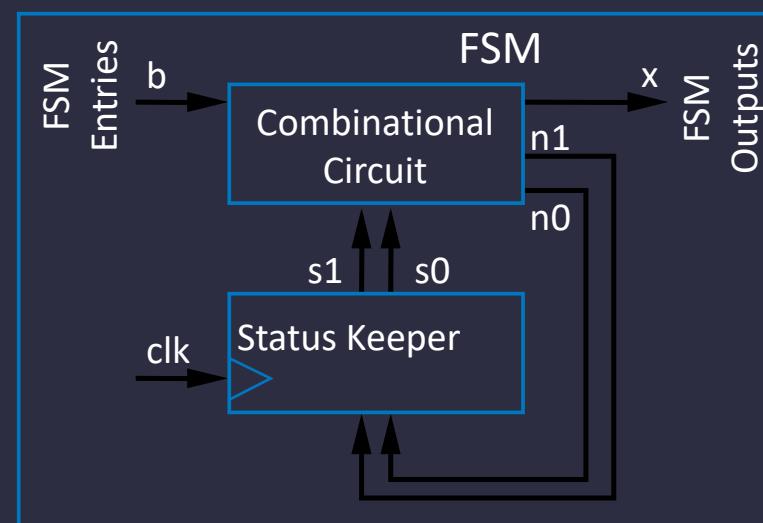
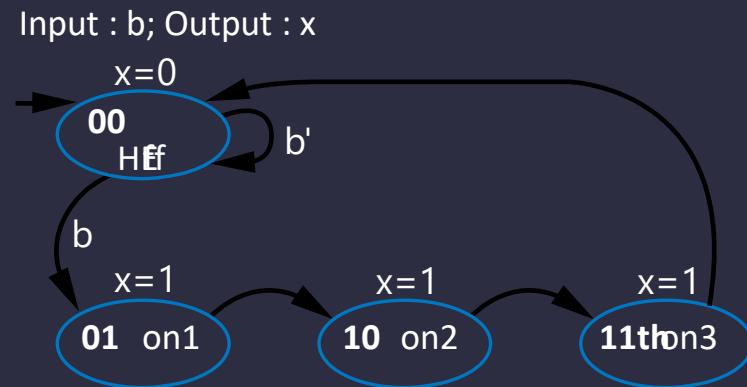
# FSM Architecture

- How is FSM implemented as a sequential circuit ?
  - Status register - to keep the current state
  - Combinational circuit – to calculate output and next state



# FSM Design Example

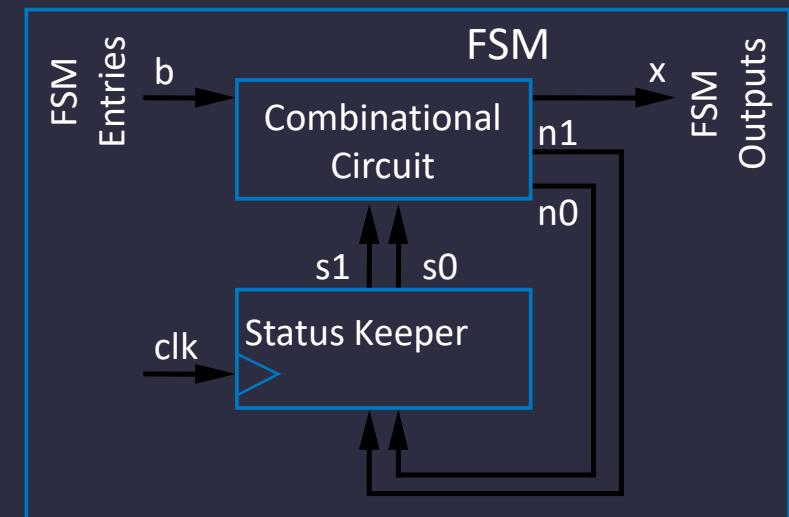
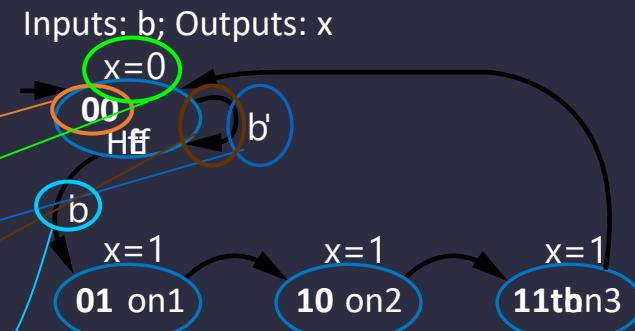
- Step 1: Analyze the requirements
- Step 2: Extract architecture
  - 2-bit state register (for 4 states )
  - Input b, output x
  - Next status signals n<sub>1</sub>, n<sub>0</sub>
- Step 3: Encode the states



# FSM Design Example

- Step 4: Create a Truth Table

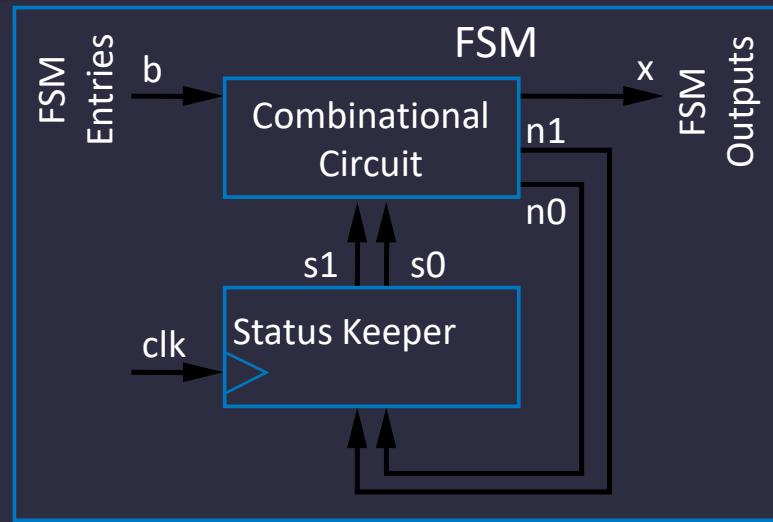
	Entries			Outputs		
	$s_1$	$s_0$	$b$	$x$	$n_1$	$n_0$
$Off$	0	0	0	0	0	0
	0	0	1	0	0	1
$On1$	0	1	0	1	1	0
	0	1	1	1	1	0
$On2$	1	0	0	1	1	1
	1	0	1	1	1	1
$On3$	1	1	0	1	0	0
	1	1	1	1	0	0



# FSM Design Example

- Step 5: Combinational Circuit Implementation

	Entries			Outputs		
	s1	s0	b	x	n1	n0
Off	0	0	0	0	0	0
	0	0	1	0	0	1
On1	0	1	0	1	1	0
	0	1	1	1	1	0
On2	1	0	0	1	1	1
	1	0	1	1	1	1
On3	1	1	0	1	0	0
	1	1	1	1	0	0



$$x = s1 \mid s0$$

$$n1 = s1's0b' \mid s1's0b \mid s1s0'b' \mid s1s0'b$$

$$n1 = s1's0 \mid s1s0'$$

$$n0 = s1's0'b \mid s1s0'b' \mid s1s0'b$$

$$n0 = s1's0'b \mid s1s0'$$

# FSM Design Example

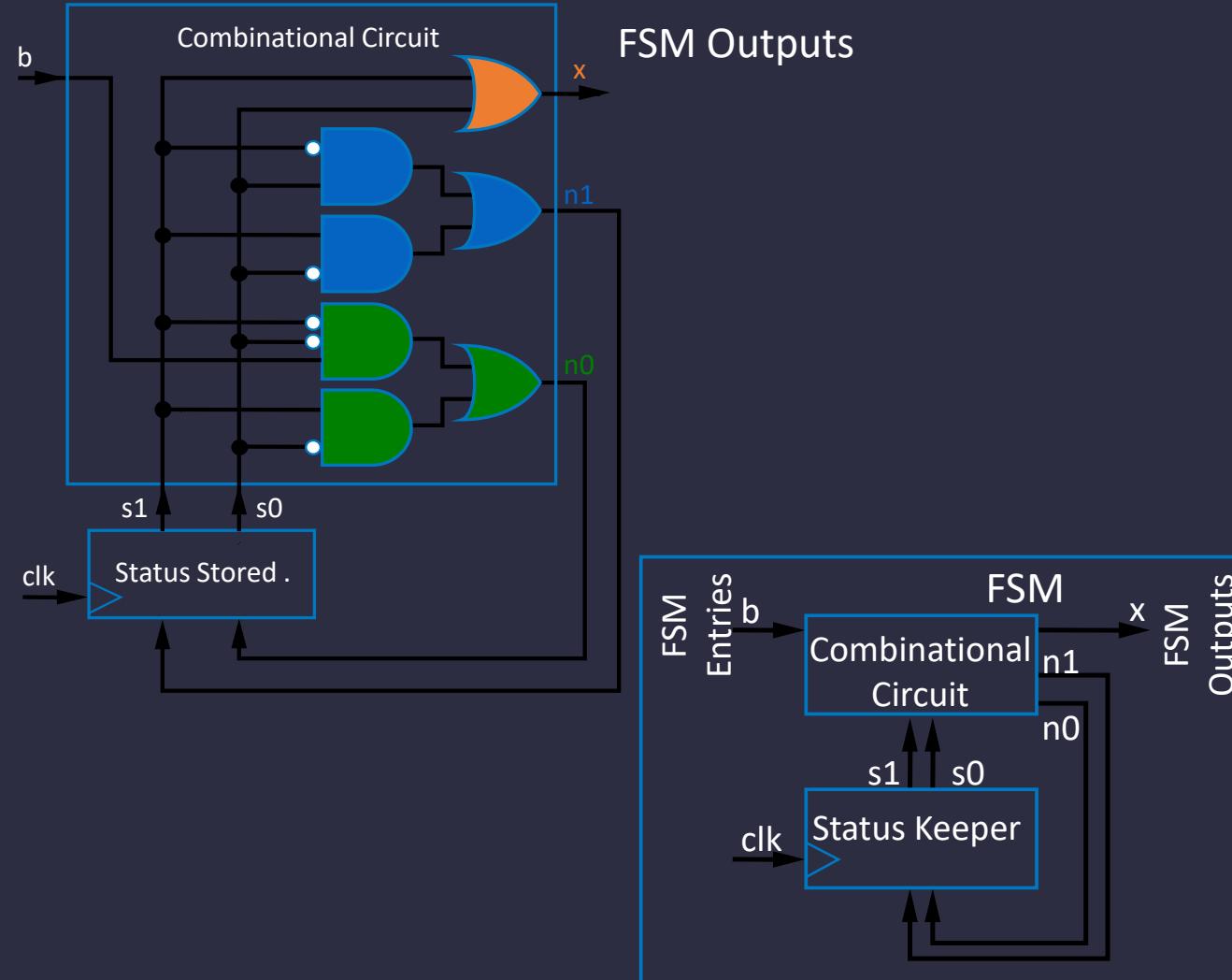
- Step 5: Build the combinational circuit

Entries			Output		
s1	s0	b	x	n1	n0
Off	0	0	0	0	0
	0	0	1	0	1
On1	0	1	0	1	1
	0	1	1	1	0
On2	1	0	0	1	1
	1	0	1	1	1
On3	1	1	0	1	0
	1	1	1	1	0

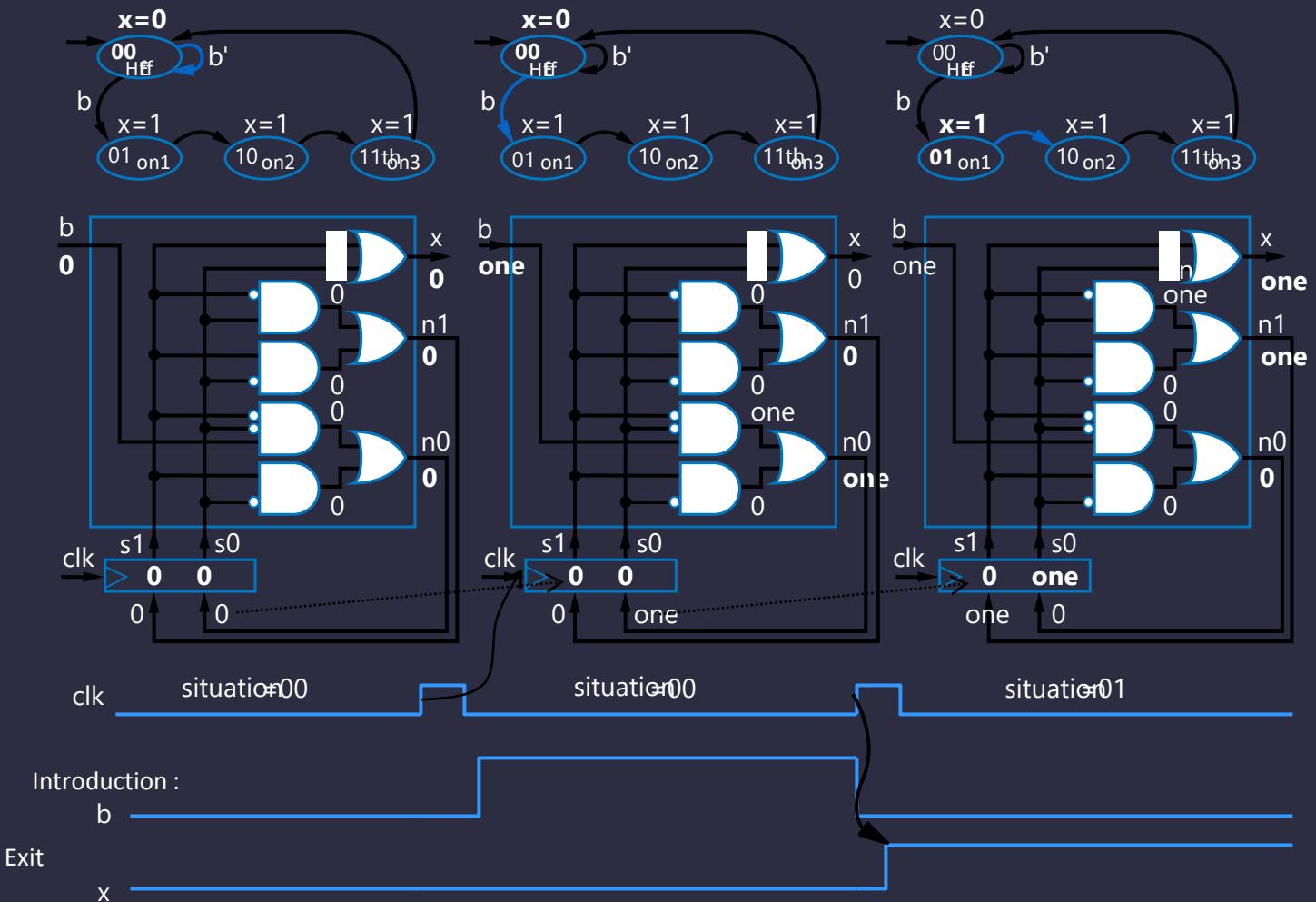
$$x = s_1 \mid s_0$$

$$n_1 = s_1's_0 \mid s_1s_0'$$

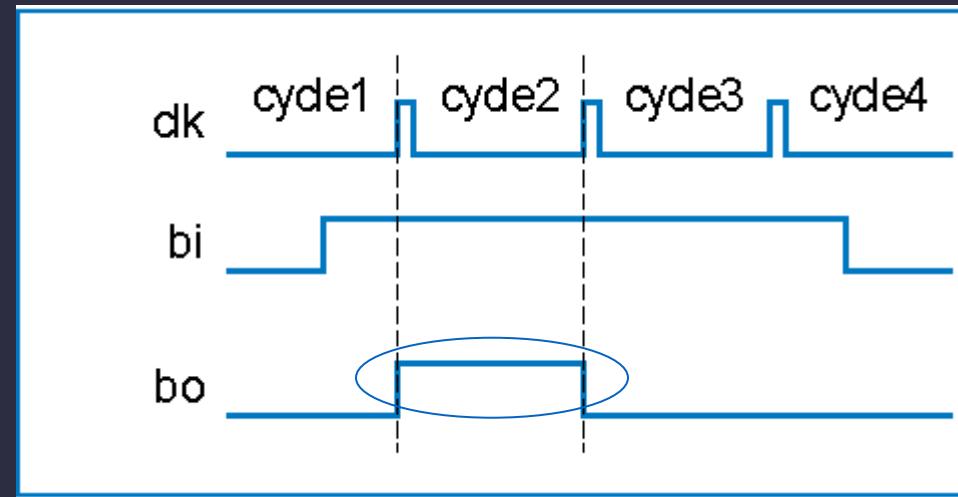
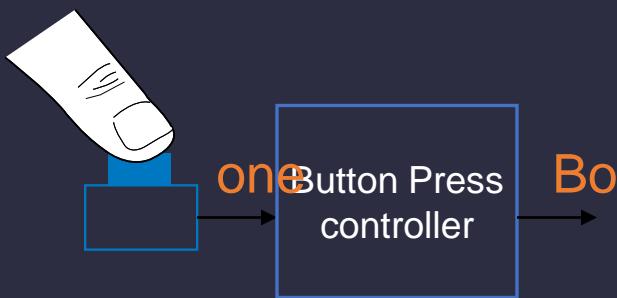
$$n_0 = s_1's_0'b \mid s_1s_0'$$



# Internal Structure of



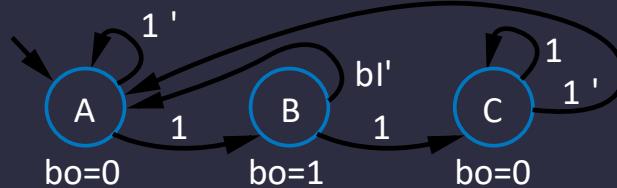
# Button Filter Module Example



- cycle when the button is pressed A design that produces a pulse is desired.

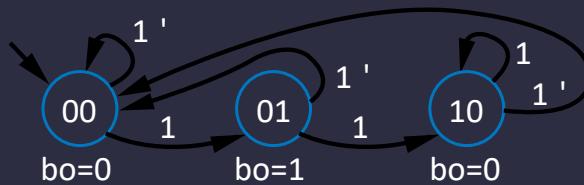
# Button Filter Module Example

FSM Inputs : bi; FSM outputs : bo

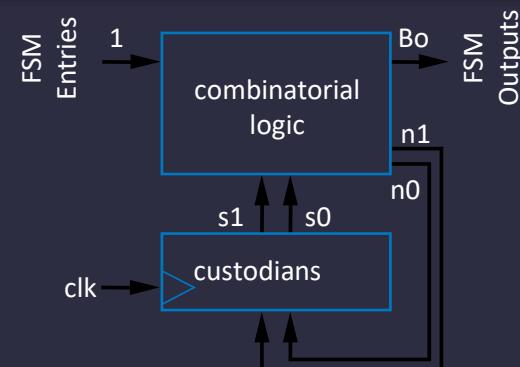


Step 1: FSM

FSM input : bi; FSM output : bo



Step 3: State coding

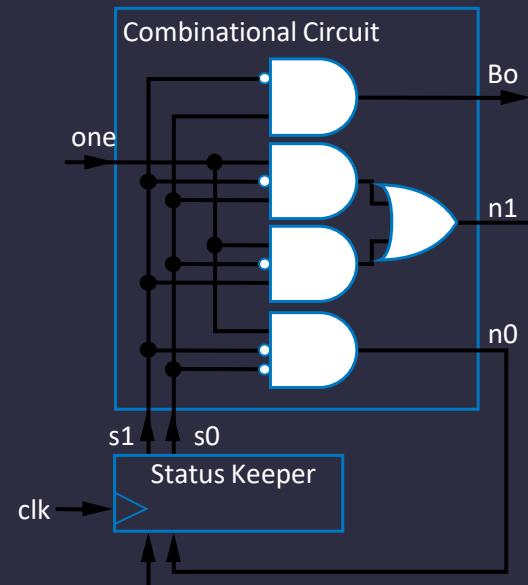


Step 2: Build architecture

Entries			Outputs		
s1	s0	bi	n1	n0	bo
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

Step 4: Status Chart

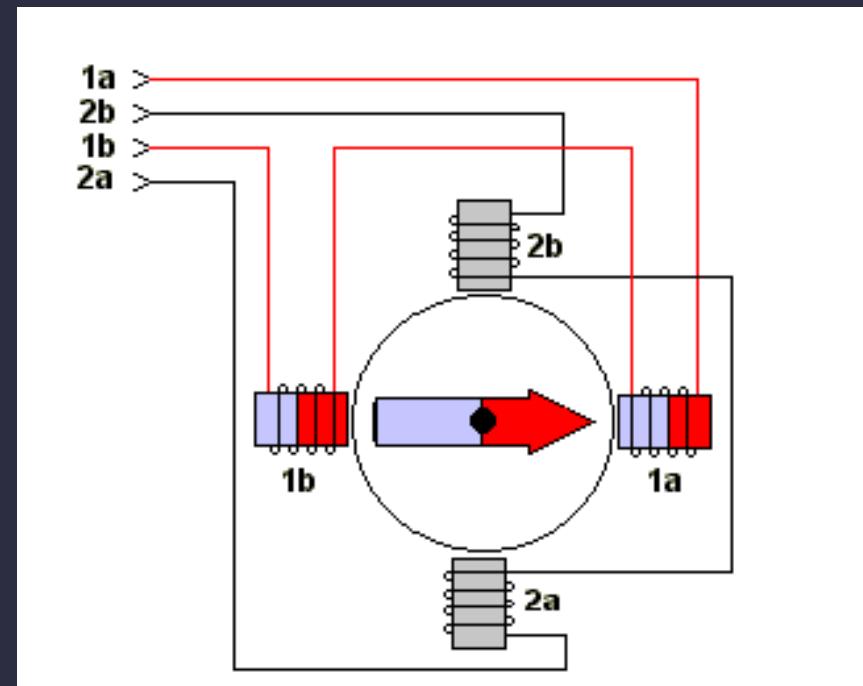
$$\begin{aligned} n1 &= s1's0bi' + s1s0bi \\ n0 &= s1's0'bi \\ bo &= s1's0bi' + s1's0bi = s1s0 \end{aligned}$$



Step 5: Build Circuit

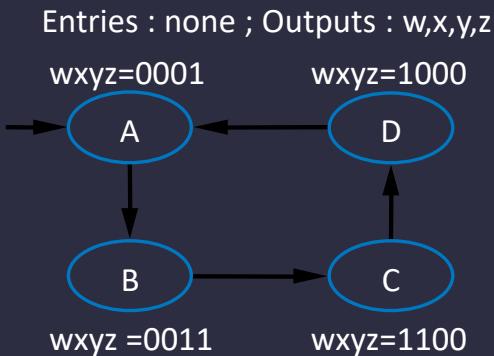
# FSM Example

- Design an FSM that outputs 0001, 0011, 1100, 1000 sequentially.
  - For example, for stepper motor control



# FSM Example

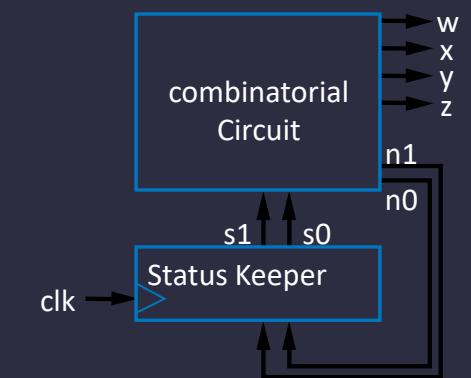
- Design an FSM that outputs 0001, 0011, 1100, 1000 sequentially.
  - For example, for stepper motor control



Step 1: Creating the FSM

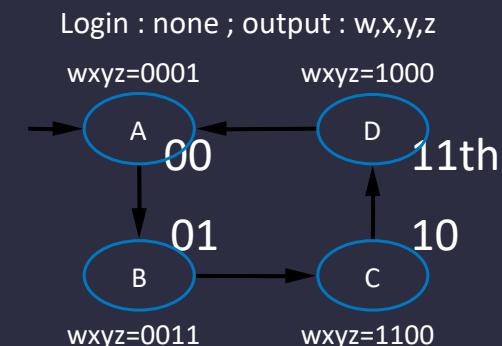
Entries	Outputs					
s1 s0	w	x	y	z	n1	n0
A 0 0	0	0	0	1	0	1
B 0 1	0	0	1	1	1	0
C 1 0	1	1	0	0	1	1
D 1 1	1	0	0	0	0	0

Step 4: Create a Truth Table

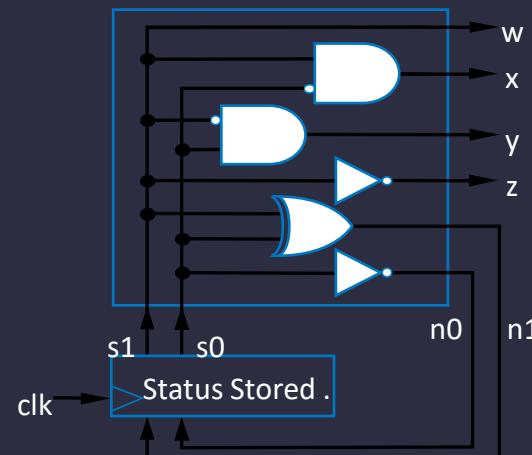


Step 2: Creating Architecture

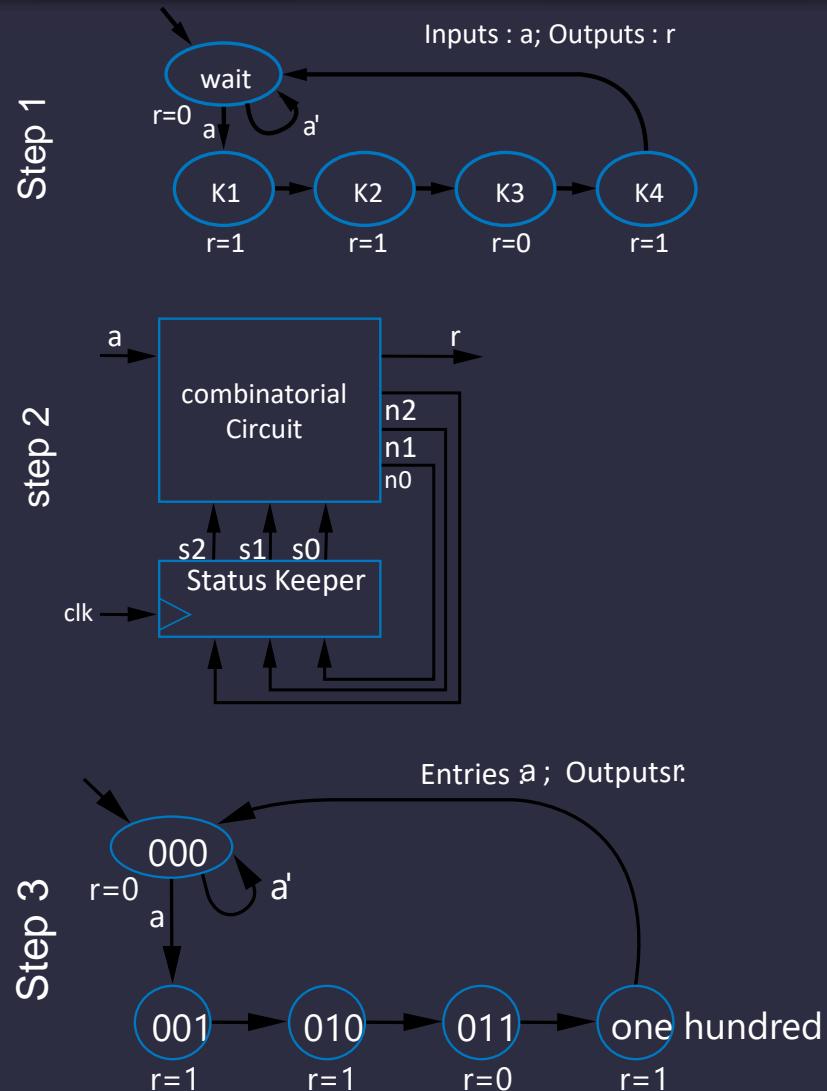
$$\begin{aligned}
 w &= s1 \\
 x &= s1s0' \\
 y &= s1's0 \\
 z &= s1' \\
 n1 &= s1 x \text{ or} \\
 s0 \\
 n0 &= s0'
 \end{aligned}$$



Step 3: State Coding



# FSM Example, Car Key

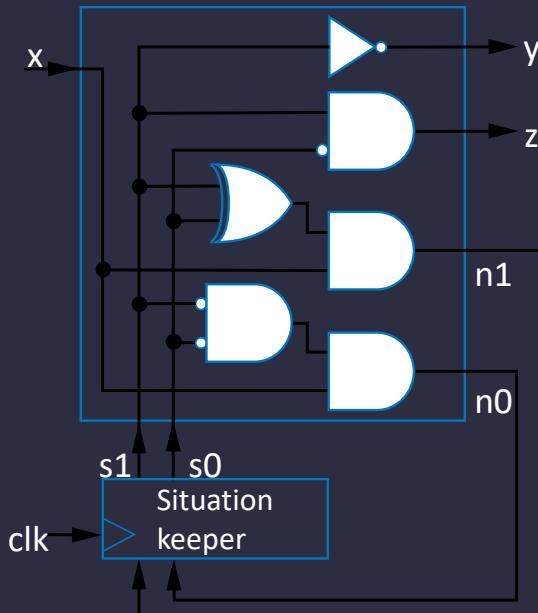


	Entries				Outputs			
	s2	s1	s0	a	r	n2	n1	n0
wait	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	1
K1	0	0	1	0	1	0	1	0
	0	0	1	1	1	0	1	0
K2	0	1	0	0	1	0	1	1
	0	1	0	1	1	0	1	1
K3	0	1	1	0	0	1	0	0
	0	1	1	1	0	1	0	0
K4	1	0	0	0	1	0	0	0
	1	0	0	1	1	0	0	0
-	1	0	1	0	0	0	0	0
-	1	0	1	1	0	0	0	0
-	1	1	0	0	0	0	0	0
-	1	1	0	1	0	0	0	0
-	1	1	1	0	0	0	0	0
-	1	1	1	1	0	0	0	0

Step 4

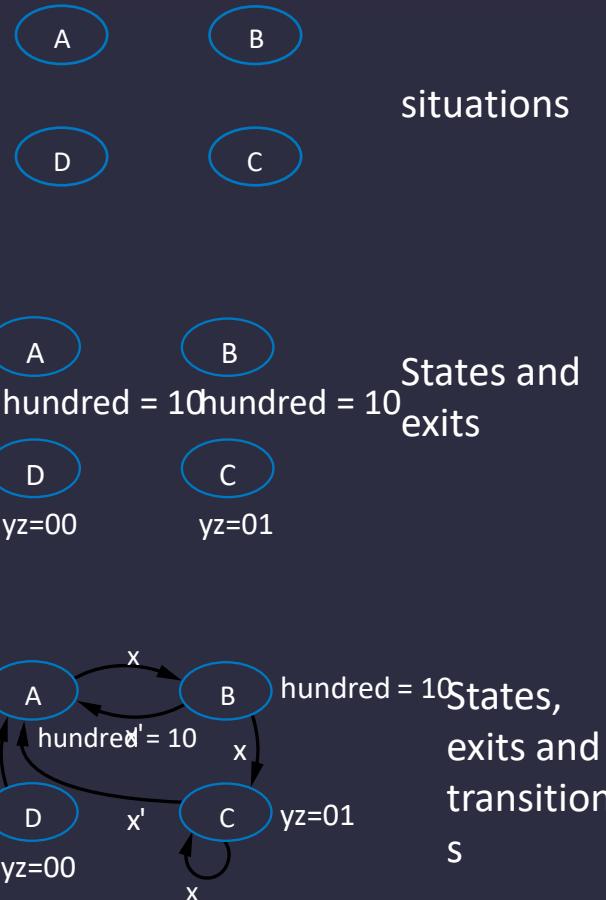
# Reverting to

What does this circuit do?



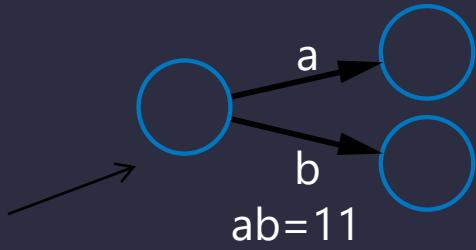
$$\begin{aligned}
 y &= s_1' \\
 z &= s_1 s_0' \\
 n_1 &= (s_1 x \text{ or } s_0) \\
 )x \\
 n_0 &= (s_1' * s_0')x
 \end{aligned}$$

Entries			Outputs				
	$s_1$	$s_0$	$x$	$n_1$	$n_0$	$y$	$z$
$A$	0	0	0	0	0	1	0
	0	0	1	0	1	1	0
$B$	0	1	0	0	0	1	0
	0	1	1	1	0	1	0
$C$	1	0	0	0	0	0	1
	1	0	1	1	0	0	1
$D$	1	1	0	0	0	0	0
	1	1	1	0	0	0	0

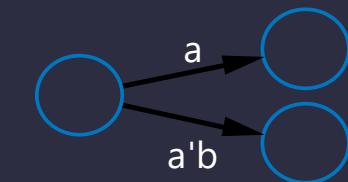
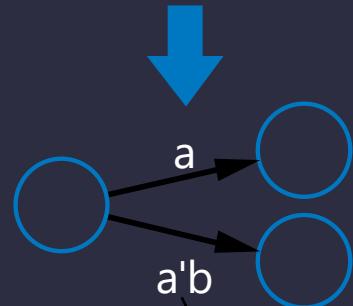


# FSM Design Errors

- In conditions in transitions between states, only one transition condition must be true at a time*



What will be the next situation?



ab=00?  
What will be the next situation?

