Definitions

Theory

Integration by parts rule. Let u and v be continuously differentiable functions on an interval I containing the interval $[a, b]$.

1. *Indefinite integral form.* We have
$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx.$$
2. *Definite integral form.* We have
$$\int_a^b u(x)v'(x)\,dx = u(x)v(x)\Big]_a^b - \int_a^b u'(x)v(x)\,dx.$$

Procedures

The art of by parts. To use the integration by parts technique on an integral of the form $\int f(x)g(x)\,dx$ proceed as follows:

1. *Who is u, and who v'?* Declare one of f and g to be u and the other to be v'. The mnemonic device LIPET ((L)og, (I)nverse trig, (P)olynomial/radical, (E)xponent, (T)rig) often leads to a useful choice of u.
2. *Assemble ingredients.* Suppose without loss of generality that we have chosen $u = f$ and $v' = g$. Then compute the derivative f' of f and compute an *antiderivative* G of g:
$$\begin{array}{ccc} u(x) = f(x) & & v'(x) = g(x) \\ & \overset{\text{compute derivative}}{\swarrow} \quad \overset{\text{compute antiderivative}}{\searrow} & \\ u'(x) = f'(x) & & v(x) = G(x) \end{array}$$
3. Apply the integration by parts rule with ingredients assembled in (2):
$$\int \underset{u}{f(x)}\underset{v'}{g(x)}\,dx = \underset{u}{f(x)}\underset{v}{G(x)} - \int \underset{u'}{f'(x)}\underset{v}{G(x)}\,dx.$$

Integration workflow. For many integral computations it will be clear whether to use a formula, substitution, or integration by parts. When it is not clear how to proceed, the following *rough* workflow might be helpful.

1. *Formula.* If possible, use an integration formula, perhaps after some simple algebraic preparation. Otherwise, move to (2).
2. *Substitution.* Evaluate whether a substitution could transform the integral into one where (1) applies. If not promising, move to (3).
3. *By parts.* Evaluate whether the integral is amenable to a by parts approach. You may want to mentally run through a couple of choices of "who is u, and who v'". If not promising, move to (4).
4. *Algebraic techniques.* Consider more creative algebraic techniques, including trigonometric identities. If applicable, return to (1).

Examples

Compute the following integrals using integration by parts. (You might explore whether the integral could also be computed using substitution.)

1. Compute $\displaystyle\int_0^1 xe^{-x}\,dx$
2. Compute $\displaystyle\int x^2e^x\,dx$
3. Compute $\displaystyle\int \ln|x|\,dx$
4. Compute $\displaystyle\int \frac{x^3}{x^2+1}\,dx$
5. Compute $\displaystyle\int \arctan x\,dx$
6. Compute $\displaystyle\int e^x\cos x\,dx$