Federated CP New Setting

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May 27, 2024

1 Idea1

For K agents each with $Z_k = \{(X_i^k, Y_i^k)\}_{i=1}^{n_k}$ sampled from distribution P^k, and point predictor f_1 based score S_i^k. If all $P^k = P$ and a new test point X, Y from P^1. For any trial data y and score S follow the procedure in "Conformal prediction with local weights: randomization enables robust guarantees"[1]:

- Find some kernel function $H(\cdot, \cdot)$, sample $\tilde{X}$ based on $H(X, \cdot)$.
- Calculate empirical function $\tilde{F} = \sum\limits_{i,k} w_i^k \delta_{S_i^k} + w\delta_S$ with weight

$$w_i^k = \frac{H(X_i^k, \tilde{X})}{\sum\limits_{i',k'} H(X_{i'}^{k'}, \tilde{X}) + H(X, \tilde{X})}, \ w = \frac{H(X, \tilde{X})}{\sum\limits_{i',k'} H(X_{i'}^{k'}, \tilde{X}) + H(X, \tilde{X})}.$$

- Conformal set is $C_\alpha(X) = \{S \le Q(1-\alpha, \tilde{F})\}$.

However for all k $P^k = P$ is not practical, potential covariate shift exists

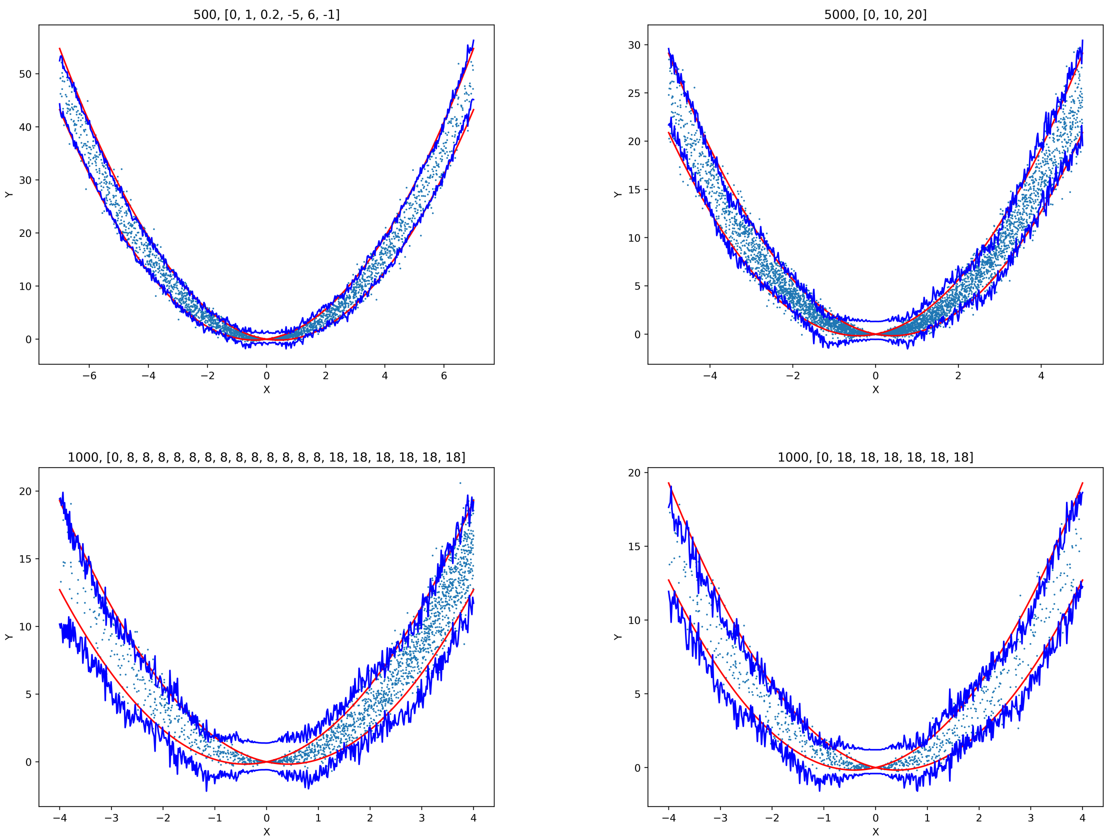
1.1 Experiment1

Assume agent $k = 1, \cdots, K$ each has n samples $X_1^k, \cdots, X_n^k$ follow $N(\mu_k, 9)$. Sythesize $Y_i^k = (X_i^k)^2 + \epsilon$, where $\epsilon \sim N(0, (ep * |X|)^2)$, ep be some parameter. Under this problem only have covariate shift.

- X has different distribution for each agents

- $EY|X$ is same for all agents
- $Y - EY|X$ has same distribution for all agents

Covariate shift has little influence on this method as the method is localized.



1.2 Experiment2

Assume agent $k = 1, \cdots, K$ each has n samples $X_1^k, \cdots, X_n^k$ follow $N(\mu_k, 9)$. Sythesize $Y_i^k = (X_i^k)^2 + \epsilon$, where $\epsilon \sim N(0, (ep * |X| + ep * \theta_k)^2)$, ep be some parameter and θ_k generated for each agent randomly between 1 and 10.

- X has different distribution for each agents
- $EY|X$ is same for all agents
- $Y - EY|X$ has different distribution for different agents

References

[1] Rohan Hore and Rina Foygel Barber. Conformal prediction with local weights: randomization enables local guarantees. *arXiv preprint arXiv:2310.07850*, 2023.