31/08/2024

Class-1

Mohammad Ashraf Hossain

Probability and Statistics

Random Variable

Experiment → Deterministic
→ Probabilistic

* Probability Distribution function for discrete random variable..

→ Continuous Probability function
→ Discrete Probability function

Random variable → Discrete R.V
→ Continuous R.V

Pdf → Probability Distribution function.

Book : Probability and Statistics for Engineers and Statistics by Ronald E Walton.

Discrete Probability Distribution → Neg Binomial
→ Binomial
→ Bernoulli
→ Poison

Continuous " " → Exponential
→ T
→ Normal

Mean & Variance Calculating

Analysis.

Hypothesis Testing

* Confidence Interval → নাইনের দিনের class এ.

02/09/2024

Class-2

Mohammad Elius

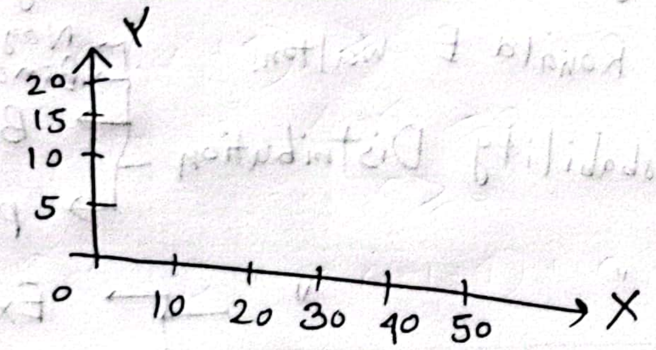
Raw Data → Data as it was collected (Ungrouped

Arrays → Data if arranged

Class Interval → Grouping.

Class Interval	Frequency	Class	Class Boundaries	fi	Mid
0-10	7	0-9	-0.5 - 9.5	35	5
10-20	8	10-19	9.5 - 19.5	120	15
20-30	20	20-29	19.5 - 29.5		25
30-40	10	30-39	29.5 - 39.5		35
40-50	5	40-49	39.5 - 49.5		45

Total 50



$d_i = x_i - A$ (A=25)
-20
-10
0
10
20

fd_i
-140
-80
0
100
100
-20

Width of the class interval => i

Arithmetic Mean

→ Long Method
→ Short Method

Arithmetic Mean $\Rightarrow \bar{X} = \frac{\Sigma X}{N}$ (Long Method)

$$\bar{X} = \frac{\sum_{i=1}^{i=N} f_i x}{\Sigma f_i}$$

Where A is assume mean

$$\bar{X} = A + \frac{\Sigma fd}{N} \Rightarrow \text{short Method}. \quad d_i = x_i - A,$$

$$\bar{X} = A + \frac{\Sigma f u_i}{N} i \rightarrow \text{Step deviation on coding Method}.$$

Where $u_i = \frac{d_i}{i}$ $\quad i =$ width (size of class interval)

$$\bar{X} = A + \frac{\Sigma fd}{N}$$

$$= 25 + \frac{-20}{50} = 25 - 0.4 = 24.6$$

u_i	fu
−2	−14
−1	−8
0	0
1	10
2	10
	−2

$$\bar{X} = 25 + \frac{-2}{50} \times 10 = 24.6$$

07/09/2021

Class-3

Ashraf Sir

A discrete random variable X is said to have a probability distribution or probability mass function $f(x)$ if the following conditions are satisfied:

i) $f(x) \geq 0$

ii) $\sum_x f(x) = 1$ → for Continuous: $\int_{min}^{max}$

iii) $P(X=x) = f(x)$

3.5 (a) $f(x) = c(x^2+4)$, for $x = 0, 1, 2, 3$

$c(x^2+4) \geq 0$

$\because x^2+4 \geq 0$

$\therefore c \geq 0$

$c(0+4) + c(1+4) + c(4+4) + c(9+4) = 1$

~~$c[26] = 1$~~ $c = \frac{1}{30}$

$\therefore f(x) = \frac{1}{30}(x^2+4)$

3.6 (a)

$$\int_{200}^{\infty} \frac{20,000}{(x+100)^3}\, dx$$

$$= \left[\frac{20,000}{-2\,(x+100)^2} \right]_{200}^{\infty}$$

$$= \frac{20,000}{2 \times (200+100)^2} = \frac{1}{9}$$

(b) $$\int_{80}^{120} \frac{20,000}{(x+100)^3}\, dx = \frac{20,000}{-2} \left[\frac{1}{(180)^2} - \frac{1}{(300)^2} \right]$$

$$= \frac{1000}{9801} = 0.1020$$

question: $f(x) = \begin{cases} \frac{20,000}{(x+100)^3}, & x>0, \\ 0, & \text{elsewhere} \end{cases}$

(a) $P(X > 200)$
↑shelf life

$$P(a < x < b) = \int_a^b f(x)\, dx$$

Definition:

Cumulative distribution function

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t) \quad \text{for } -\infty < x < \infty.$$

the point I wanna know

$$\int_{-\infty}$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{t} f(t)$$

Example 3.8

Out of 20, 3 are defective.

$\therefore F(2) = f(1) + f(2)$

$= \frac{3}{20} + \left(\frac{3}{20} \times \frac{2}{19}\right) \times$

${}^{20}C_2 \rightarrow$

$$\frac{{}^{3}C_0 \times {}^{17}C_2}{{}^{20}C_2} = P(X=0)$$

$$P(X=1) = \frac{{}^{3}C_1 \times {}^{17}C_1}{{}^{20}C_2}$$

$$P(X=2) = \frac{{}^{3}C_2 \times {}^{17}C_0}{{}^{20}C_2}$$

x	0	1	2
$f(x)$	$\frac{68}{95}$	$\frac{57}{190}$	$\frac{3}{190}$
$F(x)$	$\frac{68}{95}$		1

09/09/24

Class-9
Elius Sir

Median : 5, 7, $\boxed{10, 13}$, 16, 20

for grouped data :

$$\text{Median} = L_1 + \left(\frac{\frac{N}{2} - \Sigma f_1}{f}\right) c$$

Where L_1 is the lower class boundary of the median class.

N is the total frequency.

f is the frequency of the median.

Σf_1 is the sum of the frequencies of all classes lower than the median class.

C is the size of the median class interval.

Marks :	10-20	20-30	30-40	40-50	50-60	60-70	70-80
	12	30	34	65	45	25	18

$\frac{N}{2} = 114.5$, $L_1 = 40$,

$f = 65$ $\Sigma f_1 = 76$ $c = 10$

Mode =

2, 6, 8, 8, 10, 14 → 8

2, 6, 8, 10, 14 → No modes

2, 2, 6, 6, 8, 8, 10, 10, 14, 14 → All modes

for grouped data, Mode = $L_1 + \left(\frac{f - f_1}{2f - f_1 - f_2}\right)c$

$f = 65$

$f_1 = 65 - 34$ $\qquad f_2 = 65 - 45$

Q An incomplete distribution is given below:

Variable:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
	10	20	?	40	?	25	15

It is given that total frequency is 170 and the median value is 35. Find the missing frequencies using median formula.

$10 + 20 + x + 40 + y + 25 + 15 = 170$

$x + y = 60$

~~$40 - x = f_1$ $\quad 40 - y = f_2$~~

$30 + \left(\frac{\frac{170}{2} - (30 + x)}{40}\right) \times 10 = 35$

$x = 220$

$$\boxed{\begin{array}{l} x = 35 \\ y = 25 \end{array}}$$

Standard deviation: It is the square root of the mean of the square of the deviation from the

$$S.D = \sqrt{\frac{\Sigma f (x-\bar{x})^2}{\Sigma f}}$$

The square of S.D is called variance σ^2

11/09/2024

Class-5

$S.D = \sigma$

$$\sigma^2 = \frac{1}{N} \Sigma f (x-\bar{x})^2 \qquad \frac{\Sigma (x_i - \bar{x})^2}{N} = 0$$

$\sigma^2 \to$ Variance

(*) Find the S.D for the following distribution:

Mass in Kg.	60-62	63-65	66-68	69-71	72-74
Number of students :	5	18	42	27	8

$$\sigma^2 = \frac{1}{100} \times \{(5-42)^2 + (18-42)^2 + (27-42)^2 + (8-42)^2\}$$

$$s.d = \sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

$(A = 67)$

Masses in kg	Number of Students f	Midpoint (x)	$d = x - A$	fd	fd^2
60-62	5	61	-6	-30	
63-65	18	64	-3	-54	
66-68	42	67	0	0	
69-71	27	70	3	81	
72-74	8	73	6	48	
				$\Sigma fd = 45$	$\Sigma fd^2 = 873$

$\therefore$ $s.d = 2.92$

Coefficient of Variation (C.V)

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

(*) Two cricketers scored the following runs in several innings. Find who is better run getter and who is more consistent players.

X : 42, 17, 83, 59, 72, 76, 64, 45, 40, 32

Y : 28, 70, 31, 0, 59, 108, 82, 14, 3, 95

Ans : (i) Average : $\bar{X} = \frac{X_{total}}{N}$, $\bar{Y} = \frac{Y_{total}}{N}$

(ii)

X	$X-\bar{X}$	$(X-\bar{X})^2$
42	-11	
17	-36	
⋮	⋮	
$\Sigma X = 530$		

Cricketer Y

Y	$Y-\bar{Y}$	$(Y-\bar{Y})^2$
28	-21	441
70	21	
⋮	⋮	⋮
$\Sigma Y = 490$		$\Sigma Y^2 = 13734$

$\bar{X} = 53$

$\bar{Y} = 49$

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{N}}$$

$= 20.09$

$$CV_x = \frac{\sigma}{\bar{x}} \times 100 = 37.92\%$$

$\sigma_y = 37.06$

$$CV_y = \frac{\sigma_y}{49} \times 100$$

$= 75.63\%$

14/09/2024

Class-6 - Ashraf sir

* Joint Probability ~~funct~~ distribution

1. $f(x, y) \geq 0$ for all (x, y) $\quad f(X, Y) = P(X = x, Y = y)$
2. $\sum_x \sum_y f(x, y) = 1$
3. $P(X = x, Y = y) = f(x, y) = 3$ → z axis এ height নির্দেশ করে

xy plane এ নির্দিষ্ট জায়গা যে Area তে সম্ভাবনার মূলো ছেদ করে।

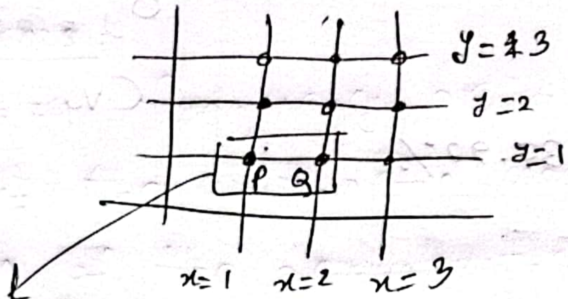
* Joint density function of continuous random variables X and Y if

1. $f(x, y) \geq 0$
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)$

Discrete:

for any region A in xy plane

$$P[(X, Y) \in A] = \sum\sum_{A} f(x, y)$$



এই 9 টা point region.

A এই region

এই region এর probability
$P+Q$

So continuous case এ পুরা region এর জন্য করব, So volume বের হবে।

Example 3.14

Blue → 3   Green → 3
Red → 2   X → num Blue   Y → num Red

(a) joint probability function $f(x, y)$

y \ X	0	1	2	3
0	3/28	9/28	3/28	0
1	3/14	3/14	0	0
2	1/28	0	0	0

$f(0,0) = \frac{1}{12}$

$P(X=0, Y=0)$

$= \frac{{}^3C_0 \times {}^3C_0 \times {}^3C_2}{{}^8C_2} = \frac{3}{56}$

np. plot (- - -, label= "X")

$$f(0,1) = P(x=0, Y=1)$$

$$= \frac{{}^3C_0 \times {}^2C_1 \times {}^3C_1}{{}^8C_2}$$

(b) $P[(X,Y) \in A]$, when A is the region $\{(x,y) \mid x+y \leq 1\}$.

* $$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify condition 2 of Definition 3.9

(b) Find $P[(x,Y) \in A]$, where $A = \{(x,y) \mid 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\}$.

$$\int_{y}\int_{x=}^{x} \frac{2}{5}(2x+3y)\,dx\,dy$$

$A(\cancel{S_2+S_1+S_0)+B})$ 18/09/2024

Class 7 - Elius Sir

Statistical inference :- is the process of using a sample to infer the properties of a population.

Moments :

For a frequency distribution, the first moment about the arithmetic mean is defined as the mean of the derivativations of items taken from their A.M.

First moment ~~the~~ about the mean $\mu_1 = \frac{\Sigma(x-\bar{x})}{N} = \frac{\Sigma d}{N} = 0$

with frequency, $\mu_1 = \frac{\Sigma f(x-\bar{x})}{N} = \frac{\Sigma fd}{N}$

$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{N}$, $\mu_2 = \frac{\Sigma f(x-\bar{x})^2}{N} = \Sigma f d^2$

$\mu_3 = \frac{\Sigma(x-\bar{x})^3}{N}$ $\mu_3 = \Sigma$ - -

$\mu_4 = \frac{\Sigma(x-\bar{x})^4}{N}$ $\mu_4 = \dots$

x

$$\mu_1' = \frac{\Sigma(x-A)}{N}, \quad \mu_1' = \frac{\Sigma f(x-A)}{N} = \frac{\Sigma f d}{N}$$

Relation between moments about the mean $\bar{K}$ and moments about any

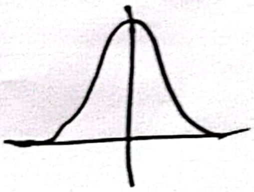
$$\mu_1 = \mu_1' - \mu_1'$$

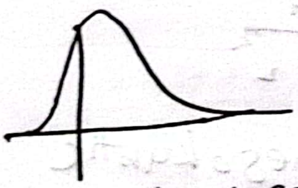
$$\mu_2 = \mu_2' - (\mu_2')^2$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

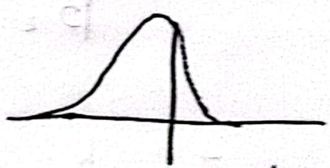
$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'\mu_1' - 3(\mu_1')^4$$

Skewness: skewness is the lack of symmetry.





+ve: skewness or right hand skewed



−ve skewness or left hand skewed

Measures of skewness

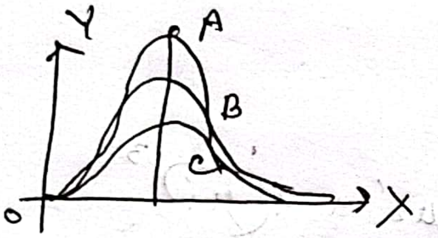
$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$\beta = 0 \rightarrow$ symmetrical

$\beta = -ve \rightarrow$ negative skewness

$\beta = +ve \rightarrow$ pos

Kurtosis : A measure of Kurtosis indicates the degree to which a curve of a frequency distribution is peaked or flat topped.



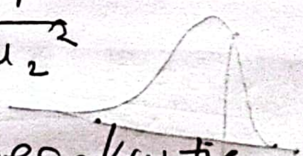
A → peaked (lepto kurtic distribution)

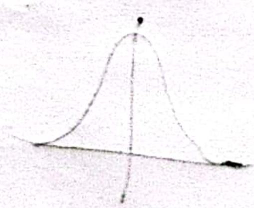
B → Normal (Mesokurtic ")

C → flat topped (platykurtic ")

Measures of Kurtosis β_2

$$\beta_2 = \frac{\mu_4'}{\mu_2^2}$$





If $\beta_2 = 3$ + mesokurtic

$\beta_2 > 3$, ~~lele~~ lepto kurtic

$\beta_2 < 3$ platykurtic distribution.

21/09/2024

Class - 8
Ashraf Sir

Marginal Distribution

① Marginal distribution of a random variable X is
$g(x) = \sum_{y} f(x,y)$ when X is discrete.
or $\int f(x,y)\,dy$ " X is continuous

② Marginal distribution of a random variably Y is $h(Y) = \sum_{x} f(x,y)$ when Y is discrete

~~h(y)~~ or $h(y) = \int f(x,y)\,dx$ when Y " continuous.

$$\sum_{y=0}^{2} f(x,y) = f(x,0) + f(x,1) + f(x,2)$$
↳ Last class math.

$\cancel{g(x) = f(0,0) + f(0,1}$

$$g(0) = f(0,0) + f(0,1) + f(0,2)$$
$$= \frac{3}{28} + \frac{3}{14} + \frac{1}{28} = \frac{15}{28}$$

Conditional distribution

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

$f(x,y) \rightarrow$ Joint distribution.

$g(x) \rightarrow$ Marginal distribution

↳ Given (x)
(At first x)

what's the probability of y occuring given x ~~distr~~ will happen)

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

Example 3.20

Given the Joint Density function

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0<x<2,\ 0<y<1, \\ 0, & \text{elseashere.} \end{cases}$$

$$g(x) = \int_0^1 \frac{x(1+3y^2)}{4}\,dy$$

$$= \frac{x}{4}\left[y+y^3\right]_0^1 = \frac{x}{2},\ 0<x<2$$

$$h(y) = \int_0^2 \frac{x(1+3y^2)}{4}\,dx$$

$$= \frac{1+3y^2}{4}\left[\frac{x^2}{2}\right]_0^2 = \frac{1+3y^2}{4}\times\frac{4}{2}$$

$$= \frac{1+3y^2}{2},\ 0<y<1$$

x variable

$$f(x|y) = \frac{f(x,y)}{\cancel{g(x)}\ h(y)} = \frac{\frac{x(1+3y^2)}{4}}{\frac{1+3y^2}{2}} = \frac{\cancel{1+3y^2}}{\cancel{2}}\ \frac{x}{2},$$

for the particular value.

$0<x<2$

$$P\left(\tfrac{1}{4}<X<\tfrac{1}{2}\,\middle|\,Y=\tfrac{1}{3}\right)$$

$$= \int_{1/4}^{1/2} \frac{x}{2}\,dx = \frac{1}{6}\left[\frac{1}{2^3}-\frac{1}{4^3}\right]$$

$$= \frac{7}{6\times 64} = \frac{7}{384}$$

$$P(a<X<b) = \int_a^b f(x)\,dx$$

Particular
Probability distribution $\Rightarrow \iint \int \rightarrow 1$

Statastical Independence

$$f(x|Y) = \frac{f(x,y)}{h(y)} \qquad f(x,y) = f(x|Y)\, h(y)$$

$g(x)$

$\int_{-\infty}^{\infty} h(y)\,dy = 1$ if we can write

$f(x,y) = g(x)\,h(y)$ then

~~it will by be margino~~ they will be statastically independent.

Previous math: $h(y)\,g(x) = f(x,y)$

$\therefore$ They are statastically independent.

(See proof in the book)

7:48 - 9:59

3.53

$$f(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2,\ 2 < y < 4 \\ 0, & \text{elsewhere} \end{cases}$$

$P(1 < Y < 3 \mid X = 1)$

$$g(x) = \int_2^4 f(x,y)\,dy = \int_2^4 \frac{6-x-y}{8}\,dy$$

$$= \frac{6-x}{8} - \frac{1}{8}\left[\frac{y^2}{2}\right]_2^4$$

$$= \frac{6-x}{8} - \frac{1}{16} \times 12 = \frac{6-x}{8} - \frac{3}{4}$$

$$h(y) = \int_0^2 \frac{6-x-y}{8}\,dx = \frac{6-y}{8}[x]_0^2 - \frac{1}{8}\left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{6-y}{4} - \frac{1}{16} \times 2^2 = \frac{6-y}{4} - \frac{1}{4}$$

$$g(x) = \int_2^4 f(x,y)\,dy$$

$$= \int_2^4 \frac{6-x-y}{8}\,dy = \frac{6-x}{8} \times (4-2) - \left[\frac{y^2}{16}\right]_2^4$$

$$= \frac{6-x}{4} - \frac{1}{16} \times 12$$

$$= \frac{6-x}{4} - \frac{3}{4} = \frac{3-x}{4}$$

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

$$= \frac{\frac{6-x-y}{8}}{\frac{3-x}{4}} = \frac{6-x-y}{2}$$

23/09/2024

Class-9

Elius Sir

Probability

If an event can occur in 'a' ways and fail to " " b " and these are equally to occur, then the probability of the event occuring is $\frac{a}{a+b}$. denoted by p and probability of not occuring is $\frac{b}{a+b}$, denoted by q. Hence $p+q=1$

$$p = \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

Conditional Probability of B, given A, denoted by $P(B|A)$, is defined by

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{if } P(A) \neq 0$$

Example:

(*) A study showed that 60 percent of managers had some business education and 45% had some engineering education. Furthermore, 15% of the managers had some business education but no engineering education. What is probability that a manager has some business education, given that he has some engineering education.

$P(A) = 0.6 \qquad P(B) = 0.45 \qquad P(A-B) = 0.15$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6-0.15}{0.45} = 1$$

Dependent Events: from the definition of conditional probabilities, if A and B are independent events.

$P(A \cap B) = P(A|B)\,P(B)$ — (I)

$P(B \cap A) = P(B/A)\,P(A)$ — (II)

The order of no significance in the intersection of two events,

since $A \cap B = B \cap A$, we get an important property of intersection,

i.e., $P(A \cap B) = P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$

Independent events : Two events A and B are said to be independent if and only if

$P(A \cap B) = P(A) P(B)$.

which implies from (i) and (ii) that

$P(A/B) = P(A)$ and $P(B/A) = P(B)$