Turing Machines

CSE 211 (Theory of Computation)

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Adapted from slides by

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Programming Techniques for Turing Machines

- A TM is exactly as powerful as a conventional computer.
- We will extend the basic model of the TM by adding features
- None of these tricks extend the basic model of the TM they are only notational conveniences

Storage in the State

- We can use the finite control not only to represent a position in the "program" of the Turing machine, but to hold a finite amount of data.
- Figure suggests this technique.
- This also suggests the idea of multiple tracks.

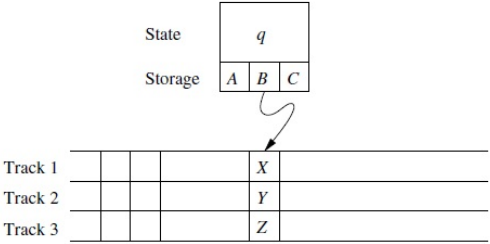
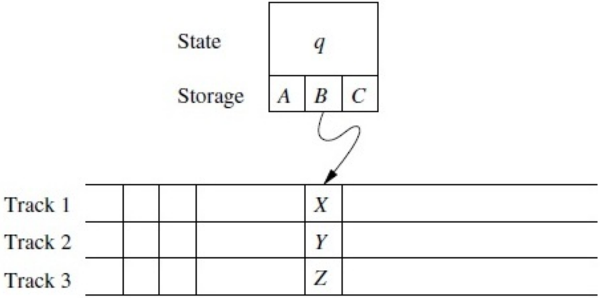


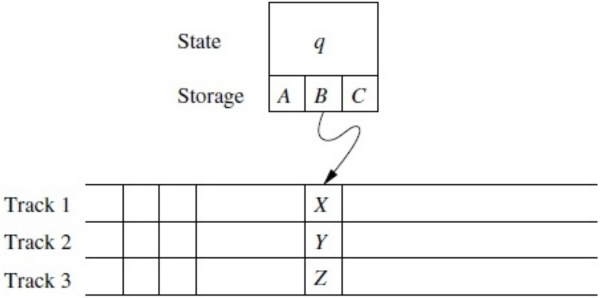
Figure 8.13: A Turing machine viewed as having finite-control storage and multiple tracks

Storage in the State



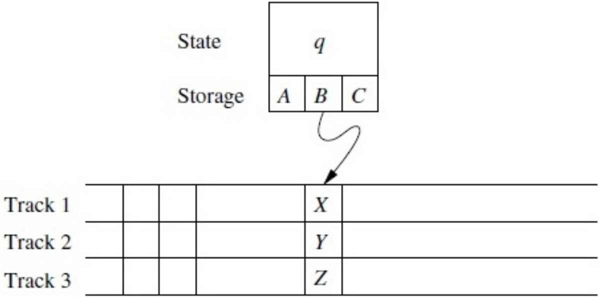
- We see the finite control consisting of not only a “control” state q, but three data elements A, B, and C.
- The technique requires no extension to the TM model.

Storage in the State



- We merely think of the state as a tuple.
- In the case of the diagram, we should think of the state as $[q, A, B, C]$.

Storage in the State



- Regarding states this way allows us to describe transitions in a more systematic way.
- This often makes the strategy behind the TM program more transparent.

Example

- We shall design a TM

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta\ , [q_0, B], \{[q_1, B]\})$$

  that remembers in its finite control the first symbol (0 or 1) that it sees.
- And checks that it does not appear elsewhere on its input.
- Thus, M accepts the language $\mathbf{01}^* + \mathbf{10}^*$.
- Accepting regular languages such as this one does not stress the ability of Turing machines.
- But it will serve as a simple demonstration.

Example-continued

- The set of states Q is $\{q_0, q_1\} \times \{0, 1, B\}$.
- That is, the states may be thought of as pairs with two components:
  a) A control portion, q_0 or q_1.
     - This remembers what the TM is doing.
     - Control state q_0 indicates that M has not yet read its first symbol.
     - q_1 indicates that it has read the symbol.
     - In q_1 it is checking that it does not appear elsewhere, by moving right and hoping to reach a blank cell.
  b) A data portion.
     - This remembers the first symbol seen, which must be 0 or 1.
     - The symbol B in this component means that no symbol has been read.

Example-continued

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- That is, the states may be thought of as pairs with two components:
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  b) A data portion.
     - This remembers the first symbol seen, which must be 0 or 1.
     - The symbol B in this component means that no symbol has been read.

Example-continued

The transition function δ of M is as follows:

1. $\delta([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$.
    - Initially, q_0 is the control state, and the data portion of the state is B.
    - The symbol scanned is copied into the second component of the state.
    - And M moves right, entering control state q_1 as it does so.

Example-continued

The transition function δ of M is as follows:

2. $\delta\left([q_1, a], \overline{a}\right) = ([q_1, a], \overline{a}, R)$.
   - Here $\overline{a}$ is the "complement" of a.
   - That is, 0 if $a = 1$ and 1 if $a = 0$.
   - In state q_1 M skips over each symbol 0 or 1 that is different from the one it has stored in its state, and continues moving right.

Example-continued

The transition function δ of M is as follows:

3. $\delta\left([q_1, a], B\right) = \left([q_1, B], B, R\right)$ for $a = 0$ or $a = 1$.
    - If M reaches the first blank, it enters the accepting state $[q_1, B]$.

Example-continued

1. $\delta\left([q_0, B], a\right) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$.
2. $\delta\left([q_1, a], \overline{a}\right) = ([q_1, a], \overline{a}, R)$.
3. $\delta\left([q_1, a], B\right) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$.

- Notice that M has no definition for $\delta\left([q_1, a], a\right)$ for $a = 0$ or $a = 1$.
- Thus, if M encounters a second occurrence of the symbol it stored initially in its finite control, it halts without having entered the accepting state.

Example-continued

$$M = (Q, \{0, 1\}, \{0, 1, B\}, \delta, [q_0, B], \{[q_1, B]\})$$

01111

1. $\delta([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
2. $\delta([q_1, a], \overline{a}) = ([q_1, a], \overline{a}, R)$
3. $\delta([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, \mathrm{B}]\,01111$

$\vdash 0[q_1, 0]\,1111$

$\vdash 01[q_1, 0]\,111$

$\vdash 011[q_1, 0]\,11$

$\vdash 0111[q_1, 0]\,1$

$\vdash 01111[q_1, 0]\,B$

$\vdash 01111B[q_1, \mathrm{B}]\,B$

Thus M reaches its accepting state and the string is accepted.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta\ , [q_0, B], \{[q_1, B]\})$$

01111

1. $\delta\ ([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
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3. $\delta\ ([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

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$\vdash 01111B[q_1, \mathrm{B}]\,B$

Thus M reaches its accepting state and the string is accepted.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta\ , [q_0, B], \{[q_1, B]\})$$

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1. $\delta\,([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
2. $\delta\,([q_1, a], \overline{a}) = ([q_1, a], \overline{a}, R)$
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$\vdash 0[q_1, 0]\,1111$

$\vdash 01[q_1, 0]\,111$

$\vdash 011[q_1, 0]\,11$

$\vdash 0111[q_1, 0]\,1$

$\vdash 01111[q_1, 0]\,B$

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Example-continued

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$[q_0, \mathrm{B}]\,01111$

$\vdash 0[q_1, 0]\,1111$

$\vdash 01[q_1, 0]\,111$

$\vdash 011[q_1, 0]\,11$

$\vdash 0111[q_1, 0]\,1$

$\vdash 01111[q_1, 0]\,B$

$\vdash 01111B[q_1, \mathrm{B}]\,B$

Thus M reaches its accepting state and the string is accepted.

Example-continued

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$[q_0, \mathrm{B}]\,01111$
$\vdash 0[q_1, 0]\,1111$
$\vdash 01[q_1, 0]\,111$
$\vdash 011[q_1, 0]\,11$
$\vdash 0111[q_1, 0]\,1$
$\vdash 01111[q_1, 0]\,B$
$\vdash 01111B[q_1, \mathrm{B}]\,B$

Thus M reaches its accepting state and the string is accepted.

Example-continued

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3. $\delta\ ([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, \mathrm{B}]\, 01111$

$\vdash 0[q_1, 0]\, 1111$

$\vdash 01[q_1, 0]\, 111$

$\vdash 011[q_1, 0]\, 11$

$\vdash 0111[q_1, 0]\, 1$

$\vdash 01111[q_1, 0]\, B$

$\vdash 01111B[q_1, \mathrm{B}]\, B$

Thus M reaches its accepting state and the string is accepted.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0, B], \{[q_1, B]\})$$

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$\vdash 0111[q_1, 0]\,1$

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Thus M reaches its accepting state and the string is accepted.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0, B], \{[q_1, B]\})$$

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$[q_0, \mathrm{B}]\,01111$

$\vdash 0[q_1, 0]\,1111$

$\vdash 01[q_1, 0]\,111$

$\vdash 011[q_1, 0]\,11$

$\vdash 0111[q_1, 0]\,1$

$\vdash 01111[q_1, 0]\,B$

$\vdash 01111B[q_1, \mathrm{B}]\,B$

Thus M reaches its accepting state and the string is accepted.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta\,, [q_0, B], \{[q_1, B]\})$$

We consider a string which does not belong to the language.

01101

1. $\delta\,([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
2. $\delta\,([q_1, a], \overline{a}) = ([q_1, a], \overline{a}, R)$
3. $\delta\,([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, \mathrm{B}]\,01101$

$\vdash 0[q_1, 0]\,1101$

$\vdash 01[q_1, 0]\,101$

$\vdash 011[q_1, 0]\,01$

Thus M it halts without having entered the accepting state.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0, B], \{[q_1, B]\})$$

We consider a string which does not belong to the language.

01101

1. $\delta([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
2. $\delta([q_1, a], \overline{a}) = ([q_1, a], \overline{a}, R)$
3. $\delta([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, B]\,01101$

$\vdash 0[q_1, 0]\,1101$

$\vdash 01[q_1, 0]\,101$

$\vdash 011[q_1, 0]\,01$

Thus M it halts without having entered the accepting state.

Example-continued

$$M = (Q, \{0, 1\}, \{0, 1, B\}, \delta, [q_0, B], \{[q_1, B]\})$$

We consider a string which does not belong to the language.

01101

1. $\delta([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
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3. $\delta([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, B]\,01101$

$\vdash 0[q_1, 0]\,1101$

$\vdash 01[q_1, 0]\,101$

$\vdash 011[q_1, 0]\,01$

Thus M it halts without having entered the accepting state.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0, B], \{[q_1, B]\})$$

We consider a string which does not belong to the language.

01101

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3. $\delta([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, B]01101$

$\vdash 0[q_1, 0]1101$

$\vdash 01[q_1, 0]101$

$\vdash 011[q_1, 0]01$

Thus M it halts without having entered the accepting state.

Example-continued

$$M = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0, B], \{[q_1, B]\})$$

We consider a string which does not belong to the language.

01101

1. $\delta([q_0, B], a) = ([q_1, a], a, R)$ for $a = 0$ or $a = 1$
2. $\delta([q_1, a], \overline{a}) = ([q_1, a], \overline{a}, R)$
3. $\delta([q_1, a], B) = ([q_1, B], B, R)$ for $a = 0$ or $a = 1$

$[q_0, B]01101$

$\vdash 0[q_1, 0]1101$

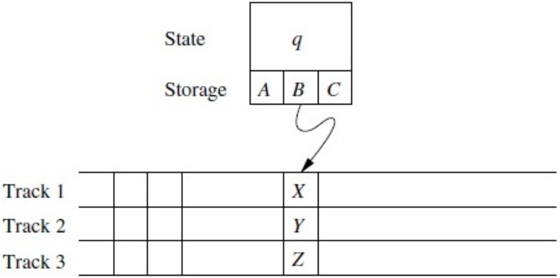
$\vdash 01[q_1, 0]101$

$\vdash 011[q_1, 0]01$

Thus M it halts without having entered the accepting state.

Multiple Tracks

- Another useful "trick" is to think of the tape of a Turing machine as composed of several tracks.
- Each track can hold one symbol.
- The tape alphabet of the TM consists of tuples, with one component for each "track."
- Thus, for instance, the cell scanned by the tape head in figure contains the symbol $[X, Y, Z]$.



Multiple Tracks

- Like the technique of storage in the finite control, using multiple tracks does not extend what the Turing machine can do.
- It is simply a way to view tape symbols and to imagine that they have a useful structure.

Example

- A common use of multiple tracks is to treat one track as holding the data and a second track as holding a mark.
- We can check off each symbol as we “use” it.
- Or we can keep track of a small number of positions within the data by marking only those positions.
- Previous examples were instances of this technique.
- But in those examples we did not think explicitly of the tape as if it were composed of tracks.

Example-continued

- In the present example, we shall use a second track explicitly to recognize the non-context-free language

$$L_{wcw} = \{ wcw \mid w \text{ is in } (0+1)^+ \}$$

Example-continued

0	1	1	c	0	1	1

Example-continued

✓						
0	1	1	c	0	1	1

Example-continued

✓				✓		
0	1	1	c	0	1	1

Example-continued

✓	✓			✓		
0	1	1	c	0	1	1

Example-continued

✓	✓			✓	✓	
0	1	1	c	0	1	1

Example-continued

✓	✓	✓		✓	✓	
0	1	1	c	0	1	1

Example-continued

✓	✓	✓		✓	✓	✓
0	1	1	c	0	1	1

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_1, B], [B, B], \{[q_9, B]\})$$

Q: The set of states is $\{q_1, q_2, \ldots, q_9\} \times \{0, 1, B\}$.

- That is, pairs consisting of a control state q_i and a data component: 0, 1, or blank.
- We again use the technique of storage in the finite control, as we allow the state to remember an input symbol 0 or 1.

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_1, B], [B, B], \{[q_9, B]\})$$

Γ: The set of tape symbols is $\{B, *\} \times \{0, 1, c, B\}$.

- The first component, or track, can be either blank or "checked," represented by the symbols B and $*$, respectively.
- We use the $*$ to check off symbols of the first and second groups of 0's and 1's.
- This eventually confirms that the string to the left of the center marker c is the same as the string to its right.
- The second component of the tape symbol is what we think of as the tape symbol itself.
- That is, we may think of the symbol $[B, X]$ as if it were the tape symbol X, for $X = 0, 1, c, B$.

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_1, B], [B, B], \{[q_9, B]\})$$

Σ: The input symbols are $[B, 0]$, $[B, 1]$ and $[B, c]$.

- This, as mentioned above, we identify with 0, 1 and c, respectively.

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_1, B], [B, B], \{[q_9, B]\})$$

δ : The transition function.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$.
   - In the initial state,
     - *M* picks up the symbol *a* (which can be either 0 or 1),
     - stores it in its finite control,
     - goes to control state q_2,
     - "checks off" the symbol it just scanned,
     - and moves right.
   - Notice that by changing the first component of the tape symbol from *B* to *, it performs the check-off.

Example-continued

δ is defined by the following rules, in which a and b each may stand for either 0 or 1.

2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$.
    - M moves right, looking for the symbol c.
    - Remember that a and b can each be either 0 or 1, independently, but can not be c.

Example-continued

δ is defined by the following rules, in which a and b each may stand for either 0 or 1.

3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$.
   - When M finds the c, it continues to move right, but changes to control state q_3.

Example-continued

δ is defined by the following rules, in which a and b each may stand for either 0 or 1.

4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$.
   - In state q_3, M continues past all checked symbols.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$.
    - If the first unchecked symbol that *M* finds is the same as the symbol in its finite control, it checks this symbol.
    - It has matched the corresponding symbol from the first block of 0's and 1's.
    - M goes to control state q_4, dropping the symbol from its finite control, and starts moving left.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$.
   - M moves left over checked symbols.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$.
   - When M encounters the symbol c, it switches to state q_5 and continues left.
   - In state q_5, M must make a decision, depending on whether or not the symbol immediately to the left of the c is checked or unchecked.
   - If checked, then we have already considered the entire first block of 0's and 1's – those to the left of the c.
   - We must make sure that all the 0's and 1's to the right of the c are also checked.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

7 (cont). $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$.

- We accept if no unchecked symbols remain to the right of the *c*.
- If the symbol immediately to the left of the *c* is unchecked,
  - we find the leftmost unchecked symbol,
  - pick it up, and
  - start the cycle that began in state q_1.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

8. $\delta\left([q_5, B], [B, a]\right) = \left([q_6, B], [B, a], L\right)$.
    - This branch covers the case where the symbol to the left of *c* is unchecked.
    - *M* goes to state q_6 and continues left, looking for a checked symbol.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$.
   - As long as symbols are unchecked, *M* remains in state q_6 and proceeds left.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

10. $\delta\left([q_6, B], [*, a]\right) = ([q_1, B], [*, a], R)$.
    - When the checked symbol is found, *M* enters state q_1 and moves right to pick up the first unchecked symbol.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

11. $\delta\left([q_5, B], [*, a]\right) = \left([q_7, B], [*, a], R\right)$.
    - Now, let us pick up the branch from state q_5.
    - In q_5 we have just moved left from the *c* and find a checked symbol.
    - We start moving right again, entering state q_7.

Example-continued

δ is defined by the following rules, in which *a* and *b* each may stand for either 0 or 1.

12. $\delta\left([q_7, B], [B, c]\right) = \left([q_8, B], [B, c], R\right)$.
    - In state q_7 we shall surely see the c.
    - We enter state q_8 as we do so, and proceed right.

Example-continued

δ is defined by the following rules, in which a and b each may stand for either 0 or 1.

13. $\delta([q_8, B], [*, a]) = ([q_8, B], [*, a], R)$.
    - M moves right in state q_8, skipping over any checked 0's or 1's that it finds.

Example-continued

δ is defined by the following rules, in which a and b each may stand for either 0 or 1.

14. $\delta([q_8, B], [B, B]) = ([q_9, B], [B, B], R)$.
    - If M reaches a blank cell in state q_8 without encountering any unchecked 0 or 1, then M accepts.
    - If M first finds an unchecked 0 or 1, then the blocks before and after the c do not match, and M halts without accepting.

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$

$011c011$

Before

State: $[q_1, B]$

B	B	B	B	B	B	B
0	1	1	c	0	1	1

M picks up the symbol 0, stores it in its finite control, goes to control state q_2, "checks off" the symbol just scanned, and moves right.

After

State: $[q_2, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before
State: $[q_2, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

M moves right, looking for the symbol *c*.

After
State: $[q_2, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

$011c011$

Before
State: $[q_2, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

M moves right, looking for the symbol c.

After
State: $[q_2, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_2, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

When M finds the c, it continues to move right, but changes to control state q_3

After

State: $[q_3, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_3, 0]$

*	B	B	B	B	B	B
0	1	1	c	0	1	1

Matched the corresponding symbol from first block. Checks this. Goes to control state q_4. Drops the symbol from finite control. Starts moving left

After

State: $[q_4, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_4, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

When M encounters the symbol c, it switches to state q_5 and continues left

After

State: $[q_5, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta\,([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta\,([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta\,([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta\,([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta\,([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta\,([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta\,([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta\,([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta\,([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta\,([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta\,([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta\,([q$ [illegible]

011c011

Before

State: $[q_5, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

This branch covers the case where the symbol to the left of c is unchecked. M goes to state q_6 and continues left, looking for a checked symbol

After

State: $[q_6, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

$011c011$

Before

State: $[q_6, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

As long as symbols are unchecked, M remains in state q_6 and proceeds left

After

State: $[q_6, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_6, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

When the checked symbol is found, M enters state q_1 and moves right to pick up the first unchecked symbol

After

State: $[q_1, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_1, B]$

*	B	B	B	*	B	B
0	1	1	c	0	1	1

M picks up the symbol 1, stores it in its finite control, goes to control state q_2, "checks off" the symbol it just scanned, and moves right

After

State: $[q_2, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

$011c011$

Before

State: $[q_2, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

M moves right, looking for the symbol *c*

After

State: $[q_2, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_2, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

When M finds the c, it continues to move right, but changes to control state q_3

After

State: $[q_3, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_3, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

M continues past all checked symbols

After

State: $[q_3, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

$011c011$

Before

State: $[q_3, 1]$

*	*	B	B	*	B	B
0	1	1	c	0	1	1

Matched the corresponding symbol from first block. Checks this. Goes to control state q_4. Drops the symbol from finite control. Starts moving left

After

State: $[q_4, B]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$

011c011

Before

State: $[q_4, B]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

M moves left over checked symbols

After

State: $[q_4, B]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_4, B]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

Encounters the symbol c, switches to state q_5 and continues left

After

State: $[q_5, B]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_5, B]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

......

......

After

State: $[q_2, 1]$

*	*	B	B	*	*	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_2, 1]$

*	*	*	B	*	*	B
0	1	1	c	0	1	1

......

......

After

State: $[q_3, 1]$

*	*	*	B	*	*	B
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_3, 1]$

*	*	*	B	*	*	B
0	1	1	c	0	1	1

Matched the corresponding symbol from first block. Checks this. Goes to control state q_4. Drops the symbol from finite control. Starts moving left

After

State: $[q_4, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) =$ $([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) =$ $([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) =$ $([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) =$ $([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) =$ $([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_4, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

. . . . . .

. . . . . .

After

State: $[q_4, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_4, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

Encounters c, switches to q_5 and continues left

After

State: $[q_5, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_5, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

Just moved left from c and find a checked symbol. Have already considered the entire block of 0's and 1's to the left of the c. Must make sure that all the 0's and 1's to the right of c are also checked. Start moving right again, entering state q_7

After

State: $[q_7, B]$

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

$011c011$

Before

State: $[q_7, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

In state q_7 we shall surely see the c. We enter state q_8 as we do so, and proceed right

After

State: $[q_8, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_8, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

M moves right in state q_8, skipping over any checked 0's or 1's that it finds

After

State: $[q_8, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_8, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

M moves right in state q_8, skipping over any checked 0's or 1's that it finds

After

State: $[q_8, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

011c011

Before

State: $[q_8, B]$

*	*	*	B	*	*	*
0	1	1	c	0	1	1

M moves right in state q_8, skipping over any checked 0's or 1's that it finds

After

State: $[q_8, B]$

*	*	*	B	*	*	*	B
0	1	1	c	0	1	1	B

1. $\delta([q_1, B], [B, a]) = ([q_2, a], [*, a], R)$
2. $\delta([q_2, a], [B, b]) = ([q_2, a], [B, b], R)$
3. $\delta([q_2, a], [B, c]) = ([q_3, a], [B, c], R)$
4. $\delta([q_3, a], [*, b]) = ([q_3, a], [*, b], R)$
5. $\delta([q_3, a], [B, a]) = ([q_4, B], [*, a], L)$
6. $\delta([q_4, B], [*, a]) = ([q_4, B], [*, a], L)$
7. $\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$
8. $\delta([q_5, B], [B, a]) = ([q_6, B], [B, a], L)$
9. $\delta([q_6, B], [B, a]) = ([q_6, B], [B, a], L)$
10. $\delta([q_6, B], [*, a]) = ([q_1, B], [*, a], R)$
11. $\delta([q_5, B], [*, a]) = ([q_7, B], [*, a], R)$
12. $\delta([q_7, B], [B, c]) =$

$011c011$

M reaches a blank cell in state q_8 without encountering any unchecked 0 or 1, it accepts

Example

- Let's design a TM to add two binary numbers. The tape initially contains

$$w_1 c w_2 c$$

- w_1 and w_2 are two binary number (assumed to be of equal length)
- The leftmost bits are LSBs for convenience
- The c's are separators
- We will write the sum to the right of the second c

Example-continued

1	0	1	c	0	1	1	c	B	B	B	B

Example-continued

✓											
1	0	1	c	0	1	1	c	B	B	B	B

Example-continued

✓				✓							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

✓	✓			✓							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$\checkmark$	$\checkmark$			$\checkmark$	$\checkmark$						
1	0	1	c	0	1	1	c	1	1	B	B

Example-continued

✓	✓	✓		✓	✓						
1	0	1	c	0	1	1	c	1	1	B	B

Example-continued

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	0	B

Example-continued

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	0	1

Example-continued

- We will store two bits and the carry in the state
- States will be 4-tuples of the form $[q, a, b, c_i]$
- We start at $[q_0, B, B, 0]$
- After reading the bit from the first number, we switch to q_1, store the bit in the state and go right
- We switch to q_2 after seeing the first c
- Once we find an unchecked bit, we switch to q_3 and store the bit in the state
- We keep going right until we see a blank. We then write the sum of the bits and the carry stored in the state

Example-continued

- We then switch to q_4 and keep going left
- We switch to q_5 when see the first c and then to q_6 when see the second c
- When we find the check mark we go to q_0 and move right
- Then either the cycle repeats
- Or if we see a c while at q_0, we are done with the numbers
- In that case we switch to q_7 and keep going right until we find a blank
- We then write the carry and switch to q_8 and halt

Example-continued

$$[q_0, B, B, 0]$$

1	0	1	c	0	1	1	c	B	B	B	B

Example-continued

$$[q_1, 1, B, 0]$$

✓											
1	0	1	c	0	1	1	c	B	B	B	B

Example-continued

$$[q_2, 1, B, 0]$$

✓											
1	0	1	c	0	1	1	c	B	B	B	B

Example-continued

$$[q_3, 1, 0, 0]$$

✓				✓							
1	0	1	c	0	1	1	c	B	B	B	B

Example-continued

$$[q_4, B, B, 0]$$

✓				✓							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$$[q_5, B, B, 0]$$

$\checkmark$				$\checkmark$							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$$[q_6, B, B, 0]$$

✓				✓							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$$[q_0, B, B, 0]$$

✓				✓							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$$[q_0, 0, B, 0]$$

✓	✓			✓							
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$$[q_3, 0, 1, 0]$$

✓	✓			✓	✓						
1	0	1	c	0	1	1	c	1	B	B	B

Example-continued

$$[q_4, B, B, 0]$$

✓	✓			✓	✓						
1	0	1	c	0	1	1	c	1	1	B	B

Example-continued

$$[q_1, 1, B, 0]$$

✓	✓	✓		✓	✓						
1	0	1	c	0	1	1	c	1	1	B	B

Example-continued

$$[q_3, 1, 1, 0]$$

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	B	B

Example-continued

$$[q_4, B, B, 1]$$

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	0	B

Example-continued

$$[q_0, B, B, 1]$$

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	0	B

Example-continued

$$[q_7, B, B, 1]$$

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	0	B

Example-continued

$$[q_8, B, B, B]$$

✓	✓	✓		✓	✓	✓					
1	0	1	c	0	1	1	c	1	1	0	1

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_0, B, B, 0], [B, B], \{[q_7, B, B, B]\})$$

where:

Q: The set of states is
$\{q_0, q_2, \ldots, q_7\} \times \{0, 1, B\} \times \{0, 1, B\} \times \{0, 1, B\}$.

- That is, pairs consisting of a control state q_i and three data components: 0, 1, or blank
- The data components store two bits from two numbers and a carry.
- We again use the technique of storage in the finite control, as we allow the state to remember an input symbol 0 or 1.

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_0, B, B, 0], [B, B], \{[q_7, B, B, B]\})$$

where:

Γ: The set of tape symbols is $\{B, *\} \times \{0, 1, c, B\}$.

- The first component, or track, can be either blank or "checked," represented by the symbols B and $*$, respectively.
- We use the $*$ to check off symbols of the first and second groups of 0's and 1's.
- The second component of the tape symbol is what we think of as the tape symbol itself.
- That is, we may think of the symbol $[B, X]$ as if it were the tape symbol X, for $X = 0, 1, c, B$.

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_0, B, B, 0], [B, B], \{[q_7, B, B, B]\})$$

where:

Σ: The input symbols are $[B, 0]$, $[B, 1]$ and $[B, c]$.

- This, as mentioned above, we identify with 0, 1 and c, respectively.

Example-continued

- The Turing machine we shall design is:

$$M = (Q, \Sigma, \Gamma, \delta\ , [q_0, B, B, 0], [B, B], \{[q_7, B, B, B]\})$$

where:

δ : The transition function.

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

1. $\delta([q_0, B, B, c_i], [B, a]) = ([q_1, a, B, c_i], [*, a], R)$.
    - In the initial state,
        - M picks up the symbol a (which can be either 0 or 1),
        - stores it in its finite control,
        - goes to control state q_1,
        - “checks off” the symbol it just scanned,
        - and moves right.
    - Notice that by changing the first component of the tape symbol from B to *, it performs the check-off.

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

2. $\delta([q_1, a, B, c_i], [B, 0/1]) = ([q_1, a, B, c_i], [B, 0/1], R)$.
    - M moves right, looking for the symbol c.
    - Remember that a and b can each be either 0 or 1, independently, but can not be c.

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

3. $\delta([q_1, a, B, c_i], [B, c]) = ([q_2, a, B, c_i], [B, c], R)$.
    - When M finds the c, it continues to move right, but changes to control state q_2.

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

4. $\delta([q_2, a, B, c_i], [*, 0/1]) = ([q_2, a, B, c_i], [*, 0/1], R)$.
   - In state q_2, M continues past all checked symbols.

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

5. $\delta([q_2, a, B, c_i], [B, b]) = ([q_3, a, b, c_i], [*, b], R)$.
    - When it sees the first unchecked symbol, it saves the bit in the state.
    - M goes to control state q_3 and keeps moving right.

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

6. $\delta([q_3, a, b, c_i], [B, 0/1/c]) = ([q_3, a, b, c_i], [B, 0/1/c], R)$.
   - It moves right over all symbols until it finds a blank

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

7. $\delta([q_3, a, b, c_i], [B, B]) = ([q_4, B, B, c_o], [B, s], L)$.
   - When it finds a blank, it writes the sum s to the tape and saves the carry out c_o in the state
   - s and c_o are given by

a	b	c_i	s	c_0
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

8. $\delta([q_4, B, B, c_i], [B, 0/1]) = ([q_4, B, B, c_i], [B, 0/1], L)$.
    - Moves over symbols until it finds c

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

9. $\delta([q_4, B, B, c_i], [B, c]) = ([q_5, B, B, c_i], [B, c], L)$.
    - Switches to q_5 when it sees c

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

10. $\delta([q_5, B, B, c_i], [B/*, 0/1]) = ([q_5, B, B, c_i], [B/*, 0/1], L)$.
    - Moves over symbols until it finds c

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

11. $\delta([q_5, B, B, c_i], [B, c]) = ([q_6, B, B, c_i], [B, c], L)$.
    - Switches to q_6 when it sees c

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

12. $\delta([q_6, B, B, c_i], [B, 0/1]) = ([q_6, B, B, c_i], [B, 0/1], L)$.
    - Moves over unchecked symbols

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

13. $\delta([q_6, B, B, c_i], [*, 0/1]) = ([q_0, B, B, c_i], [*, 0/1], R)$.
    - Switches to q_0 and starts moving right when it sees a checked symbol

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

14. $\delta([q_0, B, B, c_i], [B, c]) = ([q_7, B, B, c_i], [B, c], R)$.
    - If it sees c while at q_0, switches to q_7

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

15. $\delta\,([q_7, B, B, c_i], [B/*, 0/1/c]) =$ $([q_7, B, B, c_i], [B/*, 0/1/c], R)$.
    - Moves over everything until it sees a blank

Example-continued

δ is defined by the following rules, in which a, b and c_i each may stand for either 0 or 1.

16. $\delta\,([q_7, B, B, c_i], [B, B]) = ([q_8, B, B, B], [B, c_i], R)$.
    - When it finds a blank, writes carry to tape, switches to q_8 and halts

Subroutines

- As with programs in general, it helps to think of Turing machines as built from a collection of interacting components, or “subroutines.”
- A Turing machine subroutine is a set of states that perform some useful process.
- This set of states includes a start state and another state that temporarily has no moves, and that serves as the “return” state to pass control to whatever other set of states called the subroutine.

Subroutines

- The “call” of a subroutine occurs whenever there is a transition to its initial state.
- We note that the TM has no mechanism for remembering a “return address,” that is, a state to go to after it finishes.
- We should our design of a TM call for one subroutine to be called from several states, we can make copies of the subroutine, using a new set of states for each copy.
- The “calls” are made to the start states of different copies of the subroutine, and each copy “returns” to a different state.