EUF

Joint Entrance Examination for Postgraduate Courses in Physics

For the first semester of 2019

October 02, 2018

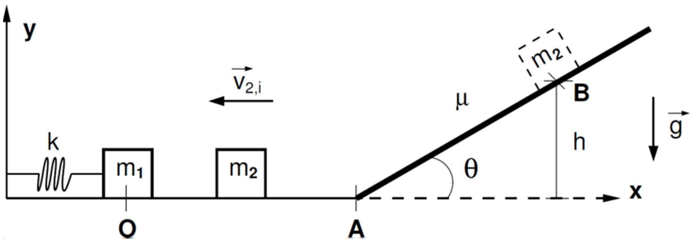
Part 1

This test contains questions on classical mechanics, quantum mechanics, modern physics and thermodynamics. All questions have the same weight.

Have a good exam!

Q1. The figure below shows a system formed by block 1, of mass $m_1 = 2m$, connected to a spring of force constant k and negligible mass, and by block 2, of mass $m_2 = m$. Initially, block 1 is at rest, the spring is relaxed, and block 2 moves towards block 1 with velocity $\vec{v}_{2,i} = -v_0\,\hat{x}$, with $v_0 > 0$. The two blocks collide elastically and, after collision block 1 starts to oscillate. There is friction between the blocks and the surface **only in the incline range AB**, and the gravitational acceleration magnitude is g.

(a) Determine, in terms of v_0, the **vectors** velocity of blocks 1 and 2 immediately after the collision ($\vec{v}_{1,f}$ and $\vec{v}_{2,f}$). Consider that the spring does not influence the collision.



(b) Determine the amplitude x_m of the oscillation of block 1 after the collision in terms of m, k e v_0.

(c) After the collision, block 2 moves towards the incline and remains at rest after reaching the point B. Determine the kinetic friction coefficient μ between block 2 and the incline range AB in terms of g, v_0, the height h and the angle θ.

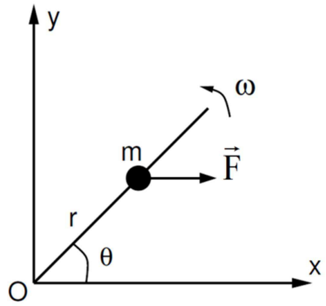
(d) Represent schematically all the forces acting on block 2 when it remains at rest at point B.

Q2. A long massless bar moves in the xy plane rotating around the z axis with constant angular velocity ω, as shown in the figure below. A particle of mass m can slide without friction along the bar, and is subjected to an external force $\vec{F} = m\gamma\hat{x}$, with γ being a positive constant.

(a) Determine the potential energy $V(\vec{r})$ associated to the force $\vec{F}$. Consider the origin O as the reference point of zero potential energy.

(b) Find the constraint equation in terms of the polar coordinates, r and θ, and of the time t. What is the physical cause of the corresponding constraint force?

(c) Write the particle's Lagrangian in terms of the coordinate r, its time derivative $\dot{r}$, and the time t. Then, determine the corresponding equation of motion.



(d) Consider the case in which $\gamma = 0$ and determine the general solution of the equation of motion obtained in (c). Then, find the radial component $r(t)$ of the particle position as a function of time. Initially, $r(t=0) = a$ and $\dot{r}(t=0) = 0$.

Q3. Consider the quantum problem of a particle of mass m moving in the xy plane inside a two-dimensional rectangular box, such that x and y coordinates are limited to the intervals $0 \le x \le a$ and $0 \le y \le b$ (the potential is zero inside the box and infinite outside).

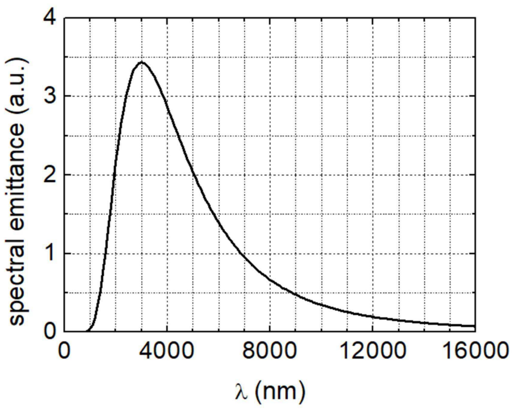
(a) Write the time-independent Schrödinger equation for the particle's wavefunction.

(b) Find the eigenfunctions and the energy eigenvalues. For this, write the solution in the form $\Psi_{n_x n_y}(x,y) = \psi_{n_x}(x)\,\phi_{n_y}(y)$, with n_x and n_y being quantum numbers belonging to the non-zero natural numbers $\mathbb{N}^*$ $\left(n_x, n_y = 1,2,3...\right)$. Normalize the eigenfunctions $\Psi_{n_x n_y}(x,y)$.

(c) Now, consider that at $t = 0$ the particle is in the state given by $\Phi(x,y) = C\,\Psi_{11}(x,y) + D\,\Psi_{12}(x,y)$, with C and D being real constants. What results could be obtained in a measurement of the particle's energy at this moment, and what are their probabilities?

(d) Is the state given in (c) stationary? If it is not, find the wavefunction $\Phi(x,y,t)$ for any time $t > 0$.

Q4. The graph shown in the figure below represents the spectral emittance, $e(\lambda)$, of a black body at a temperature T_1 as a function of the wavelength λ. The energy radiated per unit time and per unit area of the body, in the range of wavelengths from λ to $\lambda + d\lambda$, is given by $e(\lambda)d\lambda$. In the graph, λ is given in nanometers ($1\ \text{nm} = 10^{-9}\ \text{m}$) and the spectral emittance is given in arbitrary units.



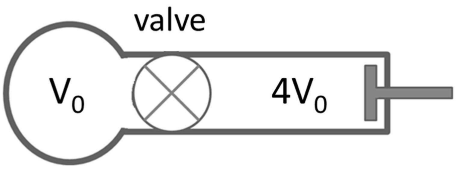
(a) Estimate the temperature T_1 from the graph.

(b) Calculate the total energy radiated per unit time and per unit area of this black body. Express the result in W/m^2.

(c) Calculate, from the graph, the approximate energy radiated per unit time and per unit area of the body, in the range of wavelengths between $6000\,\text{nm}$ and $8000\,\text{nm}$. Express the result in W/m^2.

(d) Consider now a second black body at a temperature $T_2 = 3T_1$. Determine the wavelength of maximum spectral emittance of this second body (in nm).

Q5. The figure below depicts two compartments separated by a valve. The left compartment, of volume V_0, contains 2 mols of an ideal gas at a pressure P_0. The right one, of volume $4V_0$, is initially empty. When the valve is opened, the gas experiences a free expansion (adiabatic and without work) thus occupying both compartments.



Gas	$\gamma = c_P/c_V$
Monoatomic	1.67
Diatomic	1.40
Polyatomic	1.30

(a) What is the gas pressure after the free expansion?

(b) Determine the gas entropy change in the free expansion process.

(c) After the free expansion, the gas is compressed in an adiabatic and quasi-static process until its initial volume V_0. At the end of this process, the gas pressure is $P = 5^{2/5} P_0$. Is the gas monoatomic, diatomic or polyatomic?

(d) Find the ratio U_f/U_i between the final internal energy, U_f (after the compression), and the initial one, U_i (before the free expansion).

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For the first semester of 2019

October 03, 2018

Part 2

This test contains questions on electromagnetism, quantum mechanics, modern physics and statistical mechanics. All questions have the same weight.

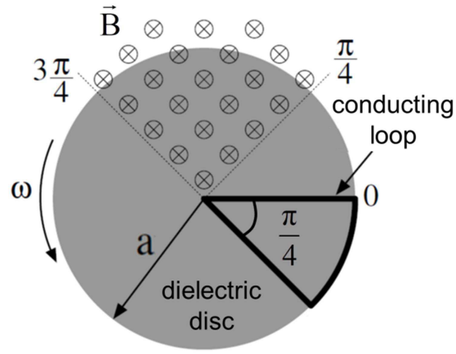
Have a good exam!

Q6. A disc of radius a made of a dielectric material rotates around its symmetry axis with constant angular velocity ω. A loop made of copper, in the form of a circular sector with angle of $\pi/4$ and same radius a, is fixed on the surface of the disc as shown in the figure below. The electric resistance of the loop is R. An external magnetic field of magnitude B, perpendicular to the disc and entering the plane of the figure, is present in the region of the space bounded by the fixed angles $\theta = \pi/4$ and $\theta = 3\pi/4$.

(a) Calculate the magnitude of the electric current induced when the loop enters or when it leaves the region of magnetic field.

(b) Considering positive the current flowing clockwise, represent in a graph the curve of the induced current as a function of time for the period of one revolution of the disc. Set $t = 0$ for the position of the loop depicted in the figure.

(c) Calculate the total energy dissipated in the loop during one cycle.



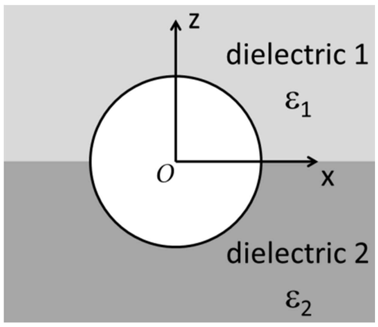
(d) Calculate the torque, with respect to the center of the disc, produced by the magnetic force on the loop when it enters the region of magnetic field. Besides the magnitude, determine the direction of the torque **vector**.

Q7. A conducting solid sphere of radius R is charged and symmetrically immersed between two dielectrics of electric permittivities ε_1 and ε_2, as illustrated in the figure below. Solving the Laplace equation, it can be shown that the electrostatic potential outside the sphere ($r > R$) is given by $V(\vec{r}) = A/r$, with A being a constant and $r = |\vec{r}|$. Express your answers in terms of A, R, ε_1, ε_2, and of the position $\vec{r}$.

(a) Calculate the electrostatic potential $V(\vec{r})$ and the electric field $\vec{E}(\vec{r})$ inside the sphere ($r < R$).

(b) Calculate the electric field $\vec{E}(\vec{r})$ and the electric displacement $\vec{D}(\vec{r})$ outside the sphere ($r > R$), in each dielectric (dielectric 1 and dielectric 2).

(c) What are the surface densities of free charge on the surface of the conducting sphere adjacent to each dielectric?



(d) Is there polarization charge at the interface between the two dielectrics? If so, what is its surface density? If not, justify your answer.

Q8. A hypothetical physical system can have its quantum states described in an orthonormal basis $\{|1\rangle, |2\rangle, |3\rangle\}$. The Hamiltonian of the isolated system is given by:

$$\hat{H}_0 = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + E_1|3\rangle\langle 3|,$$

with E_1 and E_2 being real quantities with dimensions of energy.

In the presence of an external perturbation, the Hamiltonian becomes

$$\hat{H} = \hat{H}_0 + W|1\rangle\langle 3| + W|3\rangle\langle 1|,$$

with W also being a real quantity with dimension of energy.

(a) Write the matrix that represents $\hat{H}$ in the basis $\{|1\rangle, |2\rangle, |3\rangle\}$.

(b) Find the eigenvalues $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ and the corresponding eigenvectors $(|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle)$ of $\hat{H}$.

(c) Write the non-perturbed Hamiltonian $\hat{H}_0$ in the basis $\{|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle\}$.

(d) As one can observe, there is degeneracy in the energy eigenvalues in the absence of external perturbation. For what non-zero values of W there is degeneracy in the presence of the external perturbation?

Q9. The Large Hadron Collider (LHC) accelerates beams of protons up to relativistic velocities and to energies (measured in the laboratory reference frame, S) of the order of teraelectronvolts ($1.0\ \text{TeV} = 1.0\times 10^{12}\ \text{eV}$).

(a) A proton has a total relativistic energy of $5.0\,\text{TeV}$, as measured in the laboratory reference frame S. Calculate the proton velocity (in the frame S) considering that the rest energy of the proton is $1.0\ \text{GeV} = 1.0\times 10^{9}\ \text{eV}$.

Hint: Since the proton velocity is very close to the speed of light in vacuum, c, use $v = (1-\Delta)c$ and find the value of Δ. Remember that $\sqrt{1-\varepsilon} \simeq 1 - \frac{\varepsilon}{2}$ if $\varepsilon \ll 1$.

(b) A proton A, with total relativistic energy E_A, undergoes a frontal collision with another proton B with the same energy and travelling in the opposite direction in the reference frame S. Consider that this collision produces an unknown particle X through the reaction $A + B \Rightarrow X$. Calculate the rest mass of the particle X in terms of E_A.

(c) In another experiment, a proton C, with relativistic factor γ (measured in the laboratory reference frame S) and rest mass m_0, undergoes a frontal collision with another proton D initially at rest. Consider that this collision produces a particle Y through the reaction $C + D \Rightarrow Y$. Calculate the rest mass of Y in terms of γ and m_0.

Q10. A system of N magnetic ions is in contact with a thermal reservoir at temperature T. Let σ_i, with $i = 1, 2, ..., N$, be the variable that represents the projection in the z direction of the spin of ion i in appropriate units. The variable σ_i can have the values $+1$ or -1. Consider that the number N is even. The Hamiltonian of the system is given by

$$H = -J \sum_{k=1}^{N/2} \sigma_{2k-1}\sigma_{2k} - \mu_B h \sum_{i=1}^{N} \sigma_i \, .$$

According to this Hamiltonian, each ion has a magnetic moment μ_B and is coupled to an external magnetic field h. Furthermore, we also can see that the first ion interacts only with the second one, the third only with the fourth, and so on, through the J-coupling.

(a) Calculate the partition function for the case $N = 2$, i.e., for a single pair of ions.
Hint: First, determine the energies of each possible microstate (σ_1, σ_2).

(b) Now, generalize your answer by calculating the partition function of a system with N magnetic ions. In other words, calculate the partition function of a system with $N/2$ pairs of ions.

(c) Calculate the average total magnetic moment of the system with N ions as a function of the external parameters h and T, and of the constants J and μ_B.

(d) What is the value of the average total magnetic moment in the limit of $h \to 0$. Can this system represent a ferromagnetic material? Justify your answer.