Expected value:

$$t_e = \left|\frac{\bar{x}-\mu}{s/\sqrt{n-1}}\right| \quad \text{follows t-distribution with } (26\text{-}1) = 25\text{d.f}$$

$$= 1.708$$

Inference:

Since $t_0 > t_c$, H_0 is rejected at 5% level of significance. Hence we conclude that advertisement is certainly effective in increasing the sales.

6.3 Test of significance for difference between two means:

6.3.1 Independent samples:

Suppose we want to test if two independent samples have been drawn from two normal populations having the same means, the population variances being equal. Let $x_1, x_2, \ldots x_{n_1}$ and $y_1, y_2, \ldots y_{n_2}$ be two independent random samples from the given normal populations.

Null hypothesis:

$H_0 : \mu_1 = \mu_2$ i.e. the samples have been drawn from the normal populations with same means.

Alternative Hypothesis:

$H_1 : \mu_1 \neq \mu_2$ $(\mu_1 < \mu_2$ or $\mu_1 > \mu_2)$

Test statistic:

Under the H_0, the test statistic is

$$t_0 = \left|\frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\right|$$

$$\text{where} \quad \bar{x} = \frac{\Sigma x}{n_1} \; ; \; \bar{y} = \frac{\Sigma y}{n_2}$$

$$\text{and } S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2\right] = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$$

Expected value:

$$t_e = \left| \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \right|$$ follows t-distribution with $n_1 + n_2 - 2$ d.f

Inference:

If the $t_0 < t_e$ we accept the null hypothesis. If $t_0 > t_e$ we reject the null hypothesis.

Example 3:

A group of 5 patients treated with medicine 'A' weigh 42, 39, 48, 60 and 41 kgs: Second group of 7 patients from the same hospital treated with medicine 'B' weigh 38, 42 , 56, 64, 68, 69 and 62 kgs. Do you agree with the claim that medicine 'B' increases the weight significantly?

Solution:

Let the weights (in kgs) of the patients treated with medicines A and B be denoted by variables X and Y respectively.

Null hypothesis:

$H_0 : \mu_1 = \mu_2$

i.e. There is no significant difference between the medicines A and B as regards their effect on increase in weight.

Alternative Hypothesis:

$H_1 : \mu_1 < \mu_2$ (left-tail) i.e. medicine B increases the weight significantly.

Level of significance : Let $\alpha = 0.05$

Computation of sample means and S.Ds

Medicine A

X	$x - \bar{x}$ ($\bar{x} = 46$)	$(x - \bar{x})^2$
42	-4	16
39	-7	49
48	2	4
60	14	196
41	-5	25
230	**0**	**290**

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{230}{5} = 46$$

Medicine B

Y	$y-\bar{y}$ $(\bar{y}=57)$	$(y-\bar{y})^2$
38	–19	361
42	–15	225
56	–1	1
64	7	49
68	11	121
69	12	144
62	5	25
399	**0**	**926**

$$\bar{y} = \frac{\Sigma y}{n_2} = \frac{399}{7} = 57$$

$$S^2 = \frac{1}{n_1 + n_2 - 2}\,[\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2]$$

$$= \frac{1}{10}\,[\,290 + 926] = 121.6$$

Calculation of statistic:

Under H_0 the test statistic is

$$t_0 = \left|\frac{\bar{x}-\bar{y}}{\sqrt{S^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}\right|$$

$$= \left|\frac{46-57}{\sqrt{121.6\left(\frac{1}{5}+\frac{1}{7}\right)}}\right|$$

$$= \frac{11}{\sqrt{121.6 \times \frac{12}{35}}}$$

$$= \frac{11}{6.57} = 1.7$$

Expected value:

$$t_e = \left| \frac{\bar{x} - \bar{y}}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \right|$$ follows t-distribution with (5+7–2) =10 d.f

$= 1.812$

Inference:

Since $t_0 < t_e$ it is not significant. Hence H_0 is accepted and we conclude that the medicines A and B do not differ significantly as regards their effect on increase in weight.

Example 4:

Two types of batteries are tested for their length of life and the following data are obtained:

	No of samples	Mean life (in hrs)	Variance
Type A	9	600	121
Type B	8	640	144

Is there a significant difference in the two means?

Solution:

We are given

$n_1=9$; $\bar{x}_1=600$hrs; $s_1^2=121$; $n_2=8$; $\bar{x}_2=640$hrs; $s_2^2=144$

Null hypothesis:

$H_0 : \mu_1 = \mu_2$ i.e. Two types of batteries A and B are identical i.e. there is no significant difference between two types of batteries.