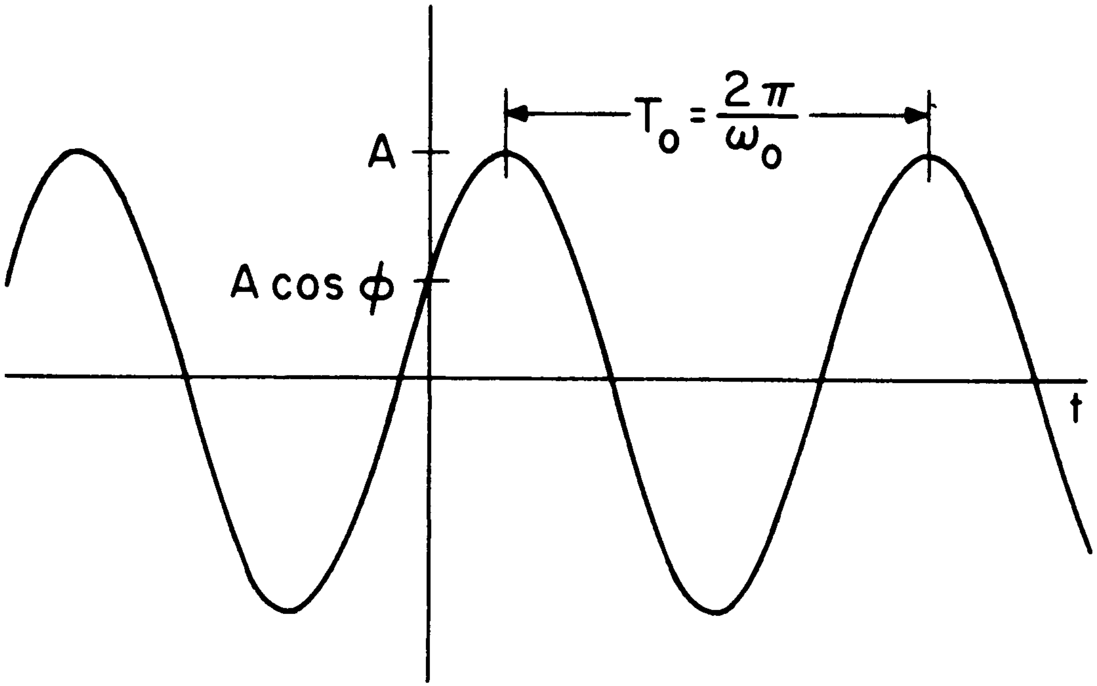
CONTINUOUS-TIME SINUSOIDAL SIGNAL

$$x(t) = A\cos(\omega_o t + \phi)$$



TRANSPARENCY 2.1
Continuous-time sinusoidal signal indicating the definition of amplitude, frequency, and phase.

• **Periodic:**

$$x(t) = x(t + T_o) \quad \text{period} \triangleq \text{smallest } T_o$$

$$A\cos[\omega_o t + \phi] = A\cos[\omega_o t + \underbrace{\omega_o T_o}_{2\pi m} + \phi]$$

$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}$$

• **Time Shift <=> Phase Change**

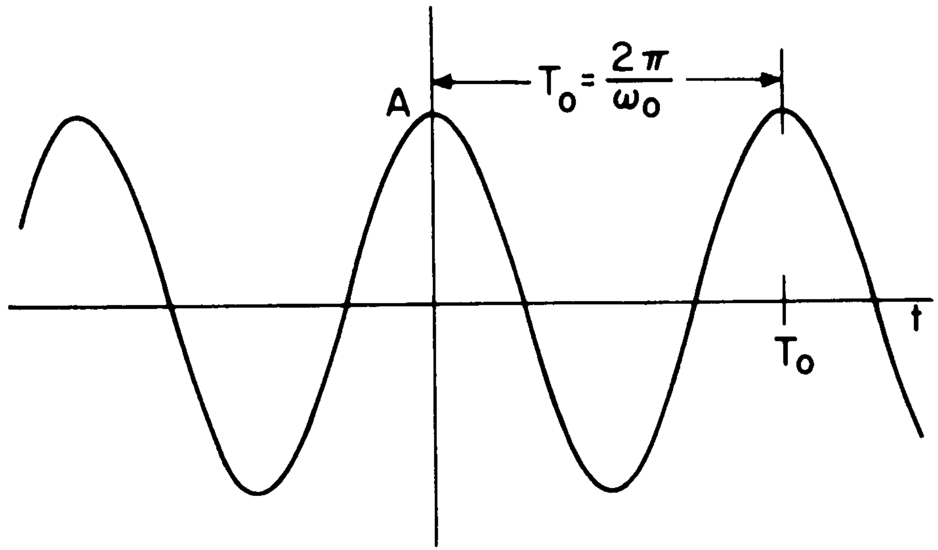
$$A\cos[\omega_o (t + t_o)] = A\cos[\omega_o t + \omega_o t_o]$$

$$A\cos[\omega_o(t + t_o) + \phi] = A\cos[\omega_o t + \omega_o t_o + \phi]$$

TRANSPARENCY 2.2
Relationship between a time shift and a change in phase for a continuous-time sinusoidal signal.

TRANSPARENCY 2.3
Illustration of the signal $A \cos \omega_0 t$ as an even signal.

$\phi = 0 \qquad x(t) = A \cos \omega_0 t$

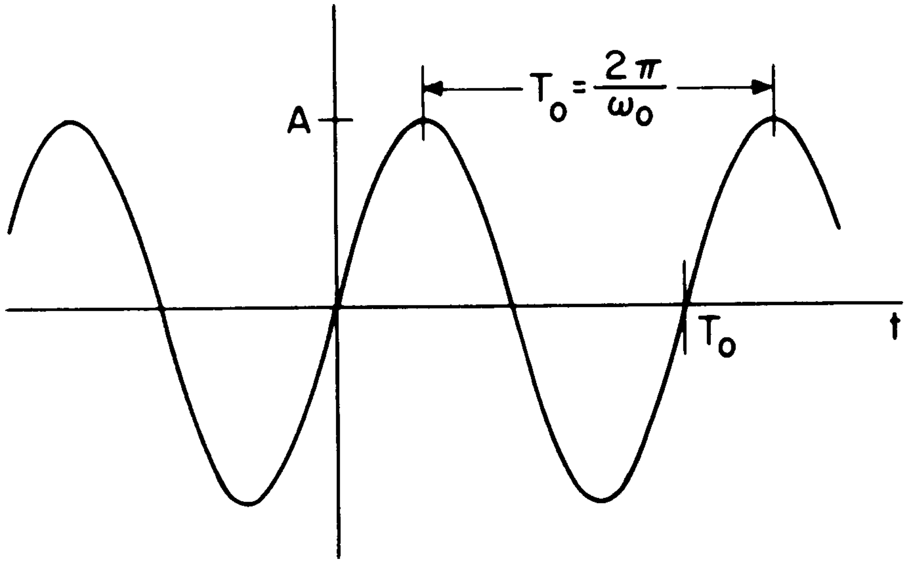


Periodic: $x(t) = x(t + T_0)$

Even: $x(t) = x(-t)$

TRANSPARENCY 2.4
Illustration of the signal $A \sin \omega_0 t$ as an odd signal.

$$\phi = -\frac{\pi}{2} \qquad x(t) = \begin{cases} A\cos\left(\omega_0 t - \frac{\pi}{2}\right) \\ A \sin \omega_0 t \\ A \cos\left[\omega_0\left(t - \frac{T_0}{4}\right)\right] \end{cases}$$



Periodic: $x(t) = x(t + T_0)$

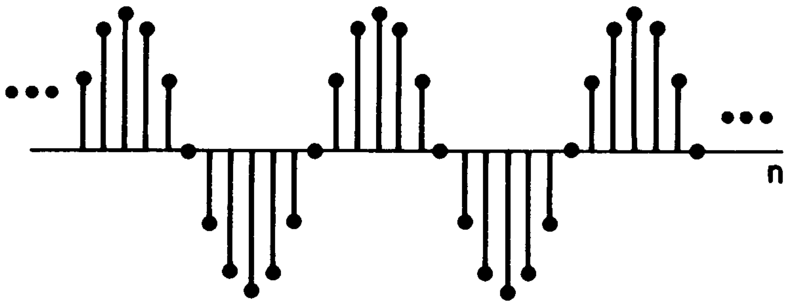
Odd: $x(t) = -x(-t)$

DISCRETE-TIME SINUSOIDAL SIGNAL

$$x[n] = A\cos(\Omega_0 n + \phi)$$

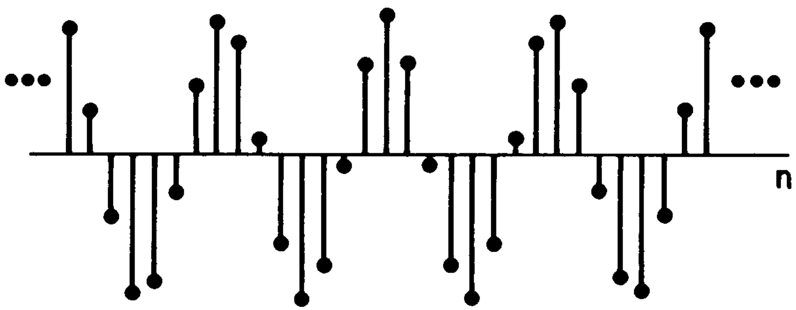
$\Omega_0 = \frac{2\pi}{12}$

$\phi = 0$



$\Omega_0 = \frac{8\pi}{31}$

$\phi = 0$



TRANSPARENCY 2.5
Illustration of discrete-time sinusoidal signals.

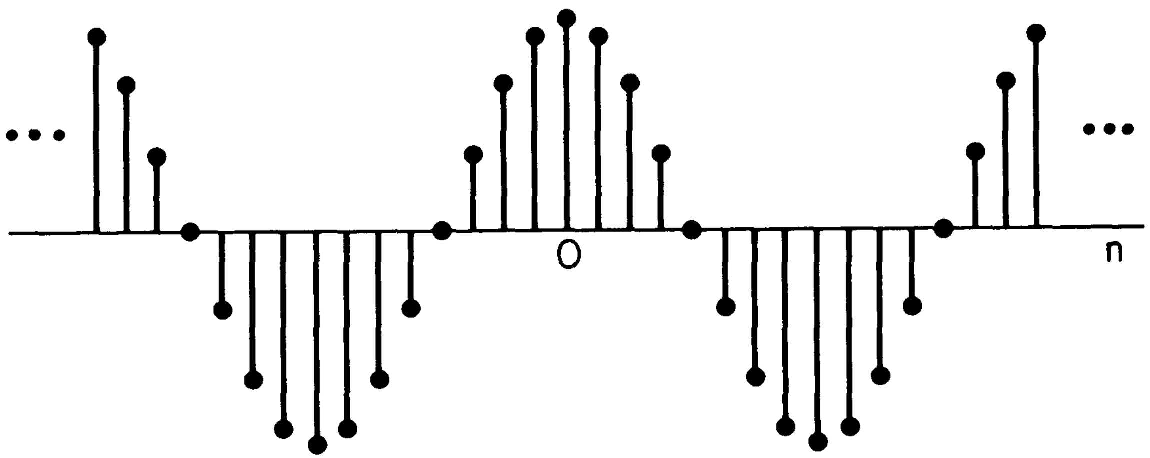
Time Shift => Phase Change

$$A \cos [\Omega_o(n + n_o)] = A \cos [\Omega_o n + \Omega_o n_o]$$

TRANSPARENCY 2.6
Relationship between a time shift and a phase change for discrete-time sinusoidal signals. In discrete time, a time shift always implies a phase change.

TRANSPARENCY 2.7
The sequence $A \cos \Omega_0 n$ illustrating the symmetry of an even sequence.

$\phi = 0 \qquad x[n] = A \cos \Omega_0 n$



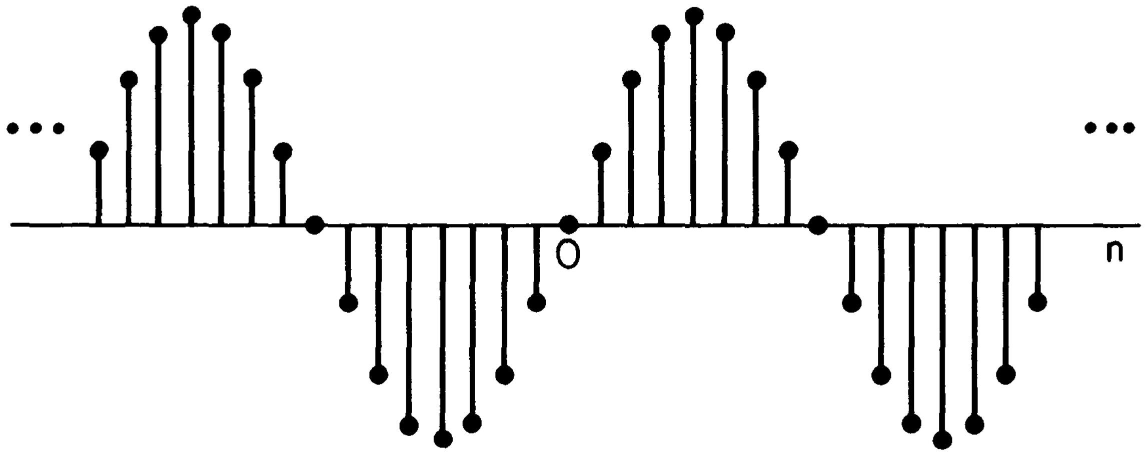
$\Omega_0 = \dfrac{\pi}{8}$

even: $\quad x[n] = x[-n]$

TRANSPARENCY
2.8
The sequence $A \sin \Omega_0 n$ illustrating the antisymmetric property of an odd sequence.

$$\phi = -\frac{\pi}{2} \qquad x[n] = \begin{cases} A\cos\left(\Omega_o n - \frac{\pi}{2}\right) \\ A \sin \Omega_o n \\ A\cos\left[\Omega_o (n - n_o)\right] \end{cases}$$

$n_o = ?$



$$\Omega_0 = \frac{\pi}{8}$$

odd: $x[n] = -x[-n]$

Time Shift => Phase Change

$$A \cos [\Omega_0(n + n_0)] = A \cos [\Omega_0 n + \Omega_0 n_0]$$

Time Shift $\overset{?}{<=}$ Phase Change

$$A \cos [\Omega_0(n + n_0)] \overset{?}{=} A \cos [\Omega_0 n + \phi]$$

TRANSPARENCY 2.9
For a discrete-time sinusoidal sequence a time shift always implies a change in phase, but a change in phase might not imply a time shift.

$$x[n] = A\cos(\Omega_o n + \phi)$$

Periodic?

$$x[n] = x[n+N] \qquad \text{smallest integer } N \triangleq \text{period}$$

$$A\cos[\Omega_o(n+N)+\phi] = A\cos[\Omega_o n + \underbrace{\Omega_o N}_{} + \phi]$$

integer multiple of 2π ?

Periodic $=> \Omega_o N = 2\pi m$

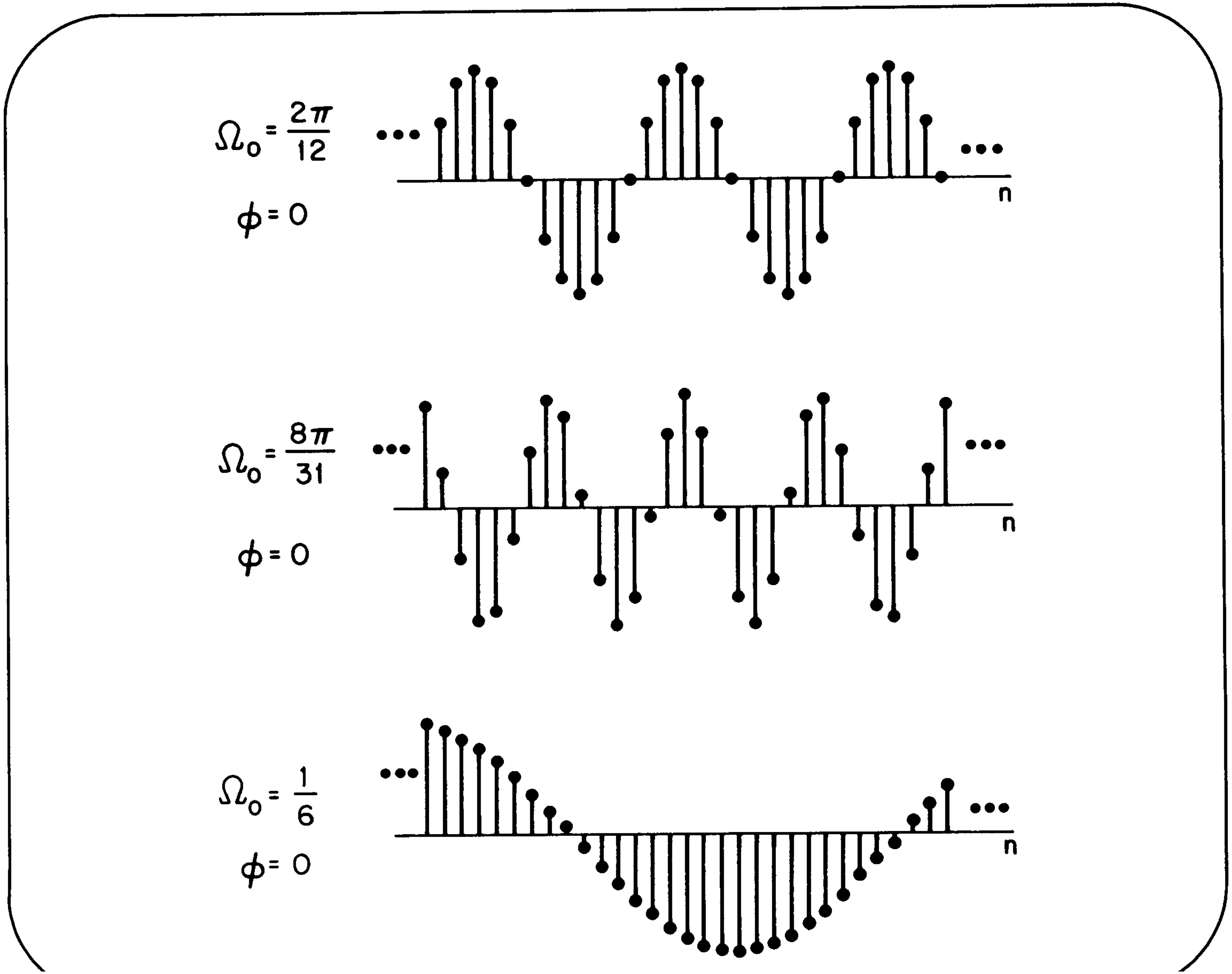
$$N = \frac{2\pi m}{\Omega_o}$$

N,m must be integers

smallest N (if any) = period

TRANSPARENCY 2.10
The requirement on Ω_0 for a discrete-time sinusoidal signal to be periodic.

TRANSPARENCY 2.11
Several sinusoidal sequences illustrating the issue of periodicity.



TRANSPARENCY 2.12
Some important distinctions between continuous-time and discrete-time sinusoidal signals.

$A\cos(\omega_0 t + \phi)$	$A\cos(\Omega_0 n + \phi)$
Distinct signals for distinct values of ω_0	Identical signals for values of Ω_0 separated by 2π
Periodic for any choice of ω_0	Periodic only if $\Omega_0 = \dfrac{2\pi m}{N}$ for some integers $N > 0$ and m

SINUSOIDAL SIGNALS AT DISTINCT FREQUENCIES:

Continuous time:

$x_1(t) = A\cos(\omega_1 t + \phi)$ If $\omega_2 \neq \omega_1$

$x_2(t) = A\cos(\omega_2 t + \phi)$ Then $x_2(t) \neq x_1(t)$

Discrete time:

$x_1[n] = A\cos[\Omega_1 n + \phi]$ If $\Omega_2 = \Omega_1 + 2\pi m$

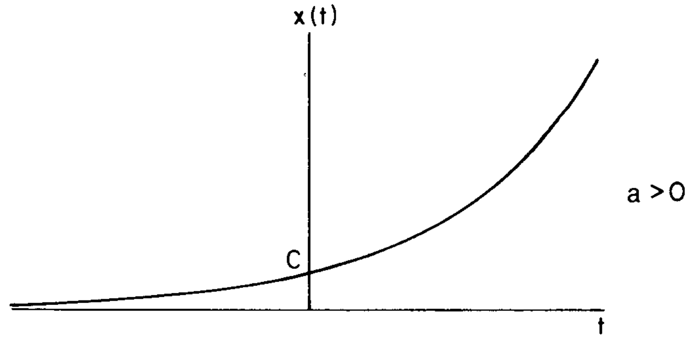
$x_2[n] = A\cos[\Omega_2 n + \phi]$ Then $x_2[n] = x_1[n]$

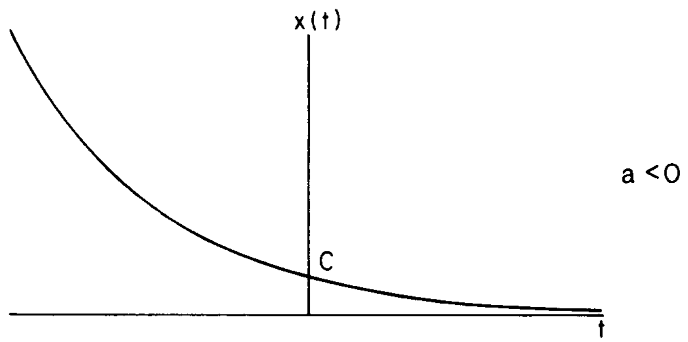
TRANSPARENCY 2.13
Continuous-time sinusoidal signals are distinct at distinct frequencies. Discrete-time sinusoidal signals are distinct only over a frequency range of 2π.

REAL EXPONENTIAL: CONTINUOUS-TIME

$x(t) = Ce^{at}$

C and a are real numbers





Time Shift <=> Scale Change

$Ce^{a(t+t_0)} = Ce^{at_0}\, e^{at}$

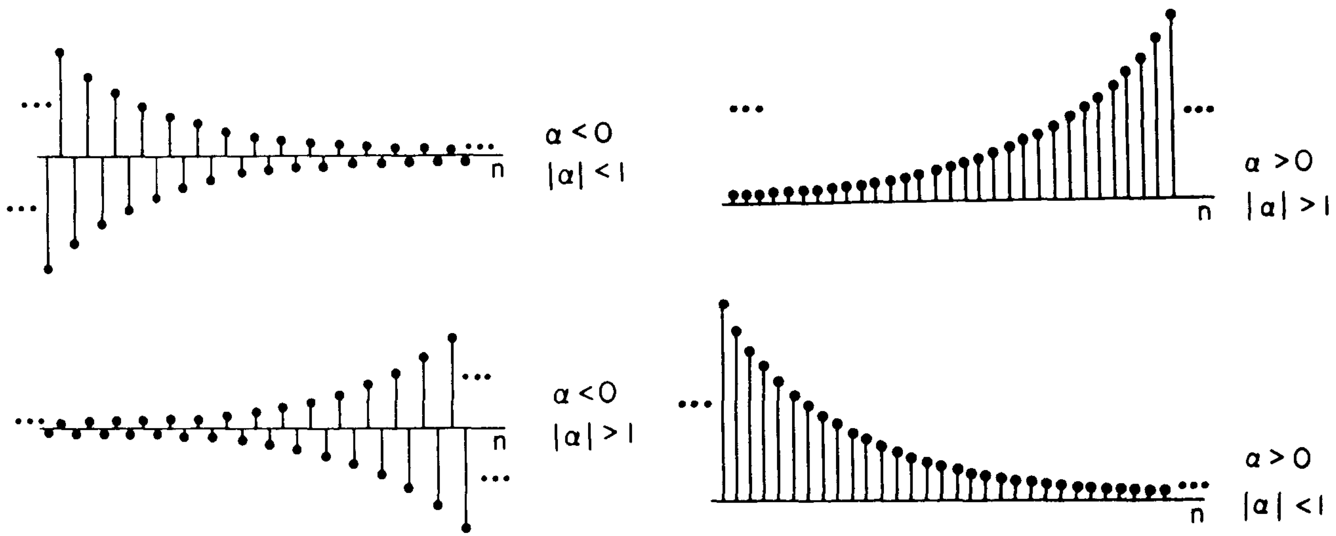
TRANSPARENCY 2.14
Illustration of continuous-time real exponential signals.

TRANSPARENCY 2.15
Illustration of discrete-time real exponential sequences.

REAL EXPONENTIAL: DISCRETE-TIME

$$x[n] = Ce^{\beta n} = C\alpha^n$$

C, α are real numbers



TRANSPARENCY 2.16
Continuous-time complex exponential signals and their relationship to sinusoidal signals.

COMPLEX EXPONENTIAL: CONTINUOUS-TIME

$$x(t) = Ce^{at}$$

C and a are complex numbers

$$C = |C|\, e^{j\theta}$$

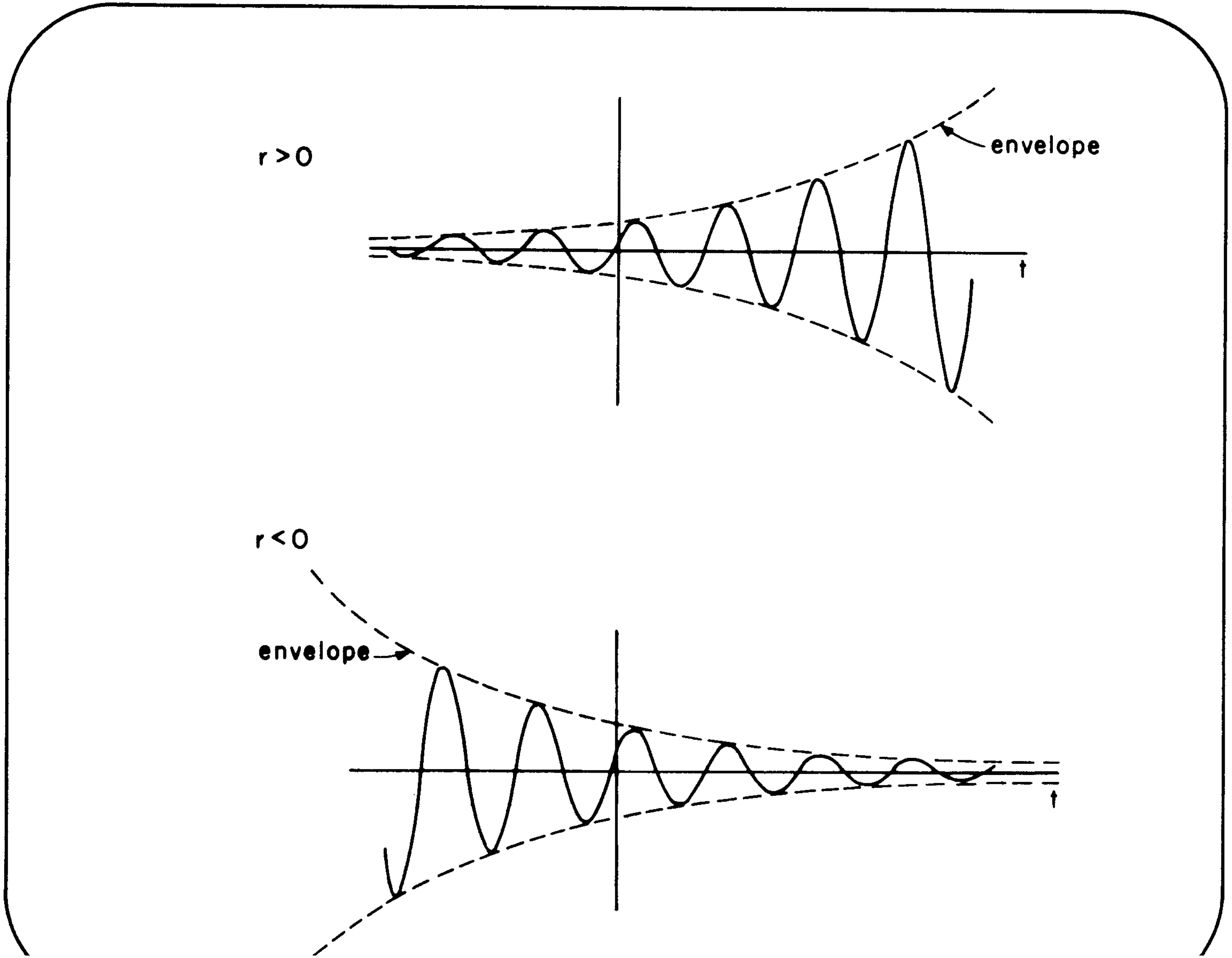
$$a = r + j\omega_o$$

$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_o)t}$$

$$= |C|\, e^{rt}\, \underbrace{e^{j(\omega_o t + \theta)}}$$

Euler's Relation: $\cos(\omega_o t + \theta) + j\sin(\omega_o t + \theta) = e^{j(\omega_o t + \theta)}$

$$x(t) = |C|\, e^{rt} \cos(\omega_o t + \theta) + j\,|C|\, e^{rt} \sin(\omega_o t + \theta)$$



TRANSPARENCY 2.17
Sinusoidal signals with exponentially growing and exponentially decaying envelopes.

COMPLEX EXPONENTIAL: DISCRETE-TIME

$$x[n] = C\alpha^n$$

C and α are complex numbers

$$C = |C|\, e^{j\theta}$$

$$\alpha = |\alpha|\, e^{j\Omega_o}$$

$$x[n] = |C|\, e^{j\theta} \left(|\alpha|\, e^{j\Omega_o}\right)^n$$

$$= |C|\, |\alpha|^n \underbrace{e^{j(\Omega_o n + \theta)}}$$

Euler's Relation: $\cos(\Omega_o n + \theta) + j \sin(\Omega_o n + \theta)$

$$x[n] = |C|\, |\alpha|^n \cos(\Omega_o n + \theta) + j\, |C|\, |\alpha|^n \sin(\Omega_o n + \theta)$$

$|\alpha| = 1$ => sinusoidal real and imaginary parts

$Ce^{j\Omega_o n}$ periodic ?

TRANSPARENCY 2.18
Discrete-time complex exponential signals and their relationship to sinusoidal signals.

TRANSPARENCY 2.19
Sinusoidal sequences with geometrically growing and geometrically decaying envelopes.