

3

Signals and Systems: Part II

In addition to the sinusoidal and exponential signals discussed in the previous lecture, other important basic signals are the unit step and unit impulse. In this lecture, we discuss these signals and then proceed to a discussion of systems, first in general and then in terms of various classes of systems defined by specific system properties.

The unit step, both for continuous and discrete time, is zero for negative time and unity for positive time. In discrete time the unit step is a well-defined sequence, whereas in continuous time there is the mathematical complication of a discontinuity at the origin. A similar distinction applies to the unit impulse. In discrete time the unit impulse is simply a sequence that is zero except at $n = 0$, where it is unity. In continuous time, it is somewhat badly behaved mathematically, being of infinite height and zero width but having a finite area.

The unit step and unit impulse are closely related. In discrete time the unit impulse is the first difference of the unit step, and the unit step is the running sum of the unit impulse. Correspondingly, in continuous time the unit impulse is the derivative of the unit step, and the unit step is the running integral of the impulse. As stressed in the lecture, the fact that it is a first difference and a running sum that relate the step and the impulse in discrete time and a derivative and running integral that relate them in continuous time should not be misinterpreted to mean that a first difference is a good “representation” of a derivative or that a running sum is a good “representation” of a running integral. Rather, for this particular situation those operations play corresponding roles in continuous time and in discrete time.

As indicated above, there are a variety of mathematical difficulties with the continuous-time unit step and unit impulse that we do not attempt to address carefully in these lectures. This topic is treated formally mathematically through the use of what are referred to as generalized functions, which is a level of formalism well beyond what we require for our purposes. The essential idea, however, as discussed in Section 3.7 of the text, is that the important aspect of these functions, in particular of the impulse, is not what its value is at each instant of time but how it behaves under integration.

In this lecture we also introduce systems. In their most general form, systems are hard to deal with analytically because they have no particular properties to exploit. In other words, general systems are simply too general. We define, discuss, and illustrate a number of system properties that we will find useful to refer to and exploit as the lectures proceed, among them memory, invertibility, causality, stability, time invariance, and linearity. The last two, linearity and time invariance, become particularly significant from this point on. Somewhat amazingly, as we'll see, simply knowing that a system is linear and time-invariant affords us an incredibly powerful array of tools for analyzing and representing it. While not all systems have these properties, many do, and those that do are often easiest to understand and implement. Consequently, both continuous-time and discrete-time systems that are linear and time-invariant become extremely significant in system design, implementation, and analysis in a broad array of applications.

Suggested Reading

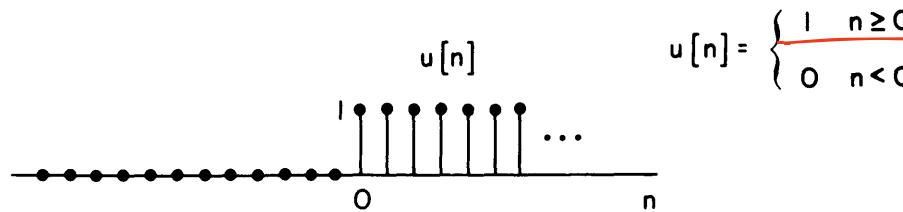
Section 2.4.1, The Discrete-Time Unit Step and Unit Impulse Sequences, pages 26–27

Section 2.3.2, The Continuous-Time Unit Step and Unit Impulse Functions, pages 22–25

Section 2.5, Systems, pages 35–39

Section 2.6, Properties of Systems, pages 39–45

UNIT STEP FUNCTION: DISCRETE-TIME



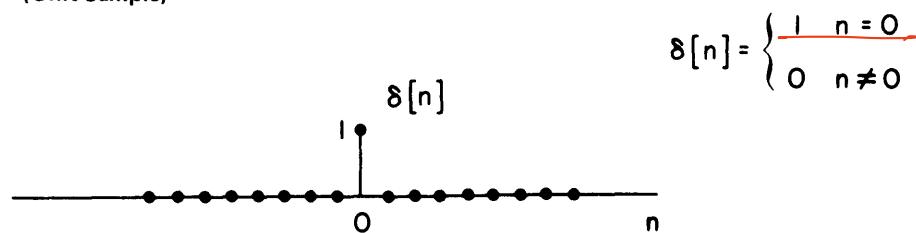
TRANSPARENCY

3.1

Discrete-time unit step and unit impulse sequences.

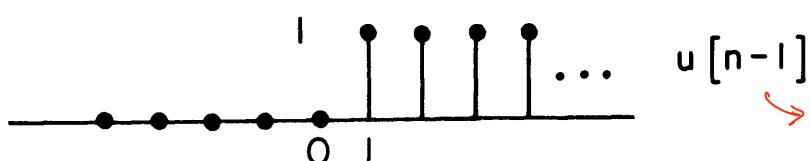
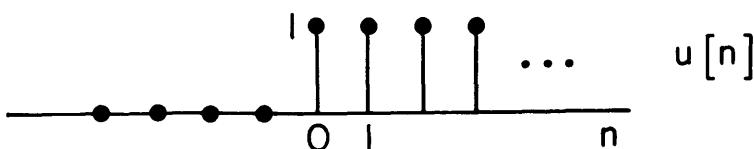
UNIT IMPULSE FUNCTION: DISCRETE-TIME

(Unit Sample)



$$\delta[n] = u[n] - u[n-1] \quad (\text{first difference of unit step function})$$

unit impulse = unit step - unit step delayed



\rightarrow unit step delayed by 1.



TRANSPARENCY

3.2

The unit impulse sequence as the first backward difference of the unit step sequence.

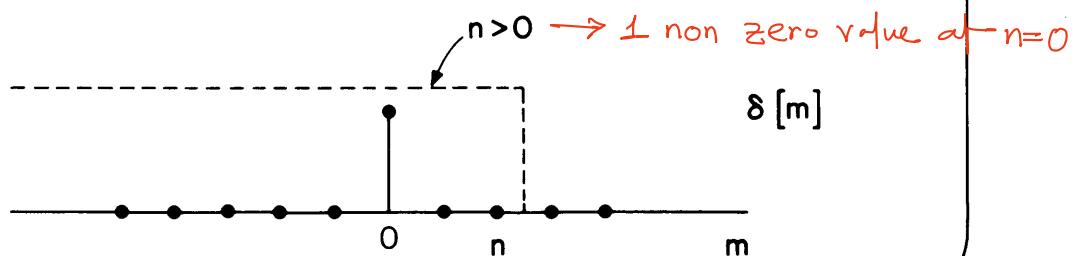
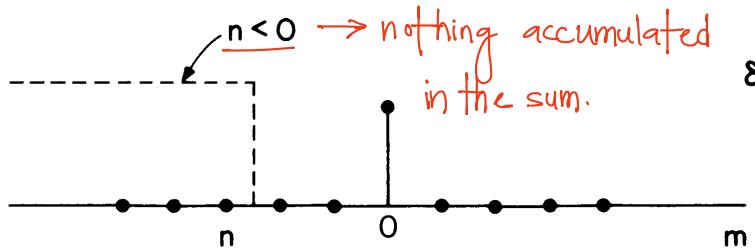
TRANSPARENCY

3.3

The unit step sequence as the running sum of the unit impulse.

$$u[n] = \sum_{m=-\infty}^n \delta[m]$$

running sum

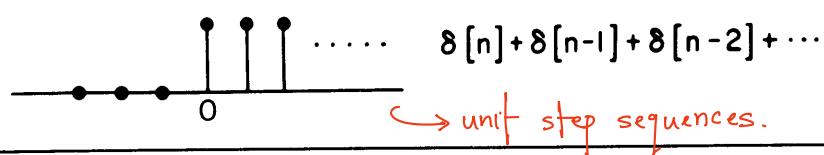
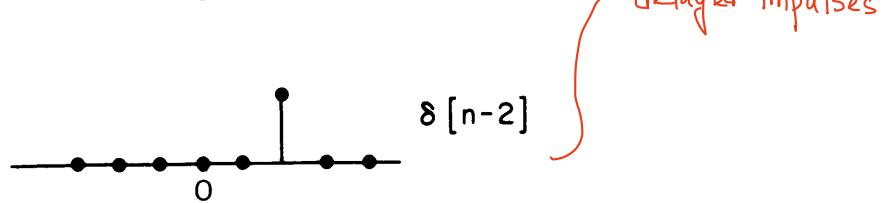
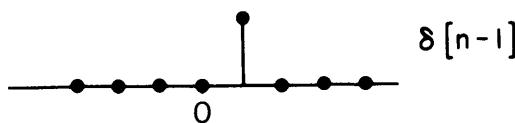
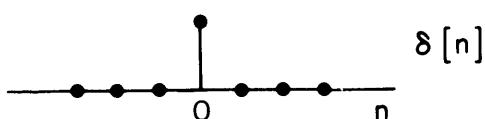


$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

TRANSPARENCY

3.4

The unit step sequence expressed as a superposition of delayed unit impulses.



একটি switch দিয়ে

bulb on/off

simulate করার জন্য use
ব্যাখ্যা

TRANSPARENCY

3.5

The continuous-time unit step function.

switching function simulate

UNIT STEP FUNCTION: CONTINUOUS-TIME

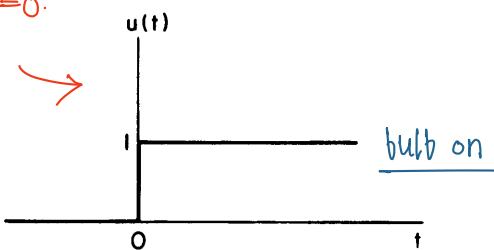
unit step function \rightarrow

discontinuous at $t=0$.

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

$t=0$ এ discontinuous

bulb এর
state simulate

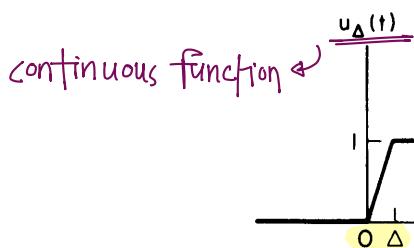


$t=0$ এ switch press করে bulb on

ব্যাখ্যা। But যখন press করছি তখন

bulb on/off জানিনা, তাই $t=0$ এ

function undefined.



$$\Delta \rightarrow 0$$

$$u_D(t) \rightarrow u(t)$$

discontinuous func differentiate করতে সাধ্যোনা, তাই অন্য একটি

continuous function এ limiting form এ ফিল্টার

$$\hookrightarrow u_\Delta(t)$$

UNIT IMPULSE FUNCTION \rightarrow unit step function পর

first derivative

$$\delta(t) = \frac{du(t)}{dt}$$

TRANSPARENCY

3.6

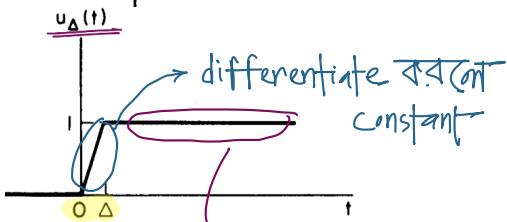
The definition of the unit impulse as the derivative of the unit step.

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

$$\begin{cases} 0 & t < 0 \\ K & 0 < t < \Delta \\ 0 & t > \Delta \end{cases}$$

$$\delta(t) = \delta_\Delta(t) \text{ as } \Delta \rightarrow 0$$

duration of discontinuity

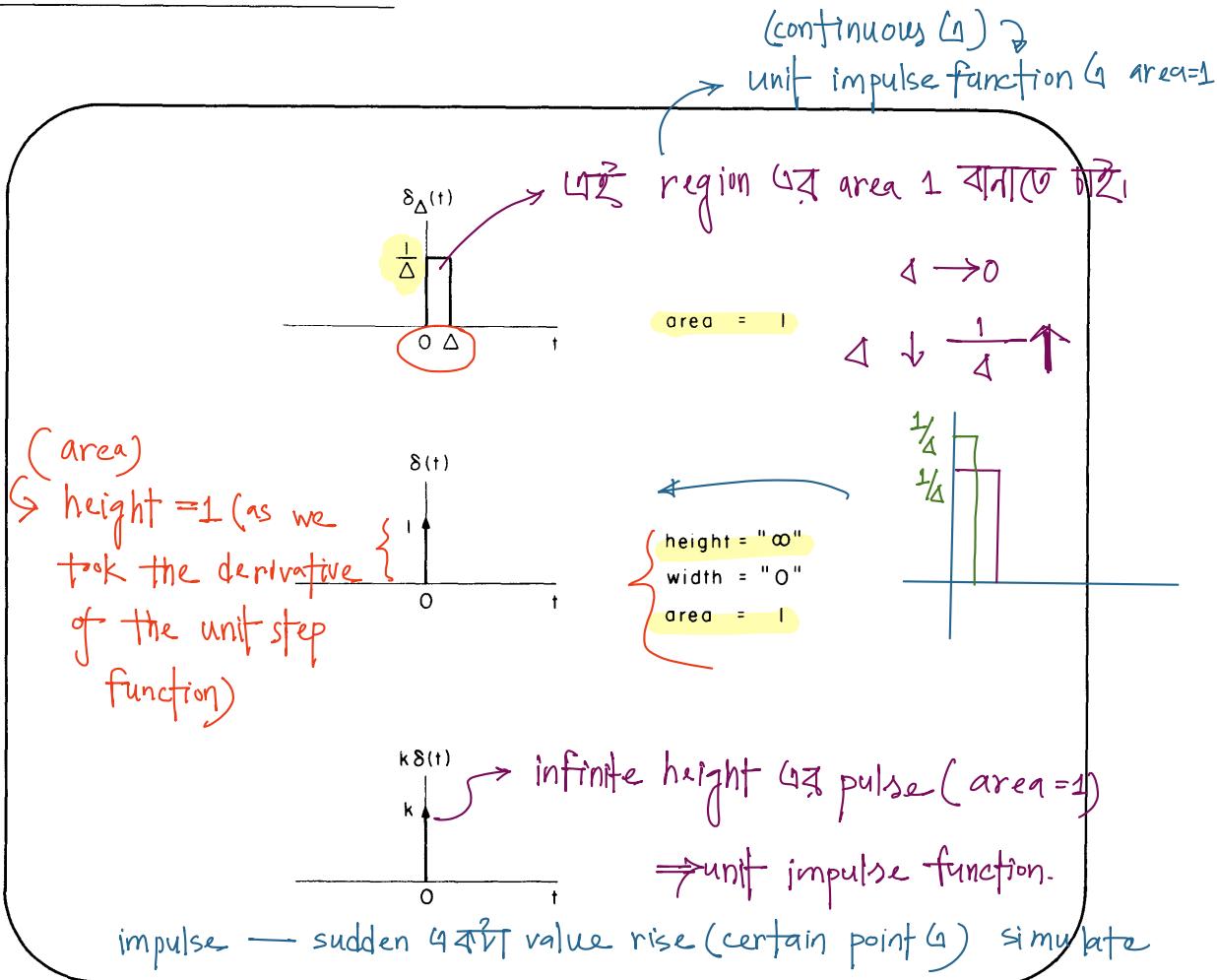


$$\frac{dI}{dt} = 0$$

TRANSPARENCY

3.7

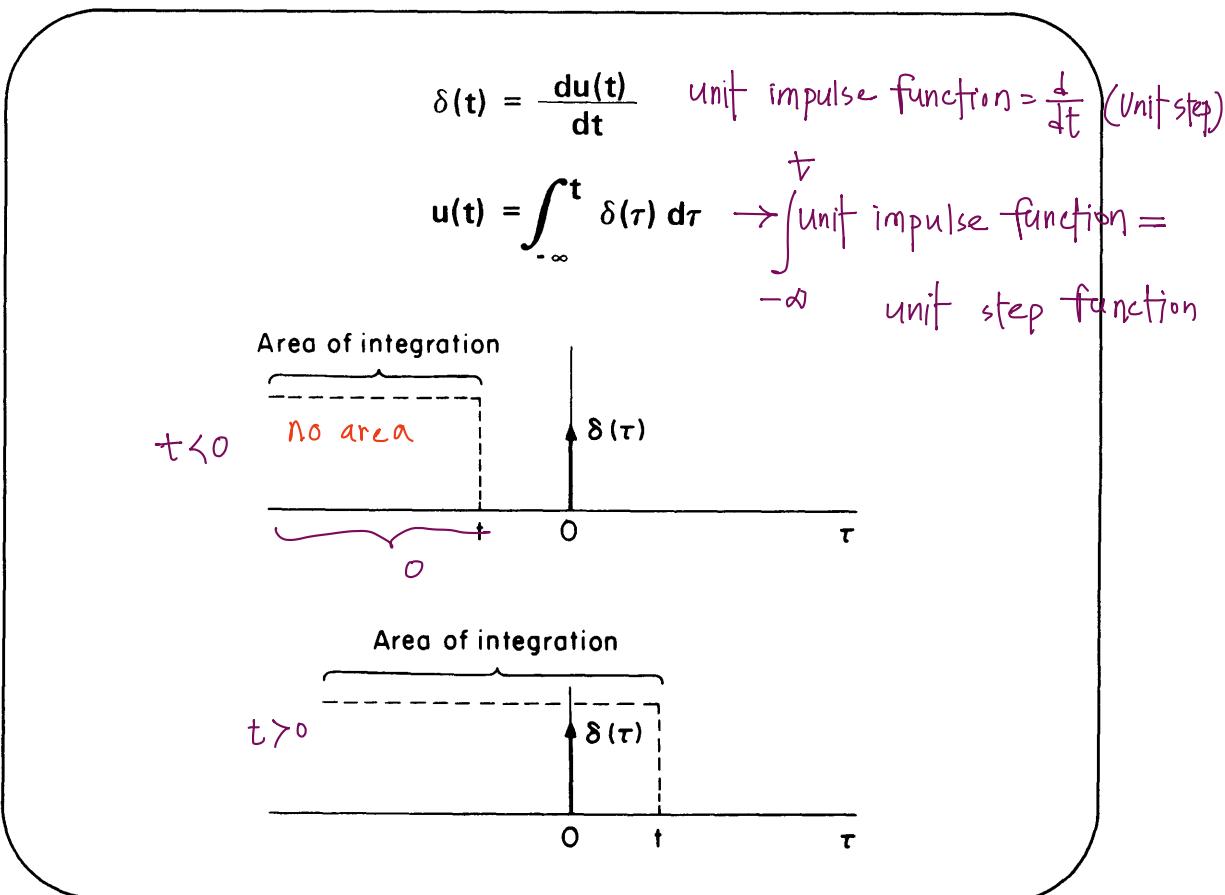
Interpretation of the continuous-time unit impulse as the limiting form of a rectangular pulse which has unit area and for which the pulse width approaches zero.



TRANSPARENCY

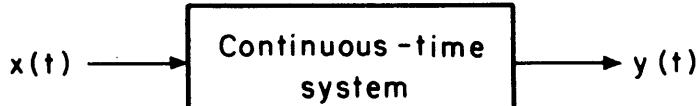
3.8

The unit step expressed as the running integral of the unit impulse.



system \rightarrow input signal $\xrightarrow{\text{থেকে}}$ o/p signal $\xrightarrow{\text{দিয়ে}}$

discrete \rightarrow discrete
continuous \rightarrow continuous



denotes transformation

$$x(t) \longrightarrow y(t)$$

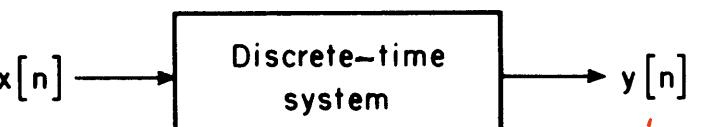
TRANSPARENCY

3.9

Definition of a system.



transformation from an i/p signal to an o/p signal.



i/p sequences

o/p sequences

$$x[n] \longrightarrow y[n]$$

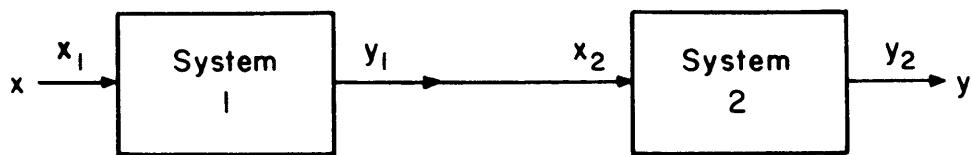
Series interconnection of system.

Cascade *একটি system এর o/p অন্তর্ভুক্ত system এর i/p হিসেবে।*

TRANSPARENCY

3.10

Interconnection of two systems in cascade.



$$x_1 \longrightarrow \underline{y_1}$$

$$\underline{x_2 = y_1} \longrightarrow y_2$$

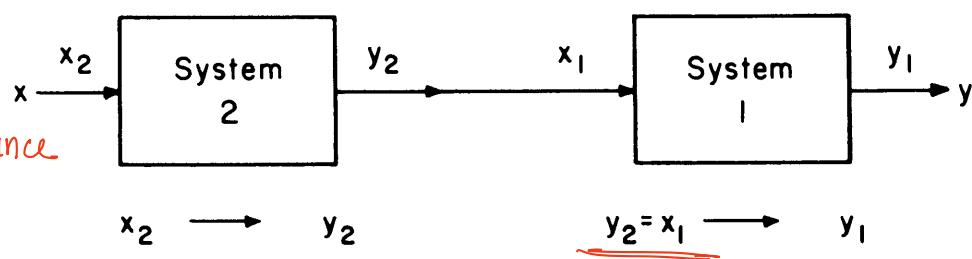
cascade র ক্ষেত্রে system র ক্ষেত্রে order important.



but linear time invariance

system এর ক্ষেত্রে

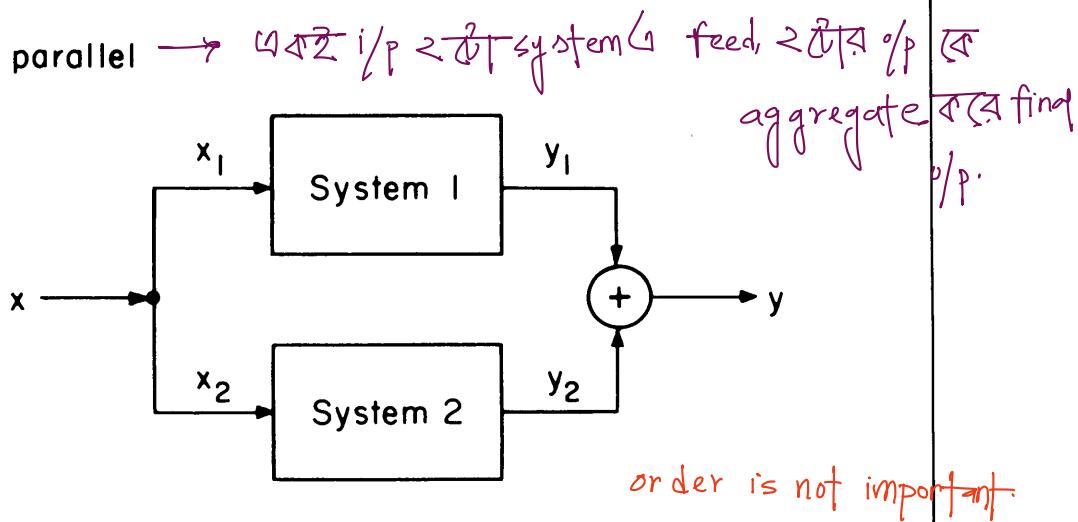
order important না।



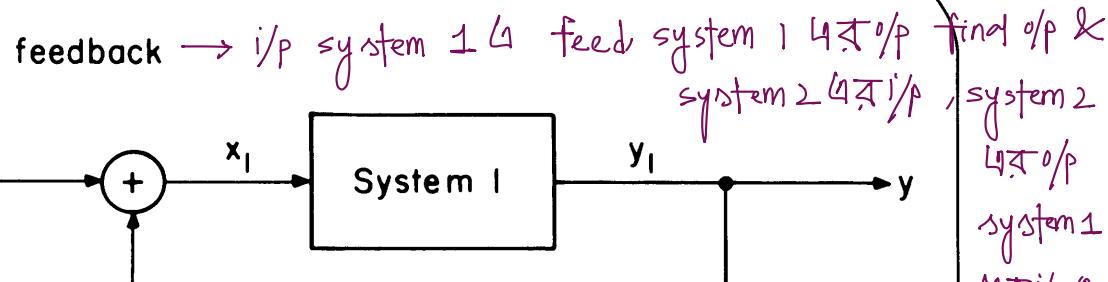
$$x_2 \longrightarrow y_2$$

$$\underline{y_2 = x_1} \longrightarrow y_1$$

TRANSPARENCY
3.11
Interconnection of two systems in parallel.



TRANSPARENCY
3.12
Feedback interconnection of two systems.



$$x_1 = x + y_2$$

$$y = y_1$$

$$x_2 = y_1$$

Properties of a system:

MARKERBOARD 3.1

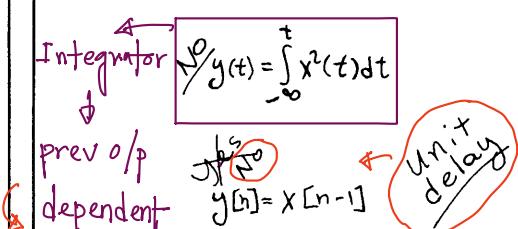
MEMORYLESS

$$y(t) @ t = t_0 \leftarrow x(t) @ t = t_0$$

$$y[n] @ n = n_0 \leftarrow x[n] @ n = n_0$$

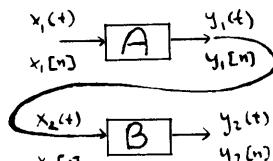
Examples

Square → $y(t) = x^2(t)$ Square
 y[n] = $x^2[n]$



(requires an input over an interval)

INVERTIBILITY



$$x_2 = y_1$$

If $B = \text{Inverse of } A$

Then $y_2 = x_1$
 Identity

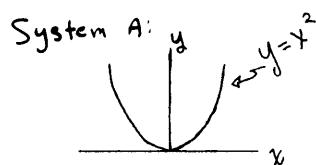
System A:

$$y_1(t) = \int_{-\infty}^t x_1(\tau) d\tau$$

Integration

$$y_2(t) = \frac{dx_2(t)}{dt}$$

Differentiator



Square

Invertible? No

Memoryless? Yes

MARKERBOARD 3.2

Causality

Output at any time depends only on input prior or equal to that time

or:

System can't anticipate "future" inputs

or:

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

If 2 i/p's are identical up until sometime, then the o/p's are identical up until that time.

If: $x_1(t) = x_2(t) \quad t < t_0$
 Then: $y_1(t) = y_2(t) \quad t < t_0$

Same for discrete Time

Example:

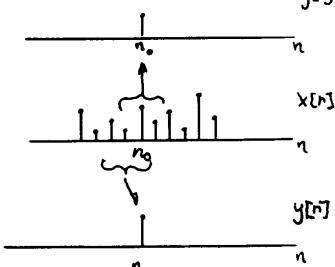
$$y[n] = \frac{1}{3} \{x[n-1] + x[n] + x[n+1]\}$$

Moving Average

$x[n]$ anticipated

Stability

For every bounded input the output is bounded



Example

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

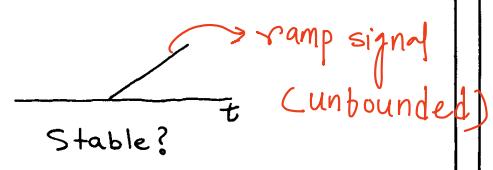
not stable



step signal

$$y[n] = \frac{1}{3} \{x[n-2] + x[n-1] + x[n]\}$$

(a u.s.d.)



ramp signal

Stable?

Unbounded

memory less: t point এর o/p মুক্তি করে point এর i/p dependent.

$$y(t) = x(t-1) \times$$

$$y(t) = x(t+1) \times$$

$$y(t) = 3x(t) \checkmark$$

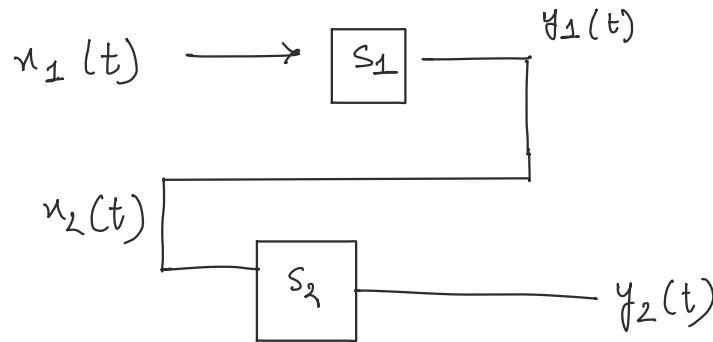
$$y(t) = x(t-1) + x(t) \times$$

Invertible:

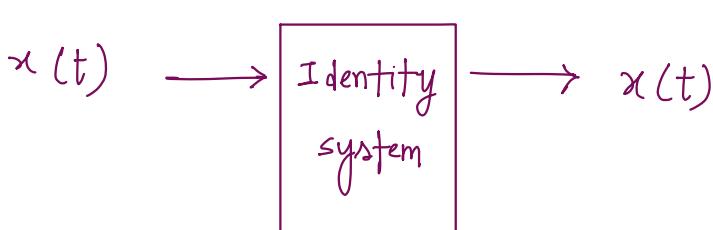
or Identity system.

i/p, o/p same হয়।

— no transformation at all.



given the o/p, there is only one i/p that could have caused it



• invertible হতে হবে।

(memory less system) Squarerr:

$$\begin{array}{rcl} -2 & \rightarrow & 4 \\ \downarrow & & \downarrow \\ 2 & \rightarrow & 4 \end{array}$$

invertible না।

↪ o/p দিলে গুরুত্ব পূর্ণ input বলতে পারতে হবে।

4 o/p এর জন্য i/p $-2/2$ হতে পারে। Original signal

বিচিত্র surely বলতে পারবেন।

যদি integrator \rightarrow invertible (its inverse is differentiator)

differentiator \rightarrow invertible হতে পারে বলাও হতে পারে।

i/p $x^n \rightarrow$ input could be x^n or $x^n + K$ (so not invertible)

Causality: $y(t) = x(t+1) + x(t)$
 \rightarrow causal না।

$$y[n] = \frac{1}{3} \{x[n-1] + x[n] + x[n+1]\} \quad \times$$

→ past value use করা যাবে but future value করা যাবে না।

Stability: i/p bounded হলে o/pও bounded হবে।

e.g. pendulum

• feedback can stabilize unstable system. But if not used correctly can destabilize stable system.

integrator \rightarrow stable না।

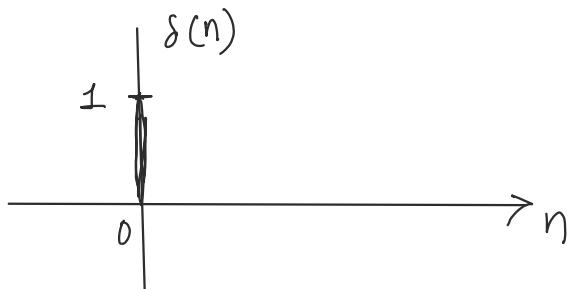
→ input bounded but o/p অস্থির হতে পারে।

CT

Time-Invariance: $x(t) \rightarrow y(t)$
 i/p o/p

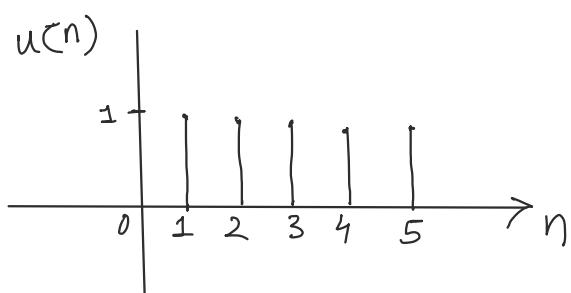
time invariant হবে যদি $x(t-t_0) \rightarrow y(t-t_0)$

i/p যেভাবে shift করবে o/pও same amount shift হবে। (change হবে o/p)



$$y[n] = \sum_{k=-\infty}^n x[k]$$

impulse to step seq.



← time invariant system
 (Accumulator)

→ accumulates past values of
 the input.

Linearity:

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

$$ax_1(t) + b x_2(t) \rightarrow ay_1(t) + b y_2(t)$$

(i/p এর linear combi.) \rightarrow (output এর linear combi)

— Linear time invariant system.

22.09.24

Time invariant system:

$$x(t) \rightarrow y(t)$$

$$x(t-t_0) \rightarrow y(t-t_0)$$

$$x[n-n_0] \rightarrow y[n-n_0]$$

$$y[n] = \sum_{k=-\infty}^n x[k]$$



i/p যাত্রুক্ত shift হয় o/p
অত্রুক্ত শিফ্ট

Linear:

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

$$ax_1(t) + b x_2(t) \rightarrow ay_1(t) + b y_2(t)$$

Linear + time invariant signal

Complex signal reproduce →

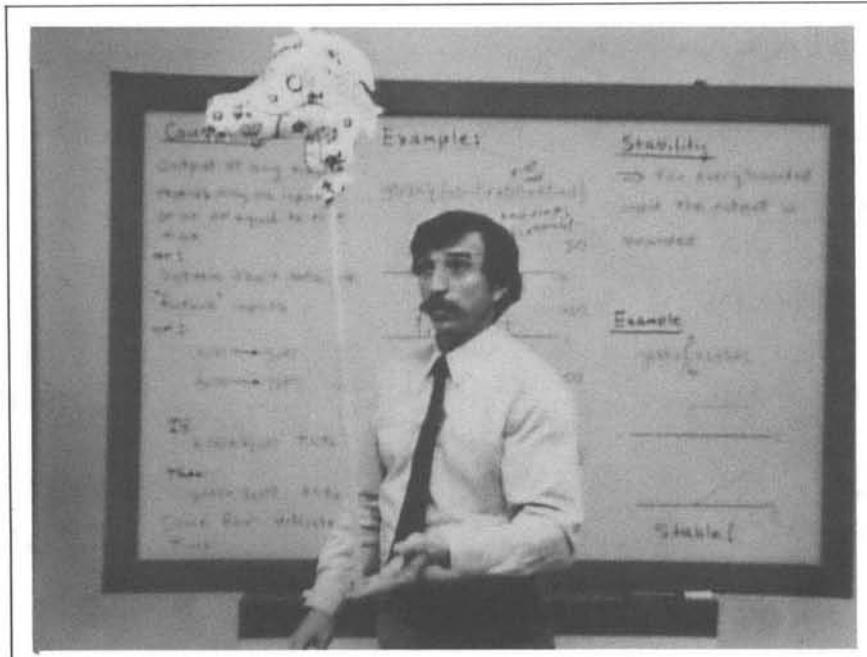
① ~~复杂~~ simple signal decompose

2 ways ↗ direct impulse (convolution)

DEMONSTRATION

3.1

Illustration of an unstable system.

**MARKERBOARD**

3.3

Time Invariance

$$C-T: \quad x(t) \rightarrow y(t)$$

Then

$$x(t-t_0) \rightarrow y(t-t_0)$$

$$D-T: \quad x[n] \rightarrow y[n]$$

$$\text{Then } x[n-n_0] \rightarrow y[n-n_0]$$

Example

$$y(t) = (\sin t)x(t)$$

$$x(t) \rightarrow (\sin t)x(t)$$

$$x(t-t_0) \rightarrow (\sin t)x(t-t_0)$$

Not equal

$$y(t-t_0) = \sin(t-t_0)x(t-t_0)$$

Example

$$y[n] = \sum_{k=-\infty}^n x[k]$$

Accumulation

Time Invariant? *No*

Time Invariant?

- yes

Linearity

$$C-T \notin D-T$$

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

Then:

$$a_1x_1(t) + b_1x_2(t) \rightarrow$$

$$a_1y_1(t) + b_1y_2(t)$$

Examples

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

yes

$$y[n] = 2x[n] + 3$$

No

But

$$y[n] = x^2[n]$$

Not

Square

if a system is linear with 2 i/p s, then it is linear with arbitrary no. of i/p s.

straight line but not linear.

MIT OpenCourseWare

<http://ocw.mit.edu>

Resource: Signals and Systems

Professor Alan V. Oppenheim

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.