Joint Entrance Examination for Postgraduate Courses in Physics

EUF

First Semester/2012

Part 1 – 4 Oct 2011

Instructions:

- **DO NOT WRITE YOUR NAME ON THE TEST. It should be identified only by your candidate number (EUFxxx).**
- This test is the **first part** of the joint entrance exam for Postgraduate Physics.

  It contains questions on: Classical Mechanics, Modern Physics, Thermodynamics and Statistical Mechanics. All questions have the same weight.
- The duration of this test is **4 hours**. Candidates must remain in the exam room for a minimum of **90 minutes**.
- The use of **calculators** or other electronic instruments is **NOT** permitted in the exam.
- **ANSWER EACH QUESTION ON THE CORRESPONDING PAGE OF THE ANSWER BOOKLET.** The sheets with answers will be reorganized for marking. If you need more answer space, use the extra sheets in the answer booklet. **Remember to write the number of the question (Q1, Q2, or . . .) and your candidate number (EUFxxx) on each extra sheet. Extra sheets without this information will not be marked.**
  Use separate extra sheets for each question. Do not detach the extra sheets.
- If you need spare paper for rough notes or calculations, use the sheets marked SCRATCH at the end of the answer booklet. **DO NOT DETACH THEM.** The scratch sheets will be discarded and **solutions written on them will be ignored.**
- Do **NOT** write **ANYTHING** on the List of Constants and Formulae provided; **RETURN IT** at the end of the test, as it will be used in the test tomorrow.

Have a good exam!

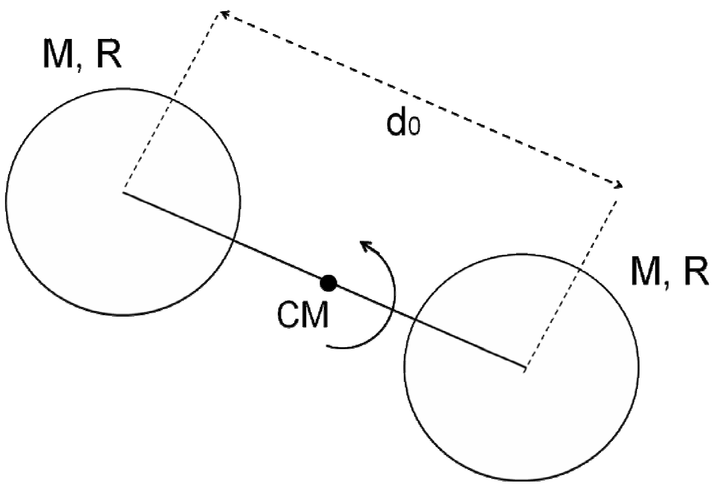
Q1. Two hollow spheres of mass M and radius R are rotating around the center of mass (CM) of the system with initial period T_0. They are maintained at a fixed distance from each other by an ideal string of length $d_0 = 8R$ connecting their centers, as shown in the figure below. At a certain moment, a motor inside one of the spheres starts to wind the string in, slowly pulling the two spheres together. Assume that the moment of inertia of the motor is very small, compared to the moment of inertia of the spheres, and that the effects of gravity may be neglected. Express your answers in terms of M, R and T_0. Note: the moment of inertia of a spherical shell relative to an axis that passes through its center is $\frac{2}{3}MR^2$.

(a) Find the angular momentum of the system relative to the center of mass before the motor is turned on.

(b) Calculate the angular velocity of rotation ω_f of the system at the time when the spheres touch each other.

(c) Determine the change in kinetic energy of the system up to this time.

(d) How much work was done by the motor to make the spheres touch?



Q2. A simple pendulum consists of a mass m suspended from a fixed point by a weightless, inextensible rod of length l. Let g be the local acceleration due to gravity and θ the angle between the pendulum and the vertical axis. Use the small angle approximation in all calculations.

(a) Write down the equation of motion for the simple pendulum, neglecting friction, and obtain its natural frequency, ω.

(b) Find $\theta(t)$ for the following initial conditions: $\theta(0) = 0$ and $\frac{d\theta}{dt}(0) = \Omega$.

(c) Obtain the equation of motion for the pendulum in the presence of a viscous frictional force given by $F_R = 2m\sqrt{gl}\,\frac{d\theta}{dt}$.

(d) In the situation described in item (c), find $\theta(t)$ for the following initial conditions: $\theta(0) = \theta_0$ and $\frac{d\theta}{dt}(0) = 0$.

Q3. Part I – Trying to observe the photoelectric effect, a scientist at the end of the XIX[th] century performs an experiment using pulses (1 ms duration) of monochromatic light, of wavelength 414 nm and three different powers, given by P_0, $3P_0$ and $5P_0$, where $P_0 = 300\ \text{keV/s}$. In this experiment, three metallic surfaces of known work function were used: Li (2.3 eV), Be (3.9 eV) and Hg (4.5 eV).

(a) Determine for which metallic surfaces and powers photoelectron emission could occur.

(b) Calculate the maximum number of photoelectrons that could be emitted by each metallic surface from the pulse with power $3P_0$.

Part II – For filling the subshells of an atom with electrons, the following rule is used: the subshells with lower value of $\boldsymbol{n} + \boldsymbol{l}$ are filled first; if two subshells have the same value of $\boldsymbol{n} + \boldsymbol{l}$, the one with lower $\boldsymbol{n}$ value is filled first.

(c) Use this rule to write the electronic configuration for Sc, which is the atom with the lowest atomic number that has an electron in a d subshell.

(d) What are the possible values of the orbital angular momentum and its z component for an electron in the d subshell of Sc?

Q4. Consider an electron confined within a one-dimensional potential well $V(x)$ given by

$$V(x) = \begin{cases} +\infty & , x < 0 \\ 0 & , 0 < x < d \\ +\infty & , x > d \end{cases}$$

(a) Write the Schrödinger equation for this electron and the boundary conditions that must be fulfilled by its wavefunctions.

(b) Find the normalized wavefunctions and the allowed energy values for this electron.

Suppose now that the electron is in the quantum state described by the following wavefunction within the well:

$$\psi(x) = \sqrt{\frac{2}{d}}\, sin\left(\frac{3\pi x}{d}\right).$$

(c) Find the quantum number n for the state occupied by this electron and its wavelength in this state.

(d) Calculate the probability of finding this electron between $x = 0$ and $x = d/6$.

Q5. Consider a system composed of two distinguishable particles, 1 and 2. Each one should be in either of two compartments, A and B. The energy of a particle is zero when it is in compartment A and ε when it is in compartment B. When the particles are both in the same compartment, there is an additional energy cost Δ. The system is in equilibrium with a heat bath at temperature T.

(a) What are the possible configurations of the system? Determine the energy of each one.

(b) Calculate the partition function Z.

(c) What is the probability of each configuration?

(d) Calculate the mean energy of the system.

(e) Find the entropy of the system in terms of Z.

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Part 2 – 5 Oct 2011

Instructions:

- **DO NOT WRITE YOUR NAME ON THE TEST. It should be identified only by your candidate number (EUFxxx).**
- This test is the **second part** of the joint entrance exam for Postgraduate Physics.

  It contains questions on: Electromagnetism, Quantum Mechanics, Thermodynamics and Statistical Mechanics. All questions have the same weight.
- The duration of this test is **4 hours**. Candidates must remain in the exam room for a minimum of **90 minutes**.
- The use of **calculators** or other electronic instruments is **NOT** permitted in the exam.
- **ANSWER EACH QUESTION ON THE CORRESPONDING PAGE OF THE ANSWER BOOKLET.** The sheets with answers will be reorganized for marking. If you need more answer space, use the extra sheets in the answer booklet. **Remember to write the number of the question (Q1, Q2, or . . .) and your candidate number (EUFxxx) on each extra sheet. Extra sheets without this information will not be marked.**
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Have a good exam!

Q6. A coaxial cable consists of a long straight inner conducting cylinder of radius a surrounded by a thin cylindrical conducting tube of radius b, concentric with the inner cylinder. The two conductors carry equal but opposite currents of intensity i.

(a) Determine the magnitude of the magnetic field in the region between the two conductors ($a < r < b$).

(b) Determine the magnitude of the magnetic field in the region outside the coaxial cable ($r > b$).

(c) Find the magnitude of the magnetic field inside the conducting cylinder ($r < a$) if the current is uniformly distributed over its cross section.

(d) Calculate the energy stored in the magnetic field per unit length of the cable.

Q7. An isolated spherical capacitor has a charge $+Q$ on the inner conductor (radius r_a) and a charge $-Q$ on the outer conductor (radius r_b). The lower half of the volume between the two conductors is then filled with a liquid of relative dielectric constant K, as indicated in the sketch of its cross section below.

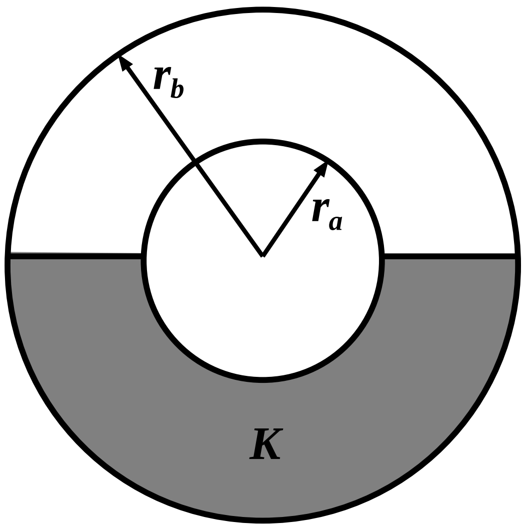
(a) Calculate the magnitude of the electric field in the volume between the two conductors as a function of the distance r measured from the center of the capacitor. Give answers for both the upper and lower halves of this volume.

(b) Determine the free surface charge density on the inner and on the outer conductors.

(c) Calculate the surface polarization charge density on the inner surface (r_a) and the outer surface (r_b) of the dielectric.

(d) What is the surface polarization charge density on the flat (horizontal) surface of the dielectric? Explain.

(e) Determine the capacitance of the system.



Q8. The time-independent Schrödinger equation for a particle of mass m in a one-dimensional harmonic oscillator potential of angular frequency ω is

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega^2x^2\right)\psi_n(x)=E_n\psi_n(x), \quad n=0,1,2,\ldots$$

A method for solving this equation consists in expressing it in terms of the operator

$$a=\frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}x+\sqrt{\frac{\hbar}{m\omega}}\frac{d}{dx}\right)$$

and its Hermitian conjugate.

(a) The ground state wave function for this oscillator satisfies the differential equation $a\,\psi_0(x)=0$. Solve this equation and determine $\psi_0(x)$ apart from a multiplicative constant.

(b) Compute this constant by normalizing $\psi_0(x)$.

(c) Find the value of the ground state energy.

(d) Assume, now, that the oscillator is perturbed by the potential

$$V(x)=V_0\exp\left(-x^2/b^2\right) \quad ,$$

where V_0 and b are real constants. Making use of first-order time-independent perturbation theory, compute the energy shift of the ground state.

Q9. A spin $\frac{1}{2}$ particle has a magnetic dipole moment $\vec{\mu}=\gamma\vec{S}$, γ being a real constant and $\vec{S}=\frac{\hbar}{2}\vec{\sigma}$ the spin operator, where

$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}, \quad \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}, \quad \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

are the Pauli matrices. If this particle is in a uniform magnetic field $\vec{B}$, the spin dynamics is governed by the Hamiltonian $H=-\vec{\mu}\cdot\vec{B}$. In what follows, assume that the magnetic field is in the direction of the axis Oz.

(a) Give the explicit form of the Hamiltonian operator as a 2 x 2 matrix, in terms of γ, $\hbar$ and B.

(b) Write the expressions for the stationary states as normalized column vectors and give their respective energies.

(c) At the initial time, $t=0$, the particle is prepared in the spin state

$$\chi(0)=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\e^{i\alpha}\end{pmatrix} \qquad (\alpha \text{ being real}).$$

What will the spin state, $\chi(t)$, be at a later time t?

(d) At this later time, a measurement is made of S_x, the spin component along Ox. What is the probability $P_+(t)$ of obtaining the value $+\hbar/2$?

Q10. Consider one mole ($n = 1$) of a monatomic ideal gas, initially in the thermal equilibrium state specified by pressure P_0 and volume V_0. This gas suffers a reversible adiabatic compression to the volume $V_0/2$. Determine:

(a) the change in the internal energy of the gas due to this compression;

(b) the change in the entropy of the gas in this compression.

After that adiabatic compression, the gas, which is thermally isolated from the rest of the universe by adiabatic walls, undergoes a completely free expansion to the original volume V_0. Determine:

(c) the change in the temperature of the gas due to this free expansion;

(d) the change in the entropy of the gas in this free expansion.