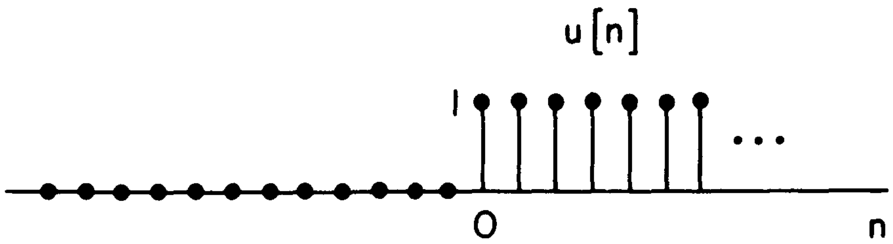
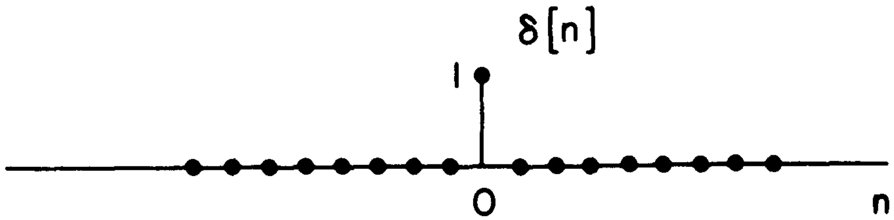
UNIT STEP FUNCTION: DISCRETE-TIME



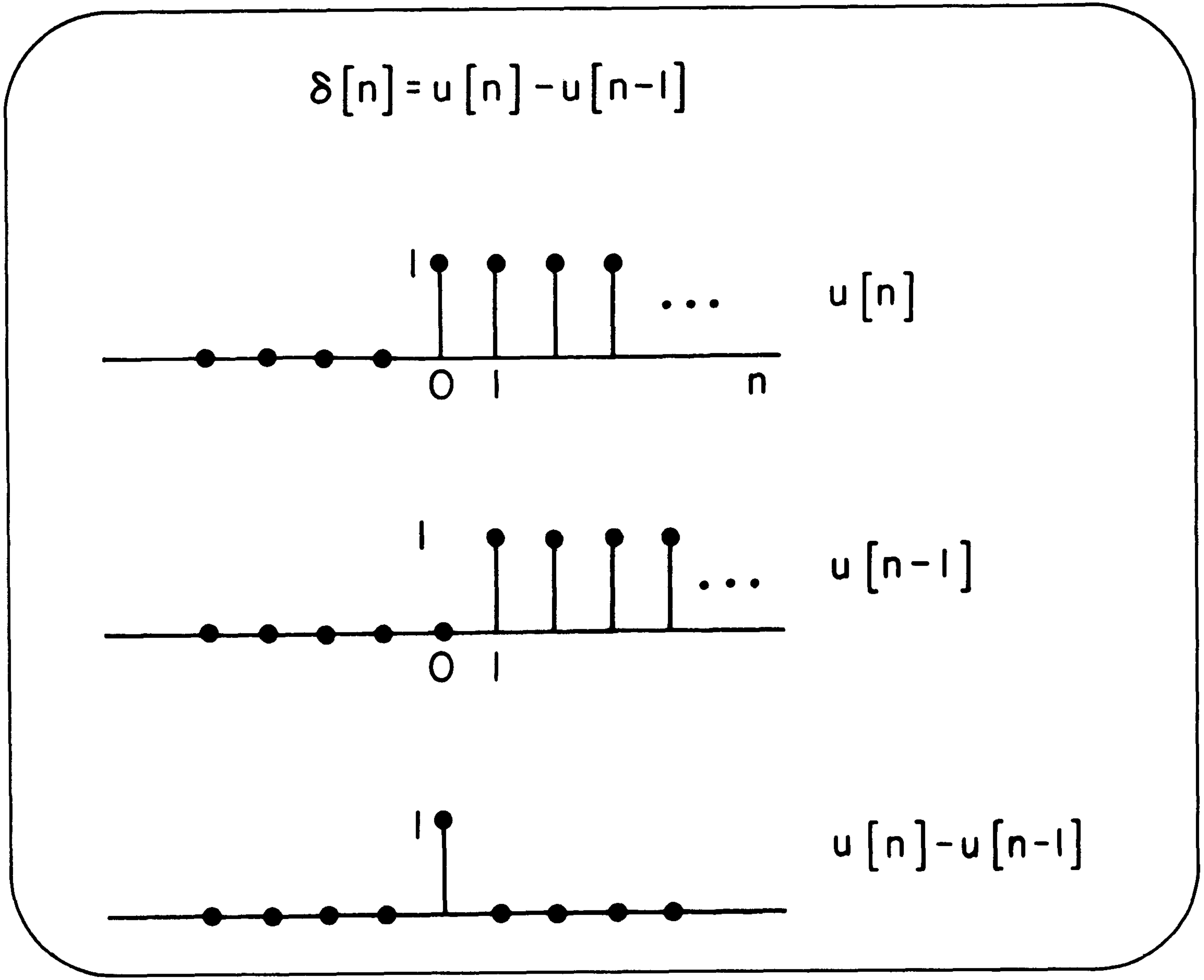
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

UNIT IMPULSE FUNCTION: DISCRETE-TIME
(Unit Sample)



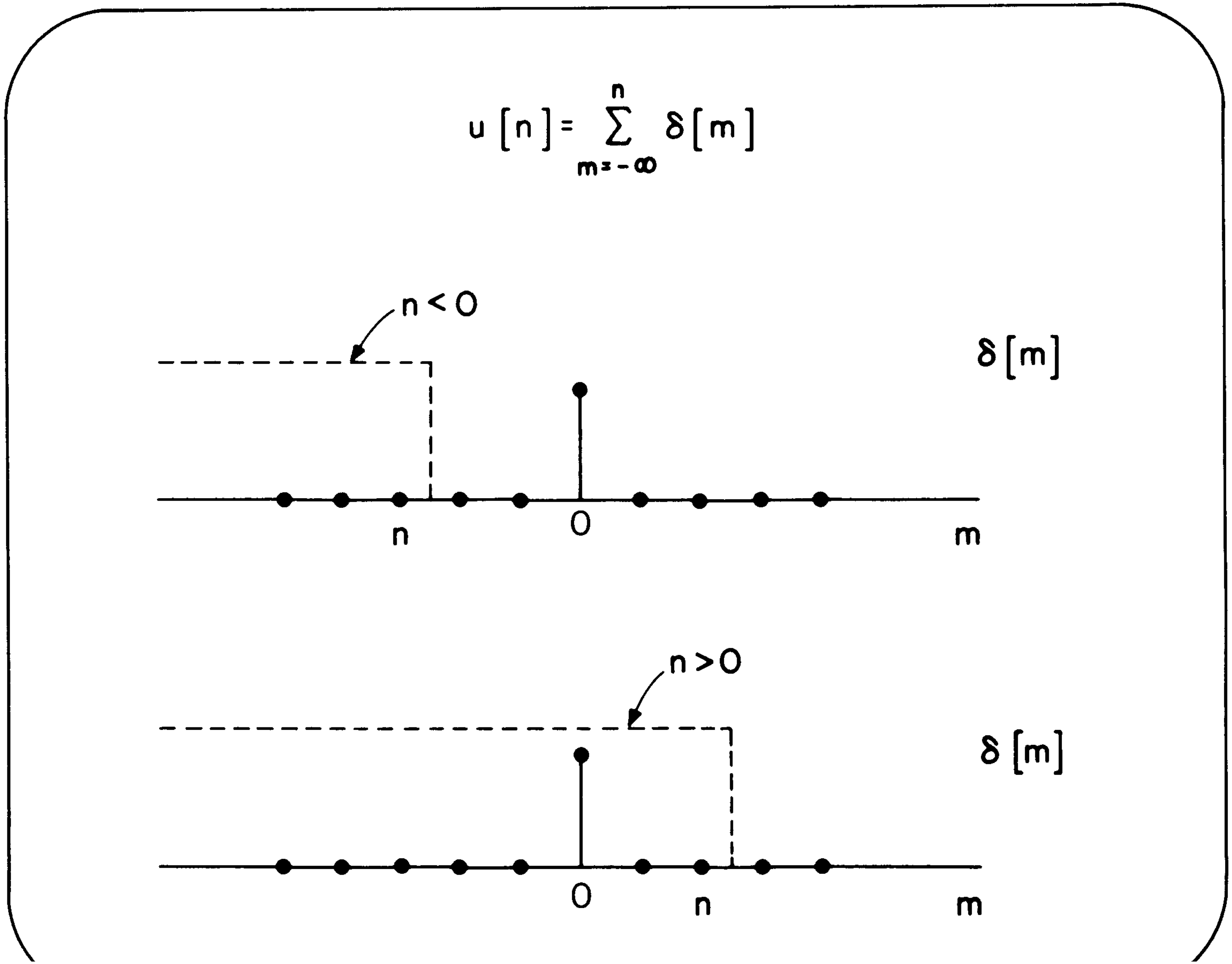
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

TRANSPARENCY 3.1
Discrete-time unit step and unit impulse sequences.



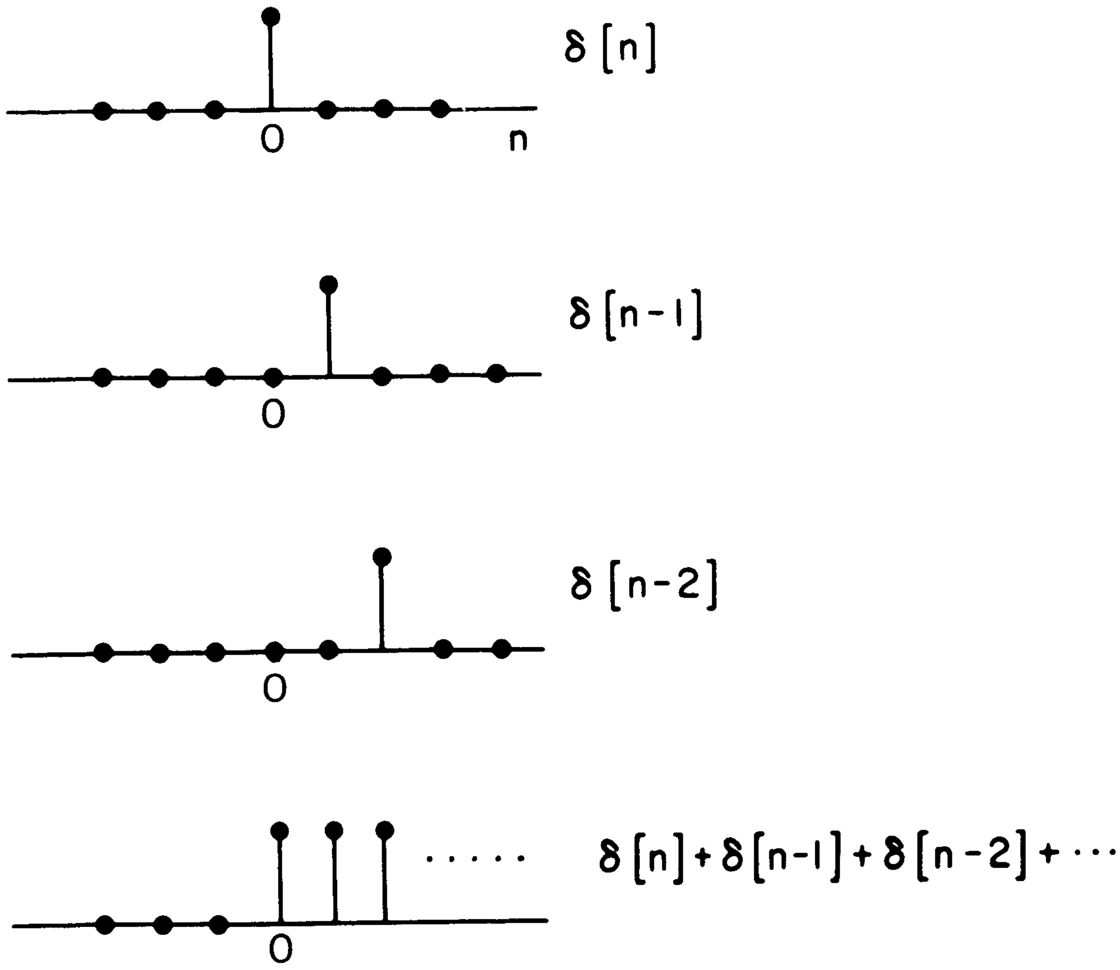
TRANSPARENCY 3.2
The unit impulse sequence as the first backward difference of the unit step sequence.

TRANSPARENCY 3.3
The unit step sequence as the running sum of the unit impulse.



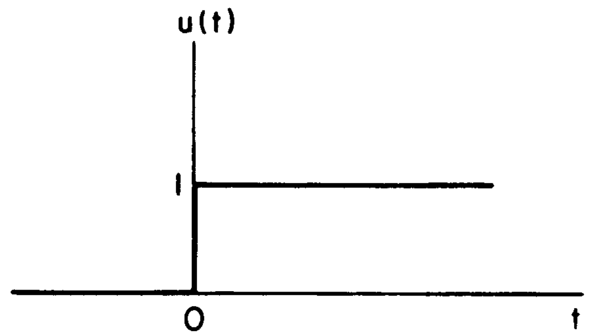
TRANSPARENCY 3.4
The unit step sequence expressed as a superposition of delayed unit impulses.

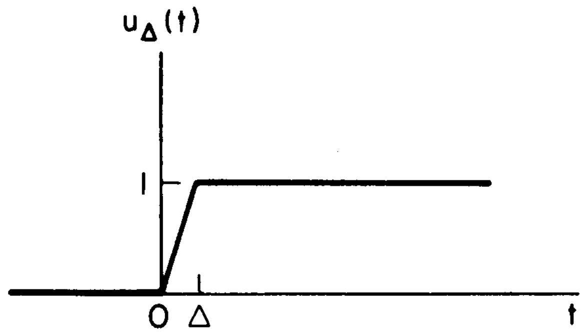
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$



UNIT STEP FUNCTION: CONTINUOUS-TIME

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$





$$u(t) = u_\Delta(t) \text{ as } \Delta \rightarrow 0$$

TRANSPARENCY 3.5
The continuous-time unit step function.

UNIT IMPULSE FUNCTION

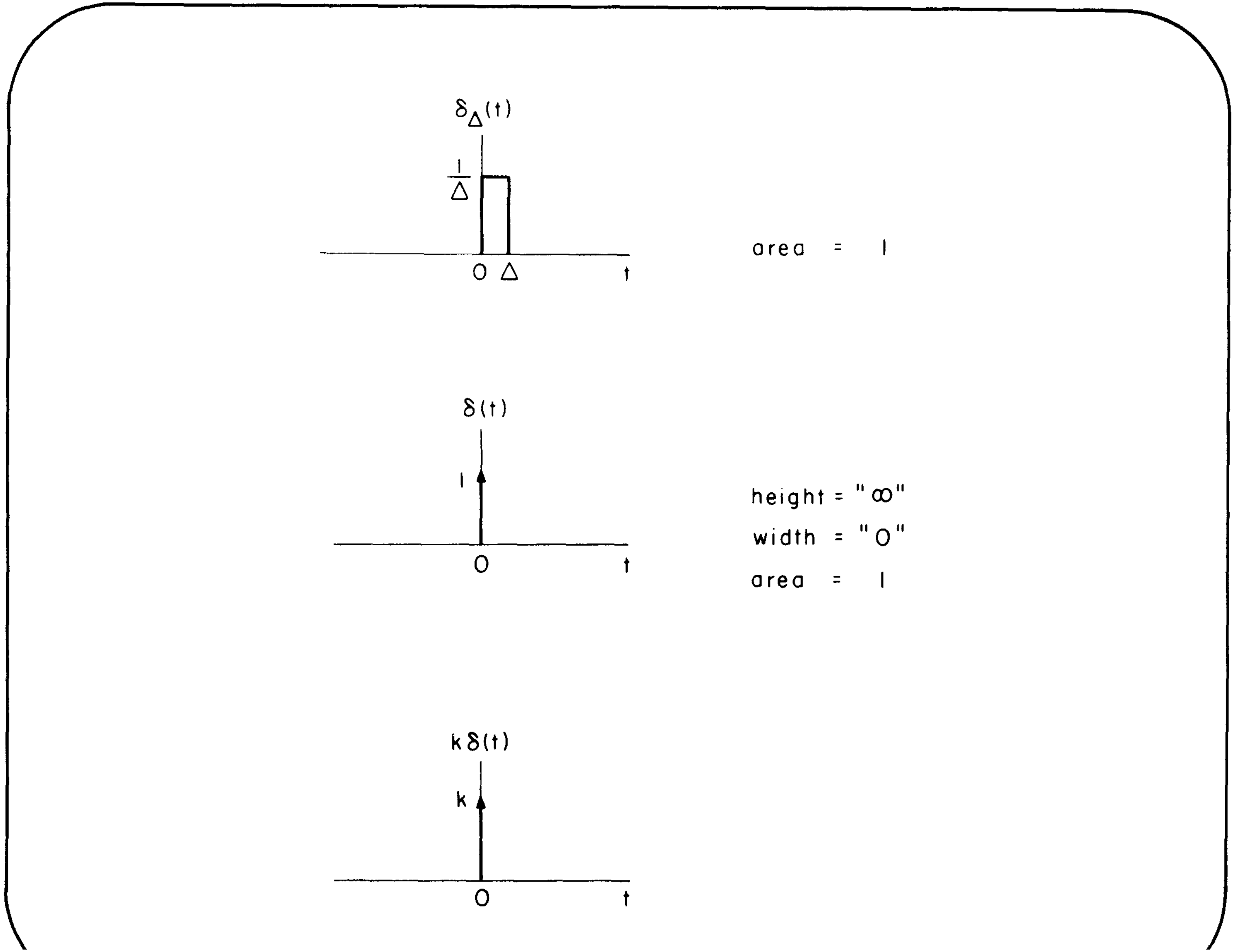
$$\delta(t) = \frac{du(t)}{dt}$$

$$\delta_{\triangle}(t) = \frac{du_{\triangle}(t)}{dt}$$

$$\delta(t) = \delta_{\triangle}(t) \text{ as } \triangle \rightarrow 0$$

TRANSPARENCY 3.6
The definition of the unit impulse as the derivative of the unit step.

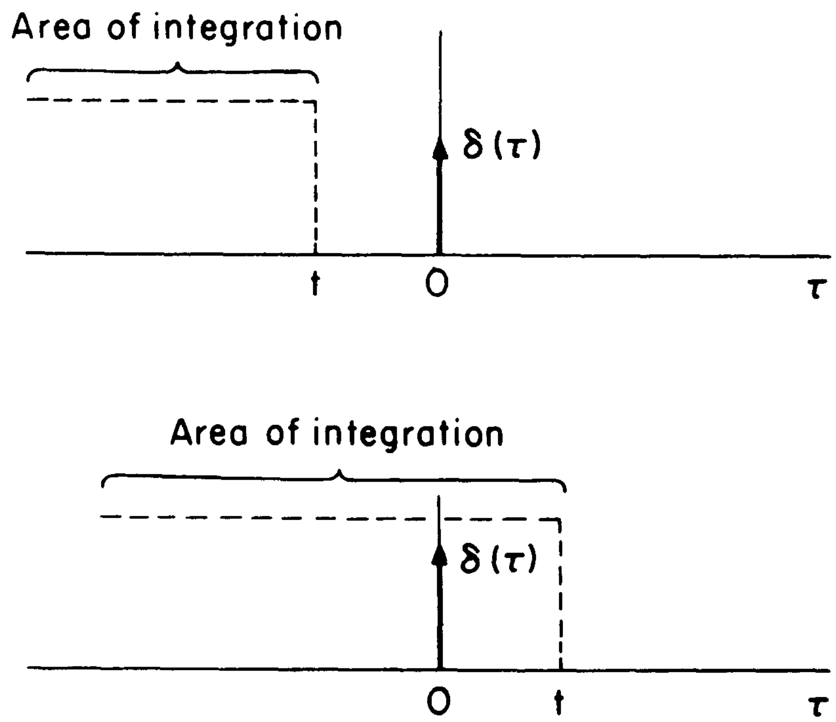
TRANSPARENCY 3.7
Interpretation of the continuous-time unit impulse as the limiting form of a rectangular pulse which has unit area and for which the pulse width approaches zero.



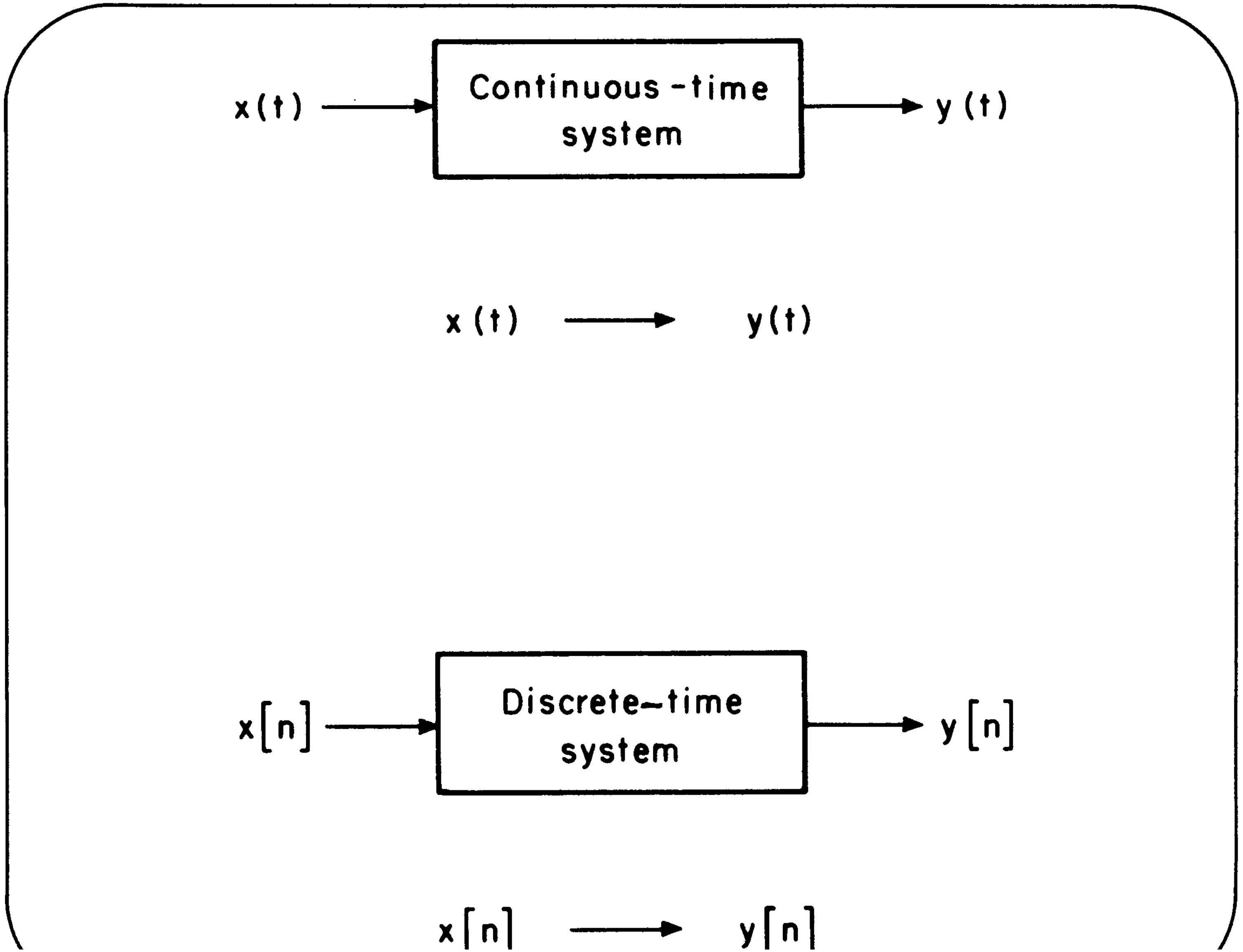
TRANSPARENCY 3.8
The unit step expressed as the running integral of the unit impulse.

$$\delta(t) = \frac{du(t)}{dt}$$

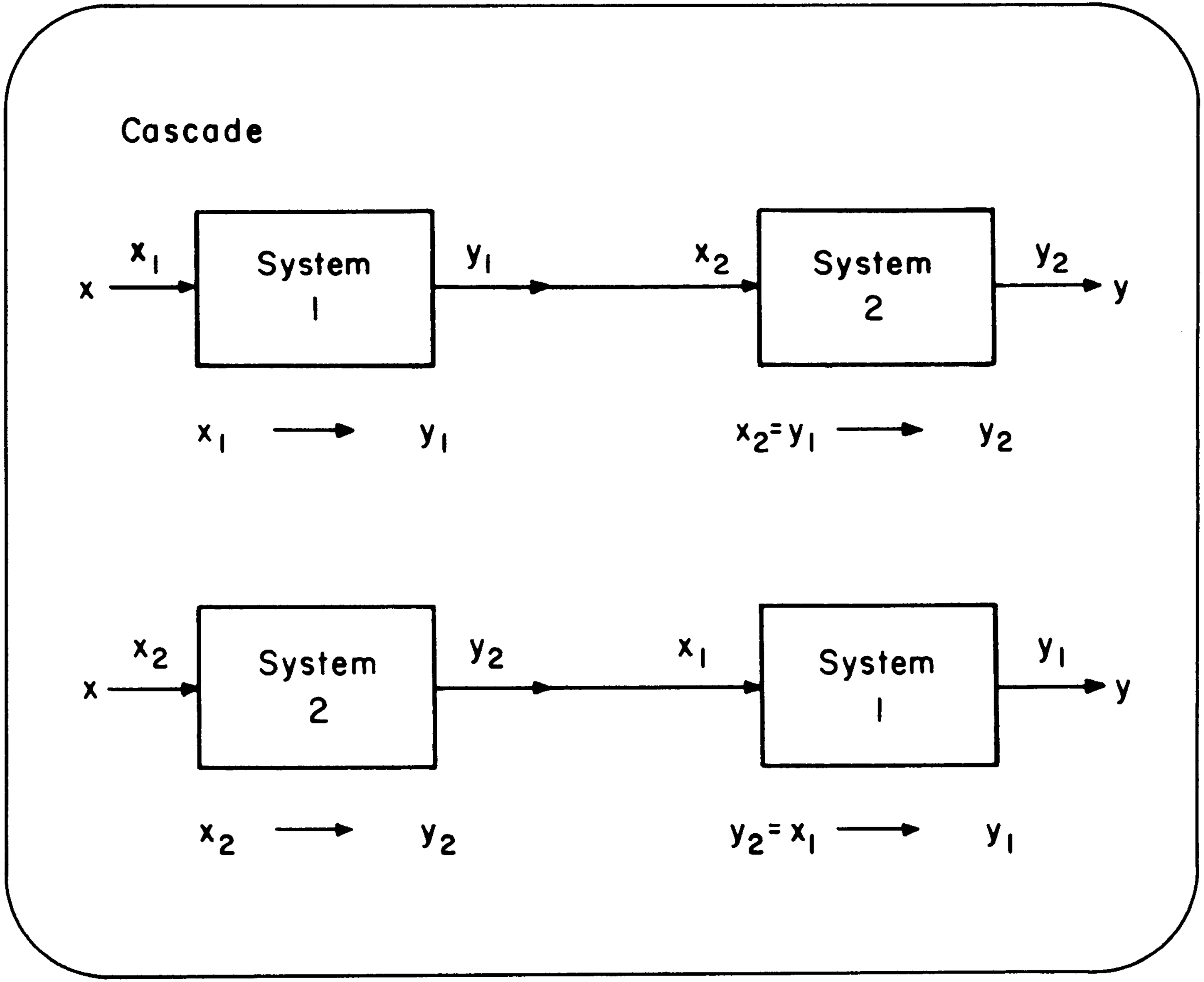
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$



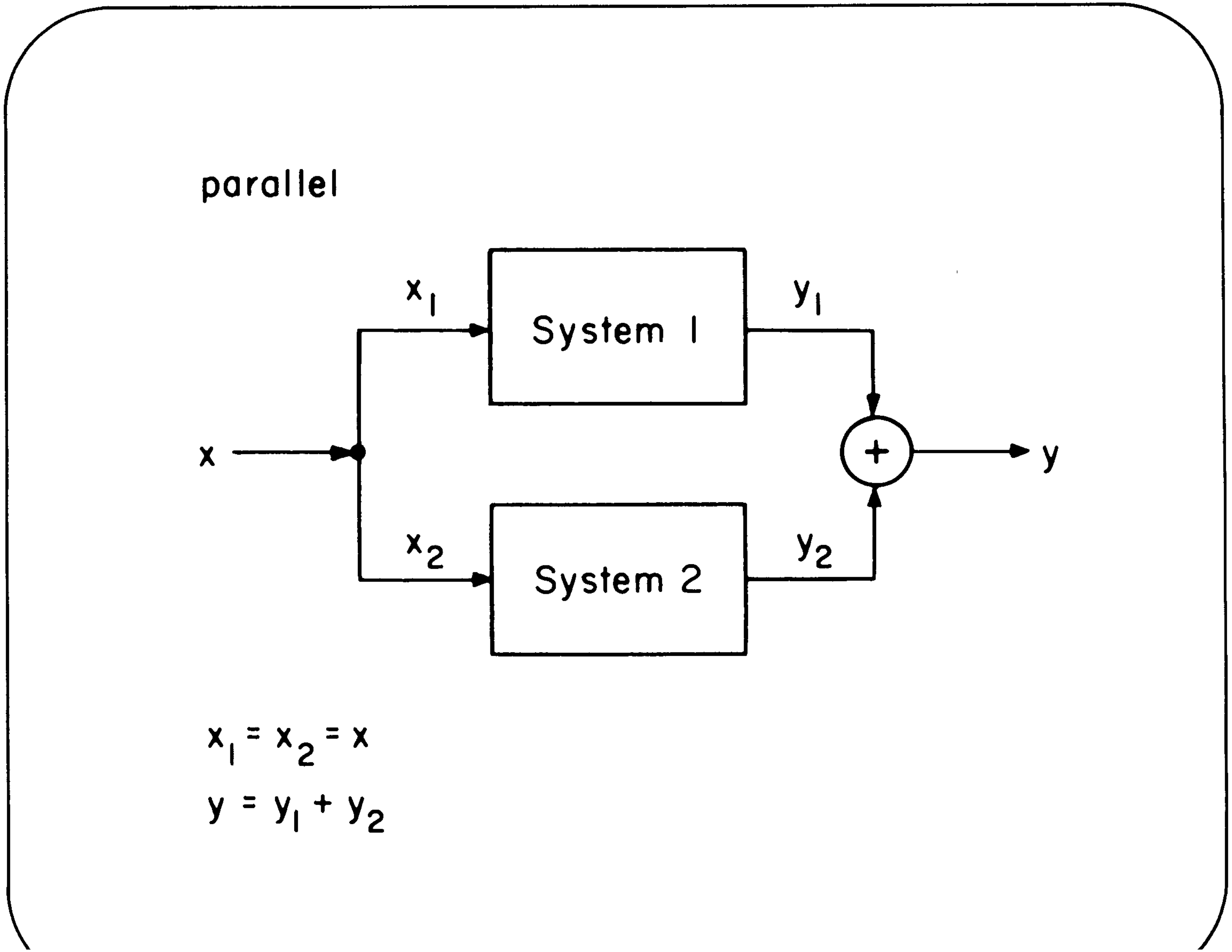
TRANSPARENCY
3.9
Definition of a system.



TRANSPARENCY 3.10
Interconnection of two systems in cascade.

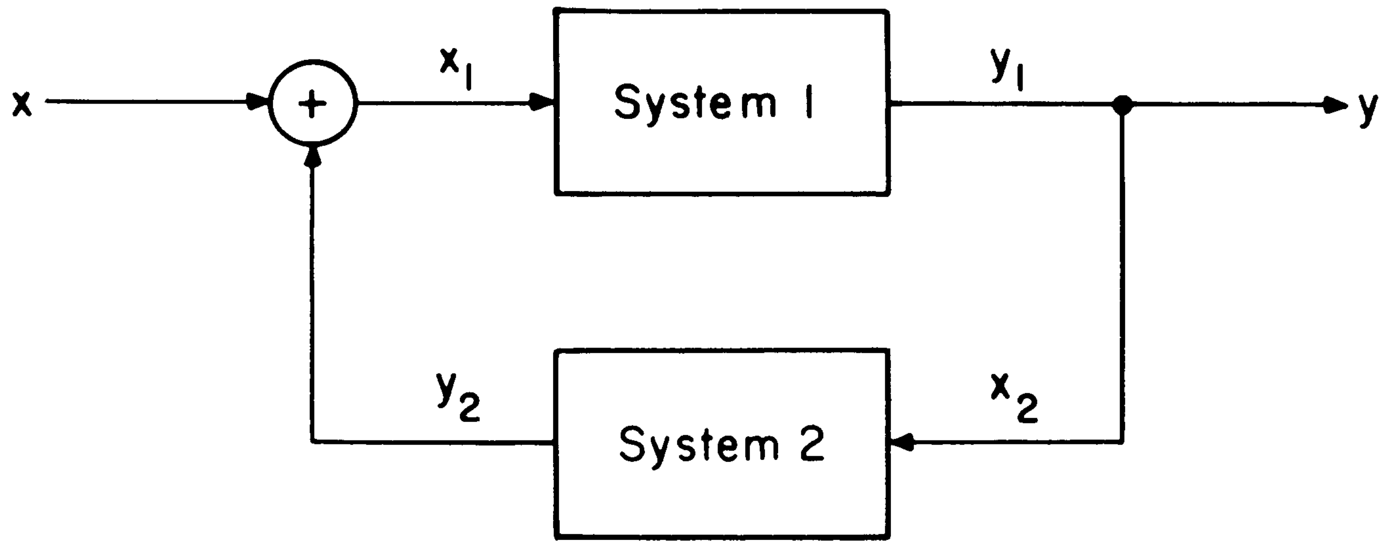


TRANSPARENCY 3.11
Interconnection of two systems in parallel.



TRANSPARENCY 3.12
Feedback interconnection of two systems.

feedback



$$x_1 = x + y_2$$

$$y = y_1$$

$$x_2 = y_1$$

MARKERBOARD 3.1

MEMORYLESS

$y(t)$ @ $t=t_0 \leftarrow x(t)$ @ $t=t_0$

$y[n]$ @ $n=n_0 \leftarrow x[n]$ @ $n=n_0$

Examples

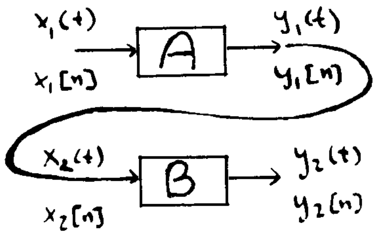
yes $y(t)=x^2(t)$ squarer

$y[n]=x^2[n]$

No $y(t)=\int_{-\infty}^{t} x^2(\tau)d\tau$

~~yes~~ No $y[n]=x[n-1]$ unit delay

INVERTIBILITY



$x_2 = y_1$

If B = Inverse of A

Then $y_2 = x_1$

Identity

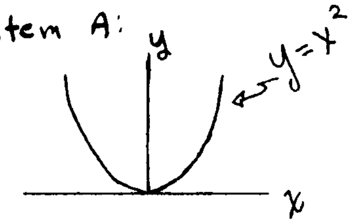
System A:

$y_1(t)=\int_{-\infty}^{t} x_1(\tau)\,d\tau$ Integrator

System A^{-1}:

$y_2(t)=\dfrac{dx_2(t)}{dt}$ differentiator

System A:



Invertible? No

Memoryless? yes

Causality

output at any time depends only on input prior or equal to that time

or:

system can't anticipate "future" inputs

or:

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

If:

$x_1(t) = x_2(t) \quad t < t_0$

Then:

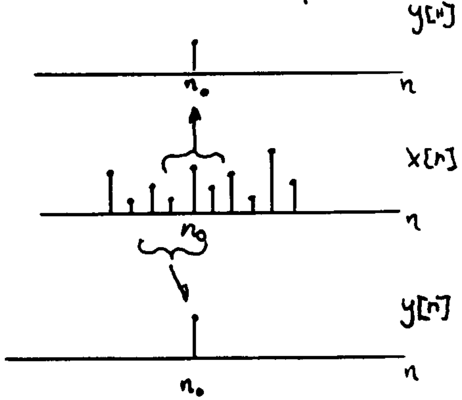
$y_1(t) = y_2(t) \quad t < t_0$

Same for discrete Time

Example:

$$y[n] = \frac{1}{3}\{x[n-1] + x[n] + \underset{\text{not}}{x[n+1]}\}$$

Moving Average



$$y[n] = \frac{1}{3}\{x[n-2] + x[n-1] + x[n]\}$$

causal

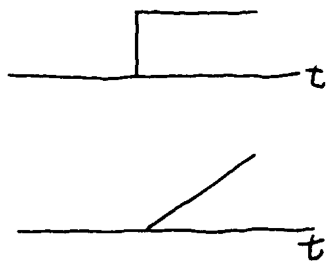
Stability

$\Rightarrow$ For every bounded input the output is bounded

Example

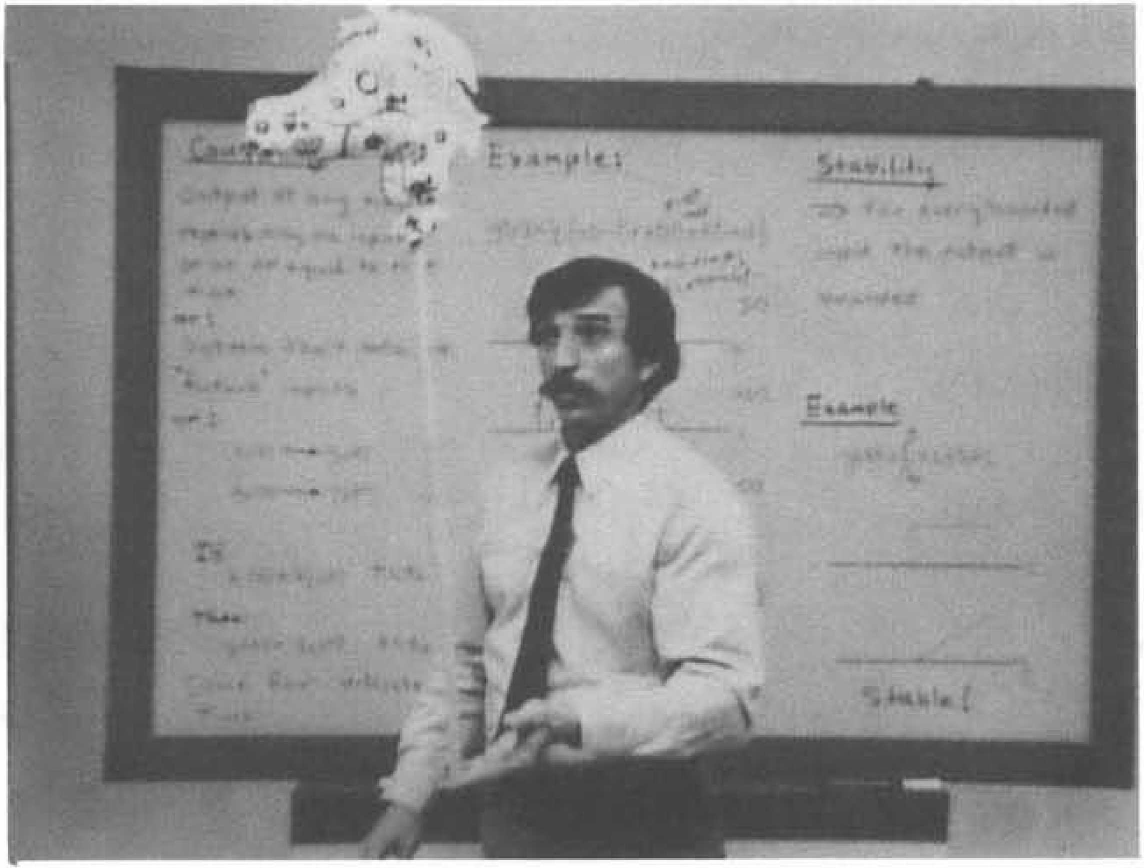
$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

not stable



Stable?

DEMONSTRATION 3.1
Illustration of an unstable system.



MARKERBOARD
3.3

Time Invariance

C-T:

$x(t) \longrightarrow y(t)$

Then

$x(t-t_0) \longrightarrow y(t-t_0)$

D-T:

$x[n] \longrightarrow y[n]$

Then

$x[n-n_0] \longrightarrow y[n-n_0]$

Example

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Accumulator

Time Invariant?

Example

$y(t) = (\sin t)\, x(t)$

$x(t) \longrightarrow (\sin t)\, x(t)$

$x(t-t_0) \longrightarrow (\sin t)\, x(t-t_0)$

$\neq$

$y(t-t_0) = \sin(t-t_0)\, x(t-t_0)$

Time Invariant? No

Linearity

C.T & D.T

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

Then:

$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t)$

Examples

$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$ yes

$y[n] = 2x[n] + 3$ No But

$y[n] = x^2[n]$ Not