L-2/T-2/CSE Date : 03/06/2008

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2006-2007

Sub : **CSE 211** (Theory of Computation)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Design a DFA accepting the set of all strings which start with 0 and has odd length, or start with 1 and has even length. Depict the designed automaton using transition diagram as well as transition table. (10)

(b) Develop a regular expression for the set of strings that consist of alternating 0's and 1's. (8)

(c) "ϕ, ϵ are languages over any alphabet" – explain. Can these two be called equivalent? Why? $(5\frac{1}{3})$

2. (a) Prove the theorem, "If $D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ is the DFA constructed from NFA $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ by the subset construction, then $L(D) = L(N)$". (10)

(b) Design a DFA to accept the language $L = \{w | w$ has an even number of 0's and an odd number of 1's$\}$. Verify that your DFA works by finding out $\hat{\delta}\{q_0, 11010\}$, where q_0 is the start state. (8)

(c) "One way to think of a nondeterministic computation is as a tree of possibilities" – elaborate this statement with necessary example. $(5\frac{1}{3})$

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3. (a) Using pumping lemma show that the language consisting of all strings with an equal number of 0's and 1's (not in any particular order) is not a regular language. (10)

(b) Describe the steps in detail in finding the equivalent DFA of an ϵ-NFA using lazy evaluation. (8)

(c) What is meant by enumeration of binary strings? Find out the 13th binary string. $(5\frac{1}{3})$

4. (a) Prove that the diagonalization language is not a recursively enumerable language. (10)

(b) Describe the languages which each of the following regular expression represents. You may assume that the alphabet Σ is $\{0, 1\}$. (8)

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Contd P/2

সেই ব্যক্তির কথার চেয়ে আর কার কথা উত্তম হবে যে আল্লাহর দিকে ডাকলো, সৎ কাজ করলো এবং ঘোষণা করলো আমি মুসলমান। –(সূরা হা-মিম-আস-সাজদাহঃ ৩৩)

Contd ... Q. No. 4(b)

i. 0^*10^*,

ii. $\Sigma^*\, j\, \Sigma^*$,

iii. $1^*\,(01^+)^*$,

iv. $(\Sigma\Sigma\Sigma)^*$,

v. $0\Sigma^*0 \cup 1\Sigma^*1 \cup 0 \cup 1$,

vi. $01 \cup 10$,

vii. $(0 \cup \epsilon)1^*$,

viii. $(01 \cup 010)^*$.

(c) Encode the Turing machine, $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, \delta, q_1, B, \{q_2\})$, where δ is given by $\{\delta(q_1, 1) = (q_3, 0, R), \delta(q_3, 0) = (q_1, 1, R), \delta(q_3, 1) = (q_2, 0, R), \delta(q_3, B) = (q_3, 1, L)\}$. (5 ⅓)

SECTION – B

There are NINE questions in this Section. Answer any SEVEN.

5. Define context-free grammar. Design context-free grammar for the language $\{a^m b^n \mid m > n, n \geq 1\}$. (10)

6. Define parse tree for a grammar. What is yield of a parse tree? Consider the grammar with start symbol D given by the following productions : (10)

$D \rightarrow TL;$

$T \rightarrow bin\_$

$L \rightarrow L, V \mid V$

$V \rightarrow I \mid I = N$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$N \rightarrow 1 \mid N1 \mid N0$

Derive:

(i) bin_ab00 = 101, b1, b1; using leftmost derivation

(ii) bin_a = 10, b = 1; using rightmost derivation.

7. Show that the following grammar is ambiguous. **(10)**

$E \rightarrow E + E \mid E * E \mid (E) \mid I$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

Remove ambiguity from the above grammar.

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8. Define pushdown automata (PDA) with an example. What is instantaneous description (ID) of a PDA? (10)

9. Draw the transition diagram of a pushdown automaton to accept palindromes of all length over the alphabet $\{0, 1\}$. (10)

10. Design a Turing machine to decide the language $\{0^n 1^n \mid n \geq 1\}$. Draw transition diagram of this Turing machine. (10)

11. Construct a Turing machine of any kind to compute exclusive OR (XOR) of two binary numbers of equal length. When the machine is started, its tape contains two binary numbers separated by a ',' (comma). The machine computes exclusive OR of two numbers and replaces the second number by it. (10)

12. What is meant by the language of a Turing machine? Define recursive language. Consider the Turing machine (10)

$$M = (\{q_0, q_1, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$$

Describe the language L(M) if δ consists of the following set of rules :

$$\delta(q_0, 0) = (q_1, 1, R);\ \delta(q_0, B) = (q_f, B, R);$$

$$\delta(q_1, 1) = (q_0, 0, R);\ \delta(q_1, B) = (q_f, B, R);$$

13. What are multitape Turing machines and nondeterministic Turing machines (NTM)? Do they accept any language not accepted by basic Turing machines? (10)

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বৈজ্ঞানিক জাবীর ইবনে হাইয়ান ইস্পাত তৈরি, ধাতুর শোধন, তরল বাষ্পীয় করণ, কাপড় ও চামড়া রঙ করা, ওয়াটার প্রুফ তৈরি করা, লোহার মরিচা প্রতিরোধক বার্ণিশ, চুলের কলপ, লেখার পাকা কালি আবিষ্কার করেন। বিজ্ঞান জগতে তার স্মৃতি অমর হয়ে আছে।

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2007-2008

Sub : **CSE 211** (Theory of Computation)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Explain the properties of parse trees with necessary examples. **(10)**

   (b) Design a deterministic finite automaton which accepts the language which consist of strings from $\{a, b\}^*$ and each a is followed by at least one b. You need to mention the design steps. The designed automaton is to be depicted using transition diagram as well as transition table. **(8)**

   (c) Explain how set-formers can be used to define a language. **($5\frac{1}{3}$)**

2. (a) Prove that if $D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ is the DFA constructed from NFA, $N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ by subset construction, then $L(D) = L(N)$. **(10)**

   (b) If $\Sigma = \{0, 1)$, which language is represented by $(0 \cup 1)^*$? Which language is represented by $(0\,\Sigma^*) \cup (\Sigma^* 1)$? **(8)**

   (c) Describe the most important role played by states in automaton. Is it possible to make this role with infinite capacity? Why? **($5\frac{1}{3}$)**

3. (a) Use pumping lemma to prove whether the language, consisting of all strings of 1's whose lengths are prime is a regular language or not. **(10)**

   (b) What is meant by ambiguity is grammars? How do we remove ambiguity from grammars? What is inherent ambiguity? **(8)**

   (c) "Nondeterminism is an inessential feature of finite automata" – explain. **($5\frac{1}{3}$)**

4. (a) Describe the grammar which represents the set of palindromes over $\{0, 1\}$. Then use induction techniques to prove validity of this grammar. **(10)**

   (b) Describe with necessary examples the differences and similarities between recursive inference and derivation with respect to context free grammars. **(8)**

   (c) "Regular expressions can define exactly the same languages that the various forms of automata describe" – then why do we need regular expressions? **($5\frac{1}{3}$)**

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আল্লাহ সর্বোত্তম বাণী নাযিল করেছেন, এমন একটি গ্রন্থ(কুরআন) যার সমস্ত অংশ সামঞ্জস্যপূর্ণ। -(সূরা আয-যুমারঃ ২৩)

CSE 211

SECTION – B

There are **NINE** questions in this Section. Answer any **SEVEN**.

5. Draw the transition diagram of a pushdown automaton to accept the language **(10)**

$$L = \{a^i . c^k (bc)^i \mid i, k \geq 0\}$$

6. "A language L has a pushdown automaton (PDA) that accepts it by final state if and only if L has a PDA that accepts it by empty stack" – justify the statement. **(10)**

7. Design a Turing machine (TM) to decide the language $\{0^n 1^n \mid n \geq 1\}$ and depict it using a transition table. Show the instantaneous descriptions (ID's) of the TM if the input tape contains 0011. **(7+3=10)**

8. Construct a Turing machine (TM) of any kind to add two binary numbers of equal lengths. When the machine is started, its tape contains two numbers separated by a ',' (comma). The machine adds the two numbers and ends with only the sum on the tape. **(10)**

9. Write short notes on the following : **(10)**

   (i) Multitape Turing machine

   (ii) Multistack machine

10. Describe how a Turing machine can simulate a computer. **(10)**

11. What are recursive, recursively enumerable (RE) and non-RE languages? Consider the Turing machine **(6+4=10)**

$$M = (\{q_0, q_1, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\})$$

Describe the language L(M) if δ consists of the following set of rules :

$$\delta(q_0, 0) = (q_0, B, R);\ \delta(q_0, 1) = (q_1, B, R);$$
$$\delta(q_1, 1) = (q_1, B, R);\ \delta(q_1, B) = (q_f, B, R);$$

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12. Define the diagonalization language. Prove that the diagonalization language is not a recursively enumerable language. **(10)**

13. What is Chomsky Normal Form (CNF) of a grammar? The following are the productions of a CNF grammar G : **(10)**

$$S \rightarrow AB \mid BC$$
$$A \rightarrow BA \mid a$$
$$B \rightarrow CC \mid b$$
$$C \rightarrow AB \mid a$$

Use the CYK algorithm to determine whether the string 'abaab' is in L(G).

নিশ্চয়ই যার অন্তরে পবিত্র কুরআনের কোন একটি অংশও নেই সে যেন একটি খালি ঘর (তিরমিযী)

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L–2/T–2 B. Sc. Engineering Examinations 2008-2009

Sub : **CSE 211** (Theory of Computation)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE.**

1. For the following languages, either design a push down automata that accepts those or prove that it is impossible to do so :

   (i) $L = \{a^n b^{2n} ; n > 0\}$ **(10)**

   (ii) $L = \{a^n b^{2n} c^n ;\ n \geq 0\}$ **(13⅓)**

2. For the following grammar

   $S \rightarrow ASB \mid \in$

   $A \rightarrow aAS \mid a$

   $B \rightarrow SbS \mid A \mid bb$

   (a) Eliminate $\in$– production (7⅓)

   (b) Eliminate any unit production **(8)**

   (c) Eliminate any useless symbol **(8)**

3. (a) Describe formal notation of the Turing Machine. (2⅓)

   (b) Design a Turing Machine that computes Bitwise XOR of two binary strings having the same length. For example, if input is 0010 # 1011, then the output should be: 1001. (Here, # is separator symbol. Make necessary assumptions) **(13)**

   (c) Simulate your Turing Machine of 3(b) for the following input 1011 # 0010. **(8)**

4. (a) Describe and sketch a proof of Rice's theorem. **(3+9)**

   (b) Argue whether the following problem is decidable: "whether the language accepted by a Turing Machine is palindromic." (3⅓)

   (c) Give an example of a language that is recursively enumerable but not recursive. **(3)**

   (d) Prove that " if both L and $\overline{L}$ are recursively enumerable then both L and $\overline{L}$ are recursive." **(5)**

আর পৃথিবীর আসমানকে আমি উজ্জ্বল প্রদীপ দিয়ে সজ্জিত করলাম এবং ভালভাবে সুরক্ষিত করে দিলাম।
–(সূরা হা-মিম-আস-সাজদাহঃ ১২)
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CSE 211

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Let L be a regular language. Prove that there exists a constant n (which depends on L) such that for every string w in L, $|w| \geq n$, we can break w into three strings, w = xyz, such that : **(10)**

   (i) $y \neq \epsilon$. (ii) $|xy| \leq n$. (iii) For all $k \geq 0$, the string xy^kz is in L.

   (b) Prove that $\{ 0^n 1 0^n \mid n \geq 1 \}$ is not a regular language. **(8)**

   (c) Define a deterministic finite automaton and a nondeterministic finite automaton. **(5⅓)**

6. (a) Design a DFA to accept the language:

   L = { w | w is a binary string having a decimal value that is a multiple of three}. **(9)**

   (b) Give an illustrative example of an NFA with n+1 states, which has no equivalent DFA with fewer than 2^n states. **(9)**

   (c) Consider the following ϵ–NFA **(5⅓)**

	ϵ	a	b	c
$\rightarrow p$	ϕ	{p}	{q}	{r}
q	{p}	{q}	{r}	ϕ
* r	{q}	{r}	ϕ	{p}

   (i) Compute the ϵ – closure of each state.

   (ii) Convert the automaton to a DFA.

7. (a) Prove that if L = L (A) for some DFA A, then there is a regular expression R such that L = L (R). **(14)**

   (b) Write regular expressions for the following languages: **(5)**

   (i) The set of strings over alphabet { a, b, c} containing at least one a and at least one c.

   (ii) The set of binary strings with no two consecutive 0's or no two consecutive 1's.

   (c) Convert the regular expression $(0 + 1)^* 1 (0 + 1)$ to an ϵ – NFA. **(4⅓)**

8. (a) Let $L = \{a^n b^n c^m d^m \mid n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$ be a context-free language. Explain that all grammars for L are ambiguous. **(9)**

   (b) Design a context-free grammar G to generate the language of regular expression $0^*1(0 + 1)^*$. Using G, give leftmost and rightmost derivations of the string 00101. **(9)**

   (c) Consider the grammar $S \rightarrow aS \mid aSbS \mid \epsilon$. Show in particular that the string aab has two - **(5⅓)**

   (i) Parse trees. (ii) Leftmost derivations. (iii) Rightmost derivations.

উমার (রা) বলেছেন, সংগঠন ছাড়া ইসলাম নেই, নেতৃত্ব ছাড়া সংগঠন নেই, আনুগত্য ছাড়া নেতৃত্ব নেই।

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L-2/T-2/CSE Date : 14/07/2011

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc.. Examinations 2009-2010

Sub : **CSE 211** (Theory of Computation)

Full Marks: 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

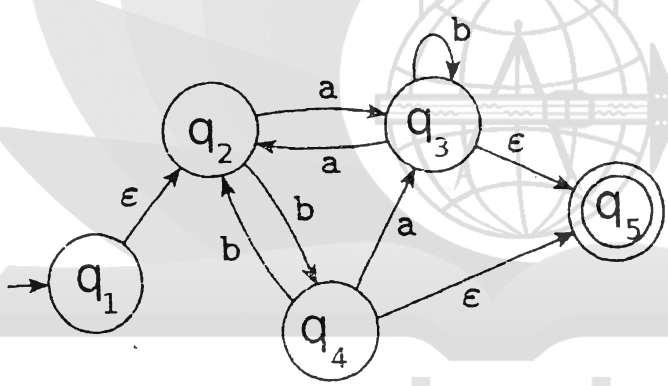
SECTION – A

There are **NINE** questions in this Section, answer any **SEVEN** questions.

1. "Nondeterminism is an inessential feature of finite automata" - explain. **(10)**

2. Design a deterministic finite automaton which accepts the language which consists of strings from {a, b}* and does not contain the character anywhere in the string consecutively three times which started the string. So, *abaab, baaabba* are accepted, but *abaaab, bbbaa* are rejected. Depict the automaton using transition diagram. **(10)**

3. Convert the NFA shown in the figure into an equivalent DFA using lazy evaluation. **(10)**



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4. We have two languages, $L_1 = 1(1\cup 0)^*000$, and $L_2 = (1(01)^*0)^*$. Find out whether each of the followings belongs to L_1 o L_2 with necessary reasoning. **(10)**
   (a) 1000.
   (b) 1000101010.
   (c) 10000111000100000.

5. Using pumping lemma show whether the language consisting of all strings of 1' s whose length is a prime is a regular language or not. **(10)**

6. Consider the grammar given below. **(10)**

আল্লাহ চোখের চুরি ও মনের গোপন কথা পর্যন্ত জানেন। –(সূরা আল মুমিনঃ ১৯)

$<pop> \rightarrow [<bop>, <pop>] \mid <bop>$

$<bop> \rightarrow <boop> \mid (<pop>)$

$<boop> \rightarrow x \mid y \mid z$

(a) What are the nonterminal and terminal symbols?

(b) What is the start symbol? Why?

(c) Draw a parse tree for the sentence (x).

7. Describe the grammar which represents the set of palindromes over $\{0, 1\}$. How do you modify the grammar when the alphabet changes to $\{0, 1, a\}$ and the length is restricted to even length only? **(10)**

8. Describe with necessary examples the differences and similarities between recursive inference and derivation with respect to context free grammars. **(10)**

9. Consider the language **(10)**

   $L = \{w \in \{a, b\}^*$: w contains equal numbers of a's and b's$\}$. Write a context-free grammar G for L. Show a leftmost derivation for the string *aabbab* using G.

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE** questions.

10. (a) Formally define Pushdown Automata (PDA). Design PDA to accept each of the following languages. **(2+4+4+4)**

    i) $\{0^n 1^n \mid n \geq 1\}$

    ii) The set of all strings of 0's and 1's with an equal number of 0's and 1's

    iii) The set of all palindromes of 0's and 1's

    (b) Suppose the PDA $P = (\{p, q\}, \{0, 1\}, \{Z, X\}, \delta, q, Z, \{p\})$ has the following transition functions **(3+6 $\frac{1}{3}$)**

    $\delta(q, 0, Z) = \{(q, XZ)\}$

    $\delta(q, 0, X) = \{(q, XX)\}$

    $\delta(q, 1, X) = \{(q, X)\}$

    $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$

    $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$

    $\delta(p, 1, X) = \{(p, XX)\}$

    $\delta(p, 1, Z) = \{(p, \epsilon)\}$

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তুমি যদি করো উপকার কর তবে সাবধান! কখনো তা তার কাছে উল্লেখ কর না। আর কেউ যদি তোমার উপকার করে তবে সাবধান! কখনো তা ভুলো না।
আব্দুল্লাহ ইবনুল মুকাফফা (র)

i) Represent the PDA *P* by a transition diagram.

ii) Convert *P* to another PDA P_1 that accepts by empty stack the same language that *P* accepts by final state.

11\. (a) Convert the following grammar to a PDA that accepts the same language by empty stack. **(5)**

$S \rightarrow AS \mid A$

$A \rightarrow 0A \mid 1B \mid 1$

$B \rightarrow 0B \mid 0$

(b) Suppose the PDA $P = (\{p, q\}, \{0, 1\}, \{Z, X\}, \delta, q, Z)$ has the following transition functions **(13)**

$\delta(q, 1, Z) = \{(q, XZ)\}$

$\delta(q, 0, X) = \{(p, X)\}$

$\delta(p, 1, X) = \{(p, \in)\}$

Convert the PDA *P* to a context free grammar.

(c) Briefly describe Deterministic Pushdown Automata (DPDA) with an appropriate example. **($5\frac{1}{3}$)**

12\. (a) Give formal definition of a Turing Machine. Write out in full a Turing Machine that scans to the right until it finds two consecutive a's and then halts. The alphabet of the Turing machine should be {a, b, ⊔, ▷} **(2+3)**

(b) Design a Turing Machine that decides the language $L = \{a^n c^n b^n d^n : n \geq 0\}$ **(7)**

(c) Give a three tape Turing machine which, when starts with two binary integers separated by a ';' on its first tape, computes their subtraction. Consider that of the two integers the first integer is greater or equal than the second integer. **($11\frac{1}{3}$)**

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13\. (a) "The operation of a Turing machine may never stop" - do you agree? Justify your answer with appropriate example. **(4)**

(b) Prove that if L is recursive language, then its complement $\overline{L}$ is also recursive. **(4)**

(c) Briefly describe Universal Turing Machine with an appropriate example. **(11)**

(d) Explain the "Halting Problem" with an appropriate example. **($4\frac{1}{3}$)**

পানির গভীরতা এবং স্রোত মাপার যন্ত্র আবিষ্কার করেছিলেন মুসলিম বৈজ্ঞানিক আব্দুল মজিদ

L-2/T-2/CSE **Date : 26/12/2012**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2010-2011

Sub : **CSE 211** (Theory of Computation)

Full Marks: 140 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **NINE** questions in this section. Answer any **SEVEN**.

1. You have designed a deterministic finite automaton (DFA) for a certain language from another nondeterministic finite automaton (NFA) using subset construction method. Do you think that the new DFA will accept/reject the same languages as the original NFA? Explain with necessary justification. **(10)**

2. Design a deterministic finite automaton which accepts the language consisting of strings from $\{a, b\}^*$ and n mod 3 = 1, where n is the number of a's in the string. **(10)**
Depict the automaton using transition diagram as well as mathematical representation.

3. Convert the NFA shown below into an equivalent DFA using lazy evaluation. **(10)**

	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
q	$\{r, s\}$	$\{t\}$
r	$\{p, r\}$	$\{t\}$
$s*$	$\emptyset$	$\emptyset$
$t*$	$\emptyset$	$\emptyset$

4. Design a deterministic finite automaton which accepts the language consisting of strings from $\{0, 1\}^*$ and the language does *not* contain the substring 110. **(7+3=10)**
Now, show the regular expression for this language.

5. State and prove the pumping lemma. **(10)**

6. Using pumping lemma show whether the language $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$ is a regular language or not. **(10)**

Contd P/2

7. Consider the grammar $G = (V, \Sigma, P, S)$, where

$V = \{a, b, S, A\}, \Sigma = (a, b),$ **(10)**

$p = \{S \rightarrow AA, A \rightarrow AAA, A \rightarrow a, A \rightarrow bA, A \rightarrow Ab\}.$

For any $m, n, p > 0$, describe with necessary explanation a derivation in G of the string $b^m ab^n ab^p$.

8. Consider the alphabet $\Sigma = \{a, b, (,), \cup, *, \emptyset\}$. Use the rules (basic and inductive) for building regular expressions to construct a context-free grammar that generates all strings in Σ^* that are regular expressions over $\{a, b\}$. **(10)**

9. Give a context-free grammar for each of the following languages. **(10)**

(a) $L_1 = \{0^n 1^m 0^m 1^n \mid n, m \geq 0\},$

(b) $L_2 = \{a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n = m + k\},$

(c) $L_3 = \{a^n b^m c^k \mid n, m, k \geq 0 \text{ and } n = 2m + 3k\},$

(d) $L_4 = \{a^n b^m \mid 0 \geq n \geq m \geq 2n\}.$

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

10. (a) Show that the set of all infinite binary sequence is uncountable. **(4)**

(b) Using the findings of Question 10(a), prove that some languages are not Turing-recognizable. **(8⅓)**

(c) Prove that a language is decidable if and only if both the language and its complement are Turing-recognizable. **(7)**

(d) Give a formal description of the halting problem. Give an example of a language that is not Turing-recognizable. **(4)**

11. (a) Describe equivalent single tape Turing machines that can be used to completely simulate the following variants of Turing machines. **(3×5=15)**

(i) Multi-tape Turing machine,

(ii) Non-deterministic Turing machine with a single-tape,

(iii) Turing machine with two-dimensional tape stretched to infinity in all directions.

(b) Describe a Turing machine that decides the following language: **(8⅓)**

$$L = \{a^{2^n} b \mid n \geq 0\}$$

Show the state diagram of the machine. Show the sequence of configurations of the machine for the input string $aaaab$.

Contd P/3

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12. (a) What is the difference between Turing-decidable and Turing-recognizable languages? What are the three conditions that must be met by a Turing Machine M if it is to recognize a string w of a language L? **(4)**

(b) What does the following Turing machine do? **(3)**

$$> R \xrightarrow{a \neq \sqcup} R \xrightarrow{b \neq \sqcup} R_{\sqcup} a R_{\sqcup} b$$

Do the Turing machines LR and RL always automate the same way? Explain.

(c) Prove that if a PDA recognizes some language L, then L is context free. **(11)**

(d) Give an informal description and state diagram for the PDA that recognizes the following CFG: **(5⅓)**

$S \rightarrow TX$
$T \rightarrow 0T0 \mid 1T1 \mid \#X$
$X \rightarrow 0X \mid 1X \mid \varepsilon$

where the starting symbol is S.

13. (a) Formally describe a PDA that recognizes the following language: **(8)**

$$L = \{w \in \{0,1\}^* \mid w = w^R\}$$

(b) Convert the PDA derived in Question 13(a), into an equivalent CFG following standard rules of conversion. **(7⅓)**

(c) Automate the following PDA for the input *1001* and *0101* separately. What does the PDA do? **(8)**

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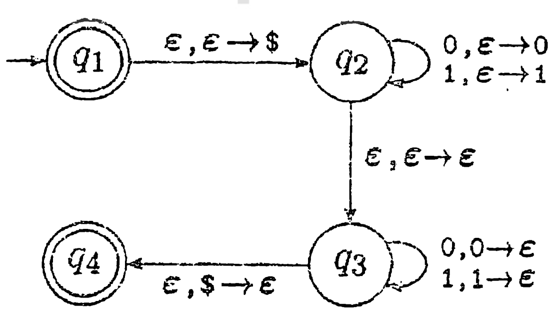

L-2/T-2/CSE

Date : 22/07/2013

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2011-2012

Sub : **CSE 211** (Theory of Computation)

Full Marks: 140 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Construct the state transition diagram for a Turning machine to compute the following subtract function: $f(x, y) = x - y$, where $x \geq y$. Assume that the numbers are represented in a modified version of unary number system where each number N is represented by repeating the symbol '1' N + 1 times. For example, the number 1 is represented as 11, the number 2 is represented as 111 and the number 0 is represented as 1. The Turing machine should enter the accept state after the computation. We don't care where you leave the head at the end of the computation. Your state diagram does not need to explicitly show the reject state or the transitions into it. **$(8\frac{1}{3})$**

   (b) Give the proof idea of the following statement: 'Every Turing machine with multiple tracks in its tape has an equivalent single track Turing machine'. **(7)**

   (c) Briefly explain an encoding technique for Turing machines. With the help of this encoding technique, show that the number of Turing machines is countable. **(6+2=8)**

2. (a) Construct the state transition diagram of a 2–tape Turing machine to decide the following language: L = {wcw^R: where $w \in \{a, b\}^*$ and w^R is the reverse string of w}. Your state diagram should be as simple as possible and it must use both tapes of the Turing machine. Your state diagram does not need to explicitly show the reject state or the transitions into it. **$(8\frac{1}{3})$**

   (b) Show that a language is recognized by a Turing Machine with a two way infinite tape if and only if it is recognized by a Turing Machine with a one way infinite tape. **(8)**

   (c) The halting problem can be described by the following language: **(7)**

   L = {<M,w> : M is a Turing machine and M halts on input string w}. Show that, the language L is Turing–acceptable.

3. (a) Give the state diagram of a Turing machine M that does the following on input string '#w' where $w \in \{0, 1\}^*$. Let n = |w| (the length of the string w). If n is even, then M converts string #w to the string #0^n (symbol 0 is repeated *n* times). If n is odd, then M converts #w to the string #1^n. The machine should enter the accept state after the conversion. We don't care where you leave the head at the end of the conversion. However, your state diagram does not need to explicitly show the reject state or the transitions into it. **$(8\frac{1}{3})$**

Contd P/2

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সেখানে এসব কাফের বলবে, "হে আমাদের রব, সেই সব জ্বিন ও মানুষ আমাদের দেখিয়ে দাও যারা আমাদের পথভ্রষ্ট করেছিলো। আমরা তাদের পদদলিত করবো, যাতে তারা লাঞ্ছিত ও অপমানিত হয়। " –(সূরা হা-মিম-আস-সাজদাহঃ ২৯)

(b) Prove the following statement: 'The union of two recursively enumerable languages is also recursively enumerable'. **(8)**

(c) Construct the state diagram of a push down automaton that accepts the following language: $L = \{w: w \in \{a, b\}^*$ and length of w is odd with middle symbol 'a'$\}$. Example strings in the language L are: aaa, bbabb, baaba, etc. **(7)**

4. (a) Construct the state diagram of a push down automaton that recognizes the following language: $L = \{a^i b^j c^k : i + j = k\}$. **(7)**

(b) Consider the following context free grammar: $CFG = (V, \Sigma, R, S)$, where $V = \{S, T, X\}, \Sigma = \{a, b\}$, the start non-terminal is S, and the rules in R are: $S \to aTXb, T \to XTS \mid \varepsilon, X \to a \mid b$. Convert the CFG to an equivalent push down automata. **(8)**

(c) Show that, the following language L is decidable: $L = \{< A, B, C >: A, B, C\}$ are DFAs over the same alphabet Σ and $L(C) = L(A) \cup L(B)\}$. **($8\frac{1}{3}$)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Give three differences between DFA and NFA with examples. **(6)**

(b) Define regular language. Prove that the class of regular languages is closed under the following operations: **(12)**

(i) *star* operation

(ii) *concatenation* operation.

(c) Give the state diagram of a DFA that recognizes the following language: **($5\frac{1}{3}$)**

$\{w \mid$ the length of w is at most 5$\}$ where $\Sigma = \{0, 1\}$

6. (a) Is there any technique to test whether a given language L is regular or not? State and prove it. **($13\frac{1}{3}$)**

(b) Prove that the following languages are not regular: **(10)**

(i) $L_1 = \{www \mid w \in \{0,1\}^*\}$

(ii) $L_2 = \{1^{n^2} \mid n \geq 0\}$

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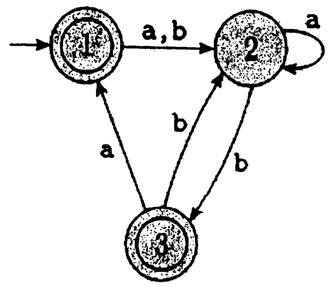
আপনি আজকে যা করছেন তা হয়ত আমি পছন্দ করিনা, কিন্তু তাই বলে আমি আপনাকে ছোট করবো না। কারণ, আগামীকালের আপনি আপনি হয়ত আজকের আমার চাইতে ভালো হবেন।

– তারিক রামাদান

7. (a) Construct an NFA that recognizes the language $(01 \cup 001 \cup 010)^*$. **(8)**

(b) Convert the NFA derived in 7(a) into an equivalent DFA. Give the state diagram of this DFA with only states reachable from the start-state. **(8)**

(c) The state diagram of a finite automaton is given below. Derive the regular expression for it. **(7⅓)**



8. (a) Formally define context-free grammar. What is ambiguous grammar? Give an example. **(6)**

(b) Design a context-free grammar for each of the following languages: **(10)**

(i) $\{0^n1^n \cup 1^n0^n \mid n \geq 0\}$

(ii) $\{w \mid w \in \{0, 1\}^*$ and w contains at least three 1's$\}$

(c) The set of rules R of a context-free grammar ({E, T, F}, {a,b,+,*,(,)}, R, E) is given below: **(7⅓)**

E → E+T | T

T → E*F | F

F → (E) | a | b

Show the steps of derivation of the expression ((a+b)*b).

Also, show the parse-tree for it.

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ম্যাঙ্গানিজ ডাই অক্সাইড থেকে প্রথম কাঁচ আবিষ্কার করেন মুসলিম বৈজ্ঞানিক আর রাজী। ইংরেজদের ইংরেজি শব্দে তার নাম - Rezes লেখা আছে।

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2012-2013

Sub : **CSE 211** (Theory of Computation)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Give formal definition of pushdown automata. Construct a pushdown automaton that recognizes the same language as recognized by the following context free grammar: **(10)**

$$S \to BS \mid A$$
$$A \to 0A \mid \varepsilon$$
$$B \to BB1 \mid 2$$

(b) Prove that a language is context free language (CFL) if and only if there exists a pushdown automaton (PDA) accepting it. **(13⅓)**

2. (a) What is the difference between Turing recognizable and decidable languages? Explain. The halting problem can be described by the following language: $L = \{< M, w > : M \text{ is a Turing machine and } M \text{ halts on input string } w\}$. Show that the language L is Turing-acceptable. **(10)**

(b) Give a formal definition of pumping lemma for context free language (CFL). Use the pumping lemma for CFL to show that the language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not CFL. **(13⅓)**

3. (a) Prove that every multi-tape Turing machine has an equivalent single-tape Turing machine. **(8)**

(b) Let $\Sigma = \{-, a, b\}$ and $L = \{w\text{–}w\text{–}w \mid w \in \{a, b\}^*\}$. Give a high level description of a Turing machine M that decides L. Draw a state diagram for M. You may assume M has a doubly infinite tape. **(7)**

(c) Give the state diagram of a Turing machine M that does the following on input string '#w' where $w \in \{0, 1\}^*$. Let $n = |w|$ (the length of the string w). If n is even, then M converts string #w to the string #0^n (symbol 0 is repeated n times). If n is odd, then M converts #w to the string #1^n (symbol 1 is repeated n times). The machine should enter the accepting state after the conversion. Do not care where the head should be at the end of conversion. Do not show the reject state and the corresponding transitions to the reject state. **(8⅓)**

4. (a) Construct the state diagram of a pushdown automaton that recognizes the following language $L = \{a^i b^j c^k : i + j = k\}$. **(8)**

Contd P/2

একজন বৃদ্ধ লোক মসজিদের ভিতর আমাকে বলল, “যে সব লোক রমজান মাসের দাসত্ব করে তারা চলে গেছে এবং যাঁরা আল্লাহর দাসত্ব করে তাঁরা আজও ইবাদত করছে”। আসুন আমরা শুধুমাত্র রমজান মাসের মুসলমান না হই।
– শাইখ খালিদ ইয়াসিন

Contd ... Q. No. 4

(b) Show that every nondeterministic Turing machine has an equivalent deterministic Turing machine. **(7)**

(c) Construct the state diagram of a pushdown automaton that accepts the following language: $L = \{ w : w \in \{a, b\}^*$ and length of w is odd with middle symbol 'a'$\}$. Few example strings in the language L are: *aaa*, *bbabb*, *baaba*. **(8 ⅓)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

Assume reasonable any missing data. Symbols carry their usual meaning.

5. (a) Consider the grammar $E \rightarrow E + E | E * E | (E) | a$. Is this grammar ambiguous? Explain with respect of $a + a * a$. **(8)**

(b) Prove that Language $\{0^n \mid n$ is a perfect square$\}$ is not regular. **(5 ⅓)**

(c) Define regular language. Prove that the class of regular language is closed under the following operations: **(10)**

(i) Star operation

(ii) Concatenation operation

6. (a) Let $L = \{a^n b^n c^m \mid n \geq 1, m \geq 1\} \cup \{a^n b^m c^m \mid n \geq 1, m \geq 1\}$ be a context free language. Explain that all grammars for L are ambiguous. **(8)**

(b) Is there any technique to test whether a given language L is regular or not? State and prove it. **(12 ⅓)**

(c) Write regular expressions for the following language: **(3)**

String of 0's and 1's having even parity, that is, an even number of 1's.

7. (a) Design a context free grammar G to generate the language of regular expression $0^*1(0 + 1)^*$. Using G, give leftmost and rightmost derivations of the string 00101. **(8)**

(b) Convert the following DFA into Regular expression **(7 ⅓)**

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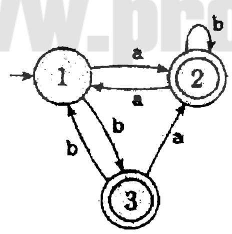

(c) Design a DFA to accept the language:

$L = \{w \mid w$ is a binary string having a decimal value that is multiple of five$\}$. **(8)**

8. (a) Give three differences between DFA and NFA with examples. **(5)**

(b) Prove that a set is regular if and only if it can be described by a regular expression. **(12 ⅓)**

(c) Prove that $\{0^n 1 0^n \mid n > 1\}$ is not a regular language. **(6)**

কিয়ামতের দিন মানব জাতিকে মথিত আটার রুটির ন্যায় লালিমাযুক্ত শ্বেতবর্ণ যমীনে একত্রিত ১১৮।
এক ব্যক্তি আহত হয়েছিল। সে আত্মহত্যা করলে আল্লাহ তায়ালা বলেন, আমার বান্দা বড় তাড়াহুড়া করল। সে নিজেই নিজেকে হত্যা করল। আমি তাঁর জন্য জান্নাত হারাম করে দিলাম। (বুখারী)

L-2/T-2/CSE Date : 06/08/2015

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **CSE 211** (Theory of Computation)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Convert the following context free grammar into an equivalent pushdown automation: (7)

$$E \to E + T \mid T$$
$$T \to T \times F \mid T/F \mid F$$
$$F \to (E) \mid a$$

(b) Construct the state diagram of a pushdown automation that recognizes the following language $L = \{a^i b^j c^k : i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$. **(7)**

(c) Prove that for any pushdown automation P, there exists a context free grammar G such that $L(P) = L(G)$. **(9⅓)**

2. (a) Build a Turning machine to recognize the language $L = \{0^N 1^N 0^N \mid N \geq 0\}$. **(7)**

(b) How can we recognize the left end of the tape of a Turing machine? Explain. **(7)**

(c) Use the pumping lemma to show that the following language are not context free: **(9⅓)**

(i) $L = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$.

(ii) $L = \{0^n \#0^{2n} \#0^{3n} \mid n \geq 0\}$.

3. (a) Answer the following questions, and explain your reasoning: **(7)**

(i) Can a Turing machine ever write the blank symbol on its tape?

(ii) Can the tape alphabet Γ be the same as the input alphabet Σ?

(iii) Can a Turing machine's head ever be in the same location in two successive steps?

(iv) Can a Turing machine contain just a single state?

(b) Let A be the language containing only the single string s, where

$$s = \begin{cases} 0 \text{ if life never will be found on Mars} \\ 1 \text{ if life will be found on Mars someday} \end{cases}$$

Is A decidable? Why or why not? For the purposes of this problem, assume that the question of whether life will be found on Mars has an unambiguous YES or NO answer. **(7)**

(c) Show that every nondeterministic Tuning machine has an equivalent deterministic Tuning machine. **(9⅓)**

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Contd P/2

CSE 211

4. (a) Consider the language $A_{DFA} = \{<B, w> \mid B \text{ is a DFA accepting } w\}$. Prove that A_{DFA} is decidable. **(7)**

   (b) Prove that the set of rational numbers is countable. **(7)**

   (c) Prove that $A_{TM} = \{<M, w> \mid M \text{ is a Turing machine accepting } w\}$. is not decidable. **($9\frac{1}{3}$)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Prove that regular languages are closed under minus (–) operation where minus (–) operation is defined as $L_1 - L_2 = \{w \mid w \in L_1 \text{ but } w \notin L_2\}$. **(8)**

   (b) Show a counter-example to disprove the following statement: If R_1 and R_2 are two regular expressions, then $(R_1 \cup R_2)^* = R^*_1 \cup R^*_2$.

   (c) Construct a DFA to recognize the following language L: **($10\frac{1}{3}$)**

   $L = \{w \in \{a, b\}^* \mid w = a^p b^q,\ p + q \text{ is even}\}$.

UPLOADED BY

6. (a) State and prove the Pumping lemma for regular languages. **($7\frac{1}{3}$)**

   (b) Let $\Sigma = \{(,)\}$ and let L be the language consisting of all strings of properly nested parenthesis. For example L contains "( ) ( )", "((( )))", "(( )( ) ((( )( )))", and ϵ, but not ")(" and "((((". Using Pumping lemma show that L is not regular. **(8)**

   (c) Construct a regular expression from the following NFA: **(8)**

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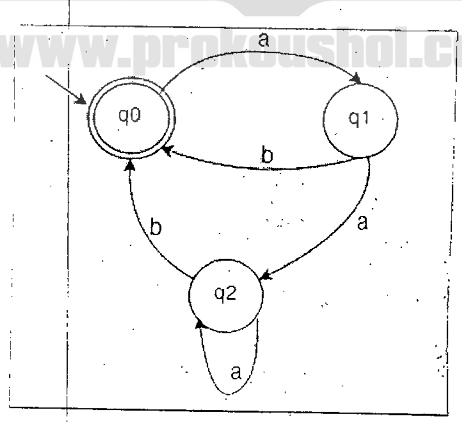
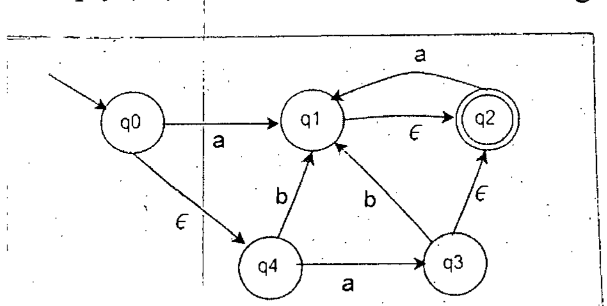


7. (a) Eliminate all empty (ϵ) – transitions from the following NFA: **(9)**



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CSE 211
Contd ... Q. No. 7

(b) Construct a context free grammar to recognize the following language L: **(8⅓)**

$L = \{w \in \{a,b,c\}^* \mid w = a^i b^j, c^k, i \geq 0, j \geq 0, k \geq 0\}.$

Draw a parse tree for the string ab^2c^3 according to your grammar.

(c) Show that the following language L is not context free: **(6)**

$L = \{w \in \{a,b,c\}^* \mid w = a^n b^{2n}, c^{3n}, n \geq 0\}.$

8. (a) Suppose L_1 and L_2 are two regular languages. Let $M_1 =< \Sigma, Q_1, q_0^2, \delta_1, F_1 >$ and $M_2 =< \Sigma, Q_2, q_0^2, \delta_2, F_2 >$ be two DFAs that recognize L_1 and L_2 respectively. Describe the steps to construct a DDA that recognize the language $L_1 \cup L_2$. **(7)**

(b) Prove that every NFA has an equivalent DFA. **(10⅓)**

(c) Show a counter-example to disprove the following statement: Every subset of a regular language is also regular. **(6)**

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L-2/T-2/CSE Date : 20/07/2016

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2014-2015

Sub : **CSE 211** (Theory of Computation)

Full Marks : 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Design a DFA M with alphabet $\Sigma = (0, 1, 2, 3, R\}$, which recognizes the language A. The language A is the set of all strings where the sum of the numbers is a multiple of 3, except that the sum is reset to 0 whenever the symbol R appears. Only a diagrammatic description of M should suffice. **(10)**

   (b) For the DFA in Question 1(a) we change the alphabet to $\Sigma = \{0, 1, 2, R\}$, and you decide to use the states to remember actual sums of the numbers. Show diagrammatically how this DFA can be constructed. What is the problem with this DFA? Explain clearly. **(13 ⅓)**

2. (a) Use pumping lemma for regular languages to find out, whether the unary language, $\left\{1^{n^2} \mid n \geq 0\right\}$, is a regular language or not. **(10)**

   (b) Design a DFA which accepts all and only the binary number strings divisible by 4. Explain how the DFA works. **(13 ⅓)**

3. (a) Design an NFA over the alphabet $\Sigma = \{a, b\}$ which accepts all and only the strings of nonzero length having nonzero even number of a's at the end. **(13 ⅓)**

   Thus, this NFA will accept, *aa*, *aaaa*, *abbaa*, *abaababaaaa*, but will reject, *aaa*, *ab*, *b*, *abbaaa* and *abaabbbb*.

   (b) Provide regular expressions generating the following languages. **(4+3+3)**

   (i) $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. The members of this language are numbers divisible by 5. A number cannot start with a 0 except for standalone 0s.

   (ii) Binary strings consisting of either an odd number of 0s (and any number of 1s) or an odd number of 1s (and any number of 0s).

   (iii) Binary strings that do not contain the sequence 101 embedded anywhere.

4. (a) Construct an NFA over the alphabet $\Sigma = \{0, 1\}$ that accepts the set of strings that contain an even number of occurrences of substring 01. Then convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state. **(7 ⅓ +4)**

   (b) State and prove pumping lemma for regular language. First, explain your ideas about the proof and then carry out the actual proof. **(12)**

Contd P/2

CSE 211

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Consider the following context free grammar: **(7)**

$S \rightarrow abScB \mid \varepsilon$

$B \rightarrow bB \mid b$

What language does it generate?

(b) Construct context free grammars to accept the following languages over $\Sigma = \{0, 1\}$: **(7)**

(i) {w | w starts and ends with the same symbol}.

(ii) {w | |w| is odd and its middle symbol is 0}.

(c) What is Chomsky normal form? convert the following context free grammar into an equivalent free grammar in Chomsky normal form: **(9 1/3)**

$A \rightarrow BAB \mid B\ \varepsilon$

$B \rightarrow 00 \mid \varepsilon$

6. (a) Explain why the context free grammar below is ambiguous: **(7)**

$S \rightarrow 0A \mid 1B$

$A \rightarrow 0AA \mid 1S \mid 1$

$B \rightarrow 1BB \mid 0S \mid 0$

(b) Show that the following language over $\Sigma = \{a, b, c\}$ is not context free: **(7)**

$L = \{a^n b^j c^k : k = jn\}$.

(c) Construct the state diagram of a pushdown automaton that recognizes the following language $L = \{wcw^R \mid w \in \{a,b\}^*$ and w^R is the reverse of w}. Trace the computation that accepts "bbcbb" by configuration. **(9 1/3)**

7. (a) Prove that for any pushdown automaton P, there exists a context free grammar G such that L(P) = L(G). **(7)**

(b) Prove that the set of infinite binary sequences is not countable. **(7)**

(c) Prove that a non-deterministic Turing machine is equivalent to a deterministic Turing machine. **(9 1/3)**

8. (a) What do you mean by decidable, Turing recognizable, and undecidable languages? **(7)**

(b) Let $L = \{w - w \mid w \in \{a, b\}^*\}$ be a language. Draw a state diagram of a Turing machine that decides L. **(7)**

(c) Prove that the language A_{TM} = {<M, w> | M is a Turing machine accepting w} is undecidable. **(9 1/3)**

L-2/T-2/CSE **Date : 25/01/2017**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2015-2016

Sub : **CSE 211** (Theory of Computation)

Full Marks: 140 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Formally define a Finite State Machine (FSM). Formally describe the FSM F1 of Figure 1.a. **($8\frac{1}{3}$)**

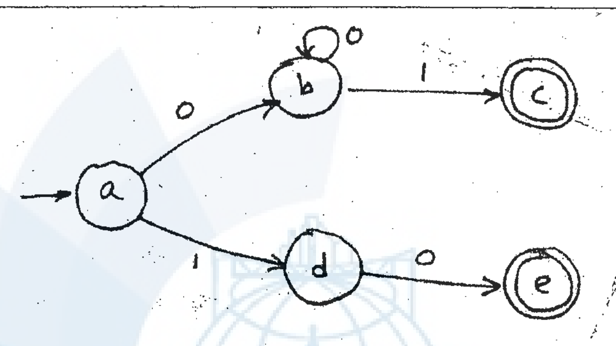

Figure 1.a: Finite State Machine, F1

(b) What language does F1 (Figure 1.a) recognize? **(4)**

(c) Suppose your friend slightly changes δ of F1 (Figure 1.a) by only adding the transition δ(d,1) = d. If the resulting FSM is F2, then what language does F2 recognize? **(4)**

(d) Another friend of yours slightly changes F1 (Figure 1.a) without changing the number of states and the number of transitions. Now, he calmly claims that the resulting FSM, F3, is such that $|L(F3)| = |L(F1)| + 1$, i.e., the cardinality of the language of F3 increases by exactly 1 (because of the change made on F1). Do you think this is possible? Justify your answer. **(4)**

(e) Suppose the cardinality of the language represented by the RE R1 is 5. Now, you have designed a DFA D1 equivalent to R1 and have written a C program implementing D1. Subsequently, you have tested the program against 1000 random different strings and for each of those strings your program returns "Accept" indicating that the DFA accepts that string. Assuming that your C implementation is correct your genius little brother claims that your DFA is wrong. Do you agree with him? Justify your answer. **(3)**

Contd P/2

CSE 211

2. (a) Design a DFA, M that takes as input a binary string and recognizes numbers that are NOT divisible by 3. For example, M rejects 1111 (because the corresponding numeric value is 15, which is divisible by 3), M accepts 111 (because the corresponding numeric value is 7, which is not divisible by 3) and so on. You need to discuss your intuition/idea behind the design and explain how it will work. Drawing the DFA only will not guarantee full marks. **($8\frac{1}{3}$)**

(b) Prove that the class of regular languages is closed under the regular operations. **(15)**

3. (a) Compute DFA and Regular Expressions for the following languages: **(18)**

   (i) L = {w | w begins with a 1 and ends with a 0}.

   (ii) L = {w | w has length at least 3 and its third symbol is a 0}.

   (iii) L = {w | w contains an even number of 0's or exactly two 1's}.

   (iv) L = {w | w is any string except 11 and 111}.

   (v) L = {w | w contains at least 2 0's and at most one 1}.

(b) Your teacher has proved that if M is a DFA that recognizes language B, swapping the accept and non-accept states in M yields a new DFA recognizing the complement of B. Now your genius brother comes up with an example that if M1 is an NFA that recognizes language C, swapping the accept and non-accept states in M1 doesn't necessarily yield a new NFA that recognizes the complement of C. Can you find such an example? **($5\frac{1}{3}$)**

4. (a) Consider the following context-free Grammar G: **($7\frac{1}{3}$)**

$R \rightarrow XRX \mid S$

$S \rightarrow aTb \mid bTa$

$T \rightarrow XTX \mid X \mid \varepsilon$

$X \rightarrow a \mid b$

Now answer all the following questions:

   (i) What are the variables of G?

   (ii) What are the terminals of G?

   (iii) Which is the start variable of G?

   (iv) Give three strings in L(G).

   (v) Give three strings not in L(G).

   (vi) Give a description of L(G).

Contd P/3

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Contd... Q. No. 4(a)

(vii) Now, write True or False for the following statements (S1-S9):

S1: $T \Rightarrow \text{aba}$.

S2: $T \overset{*}{\Rightarrow} \text{aba}$.

S3: $T \Rightarrow T$.

S4: $T \overset{*}{\Rightarrow} T$.

S5: $XXX \overset{*}{\Rightarrow} \text{aba}$.

S6: $X \overset{*}{\Rightarrow} \text{aba}$.

S7: $T \overset{*}{\Rightarrow} XX$.

S8: $T \overset{*}{\Rightarrow} XXX$.

S9: $S \overset{*}{\Rightarrow} \varepsilon$.

(b) Find context free grammars for the following languages: **(10)**

(i) The set of binary strings containing at least three 1's.

(ii) The set of odd-length binary strings having 0 at the middle.

(iii) The set of binary strings having more 0's than 1's.

(iv) $L = \{w\#x \mid w^R$ is a substring of x for $w, x \in \{0, 1\}^*\}$.

(c) Use pumping lemma to prove that the languages $A = \{0^n1^n2^n \mid n \geq 0\}$ and $B = \{0^i1^j \mid i > j\}$ are not regular. **(6)**

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SECTION-B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Suppose the PDA $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$ has the following transition function: **($8\frac{1}{3}$)**

(i) $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$

(ii) $\delta(q, 0, X) = \{(q, XX)\}$

(iii) $\delta(q, 1, X) = \{(q, X)\}$

(iv) $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$

(v) $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$

(vi) $\delta(p, 1, X) = \{(p, XX)\}$

(vii) $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$

Draw the transition diagram for this PDA.

CSE 211

Contd... Q. No. 5

(b) For the PDA described in Ques. 5(a), starting from the initial ID (q, w, Z_0), show all the reachable ID's when the input w is: **(15)**

(i) 0011

(ii) 0101

(iii) 1010

6. (a) Define a PDA that accepts the language $\{w, \in \{a, b\}^* : w$ has the same number of a's and b's$\}$. **(13)**

(b) Consider a PDA P_F that accepts a language L by final state. Is it always possible to construct another PDA P_N that accepts the same language L by empty stack? If possible, show how the construction is done. Provide definition of both P_F and P_N. If its not possible, justify why. **($10\frac{1}{3}$)**

7. (a) Design a Turing machine for the language $\{a^n b^n c^n \mid n \geq 1\}$. Also briefly describe its working principle. **(13)**

(b) Suppose, you know a particular problem P is undecidable. You want to prove another problem Q is undecidable too. Utilize P and the concept of reduction to prove Q is undecidable. **($10\frac{1}{3}$)**

8. (a) Design a Turing machine that computes the function $a \dot{-} b = max(b - a, 0)$. Initially the tape contains a, followed by b. Both a and b are represented by consecutive 1's separated by the symbol 0. After computation, the tape will contain only the output $a \dot{-} b$, represented by consecutive 1's and surrounded by infinite blanks. Provide brief description of the working principle of your designed Turing machine. **(13)**

(b) Describe how multiple tracks and multiple tapes extend the basic Turing machine. Briefly discuss the difference between these extensions. **($10\frac{1}{3}$)**

L-2/T-2/CSE **Date : 22/02/2018**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2016-2017

Sub : **CSE 211** (Theory of Computation)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. In a faraway strange planet Xustaeter, three colors of stones, red, green, and blue. When two different colored stones get in contact with each other:

   (i) these two stones are destroyed,

   (ii) two more stones of the third color are created.

   The planet fails, if, at some point, all stones are of the same color. Then, no more stone creation can take place. We want to represent the planet using a state diagram, where a state label will be a sequence of three integers – the numbers of stones of color red, green and blue. In the state diagram, an *R*-event occurs when stones of color green and blue touch and are replaced by two red stones. Analogously we define, *G*-events and *B*-events.

   (a) Represent Xustaeter when there are 3 stones in total. Identify the states in/from which the planet must fail. Similarly, identify the can't-fail and might-fail states. **(5)**

   (b) Represent Xustaeter using state diagrams when there are 4 stones in total. Again, identify the must-fail, can't-fail and might-fail states. **(10)**

   (c) From the diagrams in (b), find the symmetries in the state diagrams which will enable us to reduce the diagram sizes. Propose a reduced state diagram schemes based on this. **(6)**

   (d) Using the scheme proposed in (c), draw the state diagram for Xustaeter when there are 6 stones in total. Identify the must-fail, can't-fail and might-fail states. **(14)**

2. (a) DFA, D_1 has a rather strange alphabet $\Sigma = \{a, b, 1, 2\}$, containing both letters and digits. D_1 accepts all and only the strings which has number of a's divisible by 3 and has an even sum of the digits. As such, strings like ϵ, $aab12a1$, $aab1bbaaba2a1$, $aaba$ and 121 will be accepted. And strings like, $aaab12a1$, $aab11bbaaba2a1$, $aaaba$ and 1121 will be rejected, Draw the state diagram for D_1. **(17)**

Contd P/2

CSE 211

Contd ... Q. No. 2

(b) Convert the NFA shown in the following figure to an equivalent DFA. Showing the detailed computations is optional. **(8)**

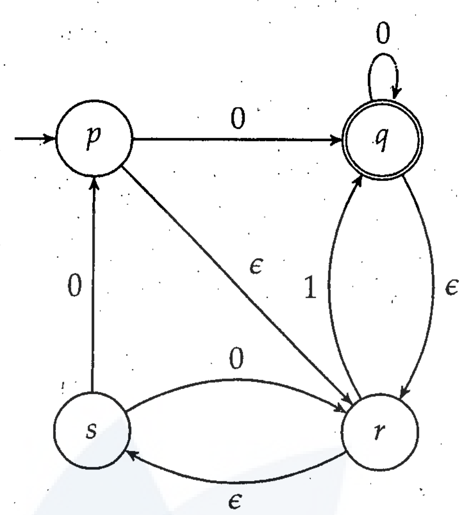


Figure for Question 2 (b).

(c) Why is the Kleene star of a null set an empty string? Explain clearly. **(10)**

3. (a) Draw state diagrams of NFAs to accept the following languages. **(5+8+7)**

(i) The set of strings over alphabet {0, 1, ..., 9} such that the final digit has appeared before.

(ii) The set of strings over alphabet {0, 1, ..., 9} such that final digit has *not* appeared before.

(iii) The set of strings of 0's and 1's such that there are at least two 0's separated by a number of positions that is a multiple of 4. In addition to the aforementioned pair of 0's, there may be other 0's in the string. Note that 0 is an allowable multiple of 4. Some of the strings accepted by this NFA are, 00, 10111010, 01010111011.

(b) Write down regular expressions for the following: **(4+4+7)**

(i) The set of strings that consist of alternating 0's and 1's.

(ii) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

(iii) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

Contd P/3

CSE 211

4. (a) We have the NFA N_1 accepting the language A. When constructing an NFA N_1^* from N_1 to accept A^*, we add an accept A, we add an additional state and make this state the new start state as well as an accepting state. You should be aware of the other edges we add to N_1*. **(10)**

   Hastif Hasty came up with the (apparently) brilliant idea of not adding the new state and making the original start state an accepting state, leaving the newly added edges unchanged when applicable. However, eventually, a flaw was detected in this proposed deign by Nutty Nitpicker. Explain clearly under which situations the idea proposed by Hasty will not work.

   (b) Prove that, "The class of regular languages, is closed under the concatenation operation". **(13)**

   (c) Using pumping Lemma for regular languages, show that the language consisting of all strings of 1's whose length is a prime is not a regular language. **(12)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0,1\}$. You only need to write the production rules using conventional notation. **(5×3=15)**

   (i) {w| the length of w is off}

   (ii) {w| w contains at least three *1s* }

   (iii) {w| the length of w is odd and its middle symbols is a 0}

   (b) Design a Turing Machine (TM) that takes as input a number N in binary and adds 1 to it. Initially the tape will contain (in its leftmost portion) a # followed by the number N in binary (e.g., #101), and the tape head will be on #. Your TM should halt after converting the N into N+1. You may replace the # with 0 or 1 if you need. Also briefly explain the working principle of the TM (For example, by drawing a flow chart or a pseudo code) **(20)**

6. (a) Design a PDA to accept the language $L = (0^n1^m | n \leq m \leq 2n)$. Provide the transition diagram of your designed PDA. No need to write the transition function. Also, show that your PDA accepts 00111 and that it rejects 0011111. **(10+5=15)**

Contd P/4

CSE 211

Contd ... Q. No 6

(b) Convert the following CFG into an equivalent CFG in Chomsky normal form. Show the steps of your conversion. **(10)**

$A \rightarrow BAB \mid B \mid \varepsilon$

$B \rightarrow 00 \mid \varepsilon$

(c) Define a Turing Machine for the set of strings with an equal number of 0's and 1's. **(10)**

7. (a) Let A be the language of strings representing undirected connected graphs, i.e., $A = \{\langle G \rangle \mid G$ is a connected undirected graph$\}$. Is it possible to design a Turing Machine that can decide A? If no, why? If yes, briefly discuss your design and demonstrate with a suitable example. **(15)**

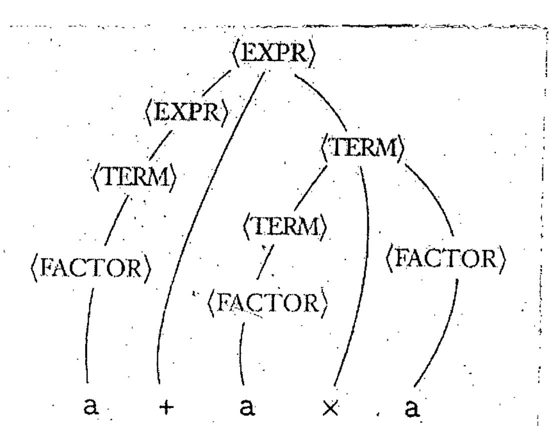
(b) Give a PDA that recognizes the language $\{WW^R \mid W \in \{0, 1\}^*\}$. Here, W^R means W written backwards. **(10)**

(c) Use the pumping lemma to show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context free. **(10)**

8. (a) Prove that, the following language is undecidable $A_{TM} = \{\langle M, w \rangle \mid M$ is a Turing Machine accepting $w\}$ **(15)**

(b) Design a context-free grammar *(CFG)* for $A = \{w \in \{0,1\}^* \mid w = w^R$ and the length of w is divisible by 4$\}$. You only need to write the production rules using conventional notation. **(10)**

(c) Give a context-free grammar, that generates the following parse tree. You only need to write the production rules using conventional notation. **(10)**



L-2/T-2/CSE **Date : 07/04/2019**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2017-2018

Sub : **CSE 211** (Theory of Computation)

Full Marks : 210 Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Define the Boolean Satisfiability Problem with necessary examples. **(6)**

(b) With the knowledge that 3-SAT problem is NP, prove that graph 3-colorability is NP-Complete. **(20)**

(c) Using the graph framework in Question (b), show how the final 3-color graph for a 3-SAT problem of,

$$\left(x_1 \vee \overline{x_2} \vee \overline{x_3}\right) \wedge \left(x_1 \vee x_3 \vee x_4\right), \text{will look like.}$$ **(9)**

2. (a) Let us say that we have a DFA D for a certain language L for an alphabet Σ. How can you readily find the DFA from D for the language, $\Sigma^* - L$? Explain why your proposed method works. Is there any scenario for which this will not work?

Does your proposed method work for NFA's as well? Explain. **(10)**

(b) Give DFA's (draw state diagrams only) accepting the following languages over the alphabet $\{0, 1\}$: **(10+7)**

(i) The set of all strings beginning with a 1 that, when interpreted as a binary integer, is a multiple of 5. For example, strings 101, 1010, and 1111 are in the language; 0, 100, 0101, and 111 are not.

(ii) The set of all strings beginning with a 1 that, when interpreted as a binary integer, is a power of 2.

(c) Using the techniques of proving closure properties of regular languages, draw the NFA for the regular expression, $(a \cup b)^* aba$. **(8)**

3. (a) Design an NFA which accepts all and only the set of strings over the alphabet $\{0, 1\}$ such that there are at least two 0's with nonzero number of characters between these two 0's and that (number of characters) is a multiple of 4. In addition to these pairs of 0's, there may be other 0's in the string. Some of the strings accepted by this NFA are, 00000, 101111010, 01000010111011. And some of the strings *not* accepted by this NFA are, 00, 10101010.

Show the NFA using state diagram as well as transition table. **(18)**

(b) After learning the pumping lemma (difficult to understand, easy to go astray when applying it), Surcharged Skye got very excited and came up with very personal opinions regarding a number of languages as follows: **(17)**

Contd P/2

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CSE 211

Contd ... Q. No. 3(b)

(i) For $L_1 = \{ww | w \in \{0, 1\}^* \}$, we choose the string $0^p 0^p$, take $x = \epsilon$, $y = (00)^k$, $k \leq \frac{p}{2}$.

Now, since $\forall i \geq 0, xy^i z \in L_1$, L_1 is regular.

(ii) For $\Sigma = \{0, 1\}$, $L_2 = \{w \in \Sigma^* | n_0(w) = 2n_1(w)\}$ ($n_a(w)$ denote the number of a's in w), we choose the string $0^{2p}1^p$, take $y = 01$. Since $\forall i \geq 0, xy^i z \in L_2$, we have failed to demonstrate that L_2 contradicts pumping lemma.

(iii) For $L_3 = \{0^n 1^n 0^n \mid n \geq 0 \}$, we chose the string $0^k 1^k 0^k$, where $k = \left\lfloor \frac{p}{3} \right\rfloor$. Now, since $\forall i \geq 0, xy^i z \notin L_3$, for all possible y's, L_3 is not regular.

Explain clearly whether you find anything wrong in each of the opinion expressed by Skye. Symbols used have got their usual meanings used in class.

4. (a) Consider the NFA given below: **(15)**

	a	b	c	ϵ
$\rightarrow p$	$\{p\}$	$\{q\}$	$\{r\}$	$\emptyset$
q	$\{q\}$	$\{r\}$	$\emptyset$	$\{p\}$
$*r$	$\{r\}$	$\emptyset$	$\{p\}$	$\{q\}$

(i) Give, with necessary explanations, general forms of all the strings of length three or less accepted by the automaton.

(ii) Convert the automaton to an equivalent DFA. Showing the detailed computations is optional.

(b) Find regular expressions for each of the following languages: **(20)**

i. $\Sigma = \{a, b, c\}$, $L = \{w \in \Sigma^* \mid w$ contains at least one occurrence of each symbol in $\Sigma\}$,

ii. $\Sigma = \{a, b\}$, $L = \{a^n b^m \mid n \geq 3, m$ is odd$\}$,

iii. $\Sigma = \{a, b\}$, $L = \{a^n b^m \mid (n + m)$ is odd$\}$,

iv. $\Sigma = \{a, b\}$, $L = \{vwv \mid v, w \in \Sigma^*, |v| = 2\}$,

v. $\Sigma = \{a, b\}$, $L = \{w \mid w \in \Sigma^*, n_a(w) \mod 3 = 0\}$, where $n_a(w)$ denotes the number of a's in w.

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SECTION – B

There are **NINE** questions in this Section. Answer any **SEVEN**.

5. Provide context-free grammars that generate the following languages (the alphabet Σ is { 0, 1 }): **(7+8)**

   (a) The set of all strings with equal number of 0s and 1s

   (b) {w|w contains at least three 1s}

6. (a) Show that the following grammar is ambiguous by giving two different parse trees for some string. **(7+8)**

   $$E \rightarrow E+E$$
   $$E \rightarrow E \times E$$
   $$E \rightarrow (E)$$
   $$E \rightarrow a$$

   (b) Remove ambiguity from the grammar given above.

7. (a) Convert the following context-free grammar into an equivalent grammar in *Chomsky normal form*. Show the steps of your conversion. Here, 0 and 1 are the terminals. **(8+7)**

   $$S \rightarrow A1A|B$$
   $$A \rightarrow 0A|1A|\varepsilon$$
   $$D \rightarrow 00$$

   (b) Determine whether 01101 is in the language of the grammar thus found using the CYK algorithm.

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8. (a) Design a PDA that recognizes the language **(15)**

   $$L = \{a^n b^n c^m d^m | n \geq 1,\ m \geq 1\}$$

   Draw its transition diagram.

9. Design a Turing machine (TM) that takes as input two numbers N_1 and N_2 of equal length in binary and computes the logical OR of the two numbers. The tape initially contains $N_1\#N_2\#$, where '#' is a tape symbol that is used as the separator. Your TM should terminate with the OR of the two numbers in binary after the second #. (You can use multiple tracks, storage in the state if you wish). **(15)**

10. Briefly explain whether the following statements are true or false: **(15)**

    (i) A one-tape Turing machine with multiple tracks can simulate a Multi-tape Turing machine.

Contd P/4

CSE 211

Contd ... Q. No. 10

(ii) A deterministic Turing machine can simulate n moves of a non-deterministic Turing machine in $P(n)$ moves, where $P(n)$ is some polynomial in n.

(iii) A deterministic Turing machine can simulate n moves of a conventional computer in $P(n)$ moves, where $P(n)$ is some polynomial in n.

11. Define the following: **(9+6)**

(i) Diagonalization language

(ii) Universal language

(iii) Halting problem

Show that one of the above is *not recursively enumerable* (*RE*).

12. The *subgraph isomorphism problem* is, given graphs G_1 and G_2, does G_1 contain a copy of G_2 as a subgraph? That is, can we find a subset of the nodes of G_1 that, together with the edges among them in G_1 forms an exact copy of G_2, when we choose the one to one correspondence between nodes of G_2 and nodes of the subgraph of G_1 properly? Prove that the *subgraph isomorphism problem* is NP-complete. **(15)**

13. (a) Define the following classes of langulages: **(6+9)**

(i) P

(ii) NP

(iii) PSPACE

(iv) NPSPACE

(b) For each of the following pairs of classes of languages, what is the widely believed relationship between the classes (whether they are equal or which one is a subset of the other)? Briefly justify your answer.

(i) P and NP

(ii) PSPACE and NPSPACE

(iii) NP and PSPACE

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L-2/T-2/CSE Date: 16/01/2021

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations (January 2020 Term)

Sub: **CSE 211** (Theory of Computation)

Full Marks: 180, Section Marks: 90, Time: 2 Hours (Sections A + B)

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Given an instance of the Dominating Set problem, desctibe how the certificate can determine whether the Dominating Set problem is in NP. (6)

   (b) After learning the pumping lemma (difficult to understand, easy to go astray when applying it), Surcharged Skye got very excited and came up with his very personal opinions regarding a number of languages as follows: (12)

   i. For $\Sigma = \{0,1\}$, $L_1 = \{ww \mid w \in \{0,1\}^*\}$, we choose the string 0^p0^p, take $x = \epsilon$, $y = (00)^k$, $k \leq \frac{p}{2}$. Now, since $\forall i \geq 0$, $xy^iz \in L_1$, L_1 is regular.

   ii. For $\Sigma = \{0,1\}$, $L_2 = \{w \in \Sigma^* \mid n_0(w) = 3n_1(w)\}$ ($n_a(w)$ denotes the number of a's in w), we choose the string $0^{3p}1^p$, take $y = 0001$. Since $\forall i \geq 0$, $xy^iz \in L_2$, we have failed to demonstrate that L_2 contradicts pumping lemma.

   Explain clearly whether you find anything wrong in each of the opinions expressed by Skye.

   (c) Consider the NFA given below: (12)

	a	b	c	ϵ
$\rightarrow p$	$\{p\}$	$\{q\}$	$\{r\}$	$\emptyset$
q	$\{q\}$	$\{r\}$	$\emptyset$	$\{p\}$
$*r$	$\{r\}$	$\emptyset$	$\{p\}$	$\{q\}$

   Give, with necessary explanations, general forms of all the strings of length three or less accepted by the automaton.

2. (a) State informally, the language accepted by the DFA shown in the figure. (7)

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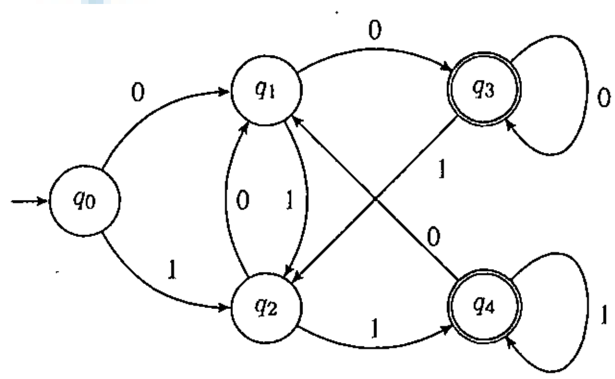

Figure for Question 2(a).

   (b) For the DFA in Question 2(a), draw the equivalent NFA for the corresponding language. (13)

   (c) For the DFA in Question 2(a), write down the regular expression for the strings that *do not* belong to the corresponding language. (10)

The first gulp from the glass of natural sciences will turn you into an atheist, but at the bottom of the glass God is waiting for you. – Heisenberg

CSE 211

3\. (a) Evaluate the following regular expressions. Show the detailed computations. (10)

i. $\emptyset^*\emptyset^*\emptyset^*\emptyset^*$

ii. $\emptyset^* \cup \{\epsilon\}$

iii. $(\emptyset^* \cup 1)\{0, 1\}$

iv. $(11)^*\emptyset(00)^*$

v. $(\emptyset^* \cup 0)\{0, 1, 11\}$

(b) We have got an NFA $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ which recognizes the language L_1. We want to construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize L_1^*. (20)

In this process, we define δ for N such that for any $q \in Q$ and any $a \in \Sigma_\epsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \epsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \epsilon \\ \{q_1\} & q = q_0 \text{ and } a = \epsilon \\ \emptyset & q = q_0 \text{ and } a \neq \epsilon \end{cases}$$

Explain clearly in plain English (stressing on the purpose) each of the conditions (at right) in the above transition function.

4\. (a) Give a description in English of the language of the regular expression: $(1 + \epsilon)(00^*1)^*0^*$. (10)

(b) Convert the NFA shown in the figure to an equivalent DFA. Show the computations. (20)

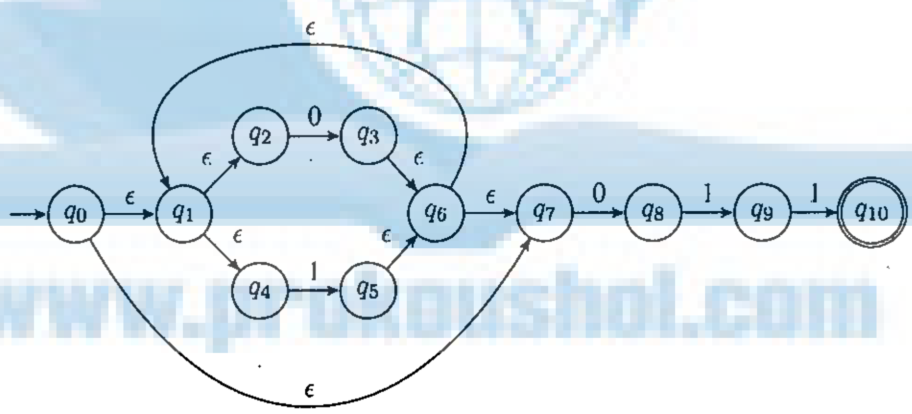

Figure for Question 4(b).

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— THE END —

L-2/T-2/CSE Date: 16/01/2021

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations (January 2020 Term)

Sub: **CSE 211** (Theory of Computation)

Full Marks: 180 Section Marks: 90 Time: 2 Hours (Sections A + B)

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – B

There are **EIGHT** questions in this section. Answer any **SIX**.

5. Give context-free grammars that generate the following languages for an alphabet, $\Sigma = \{0,1\}$: 7+8

   a) $\{w \mid w$ contains at least one 0 and at least one 1$\}$,

   b) The set of all strings with equal number of 0s and 1s.

6. Design a context-free grammar for an alphabet, $\Sigma = \{a,b,c,d\}$ that generates the language, 10+5

   $$L = \{a^n b^n c^m d^m \mid n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$$

   Show that the grammar you designed is ambiguous by giving two different parse trees for some string in the language.

7. The following are the productions of a grammar G in Chomsky Normal Form (S is the start symbol). 15

   $S \rightarrow BZ \mid CY \mid ZZ \mid YY$

   $B \rightarrow ZA$

   $C \rightarrow YA$

   $A \rightarrow ZA \mid YA \mid 0 \mid 1$

   $Z \rightarrow 0$

   $Y \rightarrow 1$.

   Determine whether 10011 is in $L(G)$ using the CYK algorithm (you need to show the table).

8. Design a pushdown automaton (PDA) that recognizes the language $\{a^i b^j c^k \mid i, j, k \geq 0$ and $(i=j$ or $i=k)\}$. Show its transition diagram. Recall that, PDAs are non-deterministic. 15

9. Design a Turing machine (TM) that takes as input two numbers, w_1 and w_2 in binary of equal lengths and computes the logical XOR of the two numbers. The tape initially contains $w_1 c w_2 c$ where 'c' is a tape symbol that is used as the separator. Your TM should terminate with the XOR of the two numbers in binary after the second c. (You can use multiple tracks and storage in the state if you wish). 15

10. Briefly explain whether the following statements are true or false: 7+8

    i) A one-tape Turing machine with multiple tracks and storage in the state can simulate a multi-tape Turing machine.

    ii) A deterministic Turing machine can simulate n steps of a conventional computer in $P(n)$ steps, where $P(n)$ is some polynomial in n.

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L-2/T-2/CSE Date: 16/01/2021

11. a) Define recursively enumerable (RE) languages and recursive languages. 7+8

b) Give one example of the following (with definitions):

i) A language that is not recursively enumerable (not RE),

ii) A language that is RE but not recursive.

12. a) State Cook's theorem. Explain why finding a polynomial time algorithm for an NP-complete problem implies P=NP. 7+8

b) Draw the Venn diagram showing the widely believed relationships (whether they are equal or which one is a subset of the other) among the classes of problems P, NP, PSPACE, NPSPACE and EXPTIME. Briefly justify your answer.

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Have those who disbelieved not considered that the heavens and the earth were a joined entity, and We separated them and made from water every living thing? Then will they not believe? [Al Quran 21:30]

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2019-2020

Sub: **CSE 211** (Theory of Computation)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE** questions.

1. (a) Using pumping lemma for regular languages we want to show that the language, $L = \{a^n \mid n \text{ is a prime number}\}$ is not regular. We start our argument with, "Given p, let $w = a^p \ldots$", where p is the pumping length. Comment on the acceptability of this. **(5)**

(b) Generally, an easy computational problem is preferable to a hard one. Describe one scenario which specifically requires computational problems that are hard, rather than the easy ones. **(15)**

(c) Design a deterministic finite automaton (DFA) which has an alphabet, $\sum\{0,1,2,R\}$. This automaton keeps a running count of the sum of the numerical input symbols it reads. However, every time it receives the R symbol, it resets the sum to 0. It accepts a string if the sum is 1 modulo 5 or 2 modulo 5. Showing only the state diagram should suffice. **(15)**

2. (a) What is the role of a state in a finite automaton? Explain briefly. **(5)**

(b) Rewrite, with necessary explanations, each of the following regular expressions as a simpler expression representing the same set. **(15)**

(i) $\emptyset^* \cup a^* \cup b^* \cup (a \cup b)^*$

(ii) $((a^*b^*)^*(b^*a^*)^*)^*$

(iii) $(a^*b)^* \cup (b^*a)^*$

(c) Design a deterministic finite automaton (DFA) which accepts the base-3 (symbols 0, 1, and 2) numbers which are not divisible by 4. Showing only the state diagram should suffice. **(15)**

3. (a) Construct the non-deterministic finite automaton NFA for the regular expression $((a^*)a)^*$ using the basic principles. **(5)**

(b) Design an NFA for the language, **(15)**

$L = \{a^n b^n \mid w \text{ has an equal number of } a\text{'s and } b\text{'s}, \ n \le 3\}$, with an alphabet, $\sum = (a,b)$. Showing only the state diagram will suffice. You may be penalized for using more states than the required minimum.

Contd P/2

(c) The NFAs, $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize the languages L_1 and L_2. You want to construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $L_1 \cup L_2$. Write down the expression for δ and explain in detail each part of the expression with necessary justifications. **(15)**

4. (a) When an NFA with five states is converted to an equivalent DFA, what is the minimum and maximum number of possible states in the DFA? Answer with necessary justifications. **(5)**

(b) For $\Sigma = \{a, b\}$. Write regular expressions for the following sets. **(15)**

(i) All strings in Σ^* whose number of a' s is divisible by three.

(ii) All strings in Σ^* with no more than three a' s.

(iii) All strings in Σ^* with exactly one occurrence of the substring *aaa*.

(c) Using pumping lemma for regular languages, prove that the language, $L = \{a^{2n}b^{3n}a^n \mid n \geq 0\}$ is not regular. **(15)**

SECTION – B

There are **NINE** questions in this section. Answer any **SEVEN** questions.

5. Give context-free grammars that generate the following languages: **(7+8=15)**

(i) The set of all strings of balanced parentheses over the alphabet $\{(,)\}$. For example, (()()), ((())), ε are in the language whereas ((()), ())),)()(are not in the language.

(ii) The set of all strings with equal number of 0s and 1s.

6. Define *ambiguous grammar* and *inherently ambiguous language*. Show that the following grammar is ambiguous: **(5+10=15)**

$$S \to AB \mid C$$
$$A \to aAb \mid ab$$
$$B \to cB \mid c$$
$$C \to aCc \mid aDc$$
$$D \to bD \mid b$$

CSE 211

7. The following are the productions of a grammar G in Chomsky Normal Form (S is the start symbol): **(10+5=15)**

$$S \rightarrow BZ \mid CY \mid ZZ \mid YY$$
$$B \rightarrow ZA$$
$$C \rightarrow YA$$
$$A \rightarrow ZA \mid YA \mid 0 \mid 1$$
$$Z \rightarrow 0$$
$$Y \rightarrow 1$$

Determine whether 01100 is in $L(G)$ using the CYK algorithm (you need to show the table). Describe the language generated by the above grammar informally.

8. Let $\Sigma = \{a, +, *, (,)\}$ and consider the language $ARITH = \{w \in \Sigma^* \mid w \text{ is a legal arithmetic expression}\}$. Design a pushdown automaton (PDA) that recognizes the language $ARITH$. Show its transition diagram. **(15)**

For example: $a + a^* a$ and $((a+a)^*(a+a)) + (a)$ are legal expressions, whereas aa^*a (two consecutive variables without an operator), $a++a$ (two consecutive operators), $+a$ (starts with an operator) and $(a+a))$ (imbalanced parentheses) are not.

9. Design a Turing machine (TM) that takes as input two numbers of equal lengths in binary w_1 and w_2, and computes the logical AND of the two numbers. The tape initially contains $w_1 c w_2 c$ where 'c' is a tape symbol that is used as the separator. Your TM should terminate with the AND of the two numbers in binary after the second c. (You can use multiple tracks and storage in states if you wish.) **(15)**

10. (a) Show that the set of languages that are accepted by final state by some pushdown automaton (PDA) is same as the set of languages that are accepted by empty stack by some PDA. **(8)**

(b) Explain how a one-tape Turing machine with multiple tracks and storage in the state can simulate a multi-tape Turing machine. **(7)**

11. Give one example from each of the following sets of languages: **(15)**

(a) Regular languages

(b) Languages that are context-free but not regular

(c) Languages that are recursive but not context-free

(d) Languages that are recursively enumerable but not recursive

(e) Languages that are not recursively enumerable

Contd P/4

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12. What is the *halting problem*? Prove that the halting problem is recursively enumerable (RE) but not recursive. **(4+11=15)**

13. Briefly explain whether the following are true or false: **(15)**

(a) If the *satisfiability* problem (SAT) can be solved in polynomial time, then P=NP.

(b) The set of languages decided by polynomial time non-deterministic Turing machines is a superset of the set of languages decided by polynomial space deterministic Turing machines.

(c) The set of languages decided by polynomial space deterministic Turing machines is same as the set of languages decided by polynomial space non-deterministic Turing machines.

L-2/T-2/CSE **Date: 3/10/2023**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2021-2022

Sub: **CSE 211** (Theory of Computation)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks

SECTION – A

There are **FOUR** questions in this section.
Answer Question No. 1 and any **TWO** from the rest. Questions No. 1 is **COMPULSORY.**

1. **[COMPULSORY]**

(a) In the password-protected login system of a computer, which of the three traditionally central areas of the Theory of Computation has been applied? Explain clearly. **(4)**

(b) Design a deterministic finite automaton (DFA) with not more than five states for the alphabet $\Sigma = \{a, b\}$, which accepts the strings that begin with a but does not contain the substring aab. Showing only the state diagram for this DFA should suffice. **(10)**

(c) If M is a deterministic finite automaton (DFA) that recognizes the language C, swapping the accepting and non-accepting states in M yields a DFA that recognizes $\overline{C}$. Explain why this technique works. Does this method work for non-deterministic finite automatons (NFA) as well? Why? Give examples. **(6+3=9)**

(d) Comment on the acceptability of the following arguments by B L Perfunctory (BLP) related to the pumping lemma for regular languages: **(9)**

(i) Consider the set of palindromes over $\{0, 1\}$. BLP supposed that the set of palindromes is regular. Let p be the pumping length. Consider the string $w = 00011000$. Clearly w is a palindrome. By the pumping lemma, there must exist strings x, y, and z satisfying the constraints of the pumping lemma. xy is entirely contained in the 0^p at the start of w. So x and y consist entirely of zeros.

Now, consider xz. By the pumping lemma, xz must be in the language. But xz can't be a palindrome. This means that the set of palindromes doesn't satisfy the pumping lemma and, thus, the set of palindromes cannot be regular.

(ii) Consider the language $\{w|w$ has an equal number of 0s and 1s$\}$. BLP supposed that the language is regular. If p is the pumping length, let w be the string $0^{\frac{p}{4}}1^{\frac{p}{4}}0^{\frac{p}{4}}1^{\frac{p}{4}}$, $w = xyz$, where for any $i \geq 0$ the string xy^iz is in the language. y will be entirely contained in the first 0s. For any length of y in the permissible range, xy^iz will have an unequal number of 0s and 1s and hence not in the given language. Thus BLP concluded that the given language cannot be regular.

Contd P/2

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Contd... Q. No. 1(d)

(iii) Consider the language $\{0^{2n} | n \geq 0\}$. BLP supposed that the language is regular. If p is the pumping length, let w be the string $w = 0^{2p}$, $w = xyz$, where for any $i \geq 0$, the string xy^iz is in the language. If $x = \epsilon$, $y = 0$, $z = 0^{2p-1}$, taking $i = 0$, we get $xy^iz = 0^{2p-1}$, which isn't in the language because its length is odd. So, the language isn't regular after all.

(e) Why are context free grammars called "context free"? **(3)**

2. (a) "Deterministic and nondeterministic finite automata recognize the same class of languages. Such equivalence is both surprising and useful". Why is this equivalence both surprising and useful? **(4)**

(b) Using the basic principles convert the regular expression $(((00)^*(11)) \cup 01)^*$ into a nondeterministic finite automata (NFA). **(10)**

(c) Consider the languages given below for an alphabet $\{a, b\}$, **(6)**

$L_1 = \{ a, aa, aaa \}$,

$L_2 = \{ a, ab, aab, aabb \}$,

$L_3 = \{$ Number of b's is a multiple of 3 $\}$

Now, comment on the size (finite/infinite) for each of the following languages. You must state the reason in each case.

i. $L_1^* \circ L_2^*$

ii. $L_1 \circ (L_1 \cup L_2)$

iii. $L_2 \circ L_3$

(d) Design a context free grammar for the language **(15)**

$\{a^ib^jc^k \mid i \neq j + k$, where $i, j, k \geq 0\}$. Merely putting forth the production rules shall not suffice. You need to explain your design idea first for your answer to be acceptable.

3. (a) Why is the Kleene star of a null set an empty sting? **(4)**

(b) Design a nondeterministic finite automaton (NFA) which accepts binary numbers which contains at least two 0s, or exactly two 1s. Showing only the state diagram for this NFA should suffice. **(10)**

(c) Convert the following non-deterministic finite automaton (NFA) into an equivalent deterministic finite automaton (DFA). You don't need to show the calculations. Showing only the state diagram for the DFA should be sufficient. **(9)**

Contd P/3

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Contd... Q. No. 3(c)

	a	b	ε
q_1	$\{q_3\}$	ϕ	$\{q_2\}$
q_2	$\{q_1\}$	ϕ	ϕ
q_3	$\{q_2\}$	$\{q_2, q_3\}$	ϕ

(d) Which one of the following regular expressions represents the language: the set of *all* binary strings having two consecutive 0s as well as two consecutive 1s? You need to provide the necessary explanation regarding why your choice is correct while the others are not. Your answer will not be accepted without proper explanations. **(12)**

A. $(0 + 1)^*0011(0 + 1)^* + (0 + 1)^*1100(0 + 1)^*$

B. $(0 + 1)^*(00(0 + 1)^*11 + 11(0 + 1)^*00)(0 + 1)^*$

C. $(0 + 1)^*00(0 + 1)^* + (0 + 1)^*11(0 + 1)^*$

D. $00(0 + 1)^*11 + 11(0 + 1)^*00$

4. (a) Why is it said that the pumping lemma for regular languages is useful for "negative proofs" only? **(4)**

(b) Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognizes L_1, and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognizes L_2. We want to construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $L_1 \cup L_2$. Now answer clearly the following with detailed explanations (must include the reason behind an action): **(12)**

i. How do we get the start state for N?

ii. How do we get the set of accept states for N?

iii. When we are considering the transitions originating from the start state of N, what are the cases that need to be considered? Why? What are the possible destination states in each case?

(c) To design a context free grammar (CFG) for the language, **(3+6=9)**

$\{a^i b^j c^k \mid i = j, \text{ or}, j = k, \text{ where } i, j, k \geq 0\}$,

A B Sloppy produced the following solutions:

i. $S \rightarrow aSb \mid bSc \mid aS \mid Sc \mid \varepsilon$

ii.

$$S_0 \rightarrow S_1 \mid S_3$$
$$S_1 \rightarrow aS_1bS_2c \mid \varepsilon$$
$$S_2 \rightarrow S_2c \mid \varepsilon$$
$$S_3 \rightarrow aS_4bS_3c \mid \varepsilon$$
$$S_4 \rightarrow aS_4 \mid \varepsilon$$

Do you find any flaw in the above solutions? Explain clearly.

Contd P/4

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Contd... Q. No. 4

(d) We have a certain grammar for an infinite language. For a string s in that language, comment with reasons on the ambiguity of s and the grammar for each of the following scenarios. **(10)**

i. The grammar generates only one parse tree for s but generates more than one parse trees for other strings in the language.

ii. The grammar generates only one parse tree for s as well for other strings in the language.

iii. The grammar generates more than one leftmost derivations for s.

SECTION – B

There are **NINE** questions in this section. Answer any **SEVEN**.

5. Convert the following context-free grammar into an equivalent grammar in *Chomsky normal form (CNF)*. Show the steps of your conversion. **(15)**

$$S \to A1A \mid B$$
$$A \to 0A \mid 1A \mid \varepsilon$$
$$B \to 1$$

6. What is an ambiguous grammar and an inherently ambiguous language? Determine whether the following grammar is ambiguous or not: **(6+9=15)**

$$S \to AB \mid C$$
$$A \to aAb \mid ab$$
$$B \to cB \mid c$$
$$C \to aCc \mid aDc$$
$$D \to bD \mid b$$

7. Let $\Sigma = \{a, +, *, (,)\}$ and consider the language ARITH = $\{w \in \Sigma^* \mid w$ is a legal arithmetic expression$\}$. Construct a pushdown automaton (PDA) that recognizes the language ARITH. Show its transition diagram. For example: $a+a*a$ and $((a+a)*(a+a))+(a)$ are legal expressions whereas $aa*a$ (two consecutive variables without an operator), $a++a$ (two consecutive operators), $+a$ (starts with an operator) and $(a+a))$ (imbalanced parentheses) are not. **(15)**

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CSE 211

8. Provide an example of a language that is not *context-free*. Illustrate that the language you provided is not context-free using the pumping lemma for context-free languages. **(5+10=15)**

9. Construct a Turing machine that takes as input a number w in *signed magnitude* form and computes the 2's *complement* of the number. Initially the tape contains a number in binary, MSB being the leftmost bit. If the MSB is 0, the number will remain the same. However, if the MSB is 1, w needs to be converted to its two's complement. Recall that you can convert a number to its two's complement by working from the least significant bit (LSB) towards the most significant bit (MSB), copying all the zeros until the first 1 is reached; then copy that 1, and flipping all the remaining bits (leave the MSB as 1). Your TM should terminate with only the two's complement of the number on the tape. (You can use multiple tracks, storages in the state if you wish). **(15)**

10. Justify whether the following statements are true or false: **(7+8=15)**

    i) A one-tape Turing machine with multiple tracks can simulate a Multi-tape Turing machine.

    ii) A conventional computer can simulate n moves of a non-deterministic Turing machine in $P(n)$ moves, where $P(n)$ is some polynomial in n.

11. What is (i) the Diagonalization language, and (ii) the Universal language? Show that the Halting problem is *not recursive*. **(6+9=15)**

12. i) Briefly explain how Cook's theorem shows that all problems in NP are reducible to the *satisfiability* problem in polynomial time. **(8+7=15)**

    ii) Explain how finding a polynomial time algorithm for any NP-complete problem implies P = NP.

13. For each of the following pairs of classes of languages, what is the widely believed relationship between the classes (whether they are equal or which one is a subset of the other)? Briefly justify your answer. **(15)**

    i) P and NP

    ii) P and PSPACE

    iii) PSPACE and NPSPACE

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