The order is of no significance in the intersection of two events, since $A \cap B = B \cap A$. Therefore, we get an important property of intersection, *viz.*,

$$P(A \cap B) = P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$$

Independent Events. Two events are said to be independent, if the probability of the occurrence of one event will not affect the probability of the occurrence of the second event. Independent events are those events whose probabilities are in no way affected by the occurrence of any other event preceding, following or occurring at the same time.

Two events A and B are said to be independent if and only if

$$P(A \cap B) = P(A)\, P(B)$$

which implies from (*i*) and (*ii*), that

$$P(A/B) = P(A)$$

and

$$P(B/A) = P(B)$$

Illustration 13. A candidate is selected for interview of management trainees for 3 companies. For the first company there are 12 candidates, for the second there are 15 candidates and for the third there are 10 candidates. What are the chances of his getting job at least at one of the company?

Solution. The probability that the candidate gets the job at least at one company = 1 − probability that the candidates does not get the job in any company.

Probability that the candidate does not get the job in the first company

$$= 1 - \frac{1}{12} = \frac{11}{12}$$

Probability that the candidate does not get the job in the second company

$$= 1 - \frac{1}{15} = \frac{14}{15}$$

Probability that the candidate does not get the job in the third company

$$= 1 - \frac{1}{10} = \frac{9}{10}$$

Since the events are independent, therefore, the probability that the candidate does not get any job in any of the three companies

$$= \frac{11}{12} \times \frac{14}{15} \times \frac{9}{10} = \frac{231}{300} = 0.77$$

Hence the required probability = 1 − 0.77 = 0.23.

Bayes' Theorem

It is associated with the name of Thomas Bayes (1702-1761) and is a theorem on probability, concerned with a method of estimating the probabilities of the causes by which an observed event may have been produced. This theorem may be stated as follows :

Let $B_1, B_2, \ldots\ldots, B_n$ be n mutually exclusive events whose union is the universe, and let A be an arbitrary event in the universe, such that $P(A) \neq 0$. Given that $P(A/B_i)$, and $P(B_i)$ $(i = 1, \ldots\ldots, n)$ are known.

$$P(B_j/A) = \frac{P(A/B_j)\, P(B_j)}{\sum_{i=1}^{n} P(B_i) P(A/B_i)} \quad \text{for } j = 1, \ldots\ldots, n.$$

This equation is called the formula for the probability of 'Causes', since it enables one to find the probability of a particular B_j, or 'Cause' by which the event A may have been brought about. It is sometimes written in another form as follows :

$$P(B_j/A) = \frac{P(A \cap B_j)}{P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)}$$