...ow large should the samples be in Exercise 13 if ...ower of our test is to be 0.95 when the true difference between thread types A and B is 8 kilograms?

...3. How large a sample is required in Exercise 6 if the power of our test is to be 0.8 when the true mean meditation time exceeds the hypothesized value by 1.2σ? Use $\alpha = 0.05$.

34. On testing

$$H_0: \mu = 14,$$
$$H_1: \mu \neq 14,$$

an $\alpha = 0.05$ level t-test is being considered. What sample size is necessary in order that the probability is 0.1 of falsely accepting H_0 when the true population mean differs from 14 by 0.5? From a preliminary sample we estimate σ to be 1.25.

35. A study was conducted at the Department of Veterinary Medicine at Virginia Polytechnic Institute and State University to determine if the "strength" of a wound from surgical incision is affected by the temperature of the knife. Eight dogs were used in the experiment. The incision was performed in the abdomen of the animals. A "hot" and "cold" incision was made on each dog and the strength was measured. The resulting data appear below.
(a) Write an appropriate hypothesis to determine if there is a significant difference in strength between the hot and cold incisions.
(b) Test the hypothesis using a paired t-test. Use a P-value in your conclusion.

Dog	Knife	Strength
1	Hot	5,120
1	Cold	8,200
2	Hot	10,000
2	Cold	8,600
3	Hot	10,000
3	Cold	9,200
4	Hot	10,000
4	Cold	6,200
5	Hot	10,000
5	Cold	10,000
6	Hot	7,900
6	Cold	5,200
7	Hot	510
7	Cold	885
8	Hot	1,020
8	Cold	460

36. Nine subjects were used in an experiment to determine if an atmosphere involving exposure to carbon monoxide has an impact on breathing capability. The data were collected by personnel in the Health and Physical Education Department at Virginia Polytechnic Institute and State University. The data were analyzed in the Statistics Consulting Center at Hokie Land. The subjects were exposed to breathing chambers, one of which contained a high concentration of CO. Several breathing measures were made for each subject for each chamber. The subjects were exposed to the breathing chambers in random sequence. The following data give the breathing frequency in number of breaths taken per minute.

Subject	With CO	Without CO
1	30	30
2	45	40
3	26	25
4	25	23
5	34	30
6	51	49
7	46	41
8	32	35
9	30	28

Make a one-sided test of the hypothesis that mean breathing frequency is the same for the two environments. Use $\alpha = 0.05$. Assume that breathing frequency is approximately normal.

10.11 One Sample: Test on a Single Proportion

Tests of hypotheses concerning proportions are required in many areas. The politician is certainly interested in knowing what fraction of the voters will favor him in the next election. All manufacturing firms are concerned about the proportion of defective items when a shipment is made. The gambler depends on a knowledge of the proportion of outcomes that he considers favorable.

We shall consider the problem of testing the hypothesis that the proportion of successes in a binomial experiment equals some specified value. That is, we are testing the null hypothesis H_0 that $p = p_0$, where p is the parameter of

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the binomial distribution. The alternative hypothesis may be one of the usual one-sided or two-sided alternatives:

$$p < p_0, \quad p > p_0, \quad \text{or} \quad p \neq p_0.$$

The appropriate random variable on which we base our decision criterion is the binomial random variable X, although we could just as well use the statistic $\hat{p} = X/n$. Values of X that are far from the mean $\mu = np_0$ will lead to the rejection of the null hypothesis. Because X is a discrete binomial variable, it is unlikely that a critical region can be established whose size is *exactly* equal to a prespecified value of α. For this reason it is preferable, in dealing with small samples, to base our decisions on P-values. To test the hypothesis

$$H_0\colon\ p = p_0,$$
$$H_1\colon\ p < p_0,$$

we use the binomial distribution to compute the P-value

$$P = P(X \leq x \text{ when } p = p_0).$$

The value x is the number of successes in our sample of size n. If this P-value is less than or equal to α, our test is significant at the α level and we reject H_0 in favor of H_1. Similarly, to test the hypothesis

$$H_0\colon\ p = p_0,$$
$$H_1\colon\ p > p_0,$$

at the α-level of significance, we compute

$$P = P(X \geq x \text{ when } p = p_0)$$

and reject H_0 in favor of H_1 if this P-value is less than or equal to α. Finally, to test the hypothesis

$$H_0\colon\ p = p_0,$$
$$H_1\colon\ p \neq p_0,$$

at the α-level of significance, we compute

$$P = 2P(X \leq x \text{ when } p = p_0) \quad \text{if } x < np_0 \quad \text{or}$$
$$P = 2P(X \geq x \text{ when } p = p_0)$$

if $x > np_0$ and reject H_0 in favor of H_1 if the computed P-value is less than or equal to α.

The steps for testing a null hypothesis about a proportion against various alternatives using the binomial probabilities of Table A.1 are as follows:

Testing a proportion: small samples

1. H_0: $p = p_0$.
2. H_1: Alternatives are $p < p_0$, $p > p_0$, or $p \neq p_0$.
3. Choose a level of significance equal to α.
4. Test statistic: Binomial variable X with $p = p_0$.
5. Computations: Find x, the number of successes, and compute the appropriate P-value.
6. Decision: Draw appropriate conclusions based on the P-value.

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Example 10.10 A builder claims that heat pumps are installed in 70% of all homes being constructed today in the city of Richmond. Would you agree with this claim if a random survey of new homes in this city shows that 8 out of 15 had heat pumps installed? Use a 0.10 level of significance.

Solution
1. H_0: $p = 0.7$.
2. H_1: $p \neq 0.7$.
3. $\alpha = 0.10$.
4. Test statistic: Binomial variable X with $p = 0.7$ and $n = 15$.
5. Computations: $x = 8$ and $np_0 = (15)(0.7) = 10.5$. Therefore, from Table A.1, the computed P-value is

$$P = 2P(X \leq 8 \quad \text{when} \quad p = 0.7) = 2\sum_{x=0}^{8} b(x; 15, 0.7)$$

$$= 0.2622 > 0.10.$$

6. Decision: Do not reject H_0. Conclude that there is insufficient reason to doubt the builder's claim. ┘

In Section 5.3, we saw that binomial probabilities were obtainable from the actual binomial formula or from Table A.1 when n is small. For large n, approximation procedures are required. When the hypothesized value p_0 is very close to 0 or 1, the Poisson distribution, with parameter $\mu = np_0$, may be used. However, the normal-curve approximation, with parameters $\mu = np_0$ and $\sigma^2 = np_0q_0$, is usually preferred for large n and is very accurate as long as p_0 is not extremely close to 0 or to 1. If we use the normal approximation, the z-value for testing $p = p_0$ is given by

$$\boxed{z = \frac{x - np_0}{\sqrt{np_0q_0}},}$$

which is a value of the standard normal variable Z. Hence, for a two-tailed test at the α-level of significance, the critical region is $z < -z_{\alpha/2}$ and $z > z_{\alpha/2}$. For the one-sided alternative $p < p_0$, the critical region is $z < -z_\alpha$, and for the alternative $p > p_0$, the critical region is $z > z_\alpha$.

Example 10.11 A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

Solution
1. H_0: $p = 0.6$.
2. H_1: $p > 0.6$.
3. $\alpha = 0.05$.
4. Critical region: $z > 1.645$.

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5. Computations: $x = 70, n = 100, np_0 = (100)(0.6) = 60$, and

$$z = \frac{70 - 60}{\sqrt{(100)(0.6)(0.4)}} = 2.04. \qquad P = P(Z > 2.04) < 0.025.$$

6. Decision: Reject H_0 and conclude that the new drug is superior.

10.12 Two Samples: Tests on Two Proportions

Situations often arise where we wish to test the hypothesis that two proportions are equal. For example, we might try to show evidence that the proportion of doctors who are pediatricians in one state is equal to the proportion of pediatricians in another state. A person may decide to give up smoking only if he or she is convinced that the proportion of smokers with lung cancer exceeds the proportion of nonsmokers with lung cancer.

In general, we wish to test the null hypothesis that two proportions, or binomial parameters, are equal. That is, we are testing $p_1 = p_2$ against one of the alternatives $p_1 < p_2$, $p_1 > p_2$, or $p_1 \neq p_2$. Of course, this is equivalent to testing the null hypothesis that $p_1 - p_2 = 0$ against one of the alternatives $p_1 - p_2 < 0$, $p_1 - p_2 > 0$, or $p_1 - p_2 \neq 0$. The statistic on which we base our decision is the random variable $\hat{P}_1 - \hat{P}_2$. Independent samples of size n_1 and n_2 are selected at random from two binomial populations and the proportion of successes $\hat{P}_1$ and $\hat{P}_2$ for the two samples is computed.

In our construction of confidence intervals for p_1 and p_2 we noted, for n_1 and n_2 sufficiently large, that the point estimator $\hat{P}_1$ minus $\hat{P}_2$ was approximately normally distributed with mean

$$\mu_{\hat{P}_1-\hat{P}_2} = p_1 - p_2 \qquad \text{and variance } \sigma^2_{\hat{P}_1-\hat{P}_2} = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}.$$

Therefore, our acceptance and critical regions can be established by using the standard normal variable

$$Z = \frac{(\hat{P}_1 - \hat{P}_2) - (p_1 - p_2)}{\sqrt{(p_1 q_1/n_1) + (p_2 q_2/n_2)}}.$$

When H_0 is true, we can substitute $p_1 = p_2 = p$ and $q_1 = q_2 = q$ (where p and q are the common values) in the preceding formula for Z to give the form

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{pq[(1/n_1) + (1/n_2)]}}.$$

To compute a value of Z, however, we must estimate the parameters p and q that appear in the radical. Upon pooling the data from both samples, the pooled estimate of the proportion p is

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2},$$

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where x_1 and x_2 are the number of successes in each of the two samples. Substituting $\hat{p}$ for p and $\hat{q} = 1 - \hat{p}$ for q, the z-value for testing $p_1 = p_2$ is determined from the formula

$$\boxed{z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}[(1/n_1) + (1/n_2)]}}.}$$

The critical regions for the appropriate alternative hypotheses are set up as before using critical points of the standard normal curve. Hence, for the alternative $p_1 \neq p_2$ at the α-level of significance, the critical region is $z < -z_{\alpha/2}$ and $z > z_{\alpha/2}$. For a test where the alternative is $p_1 < p_2$, the critical region is $z < -z_\alpha$, and when the alternative is $p_1 > p_2$, the critical region is $z > z_\alpha$.

Example 10.12 A vote is to be taken among the residents of a town and the surrounding county to determine whether a proposed chemical plant should be constructed. The construction site is within the town limits and for this reason many voters in the county feel that the proposal will pass because of the large proportion of town voters who favor the construction. To determine if there is a significant difference in the proportion of town voters and county voters favoring the proposal, a poll is taken. If 120 of 200 town voters favor the proposal and 240 of 500 county residents favor it, would you agree that the proportion of town voters favoring the proposal is higher than the proportion of county voters? Use an $\alpha = 0.05$ level of significance.

Solution Let p_1 and p_2 be the true proportion of voters in the town and county, respectively, favoring the proposal.

1. H_0: $p_1 = p_2$.
2. H_1: $p_1 > p_2$.
3. $\alpha = 0.05$.
4. Critical region: $z > 1.645$
5. Computations:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{120}{200} = 0.60$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{240}{500} = 0.48$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{120 + 240}{200 + 500} = 0.51.$$

Therefore,

$$z = \frac{0.60 - 0.48}{\sqrt{(0.51)(0.49)[(1/200) + (1/500)]}} = 2.9,$$

$$P = P(Z > 2.9) = 0.0019.$$

6. Decision: Reject H_0 and agree that the proportion of town voters favoring the proposal is higher than the proportion of county voters.

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