1 Pattern break detection

The chainladder method ranks among the most frequently applied loss reserving techniques in insurance. Originally conceived as a purely computational algorithm, there have since been various attempts to cast it in a proper stochastic framework. No matter which of these models chooses to employ, there is one particular assumption the reserving actuary cannot escape: that the development pattern observed in earlier cohorts is somehow applicable to later ones. This seems very reasonable, of course: if our models are to use the past as a guide to the future, they will be bound to postulate the existence of *some* element of constancy in the underlying data generating process.

Consider, for example, a cumulative claims process $(C_{ij})_{0\leq i\leq I, 0\leq j\leq J}$ (index i denoting the cohort and j the development period) satisfying the model assumptions of Mack's method:

$$\mathbb{E}[C_{ij} \parallel C_{i,j-1}, \ldots, C_{i1}] = \mathbb{E}[C_{ij} \parallel C_{i,j-1}] = f_{j-1}C_{i,j-1} \tag{1}$$

$$\mathrm{Var}[C_{ij} \parallel C_{i,j-1}, \ldots, C_{i1}] = \mathrm{Var}[C_{ij} \parallel C_{i,j-1}] = \sigma^2_{j-1}C_{i,j-1} \tag{2}$$

, for $i \in \{0, \ldots, I\}$ and

$$\{C_{i,0}, \ldots, C_{i,I-i}\}, \{C_{i',0}, \ldots, C_{i',I-i'}\} \text{ independent for } i \neq i' \,. \tag{3}$$

Using the rules of conditional probability, we can rewrite the first equation as

$$\mathbb{E}[\frac{C_{ij}}{C_{i,j-1}} \parallel C_{i,j-1}] = \mathbb{E}[F_{i,j-1} \parallel C_{i,j-1}] = f_{j-1} \tag{4}$$

from which it becomes clear that this is in fact a first-order stationarity assumption over the cohorts on the time series $(F_{i,j-1})_{0\leq i\leq I}$. If we further assume that the claims process is conditionally Gaussian,

$$\begin{aligned} C_{ij} \mid C_{i,j-1} = (f_{j-1}C_{i,j-1} + \sigma_{j-1}\sqrt{C_{i,j-1}}\epsilon_{i,j-1}) \mid C_{i,j-1} \\ \sim \mathcal{N}(f_{j-1}C_{i,j-1}, \sigma^2_{j-1}C_{i,j-1}) \,, \end{aligned} \tag{5}$$

with $\epsilon_{ij} \sim \mathcal{N}(0,1)$ i.i.d., then $F_{i,j-1} \mid C_{i,j-1} \sim \mathcal{N}(f_{j-1}, \sigma^2_{j-1}/C_{j-1})$ and we can consider the residuals

$$\frac{(F_{i,j-1} - f_{j-1})\sqrt{C_{i,j-1}}}{\sigma_{j-1}} \sim \mathcal{N}(0,1) \,,$$

which will be independent for i ranging over $\{0, \ldots, I\}$.