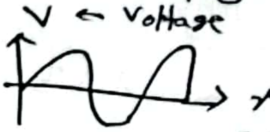
05/09/2024

Class-1 & 2

Signals → Physical Manifestation वाला ।

$f(x) = x^2 + x$



* Time most of the time independent variable.

(i) Continuous - time

(ii) Discrete

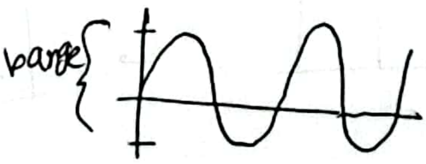
Uniformly Sampling → Collecting Data in uniform time interval

Non Uniformly Sampling

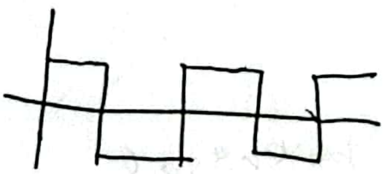
Sampling → Collecting Data

Amplitude:

(i) Continuous Amplitude

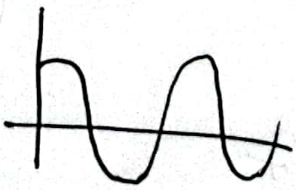


(ii) Discrete Amplitude



Continuous Signal

$x(t) = 2\cos t$



Discrete Signal → $f[n]$ → x points will be ~~discrete~~ integer ~~poin~~ values (uniformly spaced)

Continuous " → $f(t)$

$$x[n] = x(nT)$$

Amplitude Scaling

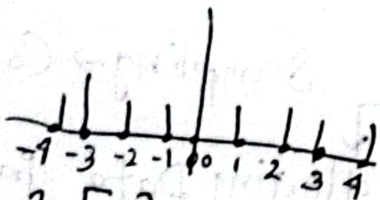
$a\,x(t) \quad x(t) * a$

$x[n], \qquad 2x[n]$

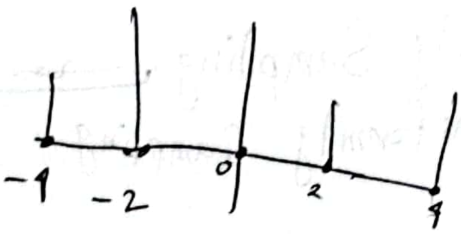
$x(bt) \quad b=1$

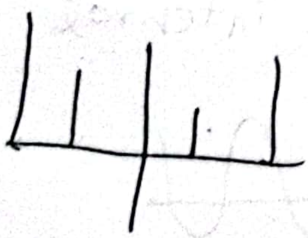
$b=2$

$b=\frac{1}{2}$



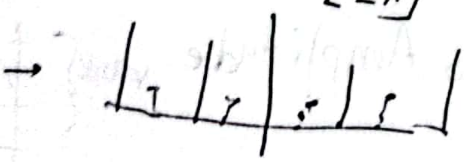
$x[n]$





$x[n]$

$x[2n]$

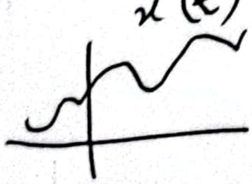


$x[n/2]$

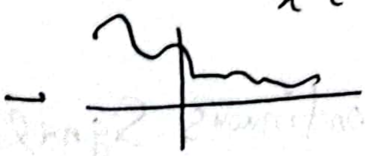
Time reversal

$x(-t)$

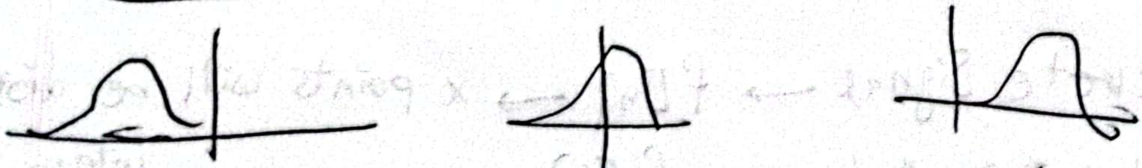
$x(t)$



$x(-t)$



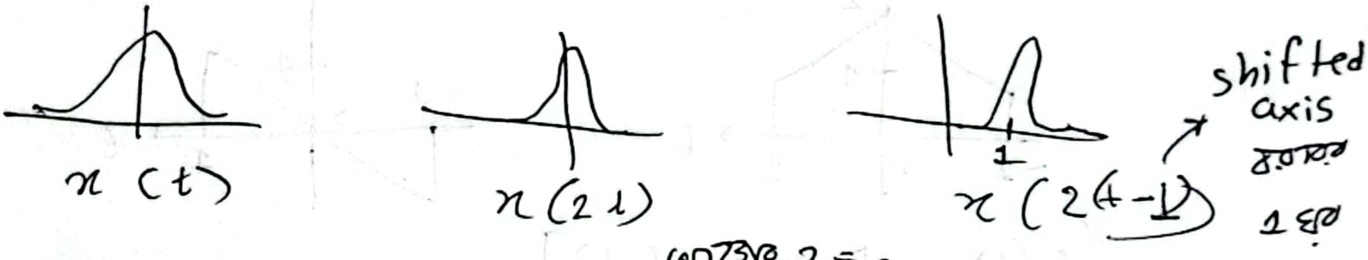
Time Shift



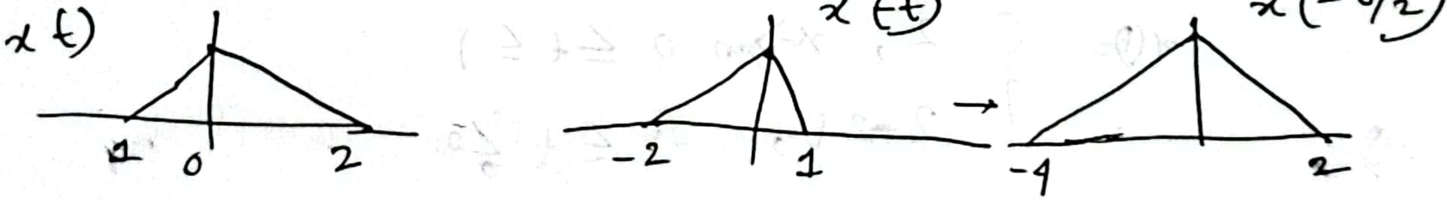
$x(t+1) \qquad x(t) \qquad x(t-1)$

Combination:

$x(2(t-1))$

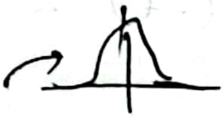


Exercise



Even & Odd Symmetry

Even → $x(t) = x(-t)$



Odd → $x(t) = -x(-t)$ →

Decomposing into even & odd

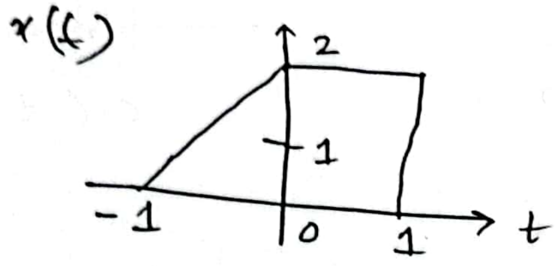
$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

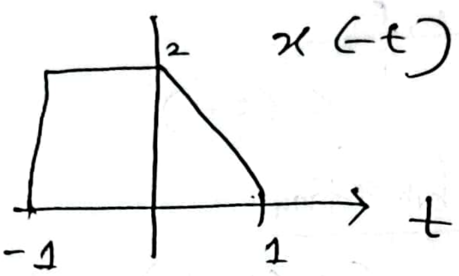
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

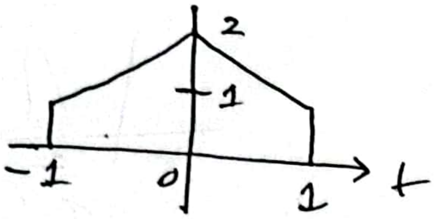
$$x_e(t) = x_e(-t)$$

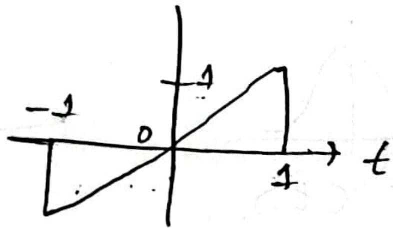
$$x_o(t) = -x_o(-t)$$

Example









$x_c(t) = \frac{1}{2}[x(t) + x(-t)]$

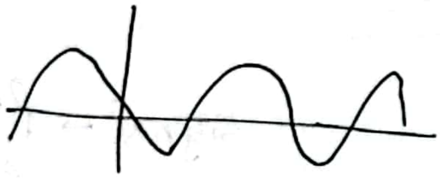
$x(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 2+2t, & -1 \le t \le 0 \end{cases}$

$x(-t) = \begin{cases} +2, & 0 \le t \le 1 \\ 2-2t, & -1 \le t \le 0 \end{cases}$

$x_e(t) = \begin{cases} \frac{1}{2}(2+2) \\ \frac{1}{2}(2+2) \end{cases}$

Continuous Time Signal

$$x(t) = A\cos(\omega_0 t + \phi)$$



Time Shift $\Leftrightarrow$ Phase Change

Time Shift $\Rightarrow$ phase change * এই নিশ্চিত হবে না

$$A\cos[\Omega_0(n+n_0)] = A\cos[\Omega_0 n + \Omega_0 n_0]$$

↳ integer হবে for uniformly signals

Time Shift $\Rightarrow$ Phase Change

Phase Change $\Rightarrow$? Time Shift.

→ ϕ এর value integer হওয়া লাগবে

$$x[n] = x[n+N] \qquad N \triangleq \text{period}$$

$$N = \frac{2\pi m}{\Omega_0} \qquad N, m \text{ must be integers.}$$

$$\Omega_0 = \frac{2\pi}{12} \rightarrow N \qquad 12 = \frac{2\pi}{\Omega_0} \qquad \therefore m = 1, \; N = 12$$

$$\Omega_0 = \frac{8\pi}{31} \qquad 31 = \frac{8\pi}{\Omega_0} \qquad 31 = \frac{2\pi \cdot 4}{\Omega_0}$$

$m = 4$

$N = 31$

$A\cos(\Omega_0 n+\phi)$

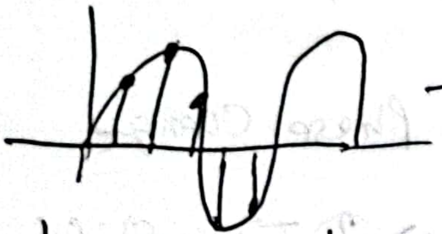
08/09/2024

Class-3

Princeton ELE 301
MIT

signal → Physical form
↳ difference from function.

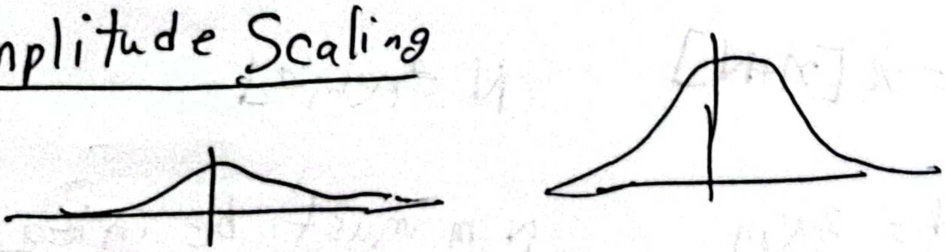
*Continuous and Discrete Time signals

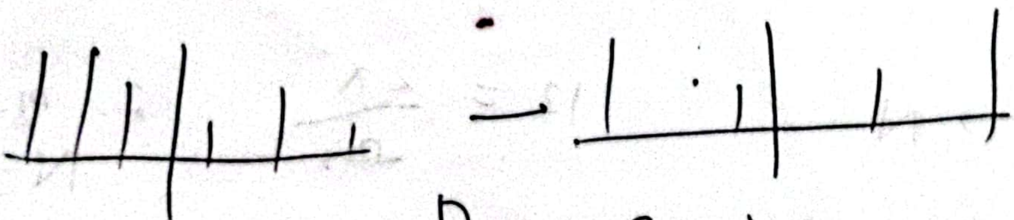


→ Sampling. (Uniformly sampled)

Uniformly Sampled → time's value integer.

Amplitude Scaling



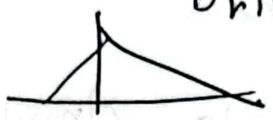


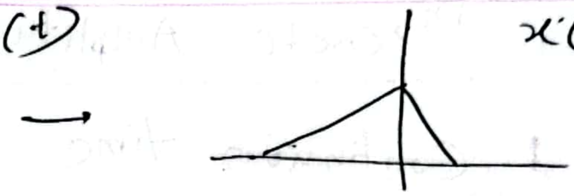
Down Sampling.

Up sampling or interpolation.

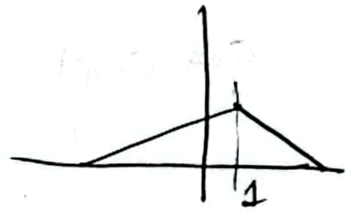
$\therefore x(t \pm 1) = x(T-1)$

Original $x(t)$





$x(-t) = x(T)$



$x(-(t-1)) = x(T-1)$

Even- Odd

$\cos t = \cos(-t)$
↳ Even

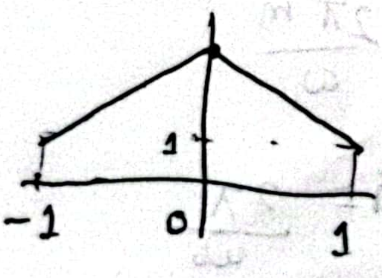
$\sin t = \sin(-t) \rightarrow$ odd function.

$$x(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 2+2t, & -1 \le t \le 0 \end{cases}$$

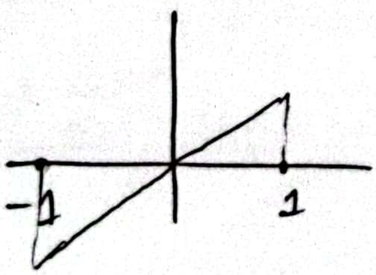
$$x(-t) = \begin{cases} 2, & -1 \le t \le 0 \\ 2-2t, & 0 \le t \le 1 \end{cases}$$

$$\therefore x_e(t) = \begin{cases} 2-t, & 0 \le t \le 1 \\ 2+t, & -1 \le t \le 0 \end{cases}$$

$$x_o(t) = \begin{cases} t, & 0 \le t \le 1 \\ t, & -1 \le t \le 0 \end{cases}$$



$x_e(t)$



Discrete Amplitude Signals

1 Continuous time Discrete Amplitude
Discrete " Continuous "
~~Con~~ Cont " " "
" " Discrete "

Causal signals

non-zero only for $t \geq 0$

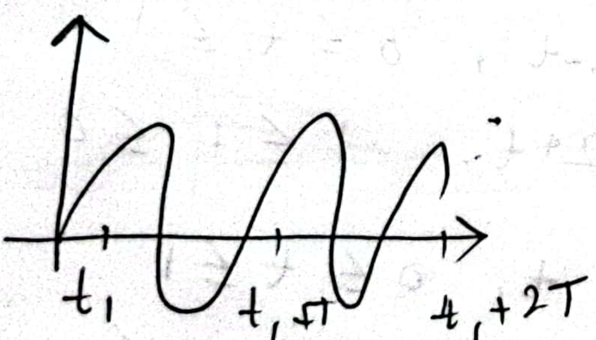
Non Casual signals

non-zero for some $t < 0$.

Continuous time sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$x(t) = x(t + T_0) \quad \rightarrow \text{Period}$$



$$A\cos(\omega_0 t + \phi)$$

$$= A\cos(\omega_0 t + \underbrace{\omega_0 T_0}_{2\pi m} + \phi)$$

$$T_0 = \frac{2\pi m}{\omega_0}$$

$$\text{period} = \frac{2\pi}{\omega_0}$$

if $m = 1$ (smallest)

time shift

$A\cos(\omega_0 t) = A\cos\{\omega_0(t+t_0)\}$

$= A\cos(\omega_0 t + \underbrace{\omega_0 t_0}_{\phi = \text{phase change}})$

11/09/24

Class 4-5

$A\cos(\omega_0 t) = A\cos(\omega_0 t+\phi) = A\cos(\omega_0 t+\omega_0 t_0)$

$\omega_0(t+t_0)$

$t_0 = \frac{\phi}{\omega_0}$

$\phi = -\frac{\pi}{2}$ $\quad x(t) = \begin{cases} A\cos(\omega t - \frac{\pi}{2}) \\ A\sin\omega t \\ A\cos\omega_0(t - \frac{T_0}{4}) \end{cases}$

$\boxed{\begin{gathered} T = \frac{2\pi}{\omega} \\ \frac{T}{4} = \frac{\pi}{2\omega} \end{gathered}}$

Discrete Time Sinusoidal signal

$x[n] = A\cos(\Omega_0 n + \phi)$

Time Shift $\Rightarrow$ Phase Change

$n \in \mathbb{Z}$

$A\cos\Omega_0 n$

$= A\cos\{\Omega_0(n+n_0)\}$

$= A\cos(\Omega_0 n + \Omega_0 n_0) = A\cos(\Omega_0 n+\phi)$

$n_0 \in \mathbb{Z} \rightarrow \phi = \Omega_0 n_0$

if $n_0 \notin \mathbb{Z}$ then Phase Change $\not\Rightarrow$ Time Shift

$\underline{\underline{2}}(t-\underline{1})$ $\qquad$ $-(t-1)$

$\Omega_0 = \frac{1}{8}$ $\qquad$ $\phi = \Omega_0 n_0$

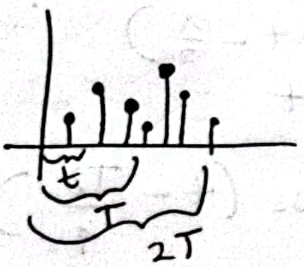
$-\frac{\pi}{2} = \frac{\pi}{8} n_0 \qquad n_0 = -4$

$x[n] = x[n+N]$

$A\cos[\Omega_0(n+N)+\phi] = A\cos[\Omega_0 n + \Omega_0 N + \phi]$

$\Omega_0 N = 2\pi m$

$N = \frac{2\pi m}{\Omega_0}$ $\qquad$ N, m integer.



$\cos x = \cos(2\pi m + x)$
$m \in Z$

$\Omega_0 N = 2\pi m$

$N = \frac{2\pi}{\Omega_0} \qquad N \in Z$

(i) $\Omega_0 = \frac{2\pi}{12}$

$12 = \frac{2\pi}{\Omega_0}$

$\boxed{N = 12 \quad \therefore \text{Periodic}}$

(ii) $\Omega_0 = \frac{8\pi}{31}$

$31 = \frac{8\pi}{\Omega_0}$

$\boxed{N = 31 \quad \therefore \text{Periodic}}$

$$x_1[n] = A\cos(\Omega_1 n + \phi)$$

$$x_2[n] = A\cos(\Omega_2 n + \phi)$$

$$= A\cos\{(\Omega_1 + 2\pi m)n + \phi\}$$

$$= A\cos(\Omega_1 n + \phi + 2\pi mn)$$

$2\pi mn$;

$m \in \mathbb{Z}$

$n \in \mathbb{Z}$

$mn \in \mathbb{Z}$

$$x_1[n] = A\cos(\Omega_1 n + \phi)$$

$$\Omega_2 = \Omega_1 + 2\pi m; \quad [m \in \mathbb{Z}]$$

<u>Real exponential</u>

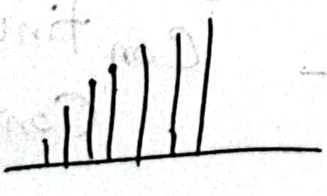
<u>Discrete time</u>

$$x[n] = ce^{\beta n} = c\alpha^n$$

$$\boxed{e^{\beta} = \alpha}$$

$$x[n] = \left(\frac{1}{2}\right)^n$$

$\alpha > 0$



$\alpha < 0$

$T \rightarrow 2\pi$

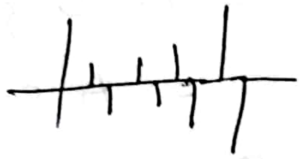
$1 \rightarrow \frac{2\pi}{T}$

$t \rightarrow 2\pi f t$

$x[n] = (-2)^n$

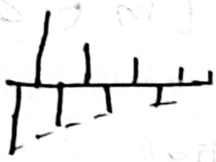
n	$(-2)^n$
0	1
1	-2
2	4

n even, $(-2)^n > 0$ else o

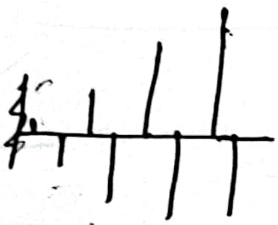


$\beta = -ve$, $\therefore e^{\beta} < 1$ $\therefore \alpha < 1$

(i) $\alpha < 0$
$|\alpha| < 1$



$\alpha < 0$
$|\alpha| > 1$



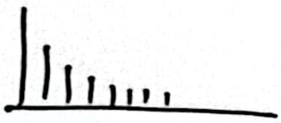
$\cos(\omega_1 t + \phi)$

$\cos(\omega_2 t + \phi)$

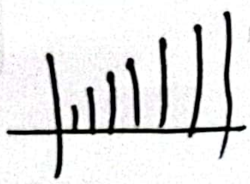
$= \cos(\omega_2 (t + t_0) + \phi)$

$= \cos(\omega_2 t + \omega_2 t_0 + \phi))$

$\alpha > 0$
$|\alpha| < 1$



$\alpha > 0$
$|\alpha| > 1$



pt-2

Continuous Exponential: Continuous Time

$x(t) = Ce^{at}.$

C and a are complex numbers

$C = |C| e^{i\theta}$ $a = r + j\omega_0$

~~$x(t) = |c| e$~~

$$x(t) = Ce^{at}$$

$$C = |c| e^{j\theta} \qquad a = r + j\omega_0$$

$$x(t) = |c| e^{j\theta} \cdot e^{(r+j\omega_0)t}$$

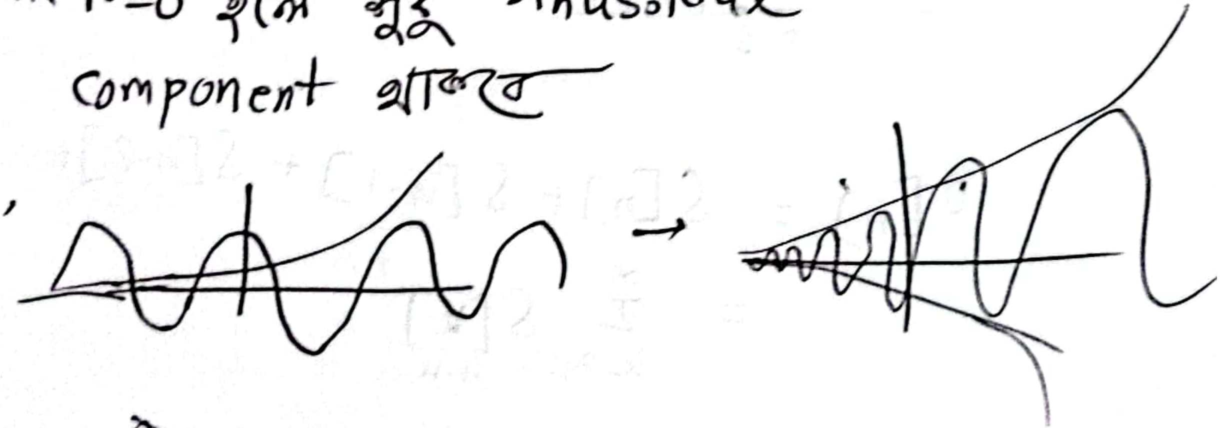
$$= |c| e^{j\theta} \cdot e^{rt} \cdot e^{j\omega_0 t}$$

$$= |c| e^{j(\theta+\omega_0 t)} \cdot e^{rt}$$

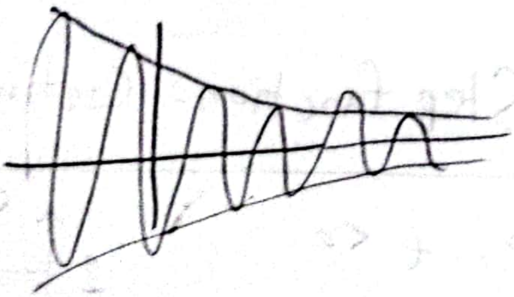
$$= |c| e^{rt} \cdot \cos(\omega_0 t + \theta) + j|c| e^{rt} \sin(\omega_0 t + \theta)$$

* $r=0$ হলে শুধু sinusoidal component থাকবে

$r>0$,

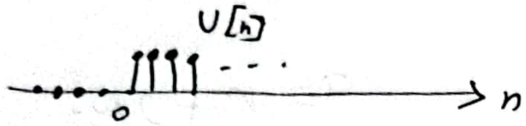


$r<0$,



* $Ce^{j\omega_0 n}$ periodic ?

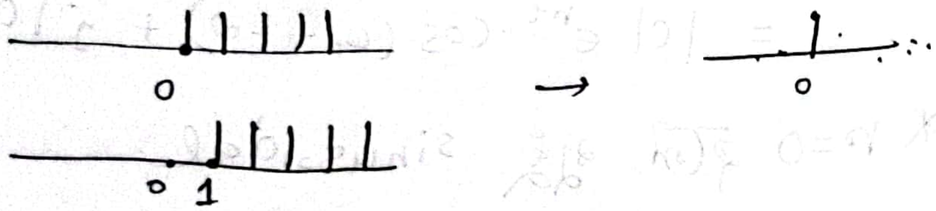
Unit Step function: Discrete time



$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Unit impulse function $\rightarrow$ $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

$$\delta[n] = u[n] - u[n-1]$$

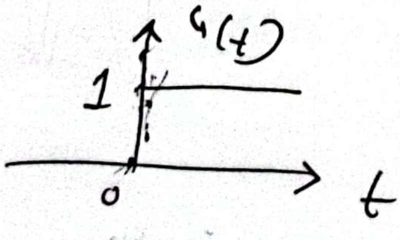


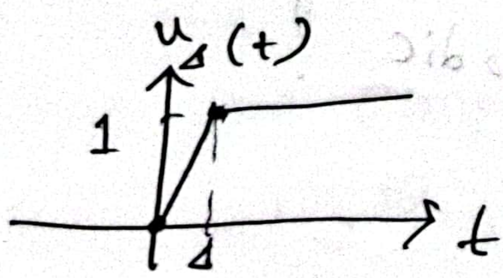
$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots$$

$$= \sum_{k=0}^{\infty} \delta[n-k]$$

Unit Step function: Continuous time

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$





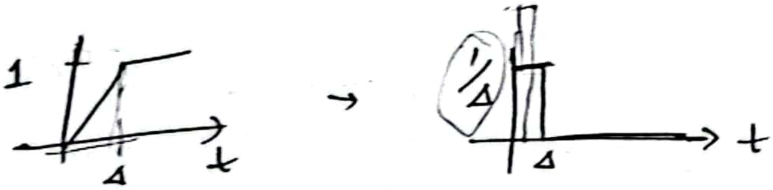
$\Delta \rightarrow 0$

$u_\Delta(t) \rightarrow u(t)$

Unit impulse function

$$\delta(t) = \frac{du(t)}{dt} \qquad \delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

$$\delta(t) = \delta_\Delta(t) \quad \text{as } \Delta \to 0$$



$\Delta \downarrow \quad \frac{1}{\Delta} \uparrow$

$$\text{Area} = \frac{1}{\Delta} \times \Delta = 1$$

height $= \infty$

width $= 0$

area $= 1$

Continuous এ Area দেখি

Discontinuous এ length দেখি

$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$