Having introduced a wealth of new derivative/integral formulas and rules, we now take a moment to give an overview of our integration techniques, and apply them in combination with some algebraic methods to solving integrals in the wild.

Theory

Derivative/antiderivative formula compendium. We collect here our various derivative/antiderivative formulas.

$$
\begin{aligned}
\frac{d}{dx}x^r = rx^{r-1} &\iff \int x^r\,dx = \frac{1}{r+1}x^{r+1} + C, r \neq -1\\
\frac{d}{dx}\sin x = \cos x &\iff \int \cos x\,dx = \sin x + C\\
\frac{d}{dx}\cos x = -\sin x &\iff \int \sin x\,dx = -\cos x + C\\
\frac{d}{dx}\tan x = \sec^2 x &\iff \int \sec^2 x\,dx = \tan x + C\\
\frac{d}{dx}\cot x = -\csc^2 x &\iff \int \csc^2 x\,dx = -\cot x + C\\
\frac{d}{dx}\sec x = \sec x\tan x &\iff \int \sec x\tan x\,dx = \sec x + C\\
\frac{d}{dx}\csc x = -\csc x\cot x &\iff \int \csc x\cot x\,dx = -\csc x + C\\
\frac{d}{dx}\ln|x| = \frac{1}{x} &\iff \int \frac{1}{x}\,dx = \ln|x| + C\\
\frac{d}{dx}\ln|\cos x| = -\tan x &\iff \int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C\\
\frac{d}{dx}\ln|\sin x| = \cot x &\iff \int \cot x\,dx = \ln|\sin x| + C\\
\frac{d}{dx}\ln|\sec x + \tan x| = \sec x &\iff \int \sec x\,dx = \ln|\sec x + \tan x| + C\\
\frac{d}{dx}\ln|\csc x + \cot x| = -\csc x &\iff \int \csc x\,dx = -\ln|\csc x + \cot x| + C\\
\frac{d}{dx}e^x = e^x &\iff \int e^x\,dx = e^x + C\\
\frac{d}{dx}a^x = (\ln a)a^x &\iff \int a^x\,dx = \frac{1}{\ln a}a^x + C\\
\frac{d}{dx}\log_a|x| = \frac{1}{(\ln a)\,x} &\iff \int \frac{1}{(\ln a)\,x}\,dx = \log_a|x| + C\\
\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} &\iff \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C\\
\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}} &\iff \int \frac{1}{\sqrt{1-x^2}}\,dx = -\arccos x + C\\
\frac{d}{dx}\arctan x = \frac{1}{1+x^2} &\iff \int \frac{1}{1+x^2}\,dx = \arctan x + C.
\end{aligned}
$$

Examples

Each of the integral computations below will combine various integral formulas, substitution, and an algebraic method.

1. Compute $\displaystyle\int \frac{1}{x^2 - 6x + 18}$

2. Compute $\displaystyle\int \frac{4x^3 + 3x + 1}{4x^2 + 1}\, dx$

   Hint. Use polynomial division with remainder.

3. Compute $\displaystyle\int \frac{1}{e^x + e^{-x}}\, dx$

4. Compute $\displaystyle\frac{3}{\sqrt{e^{2x} - 2}}\, dx$

5. Compute $\displaystyle\int_0^{\pi/3} \sin^2(2x)\cos(3x)\, dx$

   Hint. Make use of some of the following product-to-sum identities.

$$\cos\theta\cos\phi = \frac{\cos(\theta - \phi) + \cos(\theta + \phi)}{2} \tag{1}$$

$$\sin\theta\sin\phi = \frac{\cos(\theta - \phi) - \cos(\theta + \phi)}{2} \tag{2}$$

$$\sin\theta\cos\phi = \frac{\sin(\theta - \phi) + \sin(\theta + \phi)}{2} \tag{3}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \tag{4}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2} \tag{5}$$