In [1]:

```
import matplotlib
import matplotlib.pyplot as plt
import cvxopt
```

Question 1

In [2]:

```
# -*- coding: utf-8 -*-

import numpy as np

#%%
########################################################
######## X-MAC: Trade_off Energy with Delay using GT
########################################################
# Radio subsystem varaible definition

P     = 32.            # Payload [byte]
R     = 31.25          # CC2420 Radio Rate [kbyte/s = Byte/ms]
D     = 8              # number of levels
C     = 5              # neighbors size (connectivity)
N     = C*(D^2)         # number of nodes
#### BE CAREFUL:  Times are in milliseconds (ms)
Lmax  = 5000.          # Maximal allowed Delay (ms)
Emax  = 1.            # MAximal Energy Budjet (J)

L_pbl = 4.             # preamble length [byte]
L_hdr = 9. + L_pbl     # header length [byte]
L_ack = 9. + L_pbl     # ACK length [byte]
L_ps  = 5. + L_pbl     # preamble strobe length [byte]

Tal  = 0.95            # ack listen period [ms]
Thdr = L_hdr/R         # header transmission duration [ms]
Tack = L_ack/R         # ACK transmission duration [ms]
Tps  = L_ps/R          # preamble strobe transmission duration [ms]
Tcw  = 15*0.62         # Contention window size [ms]
Tcs  = 2.60            # Time [ms] to turn the radio into TX and probe the channel (carrier sense)
Tdata = Thdr + P/R + Tack # data packet transmission duration [ms]

### Sampling frequency
Fs   = 1.0/(60*30*1000)    # e.g. Min traffic rate 1 pkt/half_hour = 1/(60*30*1000) pk/ms

# Sleep period: Parameter Bounds
Tw_max  = 500.       # Maximum Duration of Tw in ms
Tw_min  = 100.       # Minimum Duration of Tw in ms
```

1-1 Finding the values of alpha1, alpha2 and alpha 3 for 1pckt/5min

We need to find the values of the frequencies

In [14]:

```
### Sampling frequency
Fs   = 1.0/(60*5*1000)    # e.g. Min traffic rate 1 pkt/ 5 min
```

In [15]:

```
d=1 #to eliminate the max in the equation
Fout=Fs*(((D**2)-(d**2)+(2*d-1))/(2*d-1)) #Node’s Output Traffic Frequency at level d
Fi=Fs*((D^2)-(d^2))/(2*d-1) #Node’s Input Traffic Frequency at level d
I= (2*d+1)/(2*d-1)
Fb=(C-I)*Fout
```

In [16]:

```
alpha1= Tcs+Tal+1.5*Tps*(((Tps+Tal)/2)+Tack+Tdata)*Fb
alpha1
```

Out[16]:

```
3.55053286912
```

In [17]:

```
alpha2=Fout/2
alpha2
```

Out[17]:

```
0.00010666666666666667
```

In [18]:

```
a1=(0.5*(Tps+Tal)+Tcs+Tal+Tack+Tdata)*Fout
a2=((1.5*Tps+Tack+Tdata)*Fi)+(0.75*Tps*Fb)
alpha3=a1+a2
alpha3
```

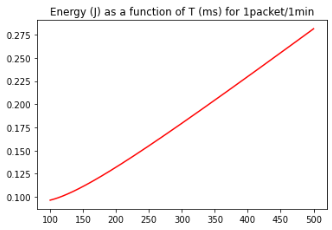
Out[18]:

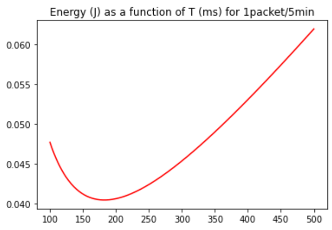
```
0.0015293333333333335
```

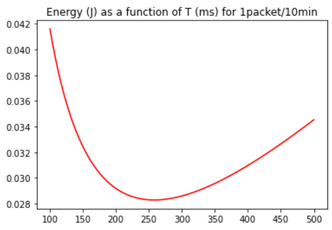
1-2 Plotting Energy as a function of T for different pckt/min rates :

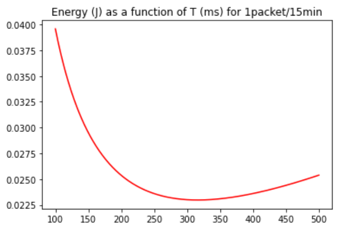
In [19]:

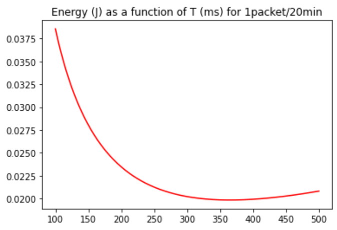
```
for x in [1,5,10,15,20,25,30]:
    Fs   = 1.0/(60*x*1000)
    d=1 #to eliminate the max in the equation
    Fout=Fs*(((D**2)-(d**2)+(2*d-1))/(2*d-1)) #Node’s Output Traffic Frequency at level d
    Fi=Fs*((D^2)-(d^2))/(2*d-1) #Node’s Input Traffic Frequency at level d
    I= (2*d+1)/(2*d-1)
    Fb=(C-I)*Fout
    alpha1= Tcs+Tal+1.5*Tps*(((Tps+Tal)/2)+Tack+Tdata)*Fb
    alpha2=Fout/2
    alpha3=(0.5*(Tps+Tal)+Tcs+Tal+Tack+Tdata)*Fout+((1.5*Tps+Tack+Tdata)*Fi)+(0.75*Tps*Fb)


    # 400 linearly spaced numbers
    Tw = np.linspace(100,500,400)

    # The function that we want to plot
    E=(alpha1/Tw)+alpha2*Tw+alpha3

    # setting the axes at the centre
    fig = plt.figure()

    # plot the function
    plt.plot(Tw,E, 'r')

    plt.title("Energy (J) as a function of T (ms) for 1packet/"+str(x)+"min")

    # show the plot
    plt.show()
```

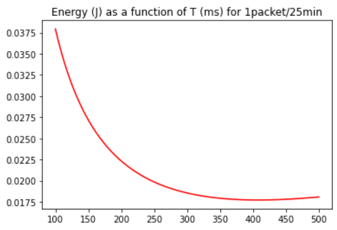


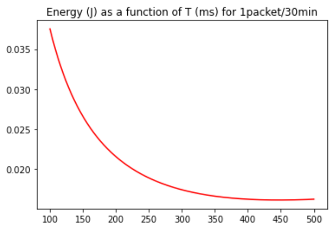












- We can see that for 1 pckt/1 min, there is nearly a linear relationship between E and Tw
- Energy doesn't grow as intensly as it was doing for big value of Tw when the packets are fewer, it even stagnates and seems constant starting from 1pckt/30min for big values of Tw.
- The decay is in Energy is less brutal when the packets are less frequent for smaller values of Tw
- We can also notice that the shape of the plot is convex.

Everything thus seems logical as the lesser are the packets and the longer is the time Tw, less energy is consumed

1-3 Finding the values of beta1 and beta 2

In [20]:

```
d=D
```

```
beta1=0
for i in range (d):
    beta1+=0.5
beta2=0
for i in range(d):
    beta2+=(Tcw/2)+Tdata
beta1,beta2
```

Out[20]:

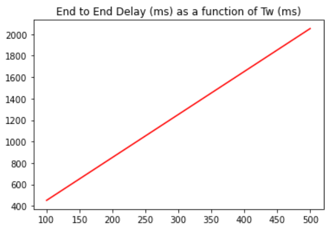
```
(4.0, 52.048)
```

The values found are the same as those found by the professor

1-4 Plotting End to End Delay (ms) as a function of Tw (ms)

In [21]:

```
Tw = np.linspace(100,500,400)


L=beta1*Tw+beta2

# setting the axes at the centre
fig = plt.figure()

# plot the function
plt.plot(Tw,L, 'r')

plt.title("End to End Delay (ms) as a function of Tw (ms)")

# show the plot
plt.show()
```

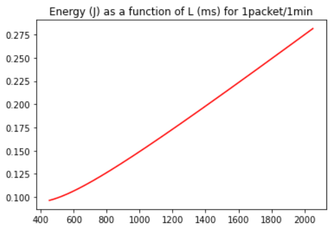


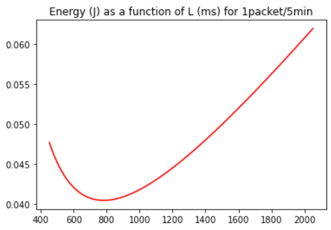
As we expected, the delay is an affine function of Tw

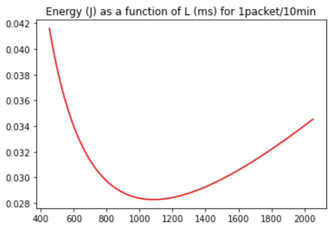
1-5 Plotting the curve E-L for different pckt/min rates

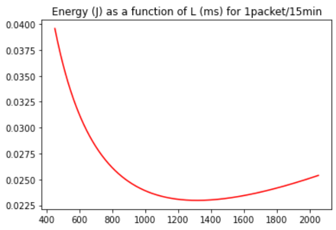
In [22]:

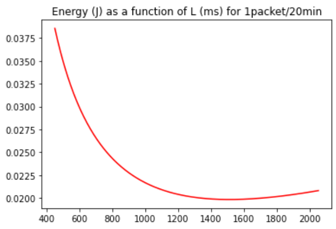
```
for x in [1,5,10,15,20,30]:
    Fs   = 1.0/(60*x*1000)
    d=1 #to eliminate the max in the equation
    Fout=Fs*(((D**2)-(d**2)+(2*d-1))/(2*d-1)) #Node’s Output Traffic Frequency at level d
    Fi=Fs*((D^2)-(d^2))/(2*d-1) #Node’s Input Traffic Frequency at level d
    I= (2*d+1)/(2*d-1)
    Fb=(C-I)*Fout
    alpha1= Tcs+Tal+1.5*Tps*(((Tps+Tal)/2)+Tack+Tdata)*Fb
    alpha2=Fout/2
    alpha3=(0.5*(Tps+Tal)+Tcs+Tal+Tack+Tdata)*Fout+((1.5*Tps+Tack+Tdata)*Fi)+(0.75*Tps*Fb)


    # 400 linearly spaced numbers
    L = np.linspace(beta1*100+beta2,beta1*500+beta2,400)

    # the function, which is y = x^2 here
    E=(alpha1*beta1/(L-beta2))+alpha2*(L-beta2)/beta1+alpha3

    # setting the axes at the centre
    fig = plt.figure()

    # plot the function
    plt.plot(L,E, 'r')

    plt.title("Energy (J) as a function of L (ms) for 1packet/"+str(x)+"min")

    # show the plot
    plt.show()
```

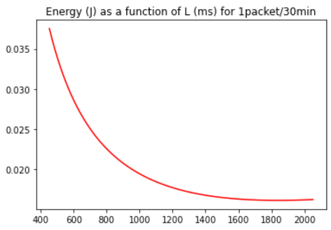












- We can see that for 1 pckt/1 min, there is neearly a linear relationship between E and L
- Energy doesn't grow as intensly as it was doing for big value of L when the packets are fewer, it even stagnates and seems constant starting from 1pckt/30min for big values of L.
- The decay is in Energy is less brutal when the packets are less frequent for smaller values of L
- We can also notice that the shape of the plot is convex.

Everything thus seems logical as the lesser are the packets and the longer is the delay, less energy is consumed. We could have guessed these shapes as we already had the shapes of E-Tw and the relationship between Tw and L is affine.

Question 2

2-1 Optimal energy for several values of L

In [23]:

```
from scipy import optimize
```

In [24]:

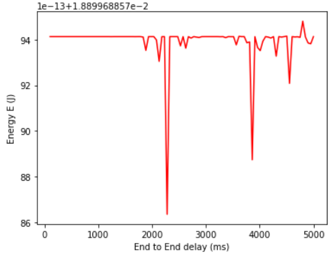
```
Tw=250
Ttx=0.5*Tw+Tack+Tdata
Etx=(Tcs+Tal+Ttx)*Fout
```

In [25]:

```
minimums=[]
dict={}
#for L_max in np.linspace(beta1*100+beta2,beta1*500+beta2,500):
for L_max in np.linspace(100,5000,100):
    def E_min(x):
        return (alpha1/x)+alpha2*x+alpha3 #x=Tw
    def f_1(x):
        return beta1*x+beta2-L_max
    def f_2(x):
        return Tw_min-x
    def f_3(x):
        return C*Etx-0.25 #I0=C
    #cons = ({'type': 'ineq','fun' : f_1},{'type': 'ineq','fun' : f_2})
    cons = ({'type': 'ineq','fun' : f_1},{'type': 'ineq','fun' : f_2},{'type': 'ineq','fun' : f_3})
    x0=250
    minimal_value=(optimize.minimize (E_min,x0,constraints= cons).fun)
    dict[(L_max-beta2)/beta1]=minimal_value
    minimums.append(minimal_value)
```

In [46]:

```
L_max = np.linspace(100,5000,100)

fig = plt.figure()
fig.subplots_adjust(top=2)
ax1 = fig.add_subplot(211)
ax1.set_ylabel('Energy E (J)')
ax1.set_xlabel('End to End delay (ms)')
plt.legend("")
plt.plot(L_max,minimums, 'r')

plt.show()
```



In [45]:
```
E_best=np.amin(minimums);E_best
```

Out[45]:
```
0.01889968857863504
```

In [47]:
```
E_average=np.average(minimums);E_average
```

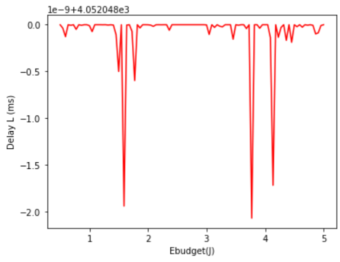
Out[47]:
```
0.018899688579392105
```

- We can see that there are some minimal energy values (inferior by 7*10(-13)J than the average) attained for some values of L_max, especially for L_max approximately equal to 2100
- This small magnitude of 10**(-13) is due to the fact that for 1pckt/30 min, energy is approximatively constant for values of L>1000ms, as we saw in the plot of 2-1
- The energy values of L>1000ms seem logical as we saw in 2-1 that the values of energy are approximatively equal to 0.018 for Fs that realises 1pckt/30 min

2-2 Optimal values of L for several values of Ebduget

In [27]:
```
minimums_2=[]
dict_2={}
for E_budget in np.linspace(0.5,5,100):
    def L_min(x):
        return beta1*x+beta2
    def f_1(x):
        return (alpha1/x)+alpha2*x+alpha3-E_budget
    def f_2(x):
        return Tw_min-x
    def f_3(x):
        return 5*Etx-0.25
    #cons = ({'type': 'ineq','fun' : f_1},{'type': 'ineq','fun' : f_2})
    cons = ({'type': 'ineq','fun' : f_1},{'type': 'ineq','fun' : f_2},{'type': 'ineq','fun' : f_3})
    x0=1000
    minimal_value=optimize.minimize (L_min,x0,constraints= cons).fun
    dict_2[E_budget]={minimal_value}
    minimums_2.append(minimal_value)
```

In [44]:
```
E_budget = np.linspace(0.5,5,100)
fig = plt.figure()
fig.subplots_adjust(top=2)
ax1 = fig.add_subplot(211)
ax1.set_ylabel('Delay L (ms)')
ax1.set_xlabel('Ebudget(J)')
plt.plot(E_budget,minimums_2, 'r')
plt.show()
```



In [36]:
```
L_best=np.amin(minimums_2);L_best
```

Out[36]:
```
4052.0479999979316
```

In [37]:
```
L_average=np.average(minimums_2);L_average
```

Out[37]:
```
4052.047999999912
```

- We can see that there are some minimal energy values attained for some values of Ebudget, especially for Ebudget approximately equal to 1.6 and 3.8 where the value of L is approximately smaller by 2*(10**-9)ms than the average

I tried with the CVXPY method in order to see if my results are logical but it showed me that there is an arethmetical error, you can see my code below. I can't see why I have this error

In [211…
```
import cvxpy as cp
from cvxpy import *
```

In [228…
```
L_max=1200
from cvxopt import matrix, solvers
A = matrix([ [beta1, -1.0 , 0.0], [0, 0, 0] ])
b = matrix([ -beta2+L_max, -Tw_min, -5*Etx+0.25 ])
c = matrix([ alpha1, alpha2 ])
sol=solvers.lp(c,A,b)
```

```
---------------------------------------------------------------------------
ArithmeticError                           Traceback (most recent call last)
~/opt/anaconda3/lib/python3.8/site-packages/cvxopt/misc.py in factor(W, H, Df)
   1428                 if type(F['S']) is matrix:
-> 1429                     lapack.potrf(F['S'])
   1430                 else:

ArithmeticError: 2

During handling of the above exception, another exception occurred:

ArithmeticError                           Traceback (most recent call last)
~/opt/anaconda3/lib/python3.8/site-packages/cvxopt/coneprog.py in conelp(c, G, h, dims, A, b, primalstart, dual
start, kktsolver, xnewcopy, xdot, xaxpy, xscal, ynewcopy, ydot, yaxpy, yscal, **kwargs)
    679         for rti in W['rti']: rti[::rti.size[0]+1 ] = 1.0
--> 680         try: f = kktsolver(W)
    681         except ArithmeticError:

~/opt/anaconda3/lib/python3.8/site-packages/cvxopt/coneprog.py in kktsolver(W)
    584         def kktsolver(W):
--> 585             return factor(W)
    586 

~/opt/anaconda3/lib/python3.8/site-packages/cvxopt/misc.py in factor(W, H, Df)
   1443                 if type(F['S']) is matrix:
-> 1444                     lapack.potrf(F['S'])
   1445                 else:

ArithmeticError: 2

During handling of the above exception, another exception occurred:

ValueError                                Traceback (most recent call last)
<ipython-input-228-618aa1f9d7b1> in <module>
      4 b = matrix([ -beta2+L_max, -Tw_min, -5*Etx+0.25 ])
      5 c = matrix([ alpha1, alpha2 ])
----> 6 sol=solvers.lp(c,A,b)

~/opt/anaconda3/lib/python3.8/site-packages/cvxopt/coneprog.py in lp(c, G, h, A, b, kktsolver, solver, primalst
art, dualstart, **kwargs)
   3007             'primal slack': pslack, 'dual slack': dslack}
   3008 
-> 3009     return conelp(c, G, h, {'l': m, 'q': [], 's': []}, A,  b, primalstart,
   3010         dualstart, kktsolver = kktsolver, options = options)
   3011 

~/opt/anaconda3/lib/python3.8/site-packages/cvxopt/coneprog.py in conelp(c, G, h, dims, A, b, primalstart, dual
start, kktsolver, xnewcopy, xdot, xaxpy, xscal, ynewcopy, ydot, yaxpy, yscal, **kwargs)
    680         try: f = kktsolver(W)
    681         except ArithmeticError:
--> 682             raise ValueError("Rank(A) < p or Rank([G; A]) < n")
    683 
    684     if primalstart is None:

ValueError: Rank(A) < p or Rank([G; A]) < n
```

Question 3

In [31]:
```
E_worst=np.amax(minimums);E_worst
```

Out[31]:
```
0.018899688579481086
```

In [32]:
```
L_worst=np.amax(minimums_2);L_worst
```

Out[32]:
```
4052.048
```

We will formalize the optimization problem:

In [34]:
```
def f_0(x):
    return np.log(np.abs(E_worst-x[0]))+np.log(np.abs(L_worst-x[1]))
def f_1(x):
    return (alpha1/x[2])+alpha2*x[2]+alpha3-E_worst
def f_2(x):
    return (alpha1/x[2])+alpha2*x[2]+alpha3-x[0]
def f_3(x):
    return beta1*x[2]+beta2-L_worst
def f_4(x):
    return beta1*x[2]+beta2-x[1]
def f_5(x):
    return Tw_min-x[2]
def f_6(x):
    return 0.25-5*Etx
#cons = ({'type': 'ineq','fun' : f_1},{'type': 'ineq','fun' : f_2})
cons = ({'type': 'ineq','fun' : f_1},{'type': 'ineq','fun' : f_2},{'type': 'ineq','fun' : f_3},{'type': 'ineq',
x0=[5,2000,300]
optimize.minimize (f_0,x0,constraints= cons)
```

Out[34]:
```
     fun: 1.1094015331840739
     jac: array([-6.76679065e+02, -4.87327576e-04,  0.00000000e+00])
 message: 'Positive directional derivative for linesearch'
    nfev: 34
     nit: 10
    njev: 6
  status: 8
 success: False
       x: array([1.74218757e-02, 2.00000000e+03, 3.00000000e+02])
```

We consequently find this result for this opimization problem