

Federated Conformal Prediction General

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1 Conformal Prediction General[1]

Definition 1.1 (Exchangeability). [3] For any r.v. x_1, \dots, x_k , we say they are exchangeable if for any permutation $\sigma : [k] \rightarrow [k]$ (bijection), $(x_1, \dots, x_k) \stackrel{d.}{=} (x_{\sigma(1)}, \dots, x_{\sigma(k)})$.

For conformal prediction two classes of targets are studied.

Definition 1.2 (Marginal Coverage). $(X, Y) \in \mathbb{R}^p \times \mathbb{R} \sim P_{XY}$ which is unknown. Given training set $Tr = \{(X_i, Y_i)\}_{i=1}^n$, and test on (X_{n+1}, Y_{n+1}) , both i.i.d.

C_α satisfies distribution-free marginal coverage at level $1 - \alpha$ if

$$P(Y_{n+1} \in C_\alpha(X_{n+1})) \geq 1 - \alpha, \quad \forall P_{XY}$$

The probability is with respect to $\{(X_i, Y_i)\}_{i=1}^{n+1}$.

Definition 1.3 (Conditional Coverage). $(X, Y) \in \mathbb{R}^p \times \mathbb{R} \sim P_{XY}$ which is unknown. Given training set $Tr = \{(X_i, Y_i)\}_{i=1}^n$, and test on (X_{n+1}, Y_{n+1}) , both i.i.d.

C_α satisfies distribution-free marginal coverage at level $1 - \alpha$ if

$$P(Y_{n+1} \in C_\alpha(X_{n+1}) \mid X_{n+1} = x) \geq 1 - \alpha, \quad \forall P_{XY}$$

The probability is with respect to $\{(X_i, Y_i)\}_{i=1}^n$ and Y_{n+1} .

2 Federated Conformal Prediction Article1

Efficient Conformal Prediction under Data Heterogeneity[2]

Idea: The marginal coverage is measured over all training data and test points. However, if there is a high variability in the coverage probability as a function of the training data, the test coverage probability may be substantially below $1 - \alpha$ for a particular training set.

Definition 2.1 (empirical miscoverage rate). $\alpha(Tr) = P(Y_{n+1} \notin C_\alpha(X_{n+1}) \mid Tr)$

References

- [1] Anastasios N Angelopoulos, Stephen Bates, et al. Conformal prediction: A gentle introduction. *Foundations and Trends® in Machine Learning*, 16(4):494–591, 2023.
- [2] Vincent Plassier, Nikita Kotelevskii, Aleksandr Rubashevskii, Fedor Noskov, Maksim Velikanov, Alexander Fishkov, Samuel Horvath, Martin Takac, Eric Moulines, and Maxim Panov. Efficient conformal prediction under data heterogeneity. In *International Conference on Artificial Intelligence and Statistics*, pages 4879–4887. PMLR, 2024.
- [3] Glenn Shafer and Vladimir Vovk. A tutorial on conformal prediction. *Journal of Machine Learning Research*, 9(3), 2008.