Produced Water Management through Optimal Operation of Produced water re-injection facility ☆☆

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Abstract

Keywords: Daily production optimization; Produced water re-injection; Economic model predictive control; Flow-weighted mean concentration

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1. Introduction

Waterflooding is an enhanced oil recovery technique widely employed in the Oil & Gas industry. Its main purpose is to increase overall oil recovery factors from a reservoir asset by injecting water at injection wells located in different reservoir zones (). The importance of waterflooding increases as a field becomes mature. Mature fields are characterized by a decline in oil production due to reserve depletion, which causes a decrease in reservoir pressure. Injecting of water through waterflooding enables one to maintain reservoir pressure while causing a sweeping effect over reminiscent oil towards production wells (). In offshore facilities, waterflooding can be implemented by either injecting ocean water or produced water. In the Norwegian Continental Shelf, a shift towards re-injecting produced water in the reservoir has been observed (). This trend follows the zero discharge policy recommended by () to reduce marine pollution caused by offshore oil and gas production. Proper operation of a produced water re-injection facility is a challenging task as there is an increasing need to improve the facility economic and environmental performance.

A MINLP model has been developed in (**?**) for the optimal operation of waterflooding strategy. However, in this work it has been considered the presence of only fixed-speed pumps (FSP).

Model predictive control (MPC) is a te

According to (),

In the traditional two-layered approach, the model employed in the real-time optimization layer should represent the process with a higher fidelity than the

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Preprint submitted to Computers and Chemical Engineering *January 10, 2022*

model in the lower layer. Contrary to the traditional approach, we showcase that the model at the MINLP layer does not necessarily need to present a higher fidelity than the one in the EMPC layer. In fact, the model at the MINLP layer should be rich enough to enable one to find the optimal steady-state integer variables. Once this is done, the EMPC layer becomes the sole responsible for keeping the process at its operational optimum.

The work is structured as follows:

- The virtual plant model as well as the main objectives are described in this section.
- The hierarchical control structure is presented in this section.
- The MINLP srategy is described in this section.
- The EMPC strategy is described in this section.

2. Literature Review

3. Notation

We refer to the set of non-negative real numbers as $\mathbb{R}_{\geq 0}$, and the set of natural numbers as $\mathbb{N}$. The time domain is defined as $t \in \mathbb{R}_{\geq 0}$. Furthermore, we define fixed-times which are pre-established time constants and are represented as $t_m \in \mathbb{R}_{\geq 0}$, $\forall m \in \mathbb{N}$, with $t_m < t_{m+1}$. System variables use the notation $\cdot(t)$, while predictive variables use $\cdot(k|t_c)$, which denotes the k-step ahead prediction at the current time $t_c \in \mathbb{R}_{\geq 0}$. We use the accent $(\bar{\cdot})$ to indicate constants. An optimal solution is indicated by the superscript $(\cdot)^*$.

4. Problem Statement

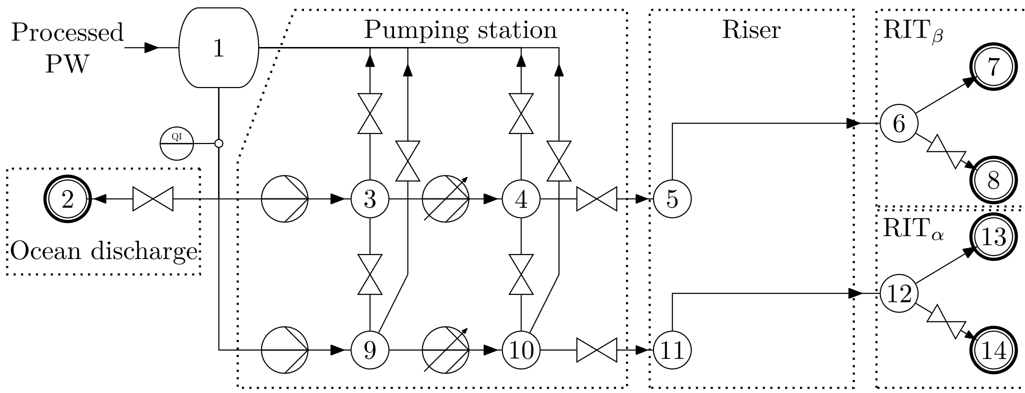
The produced water re-injection facility considered in this study is represented as a network in Figure **??**. The network is represented by a set of nodes $\mathcal{J}$ that are interconnected by a set of arcs $\mathcal{L}$. The set of nodes $\mathcal{L}$ are subdivided into a set of tanks $\mathcal{J}_T$; a set of junctions $\mathcal{J}_J$; a set of injection wells $\mathcal{J}_W$; and a set of ocean discharge $\mathcal{J}_D$. The set of arcs $\mathcal{L}$ are also subdivided into a set of valves $\mathcal{L}_V$; a set of pipelines $\mathcal{L}_P$; a set of fixed-speed pumps $\mathcal{L}_{FSP}$; and a set of variable-speed pumps $\mathcal{L}_{VSP}$. Its graphical representation can be found in Figure **??**.

One can see that this network present the following characteristics: (i) Looped layout; (ii) a pumping train is present upstream of each sink injection well; (iii) each train have fixed and variable speed pumps in series, and (iv) control valves are located upstream of each sink injection well.

The considered network can be described by the set of differential algebraic equations (DAE) shown below,

$$\dot{x}(t) = f(x(t), z(t), u(t), b(t), w(t)), \tag{1a}$$

$$0 = g(x(t), z(t), u(t), b(t), w(t)), \tag{1b}$$



with nonlinear maps given by $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_u} \times \{0,1\}^{n_b} \times \mathbb{R}^{n_w} \to \mathbb{R}^{n_x}$, and $g : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_u} \times \{0,1\}^{n_b} \times \mathbb{R}^{n_w} \to 0$. Moreover, $x \in \mathbb{R}^{n_x}$ are dynamic states, $z \in \mathbb{R}^{n_z}$ are algebraic states, $u \in \mathbb{U} \in \mathbb{R}^{n_u}$ are control inputs, $b \in \{0,1\}^{n_b}$ are binary control inputs, and $w \in \mathbb{R}^{n_w}$ are disturbances. The system represented by (1) is accompanied by a discontinuous stage cost of the form $\ell : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_u} \times \{0,1\}^{n_b} \to \mathbb{R}$.

The facility model as well as its performance metrics are represented in the following subsections.

Before introducing the facility model, we list the assumptions considered in this work.

- Produced water (PW) is considered an incompressible fluid and its temperature is assumed be constant. Moreover, oil content in PW density is neglected. Furthermore, we assume an isothermal process. As a consequence, the PW specific weight γ_i is considered constant.
- ...
- ...

4.1. Nodes - Mathematical representation

Each node is associated with a set of variables and equations that enables one to represent their behavior. Below, we present the variables that describe the set of tanks $\mathcal{L}_T$:

$$x_i = \left[\begin{array}{c} V_i \end{array} \right], \qquad \forall i \in \mathcal{J}_T \tag{2a}$$

$$z_i = \left[\begin{array}{cc} H_i & h_i \end{array} \right], \qquad \forall i \in \mathcal{J}_T \tag{2b}$$

$$p_i = \left[\begin{array}{ccc} d_i & z_i & P_i^s \end{array} \right], \forall i \in \mathcal{J}_T \tag{2c}$$

where V_i is liquid volume at node i; H_i is the hydraulic head at node i; h_i is the PW level at node i; d_i is the demand flowrate at node i; z_i is the elevation at node i; and P_i^s is the surface pressure at node i. The set of junctions $\mathcal{J}_J$ are associated with the following variables,

$$z_i = \begin{bmatrix} H_i \end{bmatrix}, \forall i \in \mathcal{J}_J \tag{3a}$$
$$p_i = \begin{bmatrix} z_i \end{bmatrix}, \forall i \in \mathcal{J}_J. \tag{3b}$$

The variables that describe the set of injection wells $\mathcal{J}_W$ are shown below,

$$z_i = \begin{bmatrix} H_i & P_i & d_i \end{bmatrix}, \forall i \in \mathcal{J}_W \tag{4a}$$
$$p_i = \begin{bmatrix} P_i^r & J_i & z_i \end{bmatrix}, \forall i \in \mathcal{J}_W \tag{4b}$$

where P_i^r *is the reservoir pressure* at node i; and J_i is the injectivity index at node i. Finally, for the set of ocean discharge $\mathcal{J}_D$, we have considered the following variables:

$$z_i = \begin{bmatrix} H_i & d_i \end{bmatrix}, \forall i \in \mathcal{J}_D \tag{5a}$$
$$p_i = \begin{bmatrix} P_i & z_i \end{bmatrix}, \forall i \in \mathcal{J}_D. \tag{5b}$$

Mass balance equations are responsible for guaranteeing that continuity holds in each node $\mathcal{L}$. We stress that only tank nodes $\mathcal{J}_T$ presents dynamics for the volume of liquid, as seen below,

$$\dot{V}_i = \sum_{\substack{i \neq j \\ j \in \mathcal{J}}} \left(q_{(i,j)} - q_{(j,i)} \right) - d_i, \qquad \forall i \in \mathcal{J}_T \tag{6a}$$
$$0 = \sum_{\substack{i \neq j \\ j \in \mathcal{J}}} \left(q_{(i,j)} - q_{(j,i)} \right), \qquad \forall i \in \mathcal{J}_J \tag{6b}$$
$$0 = \sum_{\substack{i \neq j \\ j \in \mathcal{J}}} \left(q_{(i,j)} - q_{(j,i)} \right) - d_i, \quad \forall i \in \mathcal{J}_D \cup \mathcal{J}_W \tag{6c}$$

where $q_{(i,j)}$ is the flowrate at arc (i, j). Furthermore, (6a) is the mass balance at tank nodes $\mathcal{J}_T$; (6b) is the mass balance at junction nodes $\mathcal{J}_J$; and (6c) is the mass balance at ocean discharge and well nodes $\mathcal{J}_D \cup \mathcal{J}_W$. Furthermore, we employ the convention that positive values of demand represents PW leaving the node, while positive is for PW entering the node.

It is considered that each node $\mathcal{J}$ holds an amount of mechanical energy. This energy is represented in terms of hydraulic head,

$$H_i = \frac{P_i^s}{\gamma_i} + \left(z_i + h_i - z_0 \right), \qquad \forall i \in \mathcal{J}_T \tag{7a}$$
$$H_i = \frac{P_i}{\gamma_i} + \left(h_i - z_0 \right), \qquad \forall i \in \mathcal{J} \setminus \mathcal{J}_T, \tag{7b}$$

in which (7a) is the hydraulic head at tank nodes $\mathcal{J}_T$; and (7b) is the hydraulic head at non-tank nodes $\mathcal{J} \setminus \mathcal{J}_T$.

A relation known as inflow performance relationship (IPR) correlates the pressure and flowrate at well nodes $\mathcal{J}_W$. The IPR can be obtained experimentally by performing injectivity tests in each particular well. For water, the IPR can be assumed to be linear.

$$P_i = P_i^r + \frac{d_i}{J_i}, \quad \forall i \in \mathcal{J}_W, \tag{8}$$

in which (8) is the IPR in well nodes $\mathcal{J}_W$.

4.2. Arcs - Mathematical representation

Each arc is represented by a set of variables and equations to describe their behavior. The variables that represent the set of vales $\mathcal{L}_V$ are given as,

$$z_{(i,j)} = \left[\begin{array}{cc} q_{(i,j)} & H^L_{(i,j)} \end{array} \right], \forall i \in \mathcal{L}_V, \tag{9a}$$

$$u_{(i,j)} = \left[\begin{array}{c} \phi_{(i,j)} \end{array} \right], \quad \forall i \in \mathcal{L}_V, \tag{9b}$$

$$p_{(i,j)} = \left[\begin{array}{c} K_{(i,j)} \end{array} \right], \quad \forall i \in \mathcal{L}_V, \tag{9c}$$

in which $\phi_{(i,j)}$ is the valve opening; and $H^L_{(i,j)}$ is the hydraulic head loss at arc (i, j). The variables that describe the set of pipelines are shown below,

$$z_{(i,j)} = \left[\begin{array}{cc} q_{(i,j)} & H^L_{(i,j)} \end{array} \right], \quad \forall i \in \mathcal{L}_P, \tag{10a}$$

$$p_{(i,j)} = \left[\begin{array}{ccc} C_{(i,j)} & \Delta s_{(i,j)} & D_{(i,j)} \end{array} \right], \forall i \in \mathcal{L}_P, \tag{10b}$$

$C_{(i,j)}$ is the Hazen-William constant at arc (i, j); $\Delta s_{(i,j)}$ is the pipeline length at arc (i, j); and $D_{(i,j)}$ is the pipeline diameter at arc (i, j). To represent the set of FSPs $\mathcal{L}_{FSP}$, we consider the following variables,

$$z_{(i,j)} = \left[\begin{array}{cccc} q_{(i,j)} & H^G_{(i,j)} & W_{(i,j)} & \eta_{(i,j)} \end{array} \right], \forall i \in \mathcal{L}_{FSP}, \tag{11a}$$

$$b_{(i,j)} = \left[\begin{array}{c} Z_{(i,j)} \end{array} \right], \quad \forall i \in \mathcal{L}_{FSP}, \tag{11b}$$

where $H^G_{(i,j)}$ is the hydraulic head gain at arc (i, j); and $Z_{(i,j)}$ is the pump on/off status at arc (i, j). Finally, the variables that describe the set of VSPs $\mathcal{L}_{VSP}$ can be found below:

$$z_{(i,j)} = \left[\begin{array}{cccc} q_{(i,j)} & H^G_{(i,j)} & W_{(i,j)} & \eta_{(i,j)} \end{array} \right], \forall i \in \mathcal{L}_{VSP} \tag{12a}$$

$$u_{(i,j)} = \left[\begin{array}{c} \omega_{(i,j)} \end{array} \right], \quad \forall i \in \mathcal{L}_{VSP} \tag{12b}$$

$$b_{(i,j)} = \left[\begin{array}{c} Z_{(i,j)} \end{array} \right], \quad \forall i \in \mathcal{L}_{VSP} \tag{12c}$$

where $\omega_{(i,j)}$ is the pump rotation at arc (i, j).

At each arc it is necessary to account for gains or losses of mechanical energy. The mechanical energy balance is performed in terms of hydraulic head and is shown below,

$$H^L_{(i,j)} = H_j - H_i, \quad \forall (i, j) \in \mathcal{L}_V \cup \mathcal{L}_P, \tag{13a}$$

$$Z_{(i,j)} H^G_{(i,j)} = Z_{(i,j)} \left(H_i - H_j\right), \forall (i, j) \in \mathcal{L}_{FSP} \cup \mathcal{L}_{VSP}, \tag{13b}$$

in which (13a) is the mechanical energy balance for valves and pipelines; and (13b) is the mechanical energy balance at FSP and VSP nodes, which must hold when pump (i,j) is on.

As PW passes through pipeline arcs $\mathcal{L}_P$, friction is exerted on the fluid by the inner pipeline wall, and part of the energy content of the fluid dissipates. For a single-phase system, the relation between hydraulic head loss and friction can be given by the Darcy-Weisbach empirical relation. However, ... Due to that, we consider in this work the Hazen-William empirical equation. Moreover, it is known that PW flows from a node with higher hydraulic head to one with lower hydraulic head. Due to that, we do not restrict the flowrate to only positive values in pipelines (**?**). One way of representing this behavior is by introducing non-smooth constraints, and is shown below:

$$H^L_{(i,j)} = \frac{10.67\gamma_{(i,j)}\Delta s_{(i,j)}}{{C_{(i,j)}}^{1.852}{d_{(i,j)}}^{4.87}} sgn(q_{(i,j)})|q_{(i,j)}|^{1.852}, \tag{14}$$

in which (14) is the Hazen-Willian equation for hydraulic loss.

Control valves enables one to regulate the flowrate of the PW. Assuming turbulent flow throug control valves, its mathematical formulation is given below (**?**),

$$q_{(i,j)} = 27.3\phi_{(i,j)}K sgn(H^L_{(i,j)})\sqrt{\frac{|H^L_{(i,j)}|g}{10^5}}, \quad \forall (i,j) \in \mathcal{L}_V \tag{15}$$

in which $sgn(H^L_{(i,j)})$ accounts for the possibility of reverse flow through valves. It is important to notice in (15) that when a valve is closed, decoupling between $H^L_{(i,j)}$ and $q_{(i,j)}$ occurs and $H^L_{(i,j)}$ is given solely by (13a).

The behavior of centrifugal pumps can be generally represented in terms of hydraulic head gain, shaft-power and hydraulic efficiency. To model these variables, several methods are present in the literature (**?**). Below, we show a compact model representation for FSPs:

$$H_{(i,j)} = Z_{(i,j)}f_{(i,j)}(q_{(i,j)}), \quad \forall (i,j) \in \mathcal{L}_{FSP} \tag{16a}$$
$$W_{(i,j)} = Z_{(i,j)}f_{(i,j)}(q_{(i,j)}), \quad \forall (i,j) \in \mathcal{L}_{FSP} \tag{16b}$$

in which (16a) is the hydraulic gain curve; and (16b) is the shaft-power curve. For proper operation of FSPs, the operational pumping flowrate is limited to a certain range as shown below:

$$q^{lb}_{(i,j)} \leq q_{(i,j)} \leq q^{ub}_{(i,j)}, \quad \forall (i,j) \in \mathcal{L}_{FSP}. \tag{16c}$$

When considering VSPs, it is possible to adjust its rotation. A compact model for representing VSPs is given below:

$$H_{(i,j)} = Z_{(i,j)}f_{(i,j)}(q_{(i,j)}, w_{(i,j)}), \quad \forall (i,j) \in \mathcal{L}_{VSP} \tag{17a}$$
$$W_{(i,j)} = Z_{(i,j)}f_{(i,j)}(q_{(i,j)}, w_{(i,j)}), \quad \forall (i,j) \in \mathcal{L}_{VSP} \tag{17b}$$

in which (17a) is the hydraulic gain curve; and (17b) is the shaft-power curve. Limitations are imposed in the VSP operational, creating an operational envelope. These limits are given by the following set of inequalities constraints.

$$g^{lb}_{(i,j)}(H_{(i,j)}) \leq q_{(i,j)} \leq g^{ub}_{(i,j)}(H_{(i,j)}), \qquad \forall (i,j) \in \mathcal{L}_{VSP} \tag{17c}$$

$$f_{(i,j)}(q_{(i,j)}, \omega^{lb}_{(i,j)}) \leq H_{(i,j)} \leq f_{(i,j)}(q_{(i,j)}, \omega^{ub}_{(i,j)}), \quad \forall (i,j) \in \mathcal{L}_{VSP} \tag{17d}$$

Another important element of a pump operation is its hydraulic efficiency. Shaft-power is not entirely converted into hydraulic power as energy can be lost to the environment in the form of heat, vibration or noise. The hydraulic efficiency of a centrifugal pump is obtained by the following equation:

$$\eta_{(i,j)} = \frac{\gamma H_{(i,j)} q_{(i,j)}}{W_{(i,j)}}, \quad \forall (i,j) \in \mathcal{L}_{FSP} \cup \mathcal{L}_{VSP}, \tag{18}$$

To avoid harmful phenomena in the pumps, an operational limitation based on the hydraulic efficiency of the pumps to avoid harmful phenomena in pumps during which (28) is imposed to avoid harmful phenomena in pumps during operation (*i.e.* $Z_{(i,j)} = 1$); (29) is imposed to

It is desired to keep the hydraulic efficiency of pumps above 92% of its best efficiency point to avoid the occurrence of harmful phenomena (**?**).

4.3. Objective

Some variables considered in this work are associated with the objective function. Those are found below,

$$p_{\mathcal{N}} = \begin{bmatrix} \gamma & g & z_0 \end{bmatrix} \tag{19}$$

$$p_{obj} = \begin{bmatrix} \$_{oil} & \$_{fuel} & \$_{CO_2} & E_d & \rho_{gt} \end{bmatrix} \tag{20}$$

$$p_{RIT} = \begin{bmatrix} \lambda_\alpha & \lambda_\beta \end{bmatrix} \tag{21}$$

$$b_{RIT} = \begin{bmatrix} Z_\alpha & Z_\beta \end{bmatrix} \tag{22}$$

where γ is the PW specific weight; g is the gravitational acceleration; z_0 is a reference elevation; λ_α and λ_β are the injection effectiveness at respectively RIT$_\alpha$ and RIT$_\beta$; $\$_{oil}$ is the price of oil; $\$_{fuel}$ is the price of fuel; $\$_{CO_2}$ is the carbon tax.

For optimal operation of the PWRI facility, some key aspects should be taken into consideration. The operational profit is given by the difference between revenue and operational expenditure (OPEX),

$$\ell(x(t), u(t), b(t)) = R + C_{OPEX} \tag{23}$$

where R is the revenue; and C_{OPEX} is the OPEX.

Revenue is based on oil production, and is directly influenced by PW injection. Based on reservoir simulators employed in the oil & gas industry, we present the marginal relationship between PW injection and oil production, which is valid only locally:

$$R_{total} = \$_{oil} \left[\lambda_\alpha Z_\alpha \left(\sum_{i \in \mathcal{J}_W \cap \mathcal{J}_\alpha} d_i \right) + \lambda_\beta Z_\beta \left(\sum_{i \in \mathcal{J}_W \cap \mathcal{J}_\beta} d_i \right) \right], \tag{24}$$

where R is the total revenue and R_{base} is the base revenue. The OPEX accounts for fuel costs and CO_2 emission taxes. One may notice the presence of binary control inputs in (24). Injection of PW in each re-injection template should be limited to a particular region as it is required to stay in conformity with reservoir management long-term objectives,

$$Z_i = \begin{cases} 1 & if q_i \in \left[q_i^{lb} q_i^{ub} \right] \\ 0 & if q_i < q_i^{lb} \end{cases} \qquad \forall i \in \{\alpha, \beta\} \tag{25}$$

We consider that each pump is coupled with a gas turbine, and we assume a constant efficiency from fuel combustion to pump shaft-power,

$$C_{opex} = (\$_{fuel} + \$_{CO_2}) \left(\sum_{(i,j) \in \mathcal{L}_{FSP} \cup \mathcal{L}_{VSP}} Z_{(i,j)} W_{(i,j)} \right) / \eta_{gt}. \tag{26}$$

In (26), binary control inputs also play a central role as fuel and CO_2 costs depends on whether the associated pump is on/off.

Dealing with the constraints presented in (**??**)

$$\min_{b(t), u(t)} \int_{t_0}^{T} \sum_{(i,j) \in \mathcal{L}_{FSP} \cup \mathcal{L}_{VSP}} Z_{(i,j)} W_{(i,j)} \tag{27}$$

$$Z_{(i,j)} \left(0.92 \eta^{BEP}{}_{(i,j)} - \eta_{(i,j)} \right) \leq 0, \quad \forall (i,j) \in \mathcal{L}_{FSP} \cup \mathcal{L}_{VSP}. \tag{28}$$

$$Z_{(i,j)} \left(d_i \right)) = 0 \tag{29}$$

$$\tag{30}$$

$$\tag{31}$$

in

5. Control structure

In the previous section, it was shown that the system to be controlled contains both integer and non-integer variables as control inputs. To maximize operational revenue, the control structure proposed in this work is based at a two hierarchical layer approach. In this work, the first layer is composed of a steady-state mixed-integer nonlinear programming (MINLP), while the second has an economic model predictive control (EMPC). The purpose of the first layer is to obtain the set of on/off pumps (*i.e.* select the integer variables). This information is passed to the second layer, in which the non-integer control inputs are chosen. The overall objective is to maximize the process revenue, while operating the pumping system in a healthy manner.

6.

An algorithm for

7. Valve model

Automatic switch valves installed in the re-injection templates can be set to open only when PW can flow towards the well. To obtain the desired behavior from the system, the following model is employed.

$$q_{(i,j)} = 27.3\, C_v Z_{(i,j)} H^{L,a}_{(i,j)} \sqrt{g/1e5}, \forall (i,j) \in \mathcal{L}_{SVO} \tag{32}$$

$${H^{L,a}_{(i,j)}}^2 = Z^a_{(i,j)} H^L_{(i,j)}, \qquad \forall (i,j) \in \mathcal{L}_{SVO} \tag{33}$$

$$q_{(i,j)} \geq 0,\; H^{L,a}_{(i,j)} \geq 0, \qquad \forall (i,j) \in \mathcal{L}_{SVO} \tag{34}$$

Control valves can be used to regulate the flowrate passing through an orifice. This flow is a function of the valve characteristics, opening, and the square root of the hydraulic head loss. A preview analysis in valves indicate that headloss is always positive. Thus, flow only occurs in one direction. Due to that, the following model is employed.

$$q_{(i,j)} = 27.3\, C_v \phi_{(i,j)} \sqrt{H^L_{(i,j)} g/1e5}, \forall (i,j) \in \mathcal{L}_{CVO} \tag{35}$$

$$q_{(i,j)} \geq 0,\; H^L_{(i,j)} \geq 0, \forall (i,j) \in \mathcal{L}_{CVO} \tag{36}$$

The re-injection template valve can be simplified as it is not desirable to have reverse flux from the wells to the network. Due to that, we consider the following equations,

$$q_{(i,j)} = 27.3\, C_v Z_{(i,j)} \sqrt{g H^{L,a}_{(i,j)}/1e5} \tag{37}$$

$$q_{(i,j)} \geq 0,\; H^{L,a}_{(i,j)} \geq 0 \tag{38}$$

$$H^{L,a}_{(i,j)} = H^L_{(i,j)}(2Z^a_{(i,j)} - 1) \tag{39}$$

The valve on/off model is approached by this

$$q_{(i,j)} = 27.3\, C_v\, Z_{(i,j)} H^a_{(i,j)} \sqrt{g/1e5} \forall i \in \mathcal{L}_{inter} \tag{40}$$

$${H^a_{(i,j)}}^2 = H^L_{(i,j)}(2Z^a_{(i,j)} - 1) \qquad \forall i \in \mathcal{L}_{inter} \tag{41}$$

$$0 \leq H^a_{(i,j)} H^L_{(i,j)} \qquad \forall i \in \mathcal{L}_{inter} \tag{42}$$

The regulated valve is approach as

$$q_{(i,j)} = 27.3\, C_v\, \phi_{(i,j)} H^a_{(i,j)} \sqrt{g/1e5} \forall i \in \mathcal{L}_{inter} \tag{43}$$

$${H^a_{(i,j)}}^2 = H^L_{(i,j)}(2Z^a_{(i,j)} - 1) \qquad \forall i \in \mathcal{L}_{inter} \tag{44}$$

$$0 \leq H^a_{(i,j)} H^L_{(i,j)} \qquad \forall i \in \mathcal{L}_{inter} \tag{45}$$

8. Pump selection

In the considered system, we have that each re-injection template is associated with a pumping train composed of fixed-speed pumps (FSP) and variable-speed pumps. The operational point of a pumping process is given by the intersection of the pump hydraulic head curve, referred to as QH curve, and the system resistance curve. Manipulation of a pump QH curve is only possible for VSPs as their rotation can be manipulated

In general, valves are used to manipulate the system resistance curve. As for the pump QH curve, it is

Due to that, it is possible to manipulate the pumping system operational poing.

Before introducing the model of a pumping train, we start by showcasing the model of individual pumps.

For a single fixed-speed pump (FSP), there exist little operational freedom

the hydraulic head and flowrate relationship is given by the HQ curve. Due to that, little operational freedom is given by FSPs. In fact, one can only turn on or off a FSP.

, which is shown below in its most general form.

$$H_{i,j} = f_{i,j}(q_{i,j}), \quad \forall (i,j) \in (FSP, train) \tag{46a}$$

where $f_{i,j} : \mathbb{R} \to \mathbb{R}$ represents the HQ curve. For proper operation of the FSP, the operational pumping flowrate is limited to a certain range,

$$q_{i,j} - q_{i,j}^{lb} \geq 0, \quad \forall (i,j) \in (FSP, train) \tag{46b}$$

$$q_{i,j}^{ub} - q_{i,j} \leq 0, \quad \forall (i,j) \in (FSP, train) \tag{46c}$$

When considering VSPs, a single curve is unable to represent its operational behavior. For variable-speed pumps there exist the possibility of adjusting the pumping operation by changing its current rotation. For a single variable-speed pump (VSP), the HQ curve is described in its general form below:

$$H_{i,j} = f_{i,j}(q_{i,j}, w_{i,j}), \quad \forall (i,j) \in (VSP, train) \tag{47a}$$

where $f_{i,j} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$. The operational VSP envelop is nonconvex, and it is given by the following set of inequalities constraints.

$$q_{i,j}^{lb} = g_{i,j}^{lb}(H_{i,j}), \qquad \forall (i,j) \in (VSP, train) \tag{47b}$$

$$q_{i,j}^{ub} = g_{i,j}^{ub}(H_{i,j}), \qquad \forall (i,j) \in (VSP, train) \tag{47c}$$

$$H_{i,j}^{lb} = f_{i,j}(q_{i,j}, \omega_{i,j}^{lb}), \quad \forall (i,j) \in (VSP, train) \tag{47d}$$

$$H_{i,j}^{ub} = f_{i,j}(q_{i,j}, \omega_{i,j}^{ub}), \quad \forall (i,j) \in (VSP, train) \tag{47e}$$

8.1. Pumping train

For a pumping system in series with a set of FSP and VSP, one has that the hydraulic head of each pumping train is given as:

$$H_j(q_j, \omega_j, b_j) = b_j \left(\sum_{i \in FS} H_{i,j}(q) + \sum_{i \in VSP} H_{i,j}(q, \omega_{i,j}) \right), \forall j \in train, \tag{48a}$$

where H_j is the hydraulic head of train j; q_j is the flowrate through train j; b_j is the on/off state of train j; $H_{i,j}$ is the hydraulic head at pump i in train j; The minimum and maximum allowed flowrate in the train is given as,

$$q_j^{lb} = b_j \max(\{q_{i,j}^{lb} \mid \forall i \in VSP\}, \{q_{i,j}^{lb}(H_{i,j}) \mid \forall i \in FP\}), \forall j \in train \tag{48b}$$

$$q_j^{ub} = b_j \min(\{q_{i,j}^{ub} \mid \forall i \in VSP\}, \{q_{i,j}^{lb}(H_{i,j}) \mid \forall i \in FP\}), \forall j \in train \tag{48c}$$

As for the minimum and maximum hydraulic head is given by

$$H_j^{ub} = b_j \left(\sum_{i \in FS} H_{i,j}(q) + \sum_{i \in VSP} H_{i,j}(q, \omega_{i,j}^{ub}) \right), \forall j \in train \tag{48d}$$

$$H_j^{lb} = b_j \left(\sum_{i \in FS} H_{i,j}(q) + \sum_{i \in VSP} H_{i,j}(q, \omega_{i,j}^{lb}) \right), \forall j \in train \tag{48e}$$

9. Results and Discussion

10. Conclusion

11. Conclusion

12. Appendix

12.1. Pumps formulation

We refer to the fixed-speed pump arcs as $\mathcal{L}_{FPS}$. An FSP is given by the following set of equations:

$$H_i - H_j = H_{(i,j)}^p \tag{49a}$$

$$H_{(i,j)}^p = \bar{A} + \bar{B}{q_{(i,j)}}^2, \tag{49b}$$

$$W_{(i,j)}^p = \bar{\gamma} H_{(i,j)}^p q_{(i,j)}, \tag{49c}$$

$$\bar{q}_{(i,j)}^{lb} \leq q_{(i,j)} \leq \bar{q}_{(i,j)}^{ub}, \tag{49d}$$

with five decision variables H_i, H_j, H^p, q, and W^p. As there are three equality constraints, the set of equations (49) gives two degrees of freedom per FSP.

With the possibility of turning on/off the FSPs, the set of equations (49) is modified as follows,

$$H_i - H_j = H_{(i,j)}^p + H_{(i,j)}^c, \tag{50a}$$

$$H_{(i,j)}^p = \bar{A}_{(i,j)} + \bar{B}_{(i,j)}{q_{(i,j)}}^2 - (1 - Z_{(i,j)})\bar{A}_{(i,j)}, \tag{50b}$$

$$W_{(i,j)}^p = \bar{\gamma} H_{(i,j)}^p q_{(i,j)} \tag{50c}$$

$$H_{(i,j)}^c Z_{(i,j)} = 0 \tag{50d}$$

$$\bar{q}_{(i,j)}^{lb} Z_{(i,j)} \leq q_{(i,j)} \leq \bar{q}_{(i,j)}^{ub} Z_{(i,j)} \tag{50e}$$

$$H_{(i,j)}^c \leq \bar{H}_{(i,j)}^{max} \tag{50f}$$

$$Z_{(i,j)} \in \{0, 1\} \tag{50g}$$

with decision variable $Z_{(i,j)}$. The introduction of $Z_{(i,j)}$ in (50e) and the addition of (50f) with $H^c_{(i,j)}$ represents the behavior of a check valve. The set of equations (50) has seven variables and three equality constraints. Thus, four degrees of freedom. As $Z_{(i,j)}$ is an integer variable, the following scenarios are possible:

$$
\begin{aligned}
&Z_{(i,j)} = 1, && Z_{(i,j)} = 0 && \text{(51a)}\\
&H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)} && H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)} && \text{(51b)}\\
&H^p_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)} {q_{(i,j)}}^2, && H^p_{(i,j)} = \bar{B}_{(i,j)} {q_{(i,j)}}^2, && \text{(51c)}\\
&W^p_{(i,j)} = \bar{\gamma} H^p_{(i,j)} q_{(i,j)}, && W^p_{(i,j)} = \bar{\gamma} H^p_{(i,j)} q_{(i,j)}, && \text{(51d)}\\
&\bar{q}^{lb}_{(i,j)} \leq q_{(i,j)} \leq \bar{q}^{ub}_{(i,j)}, && 0 \leq q_{(i,j)} \leq 0, && \text{(51e)}\\
&0 \leq H^c_{(i,j)} \leq 0 && -\bar{M}_{(i,j)} \leq H^c_{(i,j)} \leq 0, && \text{(51f)}
\end{aligned}
$$

one may notice that two of the four degrees of freedom are consumed in both scenarios, thus two degrees of freedom remains for each FSP. We notice that for the scenario $Z_{(i,j)} = 1$, the set of equations (49) is recovered, and $H^c_{(i,j)}$ is equal to zero. As for the scenario $Z_{(i,j)} = 0$, a different set of equations is formed in which $q_{(i,j)}$, $H^p_{(i,j)}$ and $W^p_{(i,j)}$ are equal to zero.

We refer to variable speed pumps as VSP $\mathcal{L}_{VSP}$.

$$H_i - H_j = H^p_{(i,j)} \tag{52a}$$

$$H^{p,ub}_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)} q^2_{(i,j)} + \bar{C}_{(i,j)} {\bar{w}^{ub}_{(i,j)}}^2, \tag{52b}$$

$$H^{p,lb}_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)} q^2_{(i,j)} + \bar{C}_{(i,j)} {\bar{w}^{lb}_{(i,j)}}^2, \tag{52c}$$

$$W^p_{(i,j)} = \gamma H^p_{(i,j)} q_{(i,j)}, \tag{52d}$$

$$q^{ub}_{(i,j)} = \bar{A}^{ub}_{(i,j)} + \bar{B}^{ub}_{(i,j)} H^p_{(i,j)}, \tag{52e}$$

$$q^{lb}_{(i,j)} = \bar{A}^{lb}_{(i,j)} + \bar{B}^{lb}_{(i,j)} H^p_{(i,j)}, \tag{52f}$$

$$H^{p,lb}_{(i,j)} \leq H^p_{(i,j)} \leq H^{p,ub}_{(i,j)}, \tag{52g}$$

$$q^{lb}_{(i,j)} \leq q_{(i,j)} \leq q^{ub}_{(i,j)}, \tag{52h}$$

with nine decision variables H_i, H_j, $H^{p,lb}_{(i,j)}$, $H^{p,ub}_{(i,j)}$, $H^p_{(i,j)}$, $q^{ub}_{(i,j)}$, $q^{lb}_{(i,j)}$, $q_{(i,j)}$, and $W^p_{(i,j)}$. A total of six equality equations are present in (52), thus giving three degrees of freedom per VSP. The possibility of turning a pump on/off enables one to describe the behavior of VSPs as follows,

$$H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)} \tag{53a}$$

$$H^{p,ub}_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)}{q_{(i,j)}}^2 + \bar{C}_{(i,j)}{\bar{w}^{ub}_{(i,j)}}^2 - (1-Z)(\bar{A}_{(i,j)} + \bar{C}_{(i,j)}{w^{ub}_{(i,j)}}^2), \tag{53b}$$

$$H^{p,lb}_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)}{q_{(i,j)}}^2 + \bar{C}_{(i,j)}{\bar{w}^{lb}_{(i,j)}}^2 - (1-Z)(\bar{A}_{(i,j)} + \bar{C}_{(i,j)}{w^{lb}_{(i,j)}}^2), \tag{53c}$$

$$W^p_{(i,j)} = \bar{\gamma} H^p_{(i,j)} q_{(i,j)}, \tag{53d}$$

$$q^{ub}_{(i,j)} = \bar{A}^{ub}_{(i,j)} + \bar{B}^{ub}_{(i,j)} H^p_{(i,j)} - (1 - Z_{(i,j)})\bar{A}^{ub}_{(i,j)} \tag{53e}$$

$$q^{lb}_{(i,j)} = \bar{A}^{lb}_{(i,j)} + \bar{B}^{lb}_{(i,j)} H^p_{(i,j)} - (1 - Z_{(i,j)})\bar{A}^{lb}_{(i,j)} \tag{53f}$$

$$H^{p,lb}_{(i,j)} \leq H^p_{(i,j)} \leq H^{p,ub}_{(i,j)}, \tag{53g}$$

$$q^{lb}_{(i,j)} \leq q \leq q^{ub}_{(i,j)} \tag{53h}$$

$$-\bar{M}_{(i,j)}(1 - Z_{(i,j)}) \leq H^c_{(i,j)} \leq 0 \tag{53i}$$

$$Z_{(i,j)} \in \{0, 1\} \tag{53j}$$

in which variables and equations of check valve are added. In the set of equations (53), there are eleven decision variables, with six equality constraints, granting a total of five degrees of freedom per VSP. For the set of equations (53) two possible scenarios are given below,

$$\begin{array}{lll}
Z = 1 & Z = 0 & \text{(54a)} \\
H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)} & H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)} & \text{(54b)} \\
H^{p,ub}_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)}{q_{(i,j)}}^2 + \bar{C}_{(i,j)}{\bar{w}^{ub}_{(i,j)}}^2, & H^{p,ub}_{(i,j)} = \bar{B}_{(i,j)} q^2_{(i,j)}, & \text{(54c)} \\
H^{p,lb}_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)}{q_{(i,j)}}^2 + \bar{C}_{(i,j)}{\bar{w}^{lb}_{(i,j)}}^2, & H^{p,lb}_{(i,j)} = \bar{B}_{(i,j)} q^2_{(i,j)}, & \text{(54d)} \\
W^p_{(i,j)} = \bar{\gamma} H^p_{(i,j)} q_{(i,j)}, & W^p = \bar{\gamma} H^p_{(i,j)} q_{(i,j)}, & \text{(54e)} \\
q^{ub}_{(i,j)} = \bar{A}^{ub}_{(i,j)} + \bar{B}^{ub}_{(i,j)} H^p_{(i,j)} & q^{ub}_{(i,j)} = \bar{B}^{ub}_{(i,j)} H^p_{(i,j)} & \text{(54f)} \\
q^{lb}_{(i,j)} = \bar{A}^{lb}_{(i,j)} + \bar{B}^{lb}_{(i,j)} H^p_{(i,j)} & q^{lb}_{(i,j)} = \bar{B}^{lb}_{(i,j)} H^p_{(i,j)} & \text{(54g)} \\
H^{p,lb}_{(i,j)} \leq H^p_{(i,j)} \leq H^{p,ub}_{(i,j)}, & H^{p,lb}_{(i,j)} \leq H^p_{(i,j)} \leq H^{p,ub}_{(i,j)}, & \text{(54h)} \\
q^{lb}_{(i,j)} \leq q_{(i,j)} \leq q^{ub}_{(i,j)} & q^{lb}_{(i,j)} \leq q_{(i,j)} \leq q^{ub}_{(i,j)} & \text{(54i)} \\
0 \leq H^c_{(i,j)} \leq 0 & -\bar{M}_{(i,j)} \leq H^c_{(i,j)} \leq 0, & \text{(54j)}
\end{array}$$

when $Z = 1$, the set of equations (53) is recovered, with two degrees of freedom for each VSP. As for $Z = 0$, both bounds in (54h) becomes active, which implies that,

$$H^p_{(i,j)} = \bar{B}_{(i,j)} q^2_{(i,j)}. \tag{55}$$

By substituting $H^p_{(i,j)}$ in (54i), one gets that,

$$\bar{B}^{lb}_{(i,j)} \bar{B}_{(i,j)} q^2_{(i,j)} \leq q_{(i,j)} \leq \bar{B}^{ub}_{(i,j)} \bar{B}_{(i,j)} q^2_{(i,j)}. \tag{56}$$

Given that $\bar{B}$ is a negative constant and that $\bar{B}^{lb}$ and $\bar{B}^{ub}$ are positive constants, if $\bar{B}^{lb} < \bar{B}^{ub}$ holds, then $q_{(i,j)} = 0$ and $H^p_{(i,j)} = 0$. Decision variables

$$\omega = \begin{bmatrix} H_i & H_j & H^p_{(i,j)} & H^{p,lb}_{(i,j)} & H^{p,ub}_{(i,j)} & H^c_{(i,j)} & W^p_{(i,j)} & q_{(i,j)} & q^{ub}_{(i,j)} & q^{lb}_{(i,j)} & Z_{(i,j)} \end{bmatrix} \tag{57}$$

The jacobian of equality constraints are given below:

$$\nabla g = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2Bq & 0 & 0 & -(A + C{w^{ub}}^2) \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2Bq & 0 & 0 & -(A + C{w^{lb}}^2) \\ 0 & 0 & -\gamma q & 0 & 0 & 0 & 1 & -\gamma H^p & 0 & 0 & 0 \\ 0 & 0 & -Bub & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -A^{ub} \\ 0 & 0 & -Blb & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -A^{lb} \end{bmatrix} \tag{58}$$

$$\nabla g = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & q & 0 & H^c & 0 & 0 & 0 \end{bmatrix} \tag{59}$$

The jacobian of equality and active constraints when $Z = 0$ is given below:

$$\nabla g(H,q) = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2Bq & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2Bq & 0 & 0 \\ 0 & 0 & (-q) & 0 & 0 & 0 & 1 & (-H_p) & 0 & 0 \\ 0 & 0 & (-B^{ub}) & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & (-B^{lb}) & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}, \tag{60}$$

if $q = 0$ and $H^p_{(i,j)} = 0$, some manipulation can be made to show that linear dependency between the seventh and eighth rows occur in (60),

$$\nabla g(0,0) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -B^{ub} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -B^{lb} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}. \tag{61}$$

Due to that, numerical issues are deemed to occur when $Z = 0$ is a parameter.

The equations that describe a switch valve is given below,

$$H_i - H_j = H^v_{(i,j)} \tag{62}$$

$$q^p_{(i,j)} = 27.3\bar{C}_v H^{v,r}_{(i,j)}\sqrt{\bar{g}/1e5} \tag{63}$$

$$q^n_{(i,j)} = -27.3\bar{C}_v H^{v,r}_{(i,j)}\sqrt{\bar{g}/1e5} \tag{64}$$

$${H^{v,r}_{(i,j)}}^2 = H^v_{(i,j)}(2Z^a_{(i,j)} - 1) \tag{65}$$

$$q_{(i,j)} = Z_{(i,j)}(q^p_{(i,j)}Z^a_{(i,j)} + q^n_{(i,j)}(1 - Z^a_{(i,j)})) \tag{66}$$

the set of equations is augmented with inequality constraints as follows,

$$Z^a_{(i,j)} \in \{0,1\}, Z_{(i,j)} \in \{0,1\}, \tag{67}$$

a one way switch valve has the addition of the following inequality constraint,

$$-q_{(i,j)} \leq 0 \tag{68}$$

The vector of decision variables for the relaxed NLP is given below,

$$\omega_{(i,j)} = \begin{bmatrix} H_i & H_j & H^v_{(i,j)} & q^p_{(i,j)} & q^n_{(i,j)} & H^{v,r}_{(i,j)} & q_{(i,j)} & Z^a_{(i,j)} & Z_{(i,j)} \end{bmatrix}^T \tag{69}$$

The gradient for the equality constraints are given below,

$$\nabla g = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & -(2Z^a_{(i,j)} - 1) & 0 & 0 & 2H^{v,r}_{(i,j)} & 0 & 2H^v_{(i,j)} & 0 \\ 0 & 0 & 0 & -Z_{(i,j)}Z^a_{(i,j)} & -Z_{(i,j)}(1 - Z^a_{(i,j)}) & 0 & 1 & -Z_{(i,j)}q^p_{(i,j)} & -q^n_{(i,j)}Z^a_{(} \end{bmatrix} \tag{70}$$

The gradient of inequality constraints are given below,

$$\nabla h = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{71}$$

if $Z^a = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2H^{v,r}_{(i,j)} & 0 & 2H^v_{(i,j)} & 0 \\ 0 & 0 & 0 & 0 & -Z_{(i,j)} & 0 & 1 & -Z_{(i,j)}q^p_{(i,j)} & 0 \end{bmatrix} \tag{72}$$

if $Z^a = 0$ and $Z = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2H^{v,r}_{(i,j)} & 0 & 2H^{v}_{(i,j)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{73}$$

if $Z^a = 0$, $Z = 0$, $H^{v,r} = 0$ and $H^v = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 27.3C_v\sqrt{g/1e5} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{74}$$

12.2. Mass balance and SOS 1

The global mass balance is naturally obtained by guaranteeing that the mass balance in each node holds.

$$0 = d_{(1)} - q_{(1,2)} \tag{75}$$

$$0 = q_{(1,2)} - d_{(2)} \tag{76}$$

$$0 = q_{(1,2)} - \bar{K}\phi_{(1,2)}\sqrt{\bar{H}^L_{(1,2)}} \tag{77}$$

$$0 \leq Z_{(1,2)}\bar{q}^{ub}_{(1,2)} - q_{(1,2)} \tag{78}$$

$$\omega = \begin{bmatrix} d_1 & q_{(1,2)} & d_2 & Z_{(1,2)} & \phi_{(1,2)} \end{bmatrix} \tag{79}$$

$$\nabla g = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -\bar{K}\sqrt{\bar{H}^L_{(1,2)}} & 0 \end{bmatrix} \quad \nabla h = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{80}$$

As the SOS1 in my application case is associated with the global mass balance, I am basically saying to the algorithm. "Hey, could you please start by dealing with the global mass balance? "

Also, we do know that children nodes in bonmin uses as a initial guess the solution of the father node. So, maybe it helps convergence when the global mass balance is forced already in the beginning.

12.3. Branch and bound

Branch and bound is a systematic approach in which the original optimization problem (P) is indirectly solved by solving several relaxed optimization sub-problems (P_i^r) in a successive manner. The algorithm starts by relaxing the integer variables ($z \in \mathcal{Z}^{n_z}$) of P into continuous variables ($z^r \in \mathcal{R}^{n_z}$), which creates the relaxed sub-problem P_0^r. An optimal solution to P_0^r is obtained, and the algorithm checks the values of z^*. If z^* contains only integer values, the algorithm stops. Otherwise, at least one z^* is non-integer and the algorithm should select a variable z_i to branch upon.

At each node a solution for the NLP relaxation is obtained. Fathoming of a subtree can be done under the following conditions: the solution is integer, there is no feasible solution, or the best solution in the subtree is worse than the incumbent solution. Incumbent solution is defined as the best integer feasible solution obtained so far at the decision tree.

Wide branching is performed by considering SOS1 based on the global mass balance.

It is possible to argue that the oil production demand dominates the objective function. By giving priority to the mass balance what can happen? 1- Incumbent solutions with good objective values are moved to the top of a decision tree, which can lead to more effective pruning. 2- Infeasibility of a sub-branch can be found earlier.

Am I doing constraint propagation/node presolving? "exploits the repeated application of logical inference rules inan attempt to derive contradictions that allow a subproblem to be pruned."

, different re-injection operational nodes should be first explored. by exploring first the integer variables associated with the mass balance.

will lead to quite incumbent solution can have a good objective values, which can cause pruning to be more effective.

12.4. Model - Fixed-speed pump

$$H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)}, \tag{81a}$$

$$H^p_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)}{q_{(i,j)}}^2 - (1 - Z_{(i,j)})\bar{A}_{(i,j)}, \tag{81b}$$

$$W^p_{(i,j)} = \bar{\gamma} H^p_{(i,j)} q_{(i,j)} \tag{81c}$$

$$H^c_{(i,j)} Z_{(i,j)} = 0 \tag{81d}$$

$$\bar{q}^{lb}_{(i,j)} Z_{(i,j)} \leq q_{(i,j)} \leq \bar{q}^{ub}_{(i,j)} Z_{(i,j)} \tag{81e}$$

$$0 \leq H^c_{(i,j)} \leq \bar{H}^{max}_{(i,j)} \tag{81f}$$

$$Z_{(i,j)} \in \{0, 1\} \tag{81g}$$

Due to SOS 1, the following scenarios are possible.

$$H_i - H_j = H^p_{(i,j)} + H^c_{(i,j)}, \tag{82a}$$

$$H^p_{(i,j)} = \bar{A}_{(i,j)} + \bar{B}_{(i,j)} {q_{(i,j)}}^2 \tag{82b}$$

$$W^p_{(i,j)} = \bar{\gamma} H^p_{(i,j)} q_{(i,j)} \tag{82c}$$

$$H^c_{(i,j)} Z_{(i,j)} = 0 \tag{82d}$$

$$\bar{q}^{lb}_{(i,j)} Z_{(i,j)} \le q_{(i,j)} \le \bar{q}^{ub}_{(i,j)} Z_{(i,j)} \tag{82e}$$

$$0 \le H^c_{(i,j)} \le \bar{H}^{max}_{(i,j)} \tag{82f}$$

$$Z_{(i,j)} = 0 \tag{82g}$$

$$\omega = \begin{bmatrix} H_i & H_j & H^p_{(i,j)} & H^c_{(i,j)} & W^p_{(i,j)} & q_{(i,j)} & q^{ub}_{(i,j)} & q^{lb}_{(i,j)} & Z_{(i,j)} \end{bmatrix} \tag{83}$$

$$\nabla g(\omega) = \begin{bmatrix} 1 \end{bmatrix} \tag{84}$$

Acknowledgement

This research is a part of BRU21 – NTNU Research and Innovation Program on Digital and Automation Solutions for the Oil and Gas Industry (www.ntnu.edu/bru21) and supported by OKEA ASA. In addition, we would like to acknowledge license partners Neptune Energy Norge AS, and Petoro AS for allowing the usage of field data. The authors would like to thank employees at OKEA Per Magne Bjellvåg, Alf Sebastian Lackner, Even Moen Kirkholt, Raymond Hellerud and Katrine Torvik for their valuable input.

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