Joint Entrance Examination for Postgraduate Courses in Physics

EUF

Second Semester 2013

Part 1 — 23 April 2013

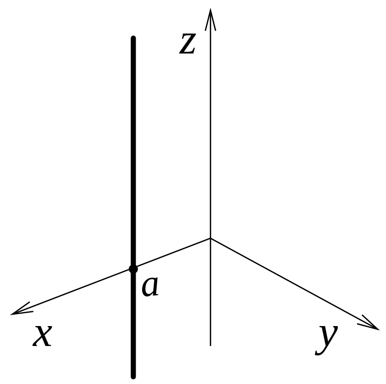
Instructions:

- **DO NOT WRITE YOUR NAME ON THE TEST.** It should be identified **only by your candidate number (EUFxxx)**.
- This test is the **first part** of the joint entrance exam for Postgraduate Physics.

  It contains questions on: Electromagnetism, Modern Physics, Thermodynamics and Statistical Mechanics. All questions have the same weight.
- The duration of this test is **4 hours**. Candidates must remain in the exam room for a minimum of **90 minutes**.
- The use of **calculators** or other electronic instruments is **NOT** permitted during the exam.
- **ANSWER EACH QUESTION ON THE CORRESPONDING PAGE OF THE ANSWER BOOKLET.** The sheets with answers will be reorganized for correction. If you need more answer space, use the extra sheets in the answer booklet. **Remember to write the number of the question (Q1, ou Q2, or . . .) and your candidate number (EUFxxx) on each extra sheet. Extra sheets without this information will be discarded. Use separate extra sheets for each question. Do not detach the extra sheets.**
- If you need spare paper for rough notes or calculations, use the sheets marked SCRATCH at the end of the answer booklet. **DO NOT DETACH THEM**. The scratch sheets will be discarded and **solutions written on them will be ignored**.
- Do **NOT** write **ANYTHING** on the list of Constants and Formulae; **RETURN IT** at the end of the test, as it will be used in the test tomorrow.

Have a good exam!

Q1. Consider an infinitely long wire placed parallel to the z-axis, intersecting the plane $z = 0$ at $x = a$ and $y = 0$, as shown in the figure. The wire has uniform linear charge density λ.



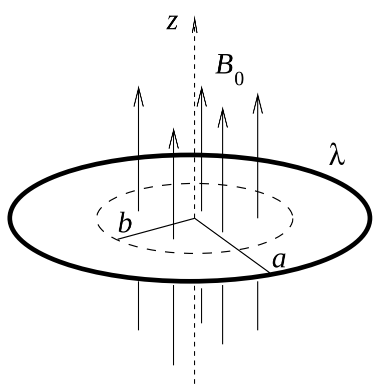
(a) Determine the electric potential $V(x,y,z)$ everywhere, in such a way that the potential is zero on the z-axis. Suggestion: the potential may be computed from the electric field of the wire, which is obtained in a simple way using Gauss's law.

(b) Consider now, in addition to the wire, an infinite (grounded) plane conductor occupying the $x = 0$ plane. Compute $V(x,y,z)$ for the region $x > 0$ in space. Suggestion: use the method of images.

(c) What is the surface charge density $\sigma(y,z)$ induced on the plane conductor at $x = 0$?

(d) Compute the integral $\int_{-\infty}^{\infty} \sigma(y,z)\, dy$ and discuss the result obtained.

Q2. A wire charged with linear electric charge density $\lambda > 0$ is glued (forming a ring) around an insulating disc of radius a, which can rotate without friction around its (vertical) axis. The length of the wire is exactly $2\pi a$. Confined to the central region of the disc, up to radius $b < a$, there is a uniform magnetic field $\mathbf{B_0}$ pointing up in the vertical direction.



(a) The magnetic field is now turned off. Obtain the expression for the torque due to the electromotive force induced in the wire, in terms of the variation of the magnetic field $d\mathbf{B}/dt$. From this result, compute the final angular momentum of the disc (magnitude and direction).

(b) Considering a moment of inertia I for the disc+wire system, compute the magnetic field (magnitude and direction) that is produced at the center of the disc by the moving charged wire in the final situation (in part (a)).

Q3. A light beam with a wavelength of 480 nm in vacuum and intensity of 10 W/m^2 is incident on a cathode of 1 cm^2 area inside a photoelectric cell. The work function of the metal is 2.2 eV. Answers should be given to two significant figures.

(a) Calculate the energy of the incident photons in Joules and electron volts.

(b) Calculate the number of photons per second incident on the metal plate.

(c) If the photoelectric conversion efficiency is 20% (only 20% of the photons cause emission of the electrons from the metal), calculate the maximum electric current through the cell when a potential difference (EMF) is applied between the cathode and the anode.

(d) Calculate the maximum wavelength of the incident photons above which the photoelectric effect does not occur.

Q4. A particle of mass m performs harmonic oscillations in one dimension in a potential $U(x) = m\omega^2 x^2/2$. Consider a particle in a state whose wavefunction is $\psi(x) = Ae^{-bx^2}$, where A and b are constants.

(a) Write the time-independent Schrödinger equation for the potential.

(b) Determine the value of b for which $\psi(x)$ is the solution of this Schrödinger equation, and the value of the energy associated with this wave function.

(c) Calculate the normalization constant A.

(d) Classically, this particle would oscillate within the symmetrical range $[-x_{\max}, x_{\max}]$, where $x_{\max} = [\hbar/m\omega]^{1/2}$. Calculate, using Quantum Mechanics, the probability of finding the particle in the interval $[-x_{\max}, x_{\max}]$. Compare this result with that expected by Classical Mechanics.

Q5. A cylinder with external walls which are impermeable, rigid, and adiabatic, closed at both ends, is equipped with an impermeable, moveable, adiabatic, and ideal (frictionless) internal wall, that divides the cylinder into two compartments (A and B). Each compartment is filled with one mole of a monoatomic ideal gas. Initially the pressure, volume and temperature (P_0,V_0,T_0) are identical on both sides of the internal wall. A given amount of heat is quasi-statically supplied to the gas in compartment A, until its pressure reaches the value $P_{\mathrm{A}} = 32P_0$.

(a) From the equations of state of the monoatomic ideal gas, $U = \frac{3}{2}NRT = \frac{3}{2}PV$ and from its entropy $S/N = \frac{3}{2}R\ln T + R\ln V + \text{constant}$, demonstrate that, during an isentropic process in a closed system, $P^3V^5 = \text{constant}$.

(b) Find the final volumes V_{A} and V_{B} of the two compartments in terms of the initial volume V_0.

(c) Find the final temperatures T_{A} and T_{B} of the two compartments in terms of the initial temperature T_0, showing that $T_{\mathrm{A}} = 15T_{\mathrm{B}}$.

(d) Find the variation of the gas entropy in each of the two compartments, ΔS_{A} and ΔS_{B}. What is the sign of the variation of the total entropy of the system?

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Second Semester 2013

Part 2 — 24 April 2013

Instructions:

- **DO NOT WRITE YOUR NAME ON THE TEST.** It should be identified **only by your candidate number (EUFxxx)**.
- This test is the **first part** of the joint entrance exam for Postgraduate Physics.

  It contains questions on: Classical Mechanics, Quantum Mechanics, Thermodynamics and Statistical Mechanics. All questions have the same weight.
- The duration of this test is **4 hours**. Candidates must remain in the exam room for a minimum of **90 minutes**.
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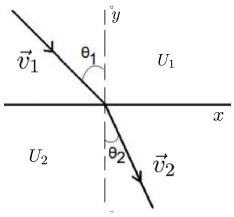
Q6. A particle of mass m moves at velocity $\vec{v}_1$ in the upper half-plane until it reaches the lower half-plane, where it propagates at velocity $\vec{v}_2$ as shown in the figure below. The following characteristics are observed experimentally: i) the particle passes from the first medium to the second medium as long as $v_1 > v_{min}$, ii) the particle moves with uniform rectilinear motion in each of the half-planes, iii) the exit angle, θ_2, differs from the entry angle, θ_1, which makes us assume that in each half-plane the particle is under the influence of different potentials U_1 and U_2.



(a) Based on the experiment, sketch the graph of the potential U as a function of y for fixed x (justify the graph).

(b) Determine v_2 in terms of v_1, m, and the potentials, U_1 and U_2. What is the velocity v_{min} above which there is the passage of the particle from medium 1 to medium 2?

(c) Determine the index of refraction $\sin\theta_1/\sin\theta_2$ in terms of m, v_1, and the potential in each medium.

Q7. A particle of mass m develops a unidimensional motion under the action of the following potential (c is a constant)

$$U(x) = \frac{1}{2}x^4 - cx^2.$$

(a) Sketch the graphs of $U(x)$ and the respective phase spaces ($\dot{x}$ versus x for all possible energies) in the following cases: i) $c > 0$, ii) $c = 0$, and iii) $c < 0$.

(b) For the total energy E, identify all possible periodic movements and their points of inversion (where the velocity is null) for each case of item (a).

(c) Determine the dependence of the period of the oscillations on the total energy E for $c = 0$.

Q8. A particle of mass m is subjected to a potential such that the Schrödinger equation in momentum space is given by ($\hbar = 1$)

$$\left(\frac{\vec{p}^{\,2}}{2m} - a\nabla_p^2\right)\bar{\psi}(\vec{p},t) = i\frac{\partial}{\partial t}\bar{\psi}(\vec{p},t)$$

where

$$\nabla_p^2 = \frac{\partial^2}{\partial p_x^2} + \frac{\partial^2}{\partial p_y^2} + \frac{\partial^2}{\partial p_z^2}.$$

(a) Write the Schrödinger equation in coordinate space.

(b) What is the potential $V(r)$, where $r = |\vec{r}|$?

(c) What is the force $\vec{F}(\vec{r})$ acting on the particle?

Q9. The spin operators of a spin-1 particle (a triplet) can be represented in the complex space C^3 by the matrices

$$\hat{S}_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \hat{S}_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \hat{S}_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Show that the commutation relations $[\hat{S}_x,\hat{S}_y] = i\hbar\hat{S}_z$, and cyclic permutations of x, y, z, are satisfied.

(b) If a measurement of z-spin component is made, what values can be obtained? What are the respective eigenvectors?

(c) If the state of the particle is given by the vector

$$|\phi\rangle = \begin{pmatrix} 1 \\ i \\ -2 \end{pmatrix},$$

what are the probabilities to obtain each of the possible results in a measurement of the z-component of the spin?

(d) Using the results obtained in item (c), what is the probability to find the particle in any of the possible states?

Q10. Consider a modified one-dimensional harmonic oscillator, defined by the Hamiltonian function

$$\mathscr{H} = \frac{p^2}{2m} + V(x),$$

where $V(x) = \frac{1}{2}m\omega^2x^2$ for $x \geq 0$, $V(x) = \infty$ for $x < 0$. The oscillator is in thermal equilibrium with a heat reservoir at temperature T.

(a) Justify, in terms of the parity of the quantum-problem eigenfunctions, why, due to the imposed conditions, only the odd integer values of n are allowed for the energy eigenvalues of this oscillator, $\epsilon_n = (n + 1/2)\hbar\omega$.

(b) For the quantum version, obtain the canonical partition function z of this oscillator and the associated Helmholtz free energy f.

(c) Find the average internal energy of this oscillator from $u = -\partial \ln z/\partial\beta$.

(d) Starting from the definition of the average internal energy in the canonical ensemble, $u \equiv \langle\epsilon_n\rangle$, demonstrate the expression $u = -\partial \ln z/\partial\beta$.

(e) Show that the classical canonical partition function of this oscillator is given by $z_{\text{class}} = (2\beta\hbar\omega)^{-1}$. Calculate the associated classical average internal energy, $u_{\text{class}} \equiv \langle\mathscr{H}\rangle_{\text{class}}$.