

The Monty Hall problem

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Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

We use a simulation to find the answer. First, we define a function to simulate a game:

```
1 set.seed(1)
2 whichDoor = function(choice, nds=3) {
3   doors = rep("goat", nds)
4   car = sample(1:nds, 1)
5   doors[car] = "car"
6   if (doors[choice] == "car") {
7     host = sample((1:nds)[-choice], 1)
8   } else if (nds == 3){
9     host = (1:nds)[-c(choice, car)]
10  } else {
11    host = sample((1:nds)[-c(choice, car)], 1)
12  }
13  if (nds == 3) {
14    switch = (1:nds)[-c(choice, host)]
15  } else {
16    switch = sample((1:nds)[-c(choice, host)], 1)
17  }
18  return(c(choice=choice, car=car, switch=switch, host=host))
19 }
20
21 # look at ten games
22 for (i in 1:10)
23   print(whichDoor(sample(1:3, 1)))
```

Here are the results for the ten games:

choice	car	switch	host
3	1	1	2
choice	car	switch	host
2	1	1	3
choice	car	switch	host

3	1	1	2
choice	car	switch	host
2	3	3	1
choice	car	switch	host
3	2	2	1
choice	car	switch	host
1	3	3	2
choice	car	switch	host
1	1	2	3
choice	car	switch	host
2	2	1	3
choice	car	switch	host
1	3	3	2
choice	car	switch	host
1	3	3	2

Now let's define a function to count the frequency from a larger number of the simulated games.

```

1 countMTH = function(n, nds=3){
2   games = replicate(n, whichDoor(sample(1:3, 1), nds=nds))
3   mean(games[1,] == games[2,])
4   mean(games[3,] == games[2,])
5   return (c(mean(games[1,] == games[2,]), mean(games[3,] == games[2,])))
6 }
7
8 # simulate 100 games to approximate the probabilities
9 probabilities = paste(countMTH(100), collapse=", ")

```

The approximate probabilities of keeping the original choice and switching are 0.41, 0.59.