

Lab 2 preparation

3.1 Geometric Jacobian (robot, q)

$$H_i^{-1}[] = \text{forward}(\text{robot})$$

for $i=1:6$

$$z_s[i] = H[i][1:3,3]$$

$$O_s[i] = H[i][1:3,4]$$

$$J_v[1] = z_s^0 \times O_s[6]$$

for $i=2:6$

$$J_v[i] = z_s[i-1] \times (O_s[6] - O_s[i-1])$$

$$J = \begin{bmatrix} J_v \\ z_s \end{bmatrix} \quad \# J = \begin{bmatrix} z_s^0 \times (O_s^0 - O_s^i) \\ z_s^i \end{bmatrix}$$

return J

3.2 Analytic Jacobian (robot, q)

$$H_0^0 = \text{forward}(\text{robot})$$

$$R_0^0 = H_0^0[1:3,1:3] = R$$

$$\phi = \text{atan2}(R_{13}, R_{23})$$

$$\theta = \text{atan2}(\sqrt{1-R_{33}^2}, R_{33})$$

$$\psi = \text{atan2}\left(\frac{R_{32}}{\sin\theta}, \frac{-R_{31}}{\sin\theta}\right)$$

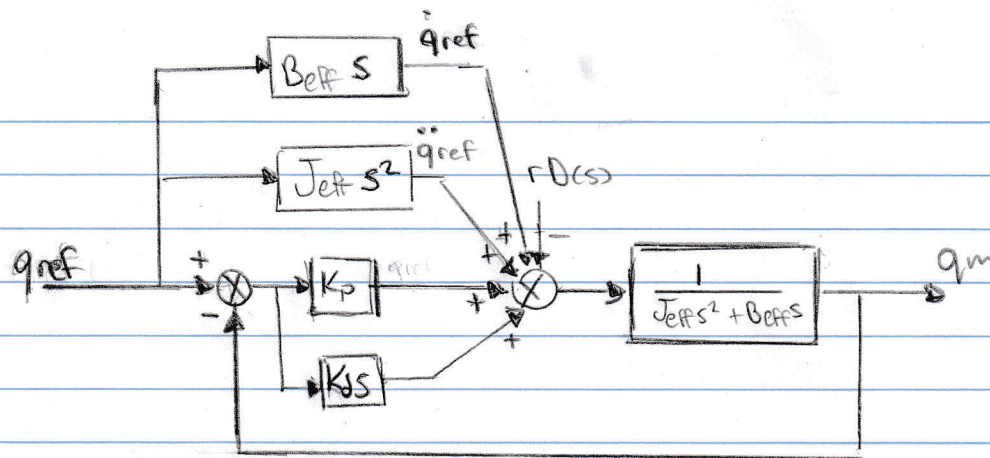
$$B = \begin{bmatrix} 1 & -s\phi & c\phi s\theta \\ 0 & c\phi & s\phi s\theta \\ 0 & 0 & c\theta \end{bmatrix}$$

$$J_a = \begin{bmatrix} I_3 & 0 \\ 0 & B^{-1} \end{bmatrix} \text{Geometric Jacobian}(\text{robot}, q)$$

return J_a

$$\psi = \text{atan2}(-R_{31}, R_{32})$$

5.



6. Using the characteristic polynomial $Js^2 + (B+K_d)s + K_p$ we can rearrange to get $s^2 + \frac{(B+K_d)}{J}s + \frac{K_p}{J} = 0$ at poles

Want this to equal $(s+2)^2 = 0$ for both poles to be -2
 $= s^2 + 4s + 4$

$$\Rightarrow \frac{B+K_d}{J} = 4 \Rightarrow B+K_d = 4J \Rightarrow K_d = 4J - B$$

$$\Rightarrow \frac{K_p}{J} = 4 \Rightarrow K_p = 4J$$

Subbing in J & B from table into K_d/K_p formulas we get

$$K_{p1} = 8e^{-4}$$

$$K_{p2} = 8e^{-4}$$

$$K_{p3} = 8e^{-4}$$

$$K_{p4} = 1.32e^{-4}$$

$$K_{p5} = 1.32e^{-4}$$

$$K_{p6} = 1.32e^{-4}$$

$$K_{d1} = -6.8e^{-4}$$

$$K_{d2} = -1.7e^{-5}$$

$$K_{d3} = -5.80e^{-4}$$

$$K_{d4} = 6.08e^{-5}$$

$$K_{d5} = 4.94e^{-5}$$

$$K_{d6} = 9.53e^{-5}$$