Modeling and Designing High Permittivity Pads for MRI

Kirsten Koolstra and Jeroen van Gemert

May 31st, 2017

Outline

- Dielectric pads
- Volume Integral Equation
- Different Discretization schemes
- Designing pads
- Perturbing Maxwell’s equations
- Reduced order modeling

accuracy

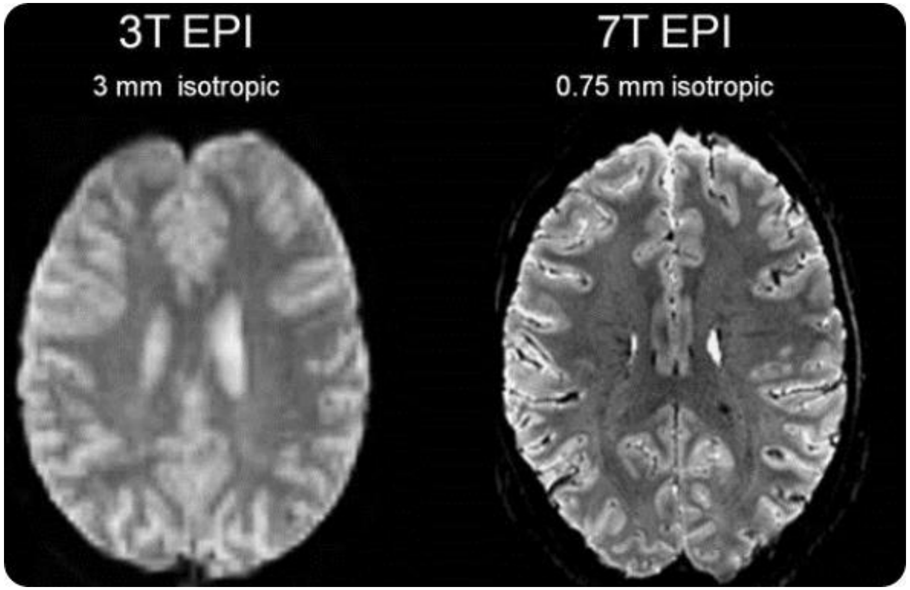
speed

Introduction

Magnetic Resonance Imaging (MRI)

$$\frac{S}{N} \propto B_0$$

www.neurensics.com/technische-specificaties



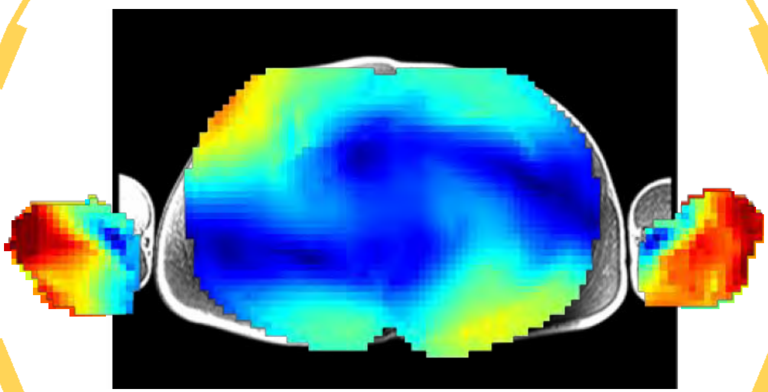
http://www.news-medical.net

Dielectric B_1^+ shimming

Magnetic Resonance Imaging (MRI)

B_0	f_L
1.5 T	64 MHz
3.0 T	128 MHz
7.0 T	300 MHz

high permittivity dielectric pad



135T

de Heer et al., Magn Res Med, 68(4), 1317-24, 2012.

Challenges

In Numerical Modeling

- Strong (localized) inhomogeneities in medium parameters
- Large computational domain due to the body model
- Accurate for low resolution!
- Fast!

Maxwell's Equations

How to solve them?

$$-\nabla \times \mathbf{H} + \sigma \mathbf{E} + j\omega\varepsilon \mathbf{E} = -\mathbf{J}^{\text{ext}}$$
$$\nabla \times \mathbf{E} + j\omega\mu \mathbf{H} = \mathbf{0}$$

Different discretization methods possible:

- Finite element method
- Finite difference method
- Method of Moments (or volume integral equation (VIE) approach)

Global approach instead of local

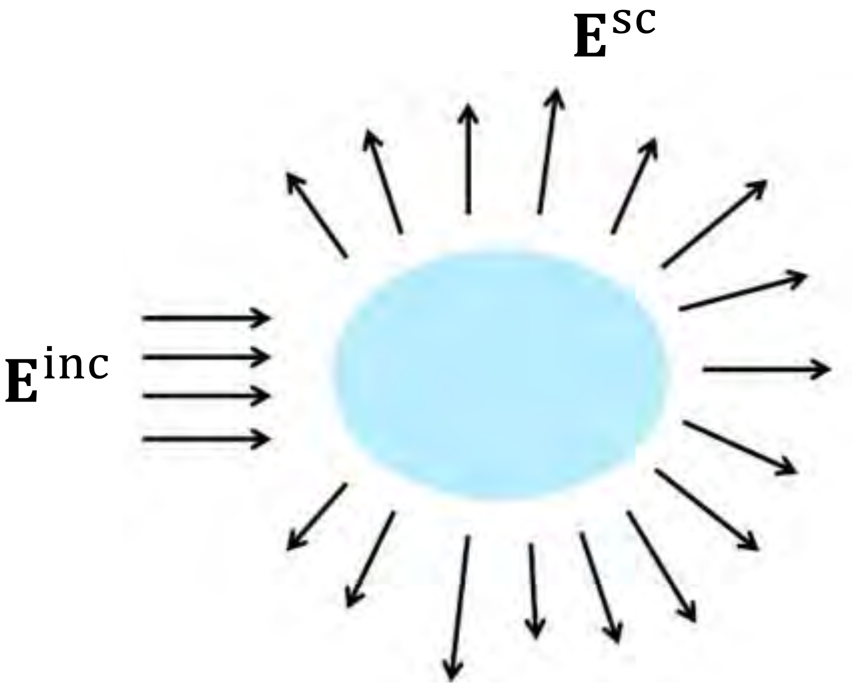
Efficient for problems with a small contrast domain

The Volume Integral Equation

$$\mathbf{E} = \mathbf{E}^{\mathrm{inc}} + \mathbf{E}^{\mathbf{sc}}$$

$$\mathbf{E} = \mathbf{E}^{\mathrm{inc}} + \left(\mathrm{k}_{\mathrm{b}}^{2} + \nabla\nabla\,\cdot\right)\mathbf{S}(\chi_e \mathbf{E})$$

$$\mathbf{S}(\mathbf{J}) = \int_{\Omega} g(\mathbf{x}' - \mathbf{x})\mathbf{J}(\mathbf{x})\mathrm{d}\mathbf{x}$$



Different Formulations

$$\mathbf{D} = \varepsilon \mathbf{E}$$

EVIE: $$\mathbf{E}^{\mathrm{inc}} = \mathbf{E} - (\mathrm{k}_{\mathrm{b}}^{2} + \nabla\nabla\cdot)\mathbf{S}(\chi_e \mathbf{E})$$

DVIE: $$\mathbf{E}^{\mathrm{inc}} = \frac{1}{\varepsilon}\mathbf{D} - (\mathrm{k}_{\mathrm{b}}^{2} + \nabla\nabla\cdot)\mathbf{S}(\frac{\chi_e}{\varepsilon}\mathbf{D})$$

The Volume Integral Equation

$$\mathbf{E}^{\text{inc}} = \mathbf{E} - \left(\mathrm{k}_\mathrm{b}^2 + \nabla\nabla\cdot\right)\mathbf{S}(\chi_e \mathbf{E})$$

$$\Big\Downarrow \; 2D$$

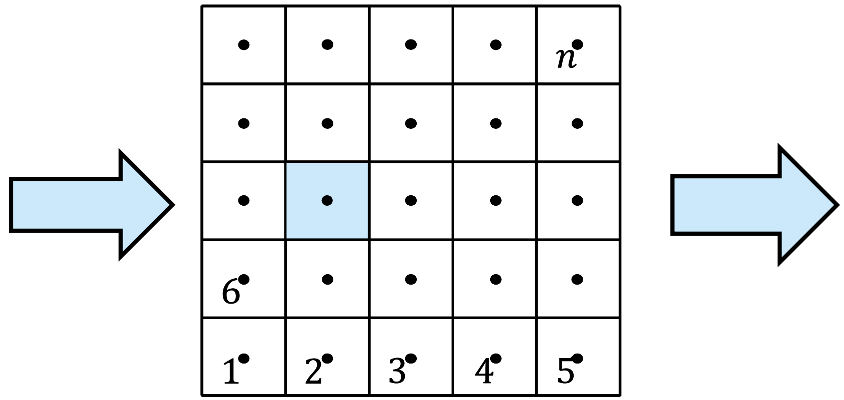
$$\begin{bmatrix} E_x^{\text{inc}} \\ E_y^{\text{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k}_\mathrm{b}^2 + \nabla\nabla\cdot\right)\begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

- The x- and y-components of the electric field are coupled via the $\nabla\nabla\cdot$ operator.

- The vector potential $\boldsymbol{S}$ depends on the material parameters.

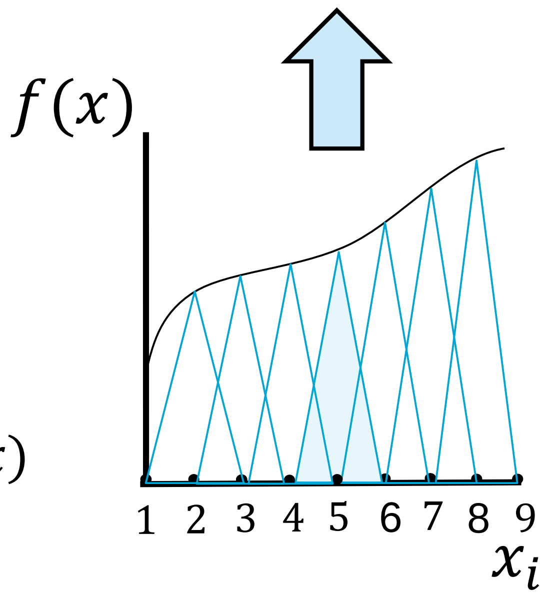
The Method of Moments

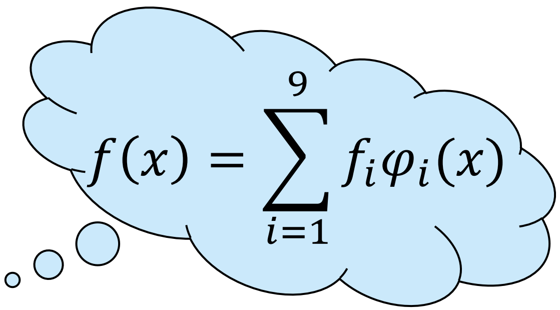
$\mathcal{L}u = f$



$A\mathbf{x} = \mathbf{b}$

1. Specify $\varphi_i(x)$
2. Find f_i for all i
3. Reconstruct $f(x)$





The Method of Moments

Expansions

$$\begin{bmatrix} E_x^{\mathrm{inc}} \\ E_y^{\mathrm{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k}_\mathrm{b}^2 + \nabla\nabla\cdot\right) \begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

The Method of Moments

Expansions

$$\begin{bmatrix} E_x^{\text{inc}} \\ E_y^{\text{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k}_{\mathrm{b}}^2 + \nabla\nabla\cdot\right)\begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

is solved via expanding

$$E_x(\boldsymbol{x}) = \sum_{i=1}^{n} e_i^x \psi_i^x(\boldsymbol{x})$$

$$E_y(\boldsymbol{x}) = \sum_{i=1}^{n} e_i^y \psi_i^y(\boldsymbol{x})$$

The Method of Moments

Expansions

$$\begin{bmatrix} E_x^{\mathrm{inc}} \\ E_y^{\mathrm{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k}_\mathrm{b}^2 + \nabla\nabla\cdot\right)\begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

is solved via expanding

$$E_x(\boldsymbol{x}) = \sum_{i=1}^{n} e_i^x \psi_i^x(\boldsymbol{x}) \qquad S_x(\boldsymbol{x}) = \sum_{i=1}^{n} s_i^x \psi_i^x(\boldsymbol{x})$$

$$E_y(\boldsymbol{x}) = \sum_{i=1}^{n} e_i^y \psi_i^y(\boldsymbol{x}) \qquad S_y(\boldsymbol{x}) = \sum_{i=1}^{n} s_i^y \psi_i^y(\boldsymbol{x})$$

The Method of Moments

Expansions

$$\begin{bmatrix} E_x^{\mathrm{inc}} \\ E_y^{\mathrm{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k}_\mathrm{b}^2 + \nabla\nabla\cdot\right) \begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

$$\Downarrow$$

$$A \begin{bmatrix} \boldsymbol{e}^x \\ \boldsymbol{e}^y \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^x \\ \boldsymbol{b}^y \end{bmatrix}$$

$$\nabla\nabla\cdot\mathbf{S} = \begin{bmatrix} \frac{\partial}{\partial x}\frac{\partial}{\partial x} S_x + \frac{\partial}{\partial x}\frac{\partial}{\partial y} S_y \\ \frac{\partial}{\partial y}\frac{\partial}{\partial x} S_x + \frac{\partial}{\partial y}\frac{\partial}{\partial y} S_y \end{bmatrix}$$

- How do we incorporate the operator $\mathbf{S}$ in the matrix A?
- How do we deal with the derivative terms?

The Method of Moments

Fast Fourier Transform

Remember,

$$\mathbf{S}(\chi_e\mathbf{E})(\mathbf{x}') = \int_\Omega g(\mathbf{x}'-\mathbf{x})\chi_e(\mathbf{x})\mathbf{E}(\mathbf{x})\mathrm{d}\mathbf{x} = g * \chi_e\mathbf{E}$$

And

$$\mathcal{F}\{\mathbf{S}\} = \mathcal{F}\{g * \chi_e\mathbf{E}\} = \mathcal{F}\{g\}\mathcal{F}\{\chi_e\mathbf{E}\}$$

$$\Longrightarrow \mathbf{S} = \mathcal{F}^{-1}\big\{\mathcal{F}\{g\}\mathcal{F}\{\chi_e\mathbf{E}\}\big\}.$$

So, use fast Fourier transform (FFT) algorithms to incorporate $\mathbf{S}$ in the matrix A!

The Method of Moments

Expansions

$$\begin{bmatrix} E_x^{\text{inc}} \\ E_y^{\text{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k}_\mathrm{b}^2 + \nabla\nabla\cdot\right)\begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

$$\Downarrow$$

$$A\begin{bmatrix} \boldsymbol{e}^x \\ \boldsymbol{e}^y \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^x \\ \boldsymbol{b}^y \end{bmatrix}$$

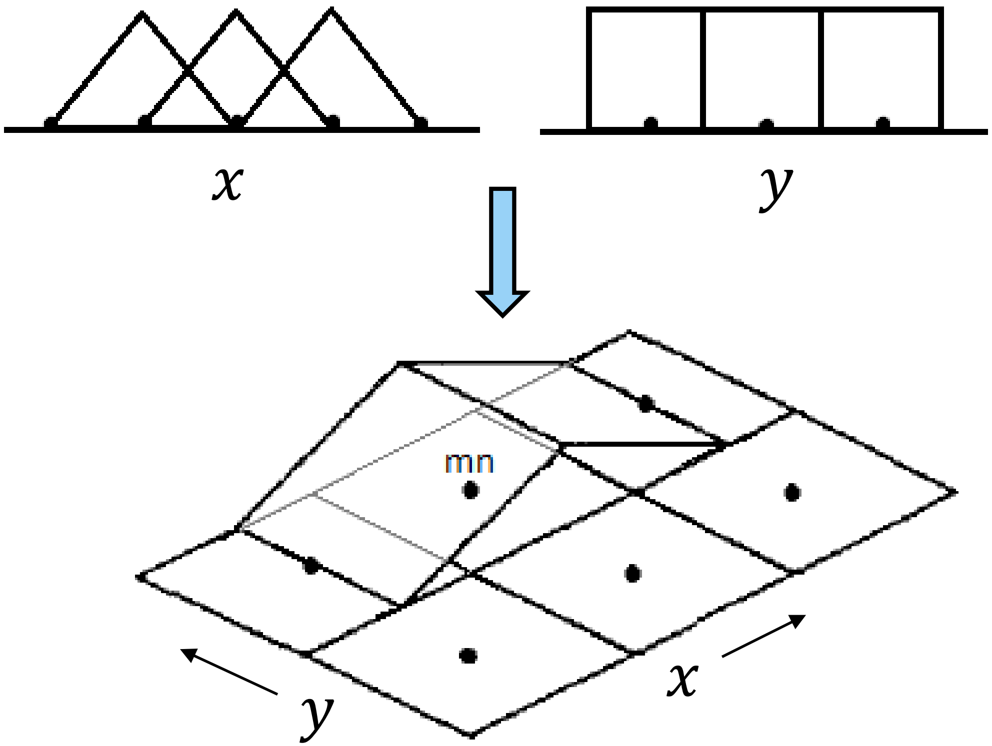
$$\nabla\nabla\cdot\mathbf{S} = \begin{bmatrix} \frac{\partial}{\partial x}\frac{\partial}{\partial x}S_x + \frac{\partial}{\partial x}\frac{\partial}{\partial y}S_y \\ \frac{\partial}{\partial y}\frac{\partial}{\partial x}S_x + \frac{\partial}{\partial y}\frac{\partial}{\partial y}S_y \end{bmatrix}$$

- How do we incorporate the operator S in the matrix A?
- How do we deal with the derivative terms?

The Method of Moments

Basis Functions: Rooftop

$$E_x(\boldsymbol{x}) = \sum_{i=1}^{n} e_i^x \psi_i^x(\boldsymbol{x})$$



The Method of Moments

Expansions

$$\begin{bmatrix} E_x^{\mathrm{inc}} \\ E_y^{\mathrm{inc}} \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} - \left(\mathrm{k_b^2} + \nabla\nabla\,\cdot\right) \begin{bmatrix} S_x(\chi_e E_x) \\ S_y(\chi_e E_y) \end{bmatrix}$$

$$\Downarrow$$

$$A \begin{bmatrix} \boldsymbol{e}^x \\ \boldsymbol{e}^y \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}^x \\ \boldsymbol{b}^y \end{bmatrix}$$

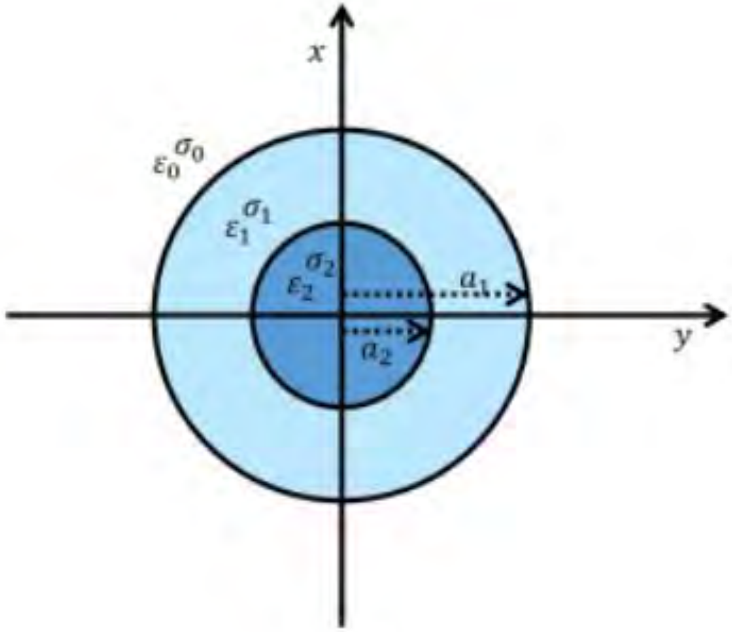
$$\nabla\nabla \cdot \mathbf{S} = \begin{bmatrix} \frac{\partial}{\partial x}\frac{\partial}{\partial x} S_x + \frac{\partial}{\partial x}\frac{\partial}{\partial y} S_y \\ \frac{\partial}{\partial y}\frac{\partial}{\partial x} S_x + \frac{\partial}{\partial y}\frac{\partial}{\partial y} S_y \end{bmatrix}$$

- ✓ How do we incorporate the operator S in the matrix A?
- ✓ How do we deal with the derivative terms?

Benchmark Problem

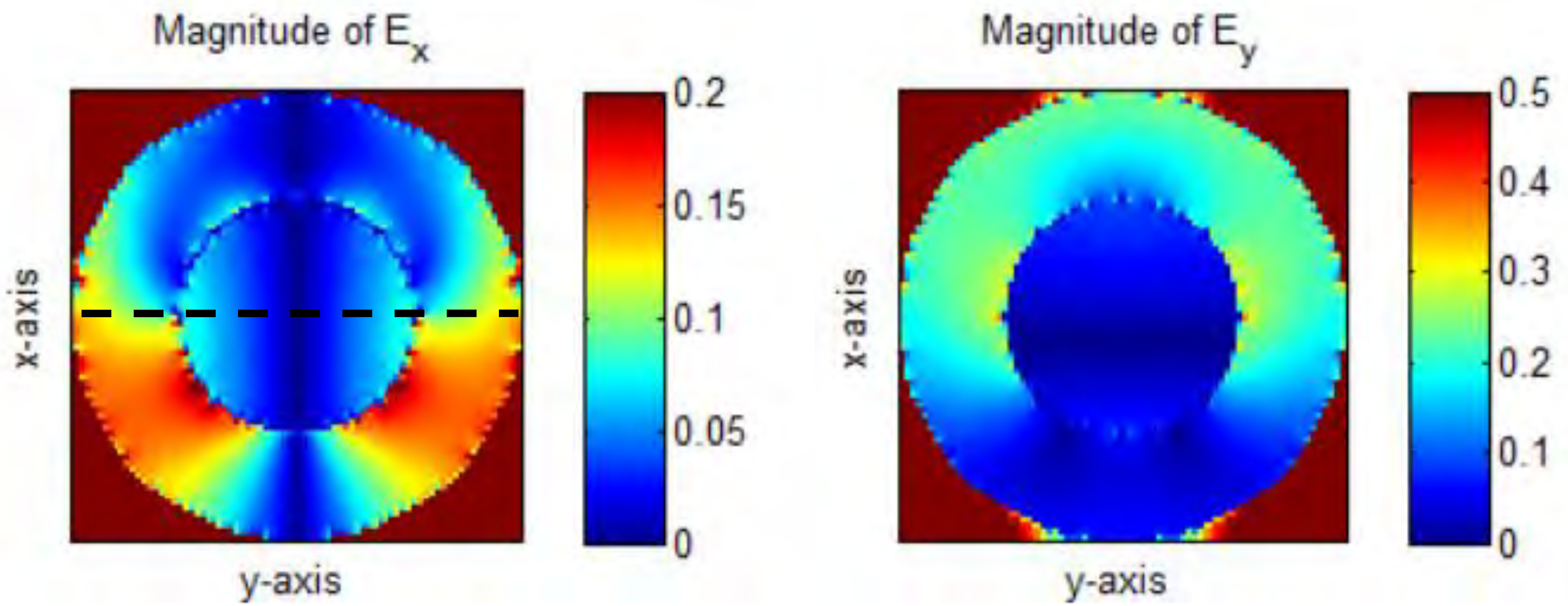
Scattering on a Two-Layer Conducting Cylinder

- TE-polarization
- $f = 100$ MHz
- Plane wave incident field
- Muscle/fat tissue



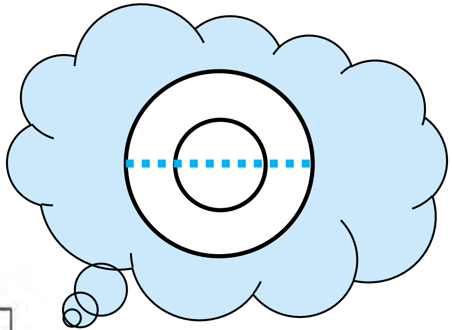
Results

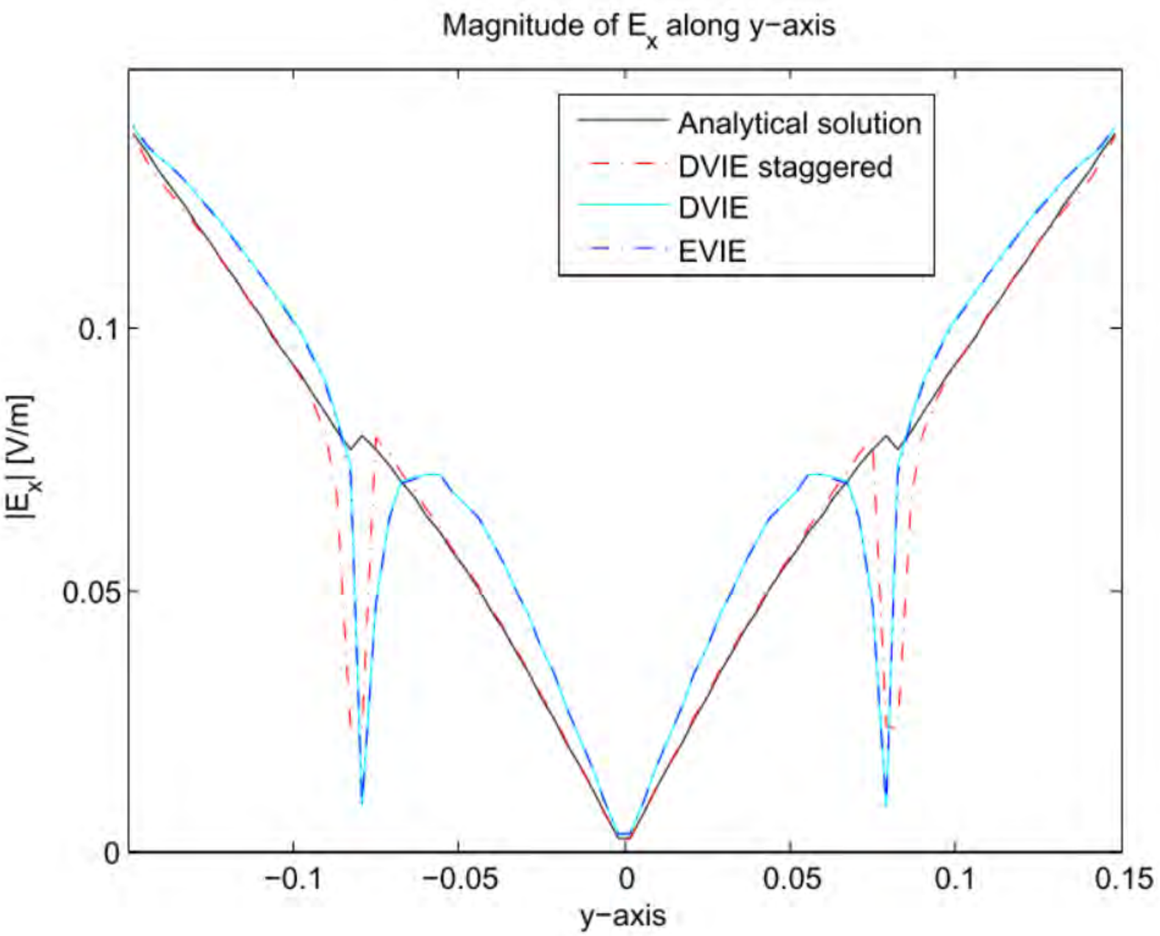
Scattering on a Two-Layer Conducting Cylinder



Results

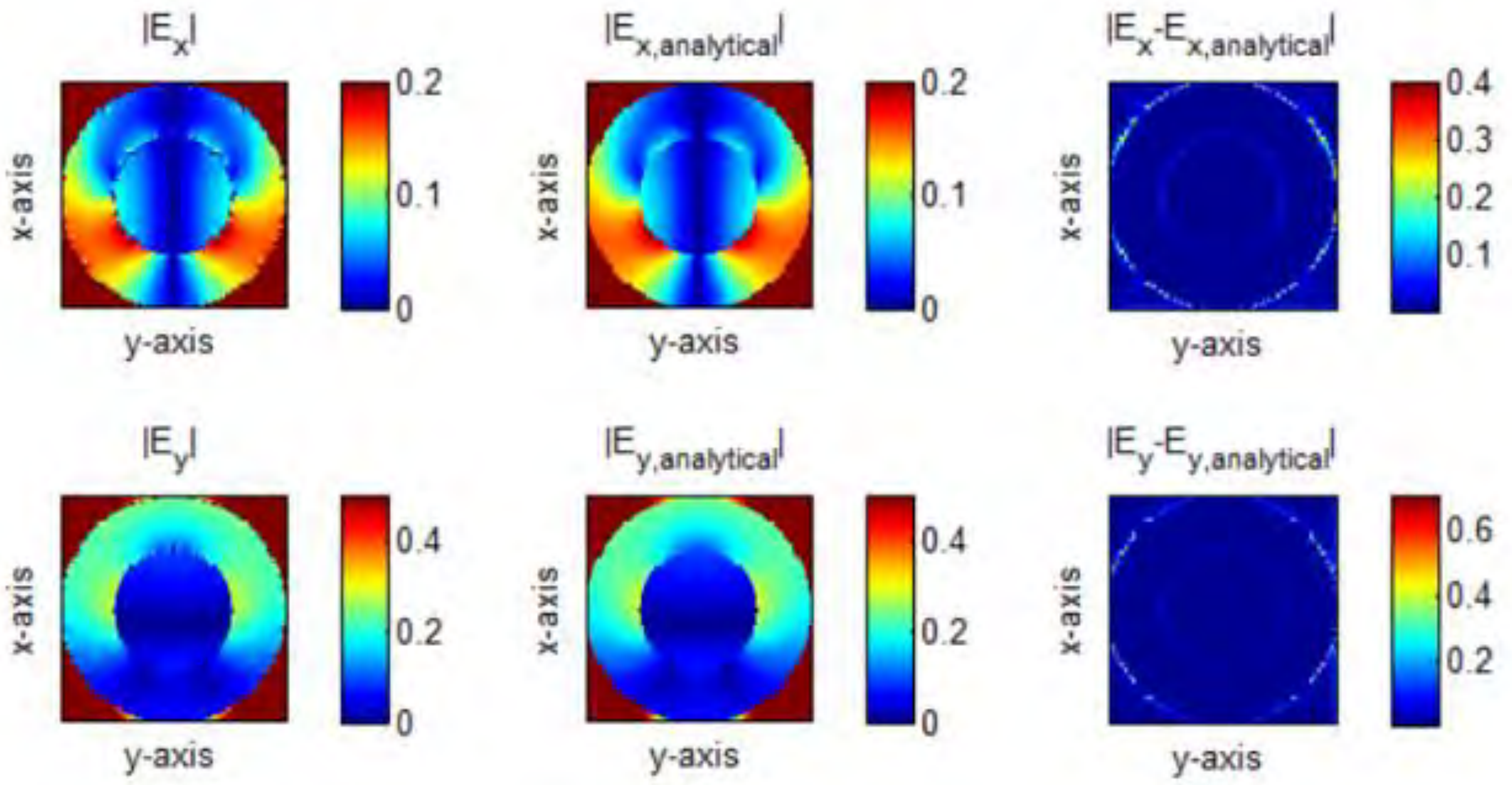
Comparison of EVIE and DVIE





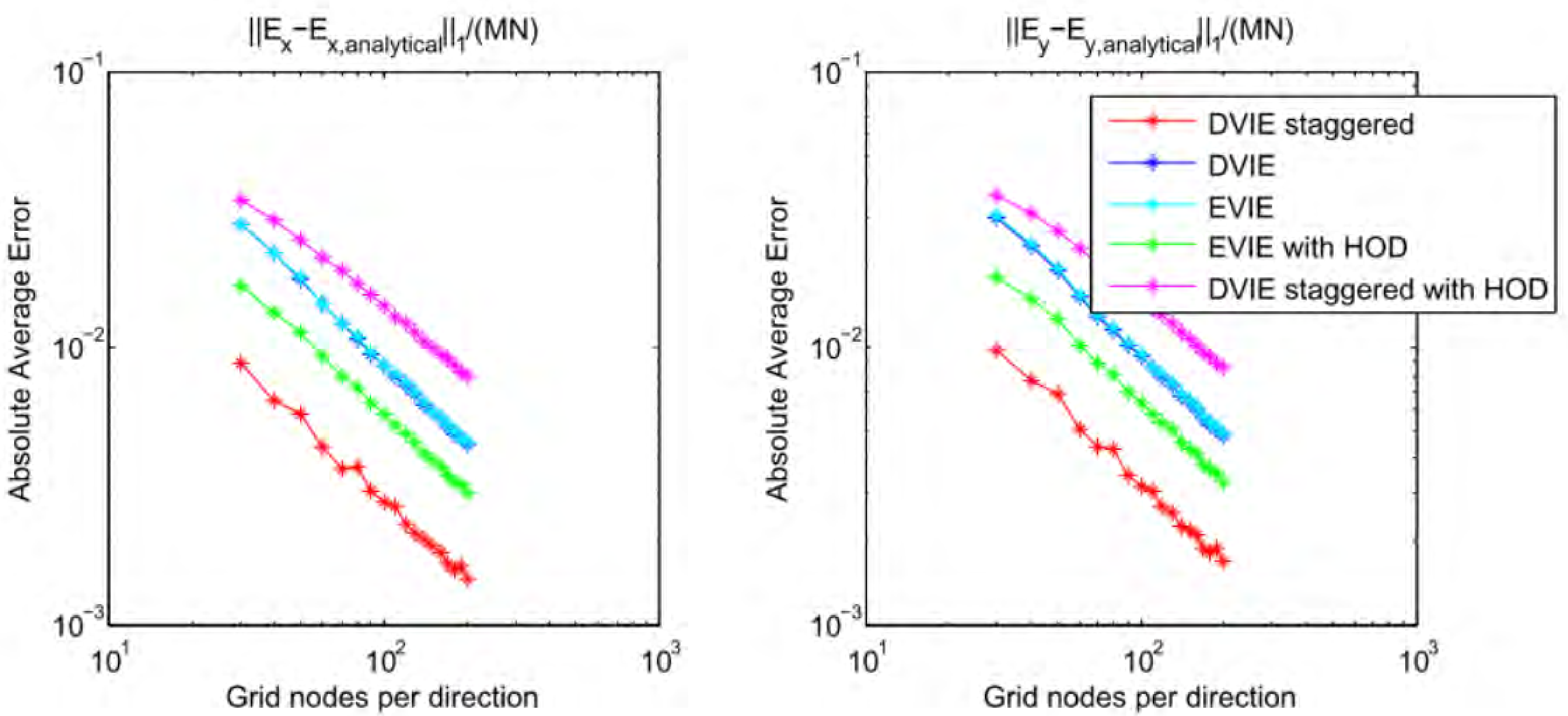
Results

Scattering on a Two-Layer Conducting Cylinder



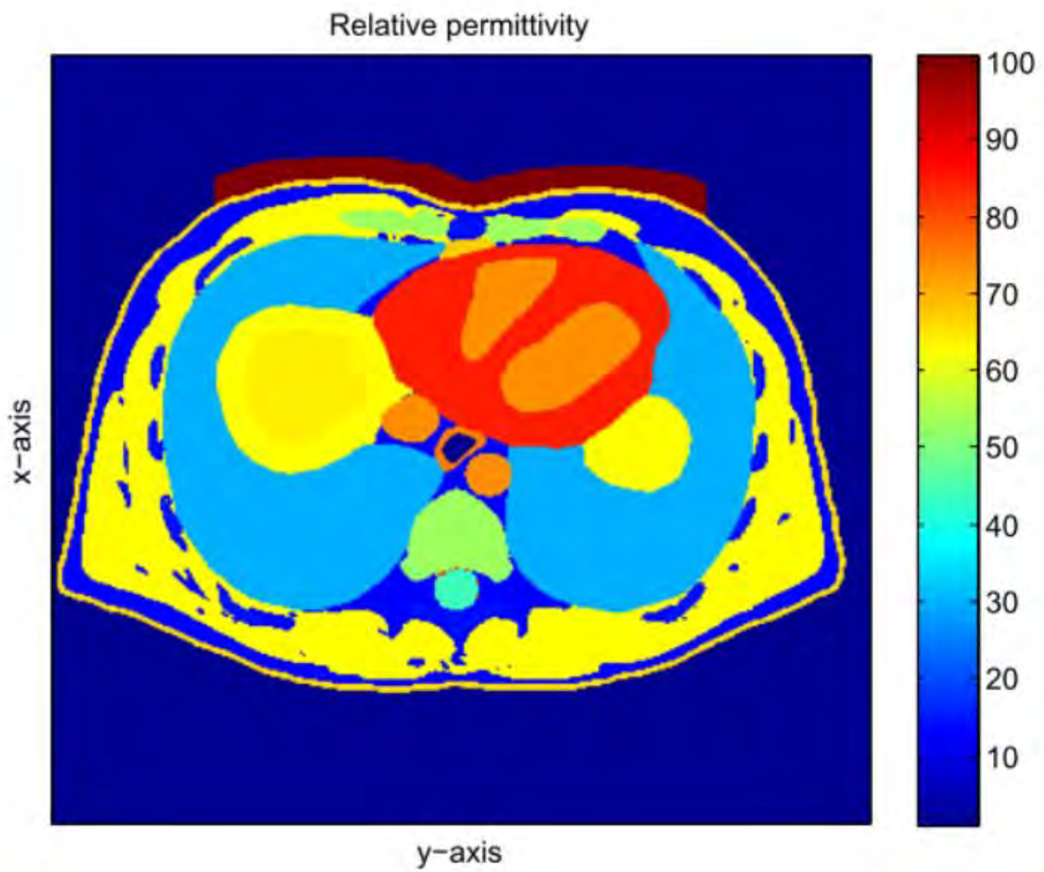
Results

Global Error Propagation



Human Body Simulations

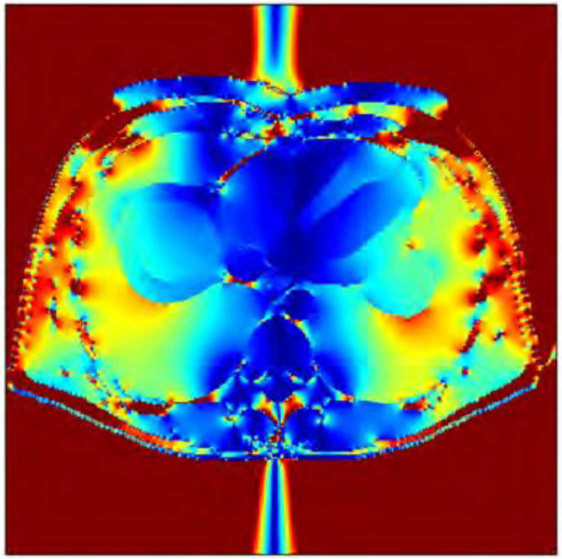
Scattering on a Human Body with Dielectric Pad



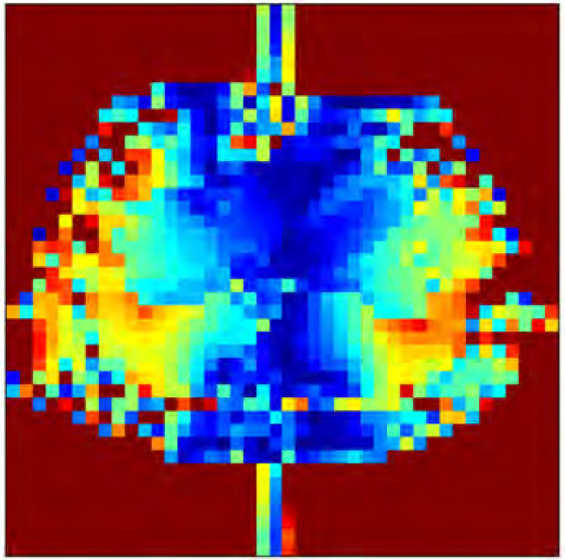
Human Body Simulations

Comparison of the staggered and non-staggered grid

High resolution

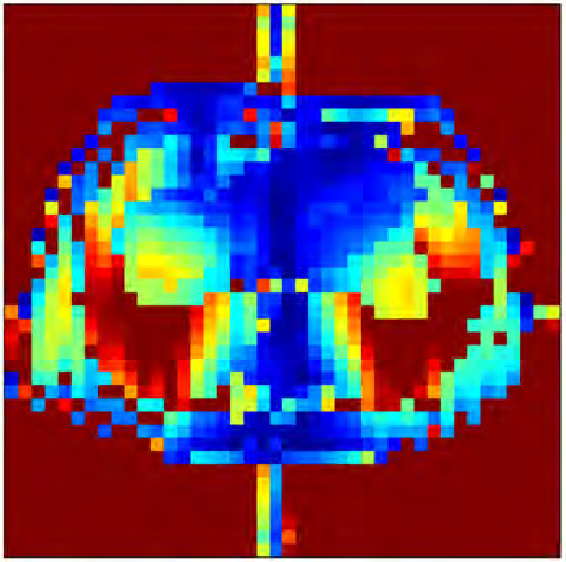


Low resolution



Staggered grid

Low resolution



Non-staggered grid

Outline

- Dielectric pads
- Volume Integral Equation
- Different Discretization schemes

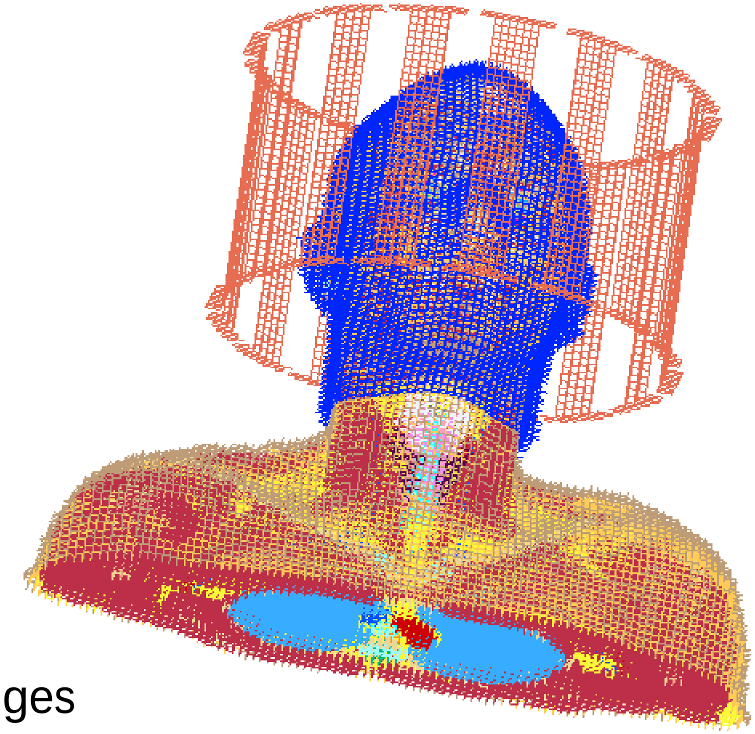
accuracy

- Designing pads
- Perturbing Maxwell's equations
- Reduced order modeling

speed

Simulating dielectrics

Useful properties



- Many trial-and-error simulations
- Choose optimum pad afterwards
- Time consuming (days)
- Properties
  - For every simulation only the pad changes
  - Pad close to ROI
  - Pad design domain is small w.r.t. computational domain

Designing dielectrics

Minimization

- Define a cost function

$$C(\text{pad}) = \frac{1}{2} \frac{\|\mathsf{b}_1^{+;\text{simulated}}(\text{pad}) - \mathsf{b}_1^{+;\text{desired}}\|_2^2}{\|\mathsf{b}_1^{+;\text{desired}}\|_2^2}$$

- IN: desired B_1^+ field as target field in region of interest
- OUT: properties dielectric pad

Maxwell's Equations

Solve for the fields

$$-\nabla \times \mathbf{H} + \sigma\mathbf{E} + \mathrm{j}\omega\varepsilon\mathbf{E} = -\mathbf{J}^{\mathrm{ext}}$$

$$\nabla \times \mathbf{E} + \mathrm{j}\omega\mu\mathbf{H} = \mathbf{0}$$

- Finding electromagnetic fields amounts to solving for f

$$\mathsf{Df} = -\mathsf{q}$$

- With EM fields and sources

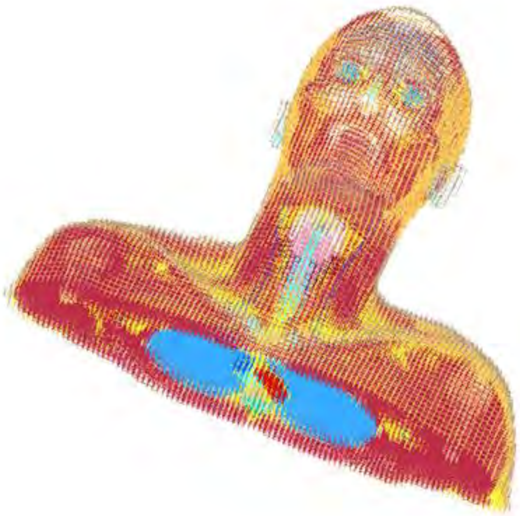
$$\mathsf{f} = \begin{bmatrix} \mathsf{e}_x \\ \mathsf{e}_y \\ \mathsf{e}_z \\ \mathsf{h}_x \\ \mathsf{h}_y \\ \mathsf{h}_z \end{bmatrix} \qquad \mathsf{q} = \begin{bmatrix} \mathsf{j}_x \\ \mathsf{j}_y \\ \mathsf{j}_z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Maxwell's Equations

Perturbation

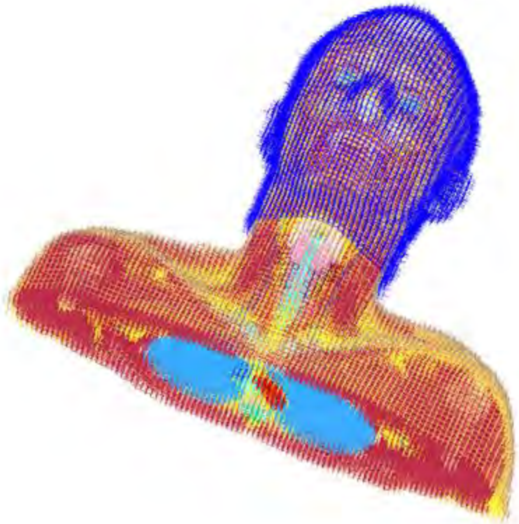
<u>Antennas + body</u>

$$\mathsf{D}\mathsf{f} = -\mathsf{q}$$



<u>Antennas + body + pad</u>

$$\left(\mathsf{D} + \mathsf{S}\mathsf{X}_{\mathrm{pad}}\mathsf{S}^{T}\right)\mathsf{f} = -\mathsf{q}$$



Maxwell's Equations

Woodbury-identity

- Solving the system

$$\mathsf{f} = -\left(\mathsf{D} + \mathsf{S}\mathsf{X}_{\mathrm{pad}}\mathsf{S}^T\right)^{-1}\mathsf{q}$$

- Perturbing matrix inverse using Woodbury-identity

$$\boxed{\mathsf{f}} = \boxed{-\mathsf{D}^{-1}\mathsf{q}} + \boxed{\mathsf{D}^{-1}\mathsf{S}}\left(\mathsf{I}_P + \boxed{\mathsf{X}_{\mathrm{pad}}}\boxed{\mathsf{S}^T\mathsf{D}^{-1}\mathsf{S}}\right)^{-1}\boxed{\mathsf{X}_{\mathrm{pad}}}\boxed{\mathsf{S}^T\mathsf{D}^{-1}\mathsf{q}}$$

- Which electromagnetically speaking represents

$$\boxed{\mathsf{b}_1^+} = \boxed{\mathsf{b}_1^{+;\mathrm{no\ pad}}} + \boxed{\mathsf{G}^{B_1^+\mathrm{J}}}\left(\mathsf{I}_P - \boxed{\mathsf{X}_{\mathrm{pad}}}\boxed{\mathsf{G}^{\mathrm{EJ}}}\right)^{-1}\boxed{\mathsf{X}_{\mathrm{pad}}}\boxed{\mathsf{e}^{\mathrm{no\ pad}}}$$

- But, we make it readable again

$$\mathsf{b}_1^+ = \mathsf{b}_1^{+;\mathrm{no\ pad}} + \mathsf{G}^{B_1^+\mathrm{J}}\mathsf{A}^{-1}\mathsf{b}$$

Maxwell's Equations

Woodbury-identity

- Solving the system

$$\mathsf{f} = -\underbrace{\left(\mathsf{D} + \mathsf{S}\mathsf{X}_{\text{pad}}\mathsf{S}^{T}\right)}_{\text{Size} \sim 10\text{^}7}{}^{-1}\mathsf{q}$$

- Perturbing matrix inverse using Woodbury-identity

$$\mathsf{f} = -\mathsf{D}^{-1}\mathsf{q} + \mathsf{D}^{-1}\mathsf{S}(\mathsf{I}_P + \mathsf{X}_{\text{pad}}\mathsf{S}^T\mathsf{D}^{-1}\mathsf{S})^{-1}\mathsf{X}_{\text{pad}}\mathsf{S}^T\mathsf{D}^{-1}\mathsf{q}$$

- Which electromagnetically speaking represents

$$\mathsf{b}_1^{+} = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+ \mathrm{J}}\left(\mathsf{I}_P - \mathsf{X}_{\text{pad}}\mathsf{G}^{\mathrm{EJ}}\right)^{-1}\mathsf{X}_{\text{pad}}\mathsf{e}^{\text{no pad}}$$

but requires ~ 30 GB of data

Size ~ 10^4

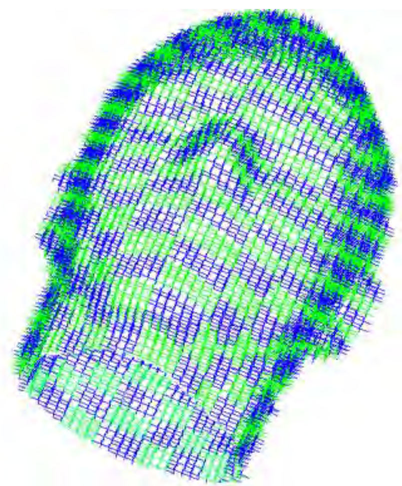
- But, we make it readable again

$$\mathsf{b}_1^{+} = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+ \mathrm{J}}\mathsf{A}^{-1}\mathsf{b}$$

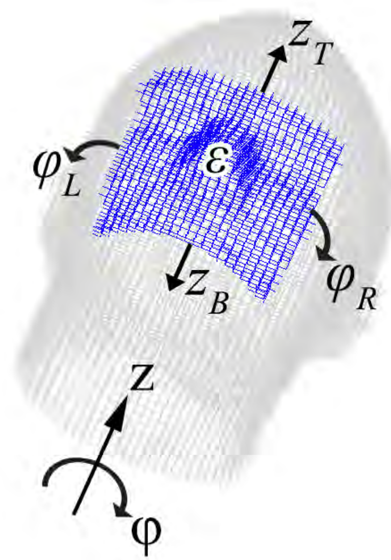
Practical considerations

Parametrization

30,000 edges



400 subdomains



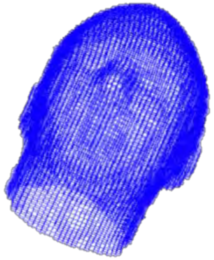
5 parameters

combine edges

enforce rectangular pads

Practical considerations

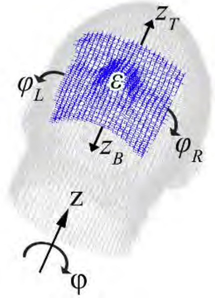
Updating equations



- Before: control every edge

$$\mathsf{b}_1^+(\mathsf{n}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+\mathsf{J}}\mathsf{A}(\mathsf{n})^{-1}\mathsf{b}(\mathsf{n})$$

- Now: only p variables to define a rectangular pad



$$\mathsf{b}_1^+(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+\mathsf{J}}\mathsf{A}(\mathsf{p})^{-1}\mathsf{b}(\mathsf{p})$$

- Still 30 GB and of same size: reduced order modeling

$$\mathsf{b}_1^+(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+\mathsf{J}}\mathsf{j}(\mathsf{p})$$

Reduced order modeling

New basis

- Original model with

$$\mathsf{b}_1^+(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+ \mathrm{J}} \mathsf{j}(\mathsf{p}) \qquad \mathsf{j}(\mathsf{p}) = \mathsf{A}(\mathsf{p})^{-1}\mathsf{b}(\mathsf{p})$$

- Find approximation for current density j(p)

$$\mathsf{j}_r(\mathsf{p}) = \alpha_1(\mathsf{p})\mathsf{u}_1 + \alpha_2(\mathsf{p})\mathsf{u}_2 + \ldots + \alpha_r(\mathsf{p})\mathsf{u}_r = \mathsf{U}_r\mathsf{a}_r(\mathsf{p})$$

- Introduces a residual

$$\mathsf{r} = \mathsf{A}(\mathsf{p})\mathsf{j}_r(\mathsf{p}) - \mathsf{b}(\mathsf{p})$$

Reduced order modeling

Galerkin condition

- Residual

$$\mathsf{r} = \mathsf{A}(\mathsf{p})\mathsf{j}_r(\mathsf{p}) - \mathsf{b}(\mathsf{p})$$

$$\mathsf{r} = \mathsf{A}(\mathsf{p})\mathsf{U}_r\mathsf{a}_r(\mathsf{p}) - \mathsf{b}(\mathsf{p})$$

- Galerkin condition

$$\mathsf{U}_r^H\mathsf{r} = 0$$

- Two equations, two unknowns:

$$\mathsf{a}_r(\mathsf{p}) = \left[\mathsf{U}_r^H\mathsf{A}(\mathsf{p})\mathsf{U}_r\right]^{-1}\mathsf{U}_r^H\mathsf{b}(\mathsf{p})$$

Reduced order modeling

Update equations

- We started with

$$\mathsf{b}_1^+(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+\mathsf{J}}\mathsf{j}(\mathsf{p})$$

- Approximated it by

$$\mathsf{b}_1^+(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+\mathsf{J}}\mathsf{U}_r^H\mathsf{a}_r(\mathsf{p})$$

- And end up with

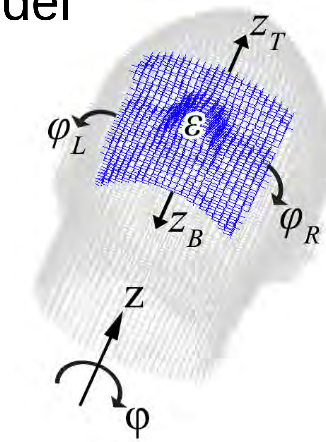
$$\mathsf{b}_1^+(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^+\mathsf{J}}\mathsf{U}_r^H \underbrace{\left[\mathsf{U}_r^H\mathsf{A}(\mathsf{p})\mathsf{U}_r\right]}_{\text{Size } r \text{ instead of } 10\text{\textasciicircum}4}^{-1}\mathsf{U}_r^H\mathsf{b}(\mathsf{p})$$

<u>What to choose for the basis U_r?</u>

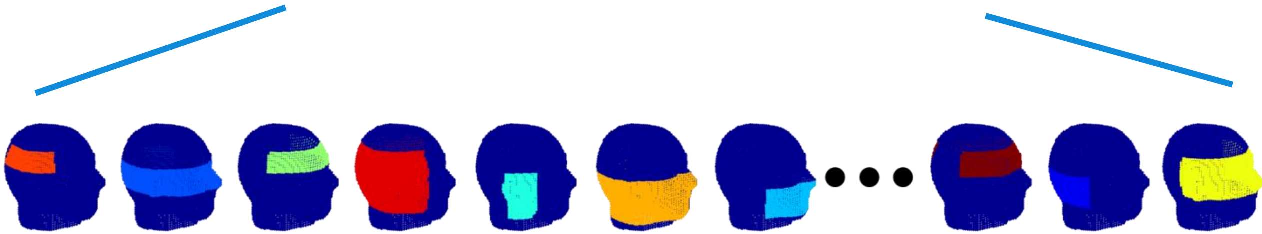
Reduced order modeling

Projection Based Model Reduction

- Create snapshots: random pad parameters in the model



$$\mathsf{S} = [\mathsf{j}_1(\mathsf{p}_1) \ldots \mathsf{j}_S(\mathsf{p}_S)]$$



- Compute SVD of S and take *r* most significant LSV

$$\mathsf{S} = \mathsf{U}\Sigma\mathsf{V}^H \qquad \mathsf{U}_r = \mathsf{U}(1:r,:)$$

Reduced order modeling

Complexity

- We take the 500 first LSV as new basis

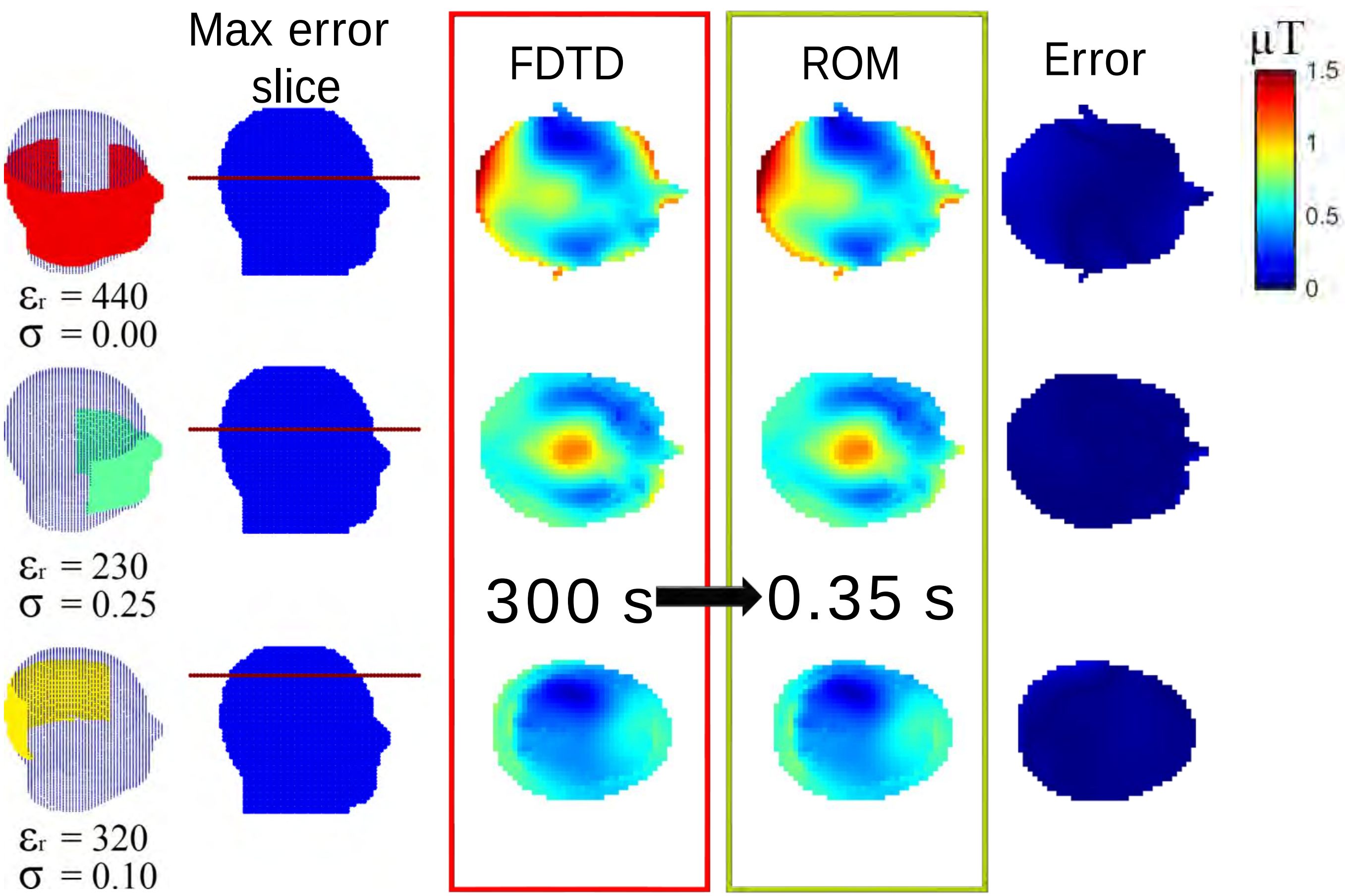
$$\mathsf{b}_1^{+;r}(\mathsf{p}) = \mathsf{b}_1^{+;\text{no pad}} + \mathsf{G}^{B_1^{+}\mathsf{J}}\mathsf{U}_r^H \underbrace{\left[\mathsf{U}_r^H \mathsf{A}(\mathsf{p}) \mathsf{U}_r\right]}_{\substack{\text{Size: } 10^4 \Rightarrow 500 \\ \text{Data: } 30\text{ GB} \Rightarrow 1\text{ GB}}}^{-1} \mathsf{U}_r^H \mathsf{b}(\mathsf{p})$$

Size: 10^4 => 500
Data: 30 GB => 1 GB

- Reduced order models introduce errors in fields

Reduced order modeling

Comparison B_1^+ fields



Pad design

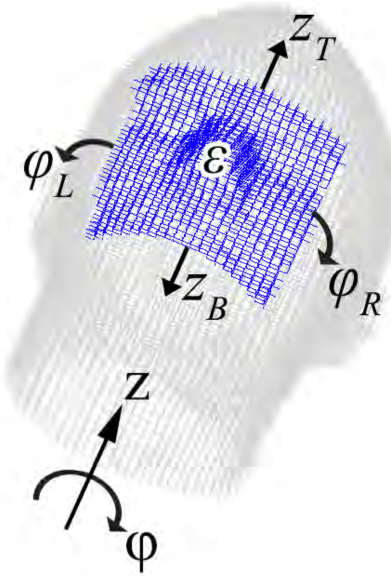
Minimization

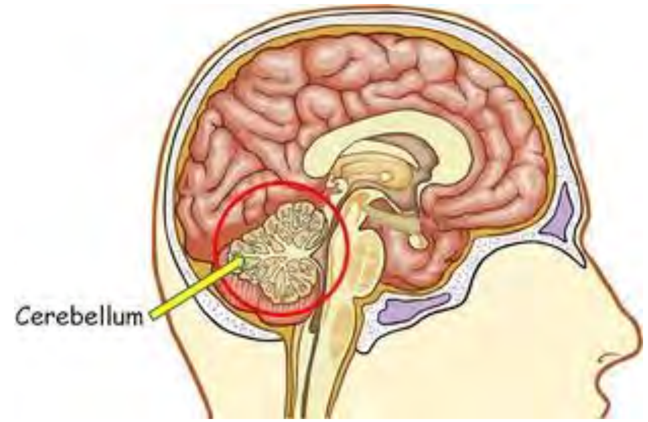
- Define a cost function

$$\mathbf{C}(\mathbf{p}) = \frac{1}{2}\frac{\|\mathbf{b}_1^{+;r}(\mathbf{p}) - \mathbf{b}_1^{+;\text{desired}}\|_2^2}{\|\mathbf{b}_1^{+;\text{desired}}\|_2^2}$$

- We set a desired B_1^+ field as target field

- Solved using Gauss-Newton approach





Pad design

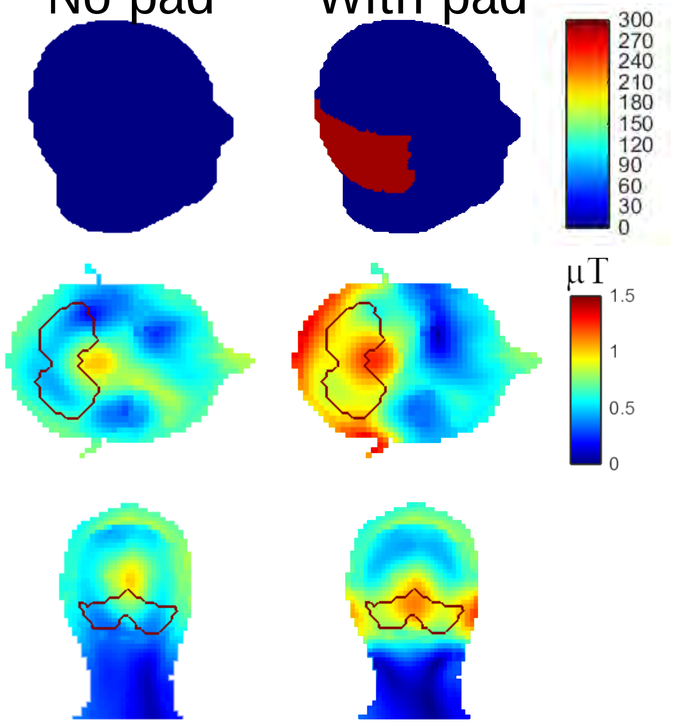
Optimization

No pad | With pad

RHS view

Transverse B_1^+

Coronal B_1^+



Iteration #

Pad found in 30 seconds:

L x W x H = 35 x 10 x 1 cm^3

$\varepsilon_r = 295$

$\sigma = 0.25$ S/m

Pad design

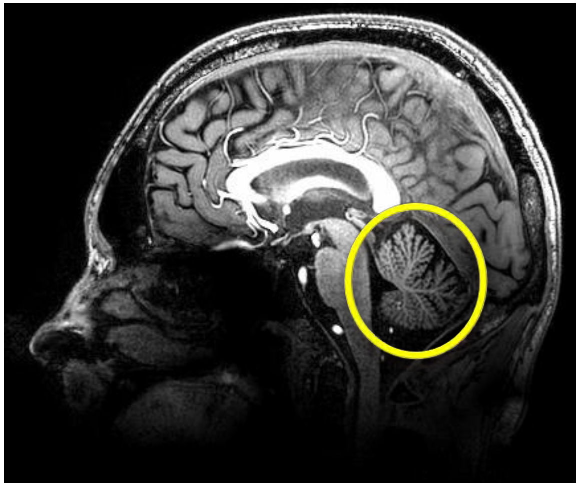
Fabrication

Results

Without pad

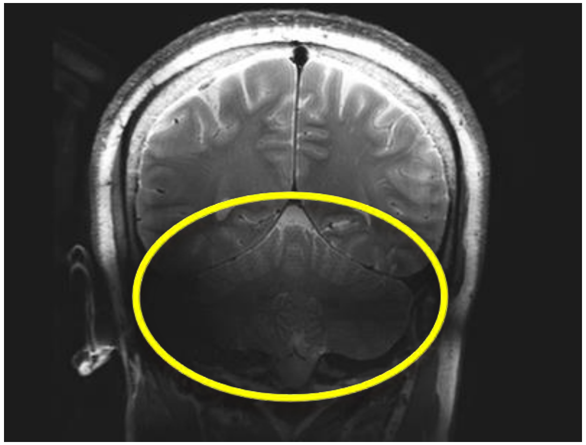
With pad

T1 GRE

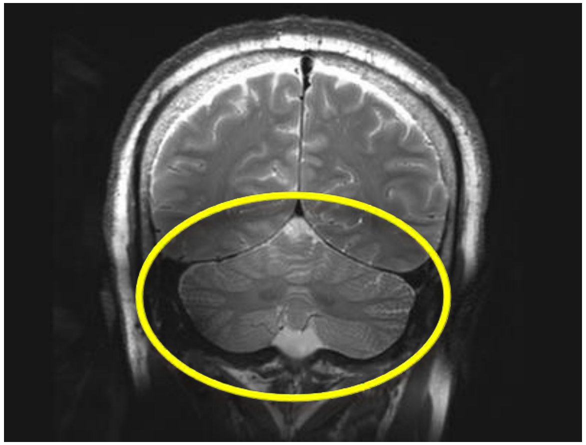


T1 GRE

T2 TSE



T2 TSE



Summary/ Conclusion

There are different methods to solve Maxwell’s equations

The VIE approach is an efficient method for solving Maxwell’s equations, which can be used to construct Green’s tensors

ROM reduces complexity with small loss in accuracy

Designing dielectrics in 30 seconds instead of days

Modeling and Designing High Permittivity Pads for MRI

Kirsten Koolstra and Jeroen van Gemert

May 31st, 2017