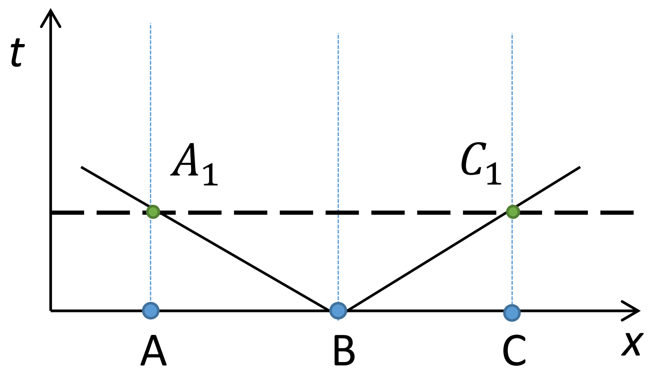
Lecture 3 and 4

Relativity of simultaneity

Lorentz-Einstein transformations

Relativity of Simultaneity

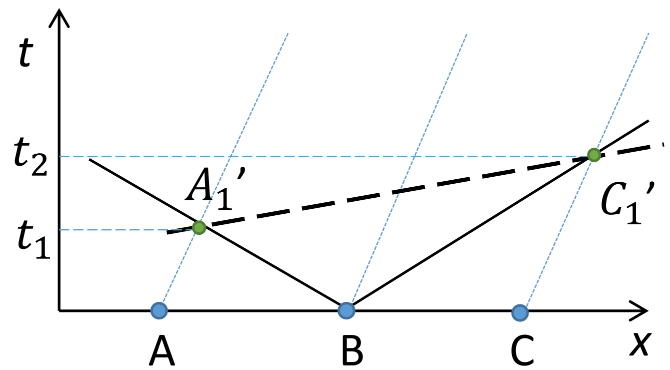
If we use this method of synchronising clocks, we find that simultaneity is *relative*, not absolute. How?



Observation stations A, B and C equally spaced on x-axis in inertial frame in which they are at rest.
Draw World Lines (graph of position vs. time).
At t = 0 send out light signal from B. Light moves at speed *c* forward and backward, with $x = x_B \pm ct$.
Arrives at A and C given by events at the intersections A_1, C_1.
Simultaneity at A and C defined by line A_1C_1 || to x-axis which joins points possessing same value of t.

Relativity of Simultaneity

Now suppose A, B and C at rest in S' which is moving at speed *v* along x-axis:



World lines are now inclined in S.
Signal sent out from B at t = 0 is described in S by $x = x_B \pm ct$.
Arrival at A and C given by intersections A_1', C_1'. These arrivals are *not* simultaneous in S.
Signal reaches A before C because A is running to meet B, while C is running away.
Signal reaches A and C at different times in S; t_1 and t_2 respectively.

Relativity of Simultaneity

Note that B is midway between A and C in both reference frames S and S'.

If we accept universality of *c* and equivalence of inertial frames, A_1' and C_1' must represent simultaneous events in S'.

Line $A_1'C_1'$ represents events which are simultaneous in S'. (An event is characterised by space and time coordinates in a given frame).

Our judgement of simultaneity depends on the particular frame of reference we use.

Relativity of Simultaneity

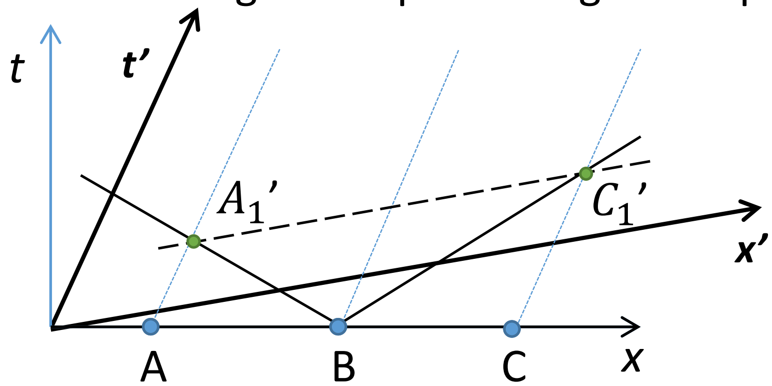
Also see section 37.2 (Young and Freedman)

Now we need to develop the method of making time and space measurements according to Special Relativity.

→Lorentz – Einstein Transformations

Lorentz – Einstein Transformations - 1

Using **Einstein's 2nd Postulate:** **"**The value of *c* is constant", we look for a transformation that gives a speed of light independent of speed of source or receiver.



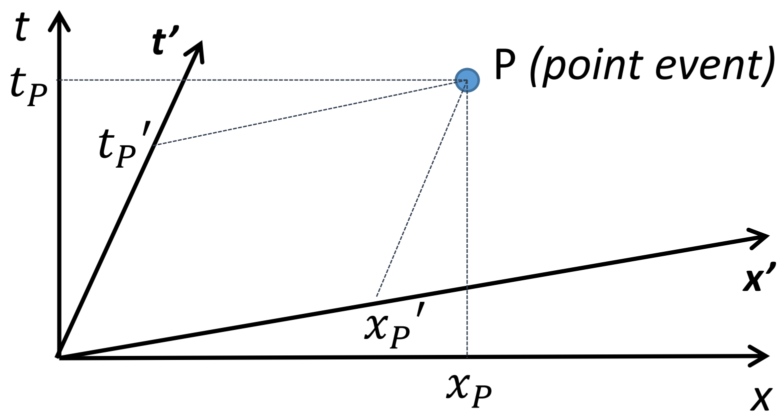
Draw *t'* axis parallel to inclined world lines, as A, B and C will be stationary in S'. Note that it coincides with *t = 0* at *t' = 0.*

Line $A_1'C_1'$ (dotted line) defines simultaneity at stations A and C in S'. Add another coordinate axis *x'* parallel to this line, crossing *t' = 0* at *x' = 0.* This is because positions along a line defining simultaneity have the same time value. The x'-axis is given by all points along line of simultaneity where *t' = 0.*

Position of origin of S' given by $x = vt$ in S if origins coincide at *t = 0.*

Lorentz – Einstein Transformations - 2

This kind of diagram shows us the kinematic transformations of SR:



Point event is given by *x, t* or *x', t'.*

x' is a linear function of *x,* likewise for *t'* and *t.*

Lorentz – Einstein Transformations - 3

From the symmetry implied by the relativity principle, we can say the form of the relationship is as follows:

$$x = ax' + bt' \qquad (1)$$
$$x' = ax - bt \qquad (2)$$

Origin of S moves at velocity $-v$ in S', also given by putting $x = 0$ in (1):

$$0 = ax' + bt'$$
$$\frac{x'}{t'} = -\frac{b}{a} = -v$$

Origin of S' moves at $+v$ in S; put $x' = 0$ in (2):

$$0 = ax - bt$$
$$\frac{x}{t} = \frac{b}{a} = v$$

Lorentz – Einstein Transformations - 4

Consider light signal moving in positive x-direction in S and S', originating at *x*, *x'* = 0.

$$x_l = ct, \quad x_l' = ct'$$

Sub for *x* and *x'* in ① and ②:

$$ct = (ac + b)t'$$
$$ct' = (ac - b)t$$

Eliminate *t* and *t'*:

$$c^2 = a^2c^2 - b^2$$

$$c^2 = a^2(c^2 - v^2) \qquad\qquad b = av$$

$$a = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Call factor $a = \gamma$, and note that $\gamma \geq 1$.

Lorentz – Einstein Transformations - 5

From ①:

$$x = ax' + bt'$$
$$x = \gamma(x' + vt') \quad ③$$

From ②:

$$x' = \gamma(x - vt) \quad ④$$

These differ from G.T. by having a factor of $\gamma \geq 1$.
As $\frac{v}{c} \to 0, \gamma \to 1$.
As $v \to c, \gamma \to \infty$.

Given ③ and ④, we can obtain:

$$t = \gamma(t' + \frac{vx'}{c^2})$$
$$t' = \gamma(t - \frac{vx}{c^2})$$

Lorentz – Einstein Transformations - 6

What about *y* and *z* transverse to the direction of relative motion?

If space is isotropic (same in all directions), all displacements transverse to the direction defined by relative motion are equivalent.

We can conclude that the appropriate transformations are:

$$y = y'$$
$$z = z'$$

If this were not true, we would have a way to detect *absolute* displacements and motions. But. this would violate the essential ideas of relativity.

The measure of transverse distance (*y* or *z)* is the same for all inertial systems that are in relative motion along *x*.

Lorentz – Einstein Transformations - 7

These are called Lorentz-Einstein Transformations are:

$$x' = \gamma(x - vt), \qquad y' = y, \qquad z' = z, \qquad t' = \gamma(t - \tfrac{vx}{c^2})$$

OR

$$x = \gamma(x' + vt'), \qquad y = y', \qquad z = z', \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

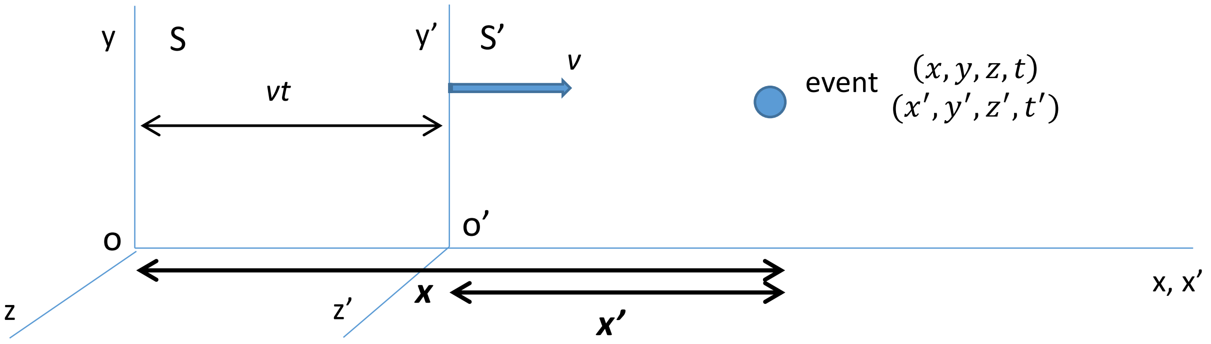
Where v is the velocity of S' as measured in S.

Note also that L.T. → Galilean Transformations as $\frac{v}{c} \to 0$.

Introduced by H.A. Lorentz to account for the null result in the Michelson-Morley experiment, but assuming the existence of a unique inertial frame provide by the ether.

Einstein discovered the equations independently through different approach (no ether).

Lorentz – Einstein Transformations - 8



Transformations give the relation between (x, y, z, t) of an event in *S to* (x', y', z', t') of the same event measured in *S'.*

Note that there is *no universal time* between frames.

Lorentz – Einstein Transformations - 9

Relativity predicts some strange effects; strange because they appear different from everyday experience, but we do not experience speeds $\geq 0.000001\ c$.

For instance, moving clocks appear to run slow, yet they keep time as accurately as any other clocks.

Objects moving at high speeds seem to contract along the direction of motion, without losing any physical matter.

Objects seem to gain mass at high speeds, without an increase in physical matter.

These strange results follows from P2; *c* is the same for all observers.

Concept Question

While John is standing of a station platform he observes a train going from left to right at a speed of 0.5c. Mary is standing at the front of the train and is sending pulses of light towards the back of the train (i.e. opposite to the direction of motion).

What is the velocity the light pulses as measured by John?

A. 0.5c to his left

B. 0.5c to his right

C. c to his left

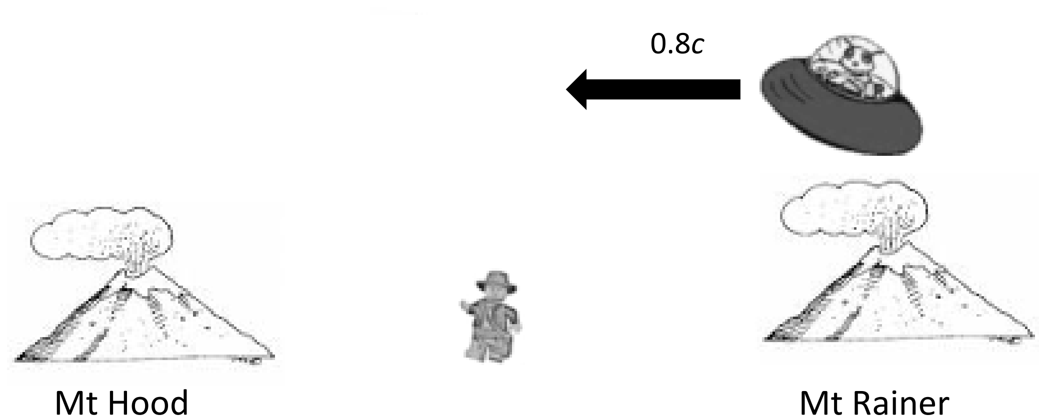
D. c to his right

E. 1.5c to his left

Concept Question

Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes.

A fast spacecraft flying with constant speed v = 0.8c from Rainier towards Hood is directly over Mt. Rainier when it erupts.



Question from:
R. E. Scherr, "An investigation of student understanding of basic concepts in special relativity," Ph.D. dissertation, Department of Physics, University of Washington, 2001 _unpublished_, _www.physics.umd.edu/perg/papers/scherr/index.html

The seismologist and the observer in the spacecraft are intelligent observers, i.e., they correct for signal travel time to determine the time of events in their reference frame. Each observer has clocks which are synchronised with all other observers in their reference frame.

Concept Question

Which of the following statements the timing of the eruptions is correct?

A. In both the seismologist’s and the spacecraft’s reference frame both volcanos erupt simultaneously.

B. In the In the seismologist’s reference frame the eruptions are simultaneous, while in the spacecraft’s reference frame Mt Rainer erupts before Mt Hood.

C. In the seismologist’s reference frame the eruptions are simultaneous, while in the spacecraft’s reference frame Mt Hood erupts before Mt Rainier.

D. In the spacecraft’s reference frame the eruptions are simultaneous, while in the seismologist's reference frame Mt Hood erupts before Mt Rainier.

Concept Question

Jim observes a large spacecraft travelling at speed of 0.7c away from him. He observes (taking into account signal delay) that the tail light and nose light of the craft flash once and simultaneously in his reference frame.

Which of the following statements about observations made in the reference frame of the spacecraft is/are true?

A. There is no way to be sure what is happening on the spacecraft.
B. The nose light flashes before the tail light
C. The lights flash simultaneously
D. The tail light flashes before the nose light