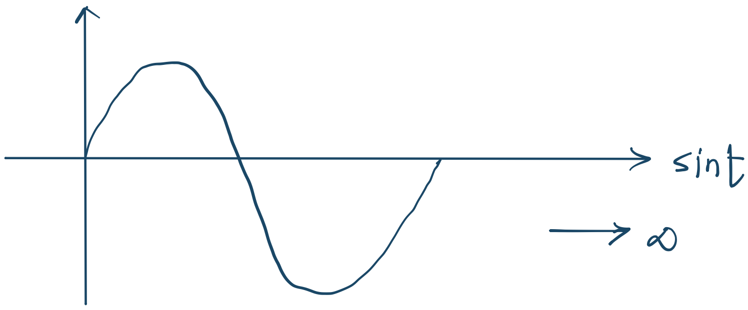
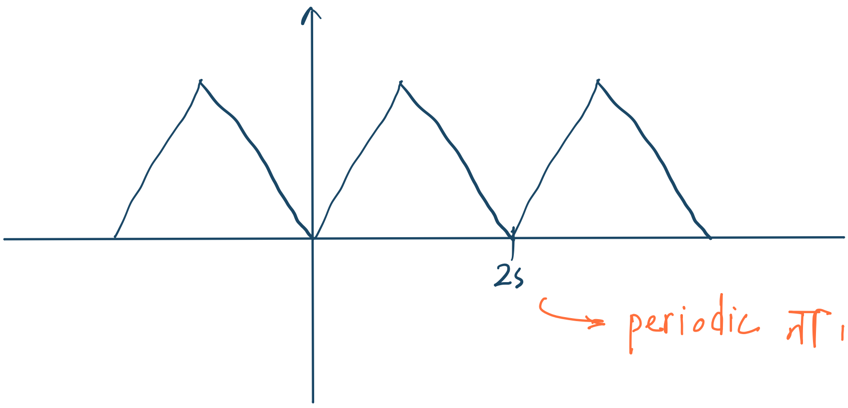
27.10.24

Periodic Waveform



$\boxed{2\pi}n$



periodic না।

$$x(t) = x(t \pm nT)$$

↪ infinite পর্যন্ত যদি $x(t)$ defined থাকে তখনই থাকবে।

periodic signal এরই শুধু fouries series হয়।

$$T_0$$

$$f_0 = \frac{1}{T_0}$$

$$\omega_0 = 2\pi f_0$$

$x(t) \longrightarrow f_0$

$nF_0 \longrightarrow 2F_0 \,,\, 3F_0 \;\ldots\; nF_0$

2nd Harmonic   3rd Harmonic

cosine / sine এর জন্য $2\omega_0 \quad 3\omega_0 \;\ldots$

\# 3 টা sequential harmonic দেয়া। কিন্তু কোনটা কত নং সেটা বলা নাই।

→ 440, 660, 880 Hz

$f_0 = ?$

↳ fundamental frequency

৩টার gcd বের করলেই পাবো।

$f_0 = 220\,Hz$

Fourier Series

$$x(t) = T_1 + T_2 + T_3 + \cdots T_{n-1} + \cdots \infty$$

$x(t) \to T_0, \omega_0, f_0$

$T_1 \to$ sinusoids $\to$ $a\cos\omega t$, $b\sin\omega t$

$x(t) \to$ periodic

so $T_1, T_2, T_3 \to$ periodic হবে।

$$T_1 = a_1 \cos\omega_1 t + b_1 \sin\omega_1 t$$

$$T_2 = a_2 \cos\omega_2 t + b_2 \sin\omega_2 t$$

$\vdots$

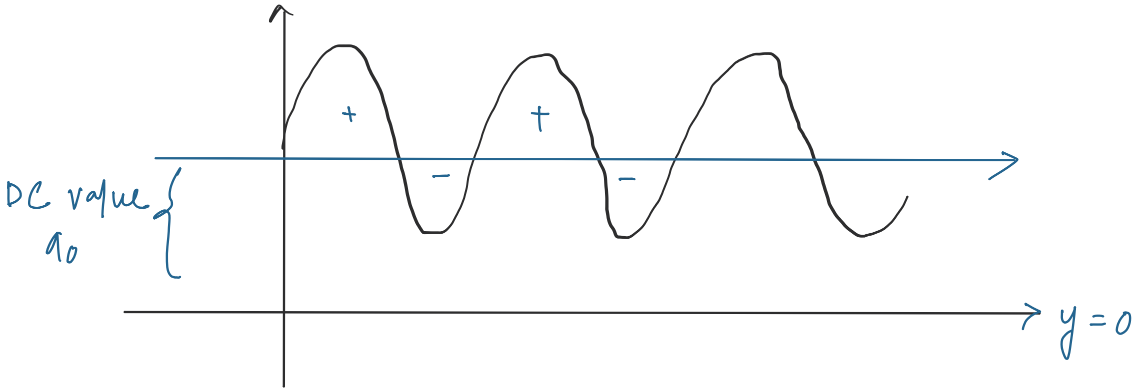
$T_1 \to$ 1st term

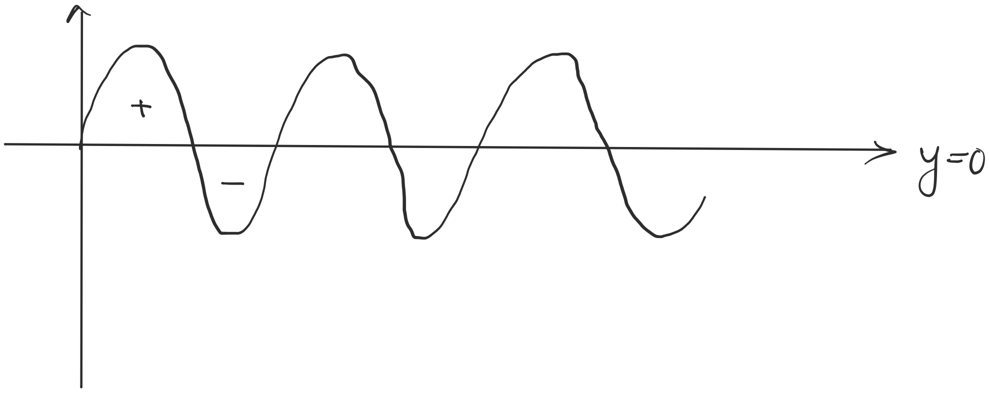
$\therefore$ ω_1 হবে $x(t)$ এর 1st harmonic (ω_0)

$T_n \to$ nth term $\to$ $n\omega_0$

$\therefore T_1 \to$ $a_1 \cos\omega_0 t$, $b_1 \sin\omega_0 t$

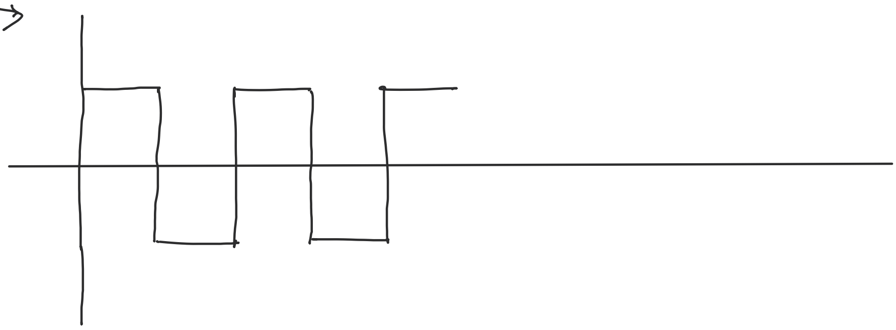
$T_2 \to$ $a_2 \cos(2\omega_0 t)$, $b_2 \sin(2\omega_0 t)$

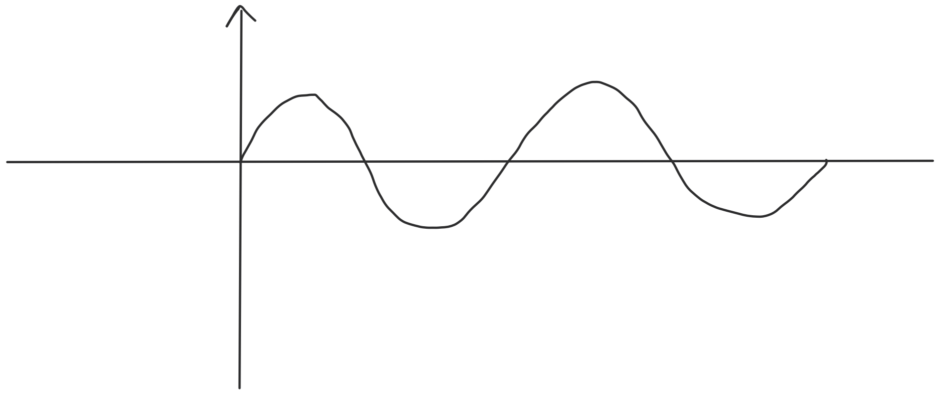




$$x(t) = \underbrace{\boxed{a_0}}_{DC} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

sum করলে





$$F_n(x) = \frac{4}{\pi} \sin x \quad (n=1)$$

n এর value যত বাড়াতে থাকবো sum square wave এর মতো হতে থাকবে।

$$x(t) = \underbrace{\boxed{a_0}}_{DC} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$a_0, a_n, b_n = ?$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n\omega_0 t)\, dt$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(n\omega_0 t)\, dt$$

$a_n = 0 \Rightarrow$ sin term থাকবে শুধু

$b_n = 0 \Rightarrow$ cos term " "

$$\int_{T_0} x(t)\, dt = \int_{T_0} a_0\, dt + \sum_{n=1}^{\infty} a_n \int_{T_0} \cos(n\omega_0 t)\, dt + 0$$

sin odd function হওয়ায় integration করে 0 পাবো

T_0	$T_0/2$
0	$-T_0/2$

$$= a_0 T_0 + \sum_{n=1}^{\infty} a_n \underbrace{\left[\frac{\sin(n\omega_0 t)}{n\omega_0}\right]_{-T_0/2}^{T_0/2}}_{\sin(n\pi) - \sin(-n\pi) = 0}$$

$$\boxed{\begin{aligned} \sin n\pi &= 0 \\ \cos n\pi &= (-1)^n \end{aligned}}$$

$$\int_{T_0} x(t)\,dt = a_0 T_0$$

$$\therefore a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$$

$$x(t) = \underbrace{\boxed{a_0}}_{DC} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$\boxed{m \neq n}$

$$\int_{T_0} x(t)\cos(m\omega_0 t)\,dt = \int_{T_0} a_0 \cos(m\omega_0 t)\,dt + \sum_{n=1}^{\infty}\int_{T_0} a_n \cos(n\omega_0 t)\cdot\cos(m\omega_0 t)\,dt + 0$$

0

$$\int_{-L}^{L} \cos(nx)\cos(mx)\,dx = \begin{cases} 0 & \text{when } n != m \\ L & \text{when } n = m \end{cases}$$

$$\int_{-L}^{L} \sin(nx)\sin(mx)\,dx = \begin{cases} 0 & \text{when } n != m \\ L & \text{when } n = m \end{cases}$$

$$n = \underbrace{1, 2, 3 \cdots\cdots}_{\text{এই part এ integrate} = 0} m, \underbrace{m+1 \cdots\cdots \infty}_{\text{এই part এ integrate} = 0}$$

$$\sum_{n=1}^{\infty} \int_{-T_0/2}^{T_0/2} a_n \cos(n\omega_0 t) \cos(m\omega_0 t)\, dt$$

$$= \frac{T_0}{2} \cdot a_{n-m}$$

$$\int_{-T_0/2}^{T_0/2} x(t) \cos(m\omega_0 t)\, dt = \frac{T_0}{2} a_n$$

$\boxed{m = n}$ হলেই শুধু non-zero.

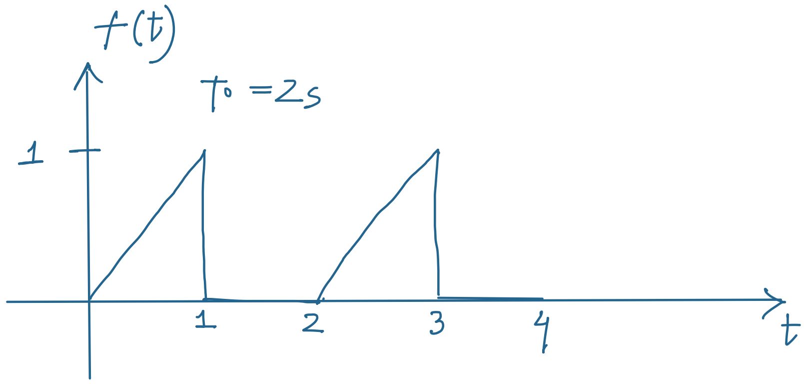
$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cos(n\omega_0 t)\, dt$$

$\rightarrow n = m$

(1) $x(t) = \begin{cases} 1.5 & \text{when} \quad 0 \le t < 1 \\ -1.5 & \text{when} \quad 1 \le t < 2 \end{cases}$

$x(t)$ is periodic. Find the coeff.

(2) Determine the fourier series representation ..



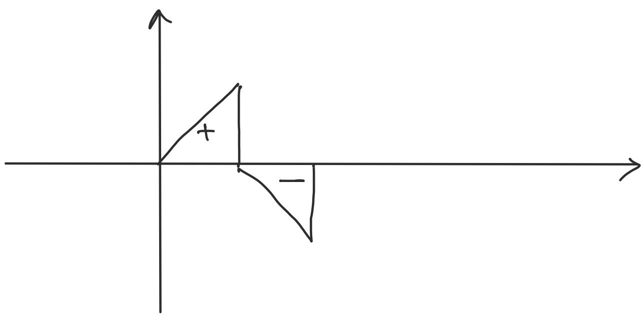
$W_0 = \frac{2\pi}{2} = \pi$

$$x(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 < t \le 2 \end{cases}$$

$$a_0 = \frac{1}{T_0} \int_0^2 x(t)\, dt$$

average value বোঝায়

$$= \frac{1}{2} \int_0^1 t\, dt = \frac{1}{4}$$



avg $= 0$ (a_0)

$$a_n = \frac{2}{2} \int_0 \underbrace{t}_{u} \underbrace{\cos(n\pi t)}_{v} \, dt$$

$$b_n = \frac{2}{2} \int \underbrace{t}_{u} \underbrace{\sin(n\pi t)}_{v} \, dt$$

change হয় বাকি part fixed

$\underline{\underline{x(t)}}$