IOP Conference Series: Materials Science and Engineering

Ground and confined underground waters and their salt content

To cite this article: N Ravshanov *et al* 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **896** 012047

IOP Conf. Series: Materials Science and Engineering **896** (2020) 012047 doi:10.1088/1757-899X/896/1/012047

Ground and confined underground waters and their salt content

N Ravshanov[1], **Sh Daliev**[2], **Z Abdullaev**[3] **and O Khafizov**[3]

[1] Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, 108 Amir Temur ave., Tashkent, 100200, Uzbekistan
[2] Samarkand State University, 15 University Blvd., Samarkand, 140104, Uzbekistan
[3] Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39 Kari Niyazov str., Tashkent, 100000, Uzbekistan

prince0102@mail.ru

Abstract. A article developed a mathematical model for monitoring and predicting changes in groundwater level and salt concentration, as well as for representing the process of flooding, salinization and waterlogging in the design of hydraulic structures. A qualitative analysis of the research work on this issue is presented. Since the process is represented by a nonlinear differential equation, a numerical algorithm based on finite-difference schemes was developed to solve it. Mathematical models of the processes of geofiltration of salt migration have been improved and computational algorithms have been developed. A numerical algorithm has been developed for calculating the problem of changing the groundwater level, taking into account active porosity, flow rate in two-layer formations.

1. Introduction

Monitoring and prediction of the salt regime of soil is largely dependent on the degree to which the basic parameters of the transport and diffusion of salts in layered porous media are determined. For a detailed and comprehensive study of the process of salt transfer in porous media, it is important to develop an adequate mathematical model that describes the basic properties of the object of study. It can be noted here that qualitative and quantitative changes in the salt content in the study area under study (in multilayer porous media) under the conditions of a stationary water-salt regime can be predicted using analytical methods. Of practical interest is the prediction of stratification and secondary salinization of soil aeration zones as a result of prolonged irrigation and rising groundwater levels over time.

For the needs of the national economy, various kinds of hydraulic structures are being built: dams on rivers to regulate the flow of water at hydroelectric power stations, reservoirs and canals. However, the construction of these structures leads in some cases to negative consequences - subsoil water supply, flooding, salinization and waterlogging of land, which causes enormous damage to the national economy.

Reclamation work to combat drowning, salinization, waterlogging requires large capital investments. Therefore, the development of effective methods for solving problems of predicting changes in groundwater levels is one of the urgent problems.

In conditions of water shortages, which are important for the national economy, the problem of water supply is especially acute in the most environmentally complex areas. Under these conditions, groundwater is one of the main sources of drinking water supply. An effective tool for determining the geofiltration parameters of groundwater intakes is an easy-to-implement numerical model, a computational algorithm, and a software package implemented on a computer. On the other hand, the negative consequences of using groundwater for irrigation are known to be inevitable. In modern conditions, it is important to predict the physicochemical properties of soils as a result of groundwater irrigation. In this case, it is important to predict the state of groundwater using a mathematical apparatus - "model - computational algorithm - computational experiment".

Prediction of changes in groundwater levels is largely dependent on the degree to which the main parameters of the external source and evaporation in layered porous media are determined. For a detailed and comprehensive study of the process of changing groundwater levels in porous media, the development of an adequate mathematical model that describes the basic properties of the object of study is important. It can be noted here that qualitative and quantitative changes in groundwater levels in the study area (in multilayer porous media) under the conditions of a stationary water-salt regime can be predicted using analytical methods.

The basics of the science of groundwater movement (hydrogeology) are associated with the names of A. Darcy, J. Dupuis, N.E. Zhurkovsky, F. Forchheimer and others. A major role in the development of mathematical methods with the intensive development of the theory and practice of groundwater movement was also played by the works of F.B. Abutalieva, E.B. Abutalieva, P.Ya. Polubarinova-Kochina, V.I.Aravina, S.N. Numerova, G.N. Kamensky, A.I. Silina-Bekchurina, P.P. Klimentova, G.B. Pykhacheva, V.A. Mironenko, I.K. Gavich et al.

The theoretical foundations of the hydrodynamic method for assessing the operational reserves of groundwater are considered in the works of F.M. Bochevera, N.N. Bindeman, N.I. Plotnikova, V.M. Shestakova, L.S. Yazvina, E.N. Bondareva, V.N. Nikolaevsky, V.I. Lavrika, V.I. Penkovsky and others

VN Shchelkachev, M. A. Guseyn-zade, V. M. Shestakov, N. N. Verigin, I. A. Charny, F made a great contribution to the development of the theory of the elastic regime in the strata and the study of the problems of fluid flow in the layers. M. Bochever et al. Among foreign researchers, M.S. Hantusha, S.E. Jacob for the first time, apparently, pointed out the need to take into account the elastic regime in a poorly permeable layer.

To date, a number of studies have been carried out to study the influence of the Aral Sea on the hydrodynamic and hydrochemical regimes of groundwater. Here you can note the work performed under the guidance of prominent Uzbek scientists like F.B.Abutaliev, N.I. Khojibaev, U. U. Umarova, I. Khabibullaev, R.N. Usmanova, Zh.S. Sidikova and others, who were mainly devoted to the study of the hydrodynamic state of the territories adjacent to the Aral Sea, the calculation of groundwater exploitation reserves, while the issues of a joint and comprehensive study of hydrodynamic and hydrochemical processes under the influence of the Aral Sea were not considered. Therefore, a comprehensive study of hydrodynamic and hydrochemical elements and groundwater regimes and their total impact on the surrounding hydrogeological environment is necessary.

The design of groundwater intakes is carried out by conducting numerous computational experiments using mathematical models of geofiltration and salt transfer in the underground hydrosphere, in order to determine the main indices and parameters of the object under study [1].

One of the effective tools for determining the geofiltration and hydrogeochemical parameters of groundwater intakes is an easily implemented mathematical apparatus - a model, a numerical algorithm and computer software.

It should be noted that numerical study of the changes in the level of ground and confined undergroundwaters and the salt content in them was carried out by many scientists; significant theoretical and applied results were obtained.

In article [2], an urgent problem is solved related to the process of changing the groundwater level and the transfer of mineral salts in soils, which describes a system of partial differential equations and various initial, internal, and boundary conditions corresponding to them. To derive a mathematical model of the process under consideration, a detailed review of scientific papers devoted to various aspects and mathematical support of the object of study is given.

Thus, if, on the one hand, the negative influence of mineralized water use for irrigation is known, on the other hand, the use of these waters becomes inevitable. Consequently, in the current situation, predicting the physicochemical properties of soils as a result of irrigation with mineralized water is of paramount importance, and here the main aspect is predicting the state of groundwater by an adequate mathematical tool - "model - numerical algorithm - software" with computational experiments.

Monitoring and prediction of the salt regime of soil to a large extent depends on the degree to which the basic parameters of salts transfer and diffusion in layered porous media are determined. For a detailed and comprehensive study of the process of salt transfer in porous media, the development of an adequate mathematical model that describes the basic properties of the object under study is important. It can be noted here that qualitative and quantitative changes in salt content in the area under study (multilayer porous media) under the conditions of a stationary water-salt regime can be predicted using analytical methods. Of practical interest is the prediction of stratification and secondary salinization of aeration zones of soil as a result of long-term irrigation and groundwater level rise over time.

In [3], a comparative analysis of calculation results obtained by numerical implementation of three mathematical models of dynamic processes in a water-saturated soil massif is presented.

Two sets of experiments were carried out in [4], taking into account various internal boundaries with a constant head. There were observed fluctuations in the groundwater level and the process of seawater intrusion in coastal multilayered aquifers. The two-dimensional model was worked out to study the seawater intrusion into coastal aquifers under the influence of tidal fluctuations and groundwater exploitation. The hydrogeological parameters of the model were calibrated using records of groundwater level and salinity.

In [5], a mathematical model of geofiltration and contaminated groundwater migration was constructed. The problem was solved in two stages. At the first stage, the natural piezometric surface of the groundwater mirror was built up at zero values of technogenic leaks. The sought for value was the activity of infiltration feed in each block of the model. At the second stage, the problem of mass transfer with set technogenic loads was solved and a forecast of pollution halos spread for a 20 year period was given. To improve the environmental situation, the creation of systematic drainage systems in the most flooded urban areas has been proposed.

Analytical models were obtained in [6] to predict groundwater pollution in isotropic and homogeneous porous media. The influence of dispersion and diffusion coefficients was included into solution of advection and dispersion equation subjected to transitional (time-dependent) boundary conditions. The deceleration coefficient and zero-order conditions were included to the statement of the problem. Analytical solutions were obtained using the Laplace integral transform method and the linear isotherm concept. Numerical solutions were obtained by explicit finite-difference methods and were compared with analytical solutions. Numerical results were analyzed for various types of geological porous formations, that is, an aquifer.

In [7], a model of the dynamics of groundwater in a stationary flow is presented, guided by Darcy's law through porous media, using it to study a 2D aquifer with a reservoir of water under constant slope consisting of homogeneous and isotropic media. A computational procedure was developed on the basis of the finite-difference method, for solving the problem both in a homogeneous isotropic and in a homogeneous anisotropic medium.

In [8], under the assumption of plane movement of groundwater in a closed aquifer, a numerical model of a two-dimensional groundwater flow in a closed aquifer with a variable h was created, the model was calculated by the finite element method. A numerical study showed that the calculated results of the finite element method were in good agreement with the calculated result of the finite difference method.

In [9], a mathematical model was developed for predicting groundwater levels in two-layer formations. The authors have considered in mathematical modeling the geofiltration process in a two-layer medium consisting of two layers: soil (with low leak-off capacity) and water.

The studies in [10] are devoted to numerical modeling of water and salt transfer process in soil. To conduct a comprehensive study, a mathematical model was proposed taking into account the colmatage of the soil pores with fine particles over time; the changes in soil permeability coefficient, water loss and filtration coefficient; changes in the initial porosity and the porosity of settled mass; an effective numerical algorithm was proposed as well.

In article [11], the problem of the process of transient groundwater filtration in a porous medium is considered an urgent problem: in the design and development of hydraulic structures, regulation of groundwater runoff, flooding, salinization and waterlogging of land, which causes enormous damage to the national economy. To develop a mathematical model of the process, the article analyzes in detail the research work associated with the bottom problem and proposes a mathematical apparatus for researching and predicting changes in the level of groundwater during the process of filtering them in porous media.

In the article [12], a fuzzy-determined mathematical model of the restoration of reserves and groundwater quality in a single-layer and two-layer structure of aquifers is proposed.

In [13], an analysis was made of freshwater reserves taking into account economic development and climate change. For accuracy of forecasting and monitoring the level of groundwater and pressure water, a physical model is given. A new method of high-precision forecasting of the groundwater level using the latest features of information and communication technology in order to make managerial decisions and respond to the situation when raising or lowering groundwater has been developed.

In the article [14], the authors used a mathematical model to perform an asymptotic analysis of the fields of excess pressure with filtration consolidation in a double relaxation system. The analysis showed that it is necessary, especially at the initial stages of consolidation, to take into account the relaxation properties of the deformed porous medium and the filtration process, which, in particular, is important in the case of sharp and significant changes in pressure. In the general case (where relaxation parameters are not assumed to be small), the dynamics of the porous filtration consolidation among can be numerically modeled in the framework of the mathematical model under consideration based on the developed algorithm.

In [15], a mathematical model of the process of salt movement during the filtration transfer of salt is developed taking into account the process of infiltration in unsaturated layered soils. To solve the problem, a solution was obtained by the finite difference method. As a result of the implementation of the problem, numerical experiments were carried out and an analysis of the results was carried out.

2. Problem Statement

For mathematical modeling of monitoring and predicting the groundwater level and hydrochemical processes, taking into account the interaction of external factors: evaporation and infiltration, we present here the studied object schematically in the following form (Fig. 1).

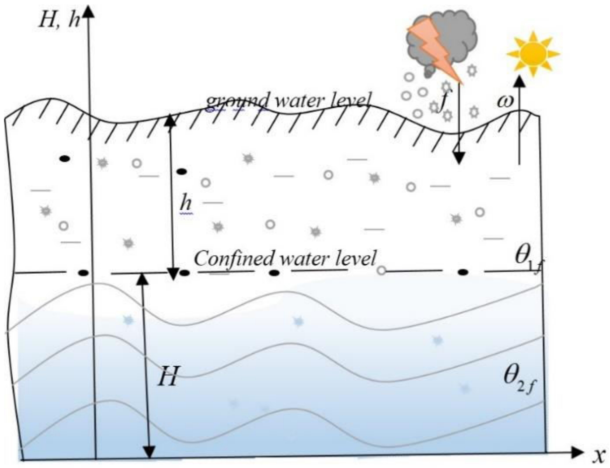

Figure 1. Schematic image of the studied object

The accepted conditions for predicting the ground water level and the changes in salt content (ground and confined aquifers) during the filtration process give reason to present the mathematical model of the object in the form of a system of nonlinear partial differential equations:

$$\left.\begin{aligned} \mu n_0 \frac{\partial h}{\partial t} &= \frac{\partial}{\partial x}\left(k_b h \frac{\partial h}{\partial x}\right) - k_b \frac{h-H}{m} + f - \omega, \\ \mu * \frac{\partial H}{\partial t} &= \frac{\partial}{\partial x}\left(T \frac{\partial H}{\partial x}\right) + k \frac{h-H}{m} - \eta Q. \end{aligned}\right\} \quad (1)$$

where $h(x,t)$, $H(x,t)$ are the ground and confined water levels; μ, μ^* - water loss coefficients; m - the separating layer capacity; k_b, k - filtration coefficients of the upper and lower layers; T - filtration conductivity of the main horizon; Q - debit; f - external source; ω - evaporation; n_0 - active porosity of soil in the respective zones; η - a coefficient to reduce the model to dimensional view.

System (1) is solved under the following initial and boundary conditions:

$$h(x,0) = h_0(x), \quad H(x,0) = H_0(x), \quad (2)$$

$$n_0 h \frac{\partial h}{\partial x}\Big|_{x=0} = -(h - h_0), \quad n_0 h \frac{\partial h}{\partial x}\Big|_{x=L} = (h - h_0), \quad (3)$$

$$\mu^* \frac{\partial H}{\partial x}\Big|_{x=0} = -(H - H_0), \quad \mu^* \frac{\partial H}{\partial x}\Big|_{x=L} = (H - H_0). \quad (4)$$

where $h_0(x)$, $H_0(x)$ are the boundary values of groundwater and confined water; initial value of groundwater and confined water levels.

The accepted filtration conditions give reason to present the mathematical model of salt transfer in the form

$$\left.\begin{aligned} \mu h \frac{\partial \theta_1}{\partial t} &= \frac{\partial}{\partial x}(D_1 h \frac{\partial \theta_1}{\partial x}) - v_x h \frac{\partial \theta_1}{\partial x} + f_1 \cdot \theta_{1f}, \\ \mu^* H \frac{\partial \theta_2}{\partial t} &= \frac{\partial}{\partial x}(D_2 H \frac{\partial \theta_2}{\partial x}) - v_x H \frac{\partial \theta_2}{\partial x} + f_2 \cdot \theta_{2f}. \end{aligned}\right\} \quad (5)$$

where $\theta_1(x,t)$, $\theta_2(x,t)$ are salt content in groundwater aquifers; v_x - components of the filtration rate; D_1, D_2 - salt diffusion coefficients; θ_{1f}, θ_{2f} - salt content fed from infiltration waters.

The system of equations (5) is solved under the following initial and boundary conditions:

$$\theta_1(x,t)\big|_{t=t_0} = (\theta_1)_0, \;\; \theta_2(x,t)\big|_{t=t_0} = (\theta_2)_0, \tag{6}$$

$$\mu h \frac{\partial \theta_1}{\partial x}\bigg|_{x=0} = -(\theta_1 - (\theta_1)_0), \;\; \mu h \frac{\partial \theta_1}{\partial x}\bigg|_{x=L} = (\theta_1 - (\theta_1)_0), \tag{7}$$

$$\mu^* H \frac{\partial \theta_2}{\partial x}\bigg|_{x=0} = -(\theta_2 - (\theta_2)_0), \;\; \mu^* H \frac{\partial \theta_2}{\partial x}\bigg|_{x=L} = (\theta_2 - (\theta_2)_0), \tag{8}$$

$$\theta_1(x,t)\big|_{x=m-0} = \theta_2(x,t)\big|_{x=m+0}, \tag{9}$$

$$D_1 h \frac{\partial \theta_1}{\partial x}\bigg|_{x=m-0} = D_2 H \frac{\partial \theta_2}{\partial x}\bigg|_{x=m+0}. \tag{10}$$

Here $\theta_{10}(x,t_0)$, $\theta_{20}(x,t_0)$ are the initial salt distributions in the ground and confined aquifers.

To solve the problem (1) and (5), introduce the dimensionless variables [10-12]:

$$h^* = \frac{h}{h_0}, \; x^* = \frac{x}{L}, \; H^* = \frac{H}{L}, \; k^* = \frac{k}{k_0}, \; k_b^* = \frac{k_b}{k_{b0}}, \; T^* = \frac{T}{T_0}, \; \tau = \frac{k_{b0} h_0}{\mu n_0 L^2} t,$$

$$\theta_1^* = \frac{\theta_1}{(\theta_1)_0}, \; \theta_2^* = \frac{\theta_2}{(\theta_2)_0}, \; D_1^* = \frac{D_1}{(D_1)_0} \; D_2^* = \frac{D_2}{(D_2)_0} \; T_0 = \frac{\mu^* k_{b0} h_0}{\mu n_0}.$$

Then, the problem (1) - (4) has the form:

$$\left.\begin{aligned} \frac{\partial h^*}{\partial \tau} &= \frac{\partial}{\partial x^*}\left(k_b^* h^* \frac{\partial h^*}{\partial x^*}\right) - \frac{L^2}{m h_0} k_b^* h^* + \frac{H_0 L^2}{m h_0^2} k_b^* H^* + \frac{L^2}{k_{b0} h_0^2}(f - \omega), \\ \frac{\partial H^*}{\partial \tau} &= \frac{\partial}{\partial x^*}\left(T^* \frac{\partial H^*}{\partial x^*}\right) + \frac{n_0 k_0 \mu L^2}{\mu^* k_{b0} H_0 m} k^* h^* - \frac{n_0 k_0 \mu L^2}{\mu^* k_{b0} h_0 m} k^* H^* - \frac{n_0 \mu L^2}{\mu^* k_{b0} H_0 h_0} \eta Q. \end{aligned}\right\} \tag{11}$$

Under initial and boundary conditions:

$$h^*(x,\tau_0) = h_0, \;\; H^*(x,\tau_0) = H_0 \;\text{ at }\; \tau = \tau_0. \tag{12}$$

$$\frac{n_0 h_0^2}{L} h^* \frac{\partial h^*}{\partial x^*}\bigg|_{x^*=0} = -(h_0 h^* - h_0), \;\; \frac{n_0 h_0^2}{L} h^* \frac{\partial h^*}{\partial x^*}\bigg|_{x^*=1} = (h_0 h^* - h_0), \tag{13}$$

$$\frac{\mu^* H_0}{L} \frac{\partial H^*}{\partial x^*}\bigg|_{x^*=0} = -(H_0 H^* - H_0), \;\; \frac{\mu^* H_0}{L} \frac{\partial H^*}{\partial x^*}\bigg|_{x^*=1} = (H_0 H^* - H_0). \tag{14}$$

Problem (5) - (10) in dimensionless form takes the following form:

$$\left.\begin{aligned}
h^*\frac{\partial\theta_1^*}{\partial\tau}&=\frac{D_{10}}{k_{b0}h_0}\frac{\partial}{\partial x^*}(D_1^*h^*\frac{\partial\theta_1^*}{\partial x^*})-\frac{L}{k_{b0}h_0}v_xh^*\frac{\partial\theta_1^*}{\partial x^*}+\frac{L^2}{k_{b0}h_0\theta_{10}}f_1\cdot\theta_{1f},\\
H^*\frac{\partial\theta_2^*}{\partial\tau}&=\frac{D_{20}}{k_0H_0}\frac{\partial}{\partial x}(D_2^*H^*\frac{\partial\theta_2^*}{\partial x^*})-\frac{L}{k_0H_0}v_xH^*\frac{\partial\theta_2^*}{\partial x^*}+\frac{L^2}{k_0H_0\theta_{20}}f_2\cdot\theta_{2f}.
\end{aligned}\right\}\tag{15}$$

Under initial and boundary conditions:

$$\theta_1^*(x,\tau)\Big|_{\tau=\tau_0}=(\theta_1)_0,\ \ \theta_2^*(x,\tau)\Big|_{\tau=\tau_0}=(\theta_2)_0,\tag{16}$$

$$\begin{aligned}
\frac{\mu h_0(\theta_1)_0}{L}h^*\frac{\partial\theta_1^*}{\partial x^*}\Big|_{x^*=0}&=-((\theta_1)_0\theta_1^*-(\theta_1)_0),\\
\frac{\mu h_0(\theta_1)_0}{L}h^*\frac{\partial\theta_1^*}{\partial x^*}\Big|_{x^*=1}&=((\theta_1)_0\theta_1^*-(\theta_1)_0),
\end{aligned}\tag{17}$$

$$\begin{aligned}
\frac{\mu^*H_0(\theta_2)_0}{L}H^*\frac{\partial\theta_2^*}{\partial x^*}\Big|_{x^*=0}&=-((\theta_2)_0\theta_2^*-(\theta_2)_0),\\
\frac{\mu^*H_0(\theta_2)_0}{L}H^*\frac{\partial\theta_2^*}{\partial x^*}\Big|_{x^*=1}&=((\theta_2)_0\theta_2^*-(\theta_2)_0),
\end{aligned}\tag{18}$$

$$(\theta_1)_0\theta_1^*(x,t)\Big|_{Lx^*=m-0}=(\theta_2)_0\theta_2^*(x,t)\Big|_{Lx^*=m+0},\tag{19}$$

$$\frac{(D_1)_0h_0(\theta_1)_0}{L}D_1^*h^*\frac{\partial\theta_1^*}{\partial x^*}\Big|_{Lx^*=m-0}=\frac{(D_2)_0H_0(\theta_2)_0}{L}D_2^*H^*\frac{\partial\theta_2^*}{\partial x^*}\Big|_{Lx^*=m+0}.\tag{20}$$

In the future, for simplicity, we will omit the "*" sign in equations. Problems (11) and (14) in dimensionless variables can be written as follows:

$$\left.\begin{aligned}
\frac{\partial h}{\partial\tau}&=\frac{\partial}{\partial x}(k_bh\frac{\partial h}{\partial x})-\frac{L^2}{mh_0}k_bh+\frac{H_0L^2}{mh_0^2}k_bH+\frac{L^2}{k_{b0}h_0^2}(f-\omega),\\
\frac{\partial H}{\partial\tau}&=\frac{\partial}{\partial x}(T\frac{\partial H}{\partial x})+\frac{n_0k_0\mu L^2}{\mu^*k_{b0}H_0m}kh-\frac{n_0k_0\mu L^2}{\mu^*k_{b0}h_0m}kH-\frac{n_0\mu L^2}{\mu^*k_{b0}H_0h_0}\eta Q.
\end{aligned}\right\}\tag{21}$$

Problem (21) has the following form:

$$\left.\begin{aligned}
\frac{1}{h}\frac{\partial h^2}{\partial\tau}&=\frac{\partial}{\partial x}(k_b\frac{\partial h^2}{\partial x})-2\xi k_bh+2\xi_1k_bH+2\xi_2(f-\omega),\\
\frac{\partial H}{\partial\tau}&=\frac{\partial}{\partial x}(T\frac{\partial H}{\partial x})+\varphi_1kH-\varphi_2kh-\varphi_3Q.
\end{aligned}\right\}\tag{22}$$

where

$$\xi=\frac{L^2}{mh_0},\ \xi_1=\frac{H_0L^2}{mh_0^2},\ \xi_2=\frac{L^2}{k_{b0}h_0^2},\ \varphi_1=\frac{n_0k_0\mu L^2}{\mu^*k_{b0}H_0m},\ \varphi_2=\frac{n_0k_0\mu L^2}{\mu^*k_{b0}h_0m},\ \varphi_3=\frac{n_0\mu L^2}{\mu^*k_{b0}H_0h_0}\eta.$$

Under initial and boundary conditions:

IOP Conf. Series: Materials Science and Engineering **896** (2020) 012047 doi:10.1088/1757-899X/896/1/012047

$$h(x,\tau_0)=h_0,\ \ H(x,\tau_0)=H_0 \quad \text{at} \quad \tau=\tau_0, \tag{23}$$

$$\frac{n_0h_0^2}{L}h\frac{\partial h}{\partial x}\bigg|_{x=0}=-(h_0h-h_0),\ \ \frac{n_0h_0^2}{L}h\frac{\partial h}{\partial x}\bigg|_{x=1}=(h_0h-h_0), \tag{24}$$

$$\frac{\mu^*H_0}{L}\frac{\partial H}{\partial x}\bigg|_{x=0}=-(H_0H-H_0),\ \ \frac{\mu^*H_0}{L}\frac{\partial H}{\partial x}\bigg|_{x=1}=(H_0H-H_0). \tag{25}$$

Problem (15) and (20) in dimensionless variables is written as follows:

$$\left.\begin{aligned} h\frac{\partial\theta_1}{\partial\tau}&=\frac{(D_1)_0}{k_{b0}h_0}\frac{\partial}{\partial x}(D_1h\frac{\partial\theta_1}{\partial x})-\frac{L}{k_{b0}h_0}v_xh\frac{\partial\theta_1}{\partial x}+\frac{L^2}{k_{b0}h_0(\theta_1)_0}f_1\cdot\theta_{1f},\\ H\frac{\partial\theta_2}{\partial\tau}&=\frac{(D_2)_0}{k_0H_0}\frac{\partial}{\partial x}(D_2H\frac{\partial\theta_2}{\partial x})-\frac{L}{k_0H_0}v_xH\frac{\partial\theta_2}{\partial x}+\frac{L^2}{k_0H_0(\theta_2)_0}f_2\cdot\theta_{2f}.\end{aligned}\right\} \tag{26}$$

Problem (26) has the following form:

$$\left.\begin{aligned} h\frac{\partial\theta_1}{\partial\tau}&=A\frac{\partial}{\partial x}(D_1h\frac{\partial\theta_1}{\partial x})-A_1v_xh\frac{\partial\theta_1}{\partial x}+A_2f_1\cdot\theta_{1f},\\ H\frac{\partial\theta_2}{\partial\tau}&=B\frac{\partial}{\partial x}(D_2H\frac{\partial\theta_2}{\partial x})-B_1v_xH\frac{\partial\theta_2}{\partial x}+B_2f_2\cdot\theta_{2f}.\end{aligned}\right\} \tag{27}$$

where

$$A=\frac{(D_1)_0}{k_{b0}h_0},\ A_1=\frac{L}{k_{b0}h_0},\ A_2=\frac{L^2}{k_{b0}h_0(\theta_1)_0},$$
$$B=\frac{(D_2)_0}{k_0H_0},\ B_1=\frac{L}{k_0H_0},\ B_2=\frac{L^2}{k_0H_0(\theta_2)_0}.$$

Under initial and boundary conditions:

$$\theta_1(x,\tau)\big|_{\tau=\tau_0}=(\theta_1)_0,\ \ \theta_2(x,\tau)\big|_{\tau=\tau_0}=(\theta_2)_0, \tag{28}$$

$$\begin{aligned}\frac{\mu h_0(\theta_1)_0}{L}h\frac{\partial\theta_1}{\partial x}\bigg|_{x=0}&=-((\theta_1)_0\theta_1-(\theta_1)_0),\\ \frac{\mu h_0(\theta_1)_0}{L}h\frac{\partial\theta_1}{\partial x}\bigg|_{x=1}&=((\theta_1)_0\theta_1-(\theta_1)_0),\end{aligned} \tag{29}$$

$$\begin{aligned}\frac{\mu^*H_0(\theta_2)_0}{L}H\frac{\partial\theta_2}{\partial x}\bigg|_{x=0}&=-((\theta_2)_0\theta_2-(\theta_2)_0),\\ \frac{\mu^*H_0(\theta_2)_0}{L}H\frac{\partial\theta_2}{\partial x}\bigg|_{x=1}&=((\theta_2)_0\theta_2-(\theta_2)_0),\end{aligned} \tag{30}$$

$$(\theta_1)_0\theta_1(x,t)\bigg|_{x=\frac{m-0}{L}}=(\theta_2)_0\theta_2(x,t)\bigg|_{x=\frac{m+0}{L}}, \tag{31}$$

$$\frac{(D_1)_0 h_0 (\theta_1)_0}{L} D_1 h \frac{\partial \theta_1}{\partial x}\bigg|_{x=\frac{m-0}{L}} = \frac{(D_2)_0 H_0 (\theta_2)_0}{L} D_2 H \frac{\partial \theta_2}{\partial x}\bigg|_{x=\frac{m+0}{L}}. \tag{32}$$

3. Solution Method

Since the posed problem describes a system of nonlinear partial differential equations, it is difficult to obtain an analytical solution.

To solve problems (22) - (25) and (27) - (32), the finite difference method is used [9,10,11,16-19]. Introduce a grid for the domain $D = \{0 \le x < L,\ 0 \le t \le T\}$, where T is the maximum time during which the process is studied. To do this, replace the continuous domain of the problem solution by a grid one:

$$\omega_{\Delta x, \Delta\tau} = \{(x_i, t_j),\ x_i = i\,\Delta x;\ i = 0,1,2,...I;\ t_j = j\tau;\ j = 0,1,2,...,J\}.$$

Next, approximate equation (22), using the implicit scheme on the grid $\omega_{\Delta x, \Delta\tau}$ in the form [9,10,11, 16-19,20]:

$$\left.\begin{aligned} &\frac{1}{h}\frac{(h^2)_i^{j+1} - (h^2)_i^j}{0.5\Delta\tau} = \frac{k_{b\ i-0.5}(h^2)_{i-1}^{j+1} - (k_{b\ i-0.5} + k_{b\ i+0.5})(h^2)_i^{j+1} + k_{b\ i+0.5}(h^2)_{i+1}^{j+1}}{\Delta x^2} - \\ &\qquad - 2\xi k_{bi}(h^2)_i^{j+1} + 2\xi_1 k_{bi} H_i^j + 2\xi_2 (f_i^j - \omega_i^j), \\ &\frac{H_i^{j+1} - H_i^j}{0.5\Delta\tau} = \frac{T_{i-0.5}H_{i-1}^{j+1} - (T_{i-0.5} + T_{i+0.5})H_i^{j+1} + T_{i+0.5}H_{i+1}^{j+1}}{\Delta x^2} + \varphi_1 k_i H_i^{j+1} - \\ &\qquad - \varphi_2 k_i h_i^{j+1} - \varphi_3 Q_i^j. \end{aligned}\right\} \tag{33}$$

Nonlinear terms of difference equation (33) are represented in the form:

$$\psi(h) \cong \psi(\tilde{h}) + (h - \tilde{h})\frac{\partial \psi(\tilde{h})}{\partial h}. \tag{34}$$

Here $\tilde{h}$ is the approximate value of function h, which is specified in the iteration process $\tilde{h} = h_{i,j}^{(s)}$, here $h_{i,j}^{(0)} = \tilde{h}_{i,j}$. Based on the linearization formula (34), the system (35) is written for the square of the level function $h^2 \approx 2\tilde{h}h - \tilde{h}^2$, and as a result we get the following:

$$\left.\begin{aligned} &\frac{1}{\tilde{h}}\frac{2\tilde{h}h_i^{j+1} - \tilde{h}^2 - (2\tilde{h}h_i^j - \tilde{h}^2)}{0.5\Delta\tau} = \frac{k_{b\ i-0.5}(2\tilde{h}h_{i-1}^{j+1} - \tilde{h}^2) - (k_{b\ i-0.5} + k_{b\ i+0.5})(2\tilde{h}h_i^{j+1} - \tilde{h}^2)}{\Delta x^2} + \\ &+ \frac{k_{b\ i+0.5}(2\tilde{h}h_{i+1}^{j+1} - \tilde{h}^2)}{\Delta x^2} - 2\xi k_{bi}(2\tilde{h}h_i^{j+1} - \tilde{h}^2) + 2\xi_1 k_{bi} H_i^j + 2\xi_2 (f_i^j - \omega_i^j), \\ &\frac{H_i^{j+1} - H_i^j}{0.5\Delta\tau} = \frac{T_{i-0.5}H_{i-1}^{j+1} - (T_{i-0.5} + T_{i+0.5})H_i^{j+1} + T_{i+0.5}H_{i+1}^{j+1}}{\Delta x^2} + \\ &+ \varphi_1 k_i H_i^{j+1} - \varphi_2 k_i h_i^{j+1} - \varphi_3 Q_i^j. \end{aligned}\right\} \tag{35}$$

Grouping equation (35):

$$\left.\begin{aligned}
&\frac{0.5\Delta\tau}{\Delta x^2}k_{b\ i-0.5}\tilde{h}h_{i-1}^{j+1}-(\frac{0.5\Delta\tau}{2\Delta x^2}(2k_{b\ i-0.5}\tilde{h}+2k_{b\ i+0.5}\tilde{h})+\Delta\tau\xi k_{bi}\tilde{h}+1)h_i^{j+1}+\\
&+\frac{0.5\Delta\tau}{\Delta x^2}k_{b\ i+0.5}\tilde{h}h_{i+1}^{j+1}=-(h_i^j+0.5\Delta\tau\xi k_{bi}\tilde{h}^2+0.5\Delta\tau\xi_1 k_{bi}H_i^j+0.5\Delta\tau\xi_2(f_i^j-\omega_i^j)),\\
&\frac{0.5\Delta\tau}{\Delta x^2}T_{i-0.5}H_{i-1}^{j+1}-(\frac{0.5\Delta\tau}{\Delta x^2}(T_{i-0.5}+T_{i+0.5})-0.5\Delta\tau\varphi_1 k_i+1)H_i^{j+1}+\\
&+\frac{0.5\Delta\tau}{\Delta x^2}T_{i+0.5}H_{i+1}^{j+1}=-(H_i^j+0.5\Delta\tau\varphi_2 k_i h_i^{j+1}+0.5\Delta\tau\varphi_3 Q_i^j).
\end{aligned}\right\} \quad (36)$$

After some transformation and grouping similar terms, the finite-difference system (36) is rewritten in the form

$$a_i h_{i-1}^{j+1}-b_i h_i^{j+1}+c_i h_{i+1}^{j+1}=-d_i, \quad (37)$$

$$a_i^1 H_{i-1}^{j+1}-b_i^1 H_i^{j+1}+c_i^1 H_{i+1}^{j+1}=-d_i^1, \quad (38)$$

where

$$a_i=\frac{0.5\Delta\tau}{\Delta x^2}k_{b\ i-0.5}\tilde{h},\ b_i=(\frac{0.5\Delta\tau}{2\Delta x^2}(2k_{b\ i-0.5}\tilde{h}+2k_{b\ i+0.5}\tilde{h})+\Delta\tau\xi k_{bi}\tilde{h}+1),\ c_i=\frac{0.5\Delta\tau}{\Delta x^2}k_{b\ i+0.5}\tilde{h},$$

$$a_i^1=\frac{0.5\Delta\tau}{\Delta x^2}T_{i-0.5},\ b_i^1=(\frac{0.5\Delta\tau}{\Delta x^2}(T_{i-0.5}+T_{i+0.5})-0.5\Delta\tau\varphi_1 k_i+1),\ c_i^1=\frac{0.5\Delta\tau}{\Delta x^2}T_{i+0.5},$$

$$d_i^1=H_i^j+0.5\Delta\tau\varphi_2 k_i h_i^{j+1}+0.5\Delta\tau\varphi_3 Q_i^j.$$

Approximating boundary conditions (23) - (25):

$$h_i^j\Big|_{j=0}=h_i^0,\ H_i^j\Big|_{j=0}=H_i^0,$$

$$\frac{n_0 h_0^2}{L}\cdot\frac{(h^2)_0^{j+1}-4(h^2)_1^{j+1}+3(h^2)_2^{j+1}}{2\Delta x}=-(h_0 h_1^{j+1}-h_0), \quad (39)$$

$$\frac{n_0 h_0^2}{L}\cdot\frac{-3(h^2)_{I-1}^{j+1}+4(h^2)_I^{j+1}-(h^2)_{I+1}^{j+1}}{2\Delta x}=(h_0 h_I^{j+1}-h_0), \quad (40)$$

$$\frac{\mu^* H_0}{L}\cdot\frac{H_0^{j+1}-4H_1^{j+1}+3H_2^{j+1}}{2\Delta x}=-(H_0 H_1^{j+1}-H_0), \quad (41)$$

$$\frac{\mu^* H_0}{L}\cdot\frac{-3H_{I-1}^{j+1}+4H_I^{j+1}-H_{I+1}^{j+1}}{2\Delta x}=(H_0 H_I^{j+1}-H_0). \quad (42)$$

As a result of transformation (34), equations (39) - (40) are transformed as follows:

$$\frac{2\tilde{h}n_0 h_0^2}{L}\cdot\frac{h_0^{j+1}-4h_1^{j+1}+3h_2^{j+1}}{2\Delta x}=-(h_0 h_1^{j+1}-h_0), \quad (43)$$

$$\frac{2\tilde{h}n_0h_0^2}{L}\cdot\frac{-3h_{I-1}^{j+1}+4h_I^{j+1}-h_{I+1}^{j+1}}{2\Delta x}=(h_0h_I^{j+1}-h_0). \tag{44}$$

The resulting systems of equations for the sought for variables are solved by the sweep method. Equations (37), (38) are solved using the sweep method, where the sweep coefficients are calculated as:

$$h_i^{j+1}=\alpha_{i+1}h_{i+1}^{j+1}+\beta_{i+1}, \tag{45}$$

$$H_i^{j+1}=\alpha_{i+1}^1H_{i+1}^{j+1}+\beta_{i+1}^1. \tag{46}$$

α_i, β_i and α_i^1, β_i^1 are the sweep coefficients

$$\alpha_{i+1}=\frac{c_i}{b_i-a_i\alpha_i},\ \beta_{i+1}=\frac{d_i+a_i\beta_i}{b_i-a_i\alpha_i},\ \alpha_{i+1}^1=\frac{c_i^1}{b_i^1-a_i^1\alpha_i^1},\ \beta_{i+1}^1=\frac{d_i^1+a_i^1\beta_i^1}{b_i^1-a_i^1\alpha_i^1}.$$

If $i=1$, then equation (37) is converted to equation (47), and as a result of simplification of equation (43), we obtain (48).

$$h_2^{j+1}=\frac{b_1}{c_1}h_1^{j+1}-\frac{a_1}{c_1}h_0^{j+1}-\frac{d_1}{c_1}. \tag{47}$$

$$h_2^{j+1}=\frac{4\tilde{h}n_0h_0^2-h_0\Delta xL}{3\tilde{h}n_0h_0^2}h_1^{j+1}-\frac{\tilde{h}n_0h_0^2}{3\tilde{h}n_0h_0^2}h_0^{j+1}+\frac{h_0\Delta xL}{3\tilde{h}n_0h_0^2}, \tag{48}$$

Comparing (47) with (48), we determine h_0^{j+1}:

$$h_0^{j+1}=\frac{4\tilde{h}n_0h_0^2c_1-h_0\Delta xLc_1-3\tilde{h}n_0h_0^2b_1}{\tilde{h}n_0h_0^2c_1-3\tilde{h}n_0h_0^2a_1}h_1^{j+1}+\frac{h_0\Delta xLc_1+3\tilde{h}n_0h_0^2c_1}{\tilde{h}n_0h_0^2c_1-3\tilde{h}n_0h_0^2a_1} \tag{47*}$$

$i=0$, equation (45) is transformed into equation (48*):

$$h_0^{j+1}=\alpha_1h_1^{j+1}+\beta_1. \tag{48*}$$

Comparing (47*) with (48*), we determine α_1 and β_1:

$$\alpha_1=\frac{4\tilde{h}n_0h_0^2c_1-h_0\Delta xLc_1-3\tilde{h}n_0h_0^2b_1}{\tilde{h}n_0h_0^2c_1-3\tilde{h}n_0h_0^2a_1},\ \beta_1=\frac{h_0\Delta xLc_1+3\tilde{h}n_0h_0^2c_1}{\tilde{h}n_0h_0^2c_1-3\tilde{h}n_0h_0^2a_1}.$$

At $i=I$ equation (37) takes the form (49), as a result of simplification of equation (44), we obtain (50):

$$h_{I+1}^{j+1}=-\frac{a_I}{c_I}h_{I-1}^{j+1}+\frac{b_I}{c_I}h_I^{j+1}-\frac{d_I}{c_I}, \tag{49}$$

$$h_{I+1}^{j+1}=-3h_{I-1}^{j+1}+\frac{4\tilde{h}n_0h_0^2-\Delta xLh_0}{\tilde{h}n_0h_0^2}h_I^{j+1}+\frac{\Delta xLh_0}{\tilde{h}n_0h_0^2}. \tag{50}$$

Comparing (49) with (50), we determine h_{I-1}^{j+1}:

$$h_{I-1}^{j+1} = -\frac{4\tilde{h}n_0h_0^2c_I - \Delta xLh_0c_I - \tilde{h}n_0h_0^2b_I}{\tilde{h}n_0h_0^2(a_I - 3c_I)}h_I^{j+1} - \frac{\Delta xLh_0c_I + \tilde{h}n_0h_0^2d_I}{\tilde{h}n_0h_0^2(a_I - 3c_I)}. \tag{51}$$

If $i = I - 1$, then equation (45) is transformed into equation (51*):

$$h_{I-1}^{j+1} = \alpha_I h_I^{j+1} + \beta_I. \tag{51*}$$

Comparing (51) with (51*), we determine h_I^{j+1}:

$$h_I^{j+1} = \frac{\Delta xLh_0c_I + \tilde{h}n_0h_0^2d_I}{\alpha_I\tilde{h}n_0h_0^2(3c_I - a_I) + 4\tilde{h}n_0h_0^2c_I - \Delta xLh_0c_I - \tilde{h}n_0h_0^2b_I} + \\ + \frac{\beta_I}{\alpha_I\tilde{h}n_0h_0^2(3c_I - a_I) + 4\tilde{h}n_0h_0^2c_I - \Delta xLh_0c_I - \tilde{h}n_0h_0^2b_I}.$$

Applying the above algorithm, find the values of α_1^1, β_1^1, H_I^{j+1}, $\bar{\alpha}_1$, $\bar{\beta}_1$, $(\theta_1)_I^{j+1}$, $\bar{\alpha}_1^1$, $\bar{\beta}_1^1$, $(\theta_2)_I^{j+1}$.

$$\alpha_1^1 = \frac{3\mu^*H_0b_1^1 - 4\mu^*H_0c_1^1 + 2\Delta xLH_0c_1^1}{\mu^*H_0(3a_1^1 - c_1^1)}, \quad \beta_1^1 = -\frac{2\Delta xLH_0c_1^1 + 3\mu^*H_0d_1^1}{\mu^*H_0(3a_1^1 - c_1^1)},$$

$$H_I^{j+1} = \frac{2\Delta xLH_0 + \mu^*H_0d_I^1 + \beta_I^1\mu^*H_0(a_I^1 - 3c_I^1)}{2\Delta xLH_0c_I^1 - 4\mu^*H_0c_I^1 + \mu^*H_0b_I^1 - \alpha_I^1\mu^*H_0(a_I^1 - 3c_I^1)}.$$

$$\bar{\alpha}_1 = \frac{3\mu h_0h_1^{j+1}\bar{b}_1 - 4\mu h_0h_1^{j+1}\bar{c}_1 + 2\Delta xL\bar{c}_1}{\mu h_0h_1^{j+1}(3\bar{a}_1 - \bar{c}_1)}, \quad \bar{\beta}_1 = -\frac{2\Delta xL\bar{c}_1 + 3\mu h_0h_1^{j+1}\bar{d}_1}{\mu h_0h_1^{j+1}(3\bar{a}_1 - \bar{c}_1)}.$$

$$(\theta_1)_I^{j+1} = \frac{\bar{\beta}_I\mu h_0h_I^{j+1}(3\bar{c}_I - \bar{a}_I) - \bar{d}_I\mu h_0h_I^{j+1} + 2\Delta xL\bar{c}_I}{4\mu h_0h_I^{j+1}\bar{c}_I - 2\Delta xL\bar{c}_I - \mu h_0h_I^{j+1}\bar{b}_I - \bar{\alpha}_I\mu h_0h_I^{j+1}(3\bar{c}_I - \bar{a}_I)}$$

$$\bar{\alpha}_1^1 = \frac{4\mu^*H_0H_1^{j+1}\bar{c}_1^1 - 2\Delta xL\bar{c}_1^1 - \bar{b}_1^1 3\mu^*H_0H_1^{j+1}}{\mu^*H_0H_1^{j+1}(\bar{c}_1^1 - 3\bar{a}_1^1)},$$

$$\bar{\beta}_1^1 = \frac{2\Delta xL\bar{c}_1^1 + \bar{d}_1^1 3\mu^*H_0H_1^{j+1}}{\mu^*H_0H_1^{j+1}(\bar{c}_1^1 - 3\bar{a}_1^1)},$$

$$(\theta_2)_I^{j+1} = \frac{\bar{\beta}_I^1\mu^*H_0H_I^{j+1}(\bar{a}_I^1 - 3\bar{c}_I^1) + 2\Delta xL\bar{c}_I^1 + \mu^*H_0H_I^{j+1}\bar{d}_I^1}{\mu^*H_0H_I^{j+1}\bar{b}_I^1 - 4\mu^*H_0H_I^{j+1}\bar{c}_I^1 + 2\Delta xL\bar{c}_I^1 - \bar{\alpha}_I^1\mu^*H_0H_I^{j+1}(\bar{a}_I^1 - 3\bar{c}_I^1)}.$$

The convergence of the iterative process is checked using the conditions:

$$\left|(h_i^j)^{(s+1)} - (h_i^j)^{(s)}\right| < \varepsilon, \quad \left|((\theta_1)_i^j)^{(s+1)} - ((\theta_1)_i^j)^{(s)}\right| < \varepsilon, \quad \left|((\theta_2)_i^j)^{(s+1)} - ((\theta_2)_i^j)^{(s)}\right| < \varepsilon.$$

Here ε is the required accuracy of the solution, S is the number of iterations, the initial iterative value is chosen equal to the solution on the previous time layer.

4. Results

Machine algorithm for solving the problem is as follows:

1st step. Input of initial (baseline) data (input of constants):

$$\mu,\ \mu^*,\ m,\ k_b,\ k,\ T,\ Q,\ f,\ \omega,\ n_0,\ \eta,\ h_0(x),\ H_0(x),$$
$$(\theta_1)_0,\ (\theta_2)_0,\ h_0\ \ L,\ \ k_0,\ \ k_{b0},\ T_0,\ t,\ (D_1)_0,\ (D_2)_0,$$

2nd step. Calculate values of specified variables:

$$h^* = \frac{h}{h_0},\ x^* = \frac{x}{L},\ H^* = \frac{H}{L},\ k^* = \frac{k}{k_0},\ k_b^* = \frac{k_b}{k_{b0}},\ T^* = \frac{T}{T_0},$$

$$\tau = \frac{k_{b0}h_0}{\mu n_0 L^2}t,\ \theta_1^* = \frac{\theta_1}{(\theta_1)_0},\ \theta_2^* = \frac{\theta_2}{(\theta_2)_0},\ D_1^* = \frac{D_1}{(D_1)_0},\ D_2^* = \frac{D_2}{(D_2)_0}\ \ T_0 = \frac{\mu^* k_{b0}h_0}{\mu n_0},$$

$$\xi = \frac{L^2}{mh_0},\ \xi_1 = \frac{H_0L^2}{mh_0^2},\ \xi_2 = \frac{L^2}{k_{b0}h_0^2},\ \varphi_1 = \frac{n_0k_0\mu L^2}{\mu^* k_{b0}H_0 m},\ \varphi_2 = \frac{n_0k_0\mu L^2}{\mu^* k_{b0}h_0 m},\ \varphi_3 = \frac{n_0\mu L^2}{\mu^* k_{b0}H_0h_0}\eta,$$

$$A = \frac{(D_1)_0}{k_{b0}h_0},\ \ A_1 = \frac{L}{k_{b0}h_0},\ \ A_2 = \frac{L^2}{k_{b0}h_0(\theta_1)_0},\ \ B = \frac{(D_2)_0}{k_0H_0},\ \ B_1 = \frac{L}{k_0H_0},\ \ B_2 = \frac{L^2}{k_0H_0(\theta_2)_0},$$

$$a_i = \frac{0.5\Delta\tau}{\Delta x^2}k_{b\ i-0.5}\tilde{h},\ \ b_i = \left(\frac{0.5\Delta\tau}{2\Delta x^2}(2k_{b\ i-0.5}\tilde{h} + 2k_{b\ i+0.5}\tilde{h}) + \Delta\tau\xi k_{bi}\tilde{h} + 1\right),$$

$$d_i = h_i^j + 0.5\Delta\tau\xi k_{bi}\tilde{h}^2 + 0.5\Delta\tau\xi_1 k_{bi}H_i^j + 0.5\Delta\tau\xi_2(f_i^j - \omega_i^j),$$

$$c_i = \frac{0.5\Delta\tau}{\Delta x^2}k_{b\ i+0.5}\tilde{h},\ \ a_i^1 = \frac{0.5\Delta\tau}{\Delta x^2}T_{i-0.5},$$

$$b_i^1 = \left(\frac{0.5\Delta\tau}{\Delta x^2}(T_{i-0.5} + T_{i+0.5}) - 0.5\Delta\tau\varphi_1 k_i + 1\right),$$

$$c_i^1 = \frac{0.5\Delta\tau}{\Delta x^2}T_{i+0.5},\ \ d_i^1 = H_i^j + 0.5\Delta\tau\varphi_2 k_i h_i^{j+1} + 0.5\Delta\tau\varphi_3 Q_i^j.$$

3rd step. Calculation of the elements of a tridiagonal transition matrix obtained by approximating differential operators to finite-difference ones.

$$\alpha_1 = \frac{4\tilde{h}n_0h_0^2c_1 - h_0\Delta xLc_1 - 3\tilde{h}n_0h_0^2b_1}{\tilde{h}n_0h_0^2c_1 - 3\tilde{h}n_0h_0^2a_1},\ \ \beta_1 = \frac{h_0\Delta xLc_1 + 3\tilde{h}n_0h_0^2c_1}{\tilde{h}n_0h_0^2c_1 - 3\tilde{h}n_0h_0^2a_1},$$

$$\alpha_1^1 = \frac{3\mu^* H_0b_1^1 - 4\mu^* H_0c_1^1 + 2\Delta xLH_0c_1^1}{\mu^* H_0(3a_1^1 - c_1^1)},\ \ \beta_1^1 = -\frac{2\Delta xLH_0c_1^1 + 3\mu^* H_0d_1^1}{\mu^* H_0(3a_1^1 - c_1^1)},$$

$$\overline{\alpha}_1 = \frac{3\mu h_0h_1^{j+1}\overline{b}_1 - 4\mu h_0h_1^{j+1}\overline{c}_1 + 2\Delta xL\overline{c}_1}{\mu h_0h_1^{j+1}(3\overline{a}_1 - \overline{c}_1)},\ \ \overline{\beta}_1 = -\frac{2\Delta xL\overline{c}_1 + 3\mu h_0h_1^{j+1}\overline{d}_1}{\mu h_0h_1^{j+1}(3\overline{a}_1 - \overline{c}_1)}.$$

$$\overline{\alpha}_1^1 = \frac{4\mu^* H_0H_1^{j+1}\overline{c}_1^1 - 2\Delta xL\overline{c}_1^1 - \overline{b}_1^1 3\mu^* H_0H_1^{j+1}}{\mu^* H_0H_1^{j+1}(\overline{c}_1^1 - 3\overline{a}_1^1)},\ \ \overline{\beta}_1^1 = \frac{2\Delta xL\overline{c}_1^1 + \overline{d}_1^1 3\mu^* H_0H_1^{j+1}}{\mu^* H_0H_1^{j+1}(\overline{c}_1^1 - 3\overline{a}_1^1)},$$

$$\alpha_{i+1} = \frac{c_i}{b_i - a_i\alpha_i},\ \ \beta_{i+1} = \frac{d_i + a_i\beta_i}{b_i - a_i\alpha_i},$$

$$\alpha_{i+1}^1 = \frac{c_i^1}{b_i^1 - a_i^1\alpha_i^1},\ \ \beta_{i+1}^1 = \frac{d_i^1 + a_i^1\beta_i^1}{b_i^1 - a_i^1\alpha_i^1},$$

$$\overline{\alpha}_{i+1} = \frac{\overline{c}_i}{\overline{b}_i - \overline{a}_i\overline{\alpha}_i},\ \ \overline{\beta}_{i+1} = \frac{\overline{d}_i + \overline{a}_i\overline{\beta}_i}{\overline{b}_i - \overline{a}_i\overline{\alpha}_i},$$

$$\bar{\alpha}_{i+1}^{1}=\frac{\bar{c}_i^1}{\bar{b}_i^1-\bar{a}_i^1\bar{\alpha}_i^1},\ \ \bar{\beta}_{i+1}^{1}=\frac{\bar{d}_i^1+\bar{a}_i^1\bar{\beta}_i^1}{\bar{b}_i^1-\bar{a}_i^1\bar{\alpha}_i^1}.$$

$$h_i^{j+1}=\alpha_{i+1}h_{i+1}^{j+1}+\beta_{i+1},$$

$$H_i^{j+1}=\alpha_{i+1}^1H_{i+1}^{j+1}+\beta_{i+1}^1$$

$$(\theta_1)_i^{j+1}=\bar{\alpha}_{i+1}(\theta_1)_{i+1}^{j+1}+\bar{\beta}_{i+1},$$

$$(\theta_2)_i^{j+1}=\bar{\alpha}_{i+1}^1(\theta_2)_{i+1}^{j+1}+\bar{\beta}_{i+1}^1,$$

4th step. We calculate the reverse run in step 3 by calculating the last set limit value:

$$H_I^{j+1}=\frac{2\Delta xLH_0+\mu^*H_0d_I^1+\beta_I^1\mu^*H_0(a_I^1-3c_I^1)}{2\Delta xLH_0c_I^1-4\mu^*H_0c_I^1+\mu^*H_0b_I^1-\alpha_I^1\mu^*H_0(a_I^1-3c_I^1)}.$$

$$(\theta_1)_I^{j+1}=\frac{\bar{\beta}_I\mu h_0h_I^{j+1}(3\bar{c}_I-\bar{a}_I)-\bar{d}_I\mu h_0h_I^{j+1}+2\Delta xL\bar{c}_I}{4\mu h_0h_I^{j+1}\bar{c}_I-2\Delta xL\bar{c}_I-\mu h_0h_I^{j+1}\bar{b}_I-\bar{\alpha}_I\mu h_0h_I^{j+1}(3\bar{c}_I-\bar{a}_I)}.$$

$$(\theta_2)_I^{j+1}=\frac{\bar{\beta}_I^1\mu^*H_0H_I^{j+1}(\bar{a}_I^1-3\bar{c}_I^1)+2\Delta xL\bar{c}_I^1+\mu^*H_0H_I^{j+1}\bar{d}_I^1}{\mu^*H_0H_I^{j+1}\bar{b}_I^1-4\mu^*H_0H_I^{j+1}\bar{c}_I^1+2\Delta xL\bar{c}_I^1-\bar{\alpha}_I^1\mu^*H_0H_I^{j+1}(\bar{a}_I^1-3\bar{c}_I^1)}.$$

5th step. Check the error of the found values by iteration:

$$\left|(h_i^j)^{(s+1)}-(h_i^j)^{(s)}\right|<\varepsilon,\ \left|((\theta_1)_i^j)^{(s+1)}-((\theta_1)_i^j)^{(s)}\right|<\varepsilon,\ \ \left|((\theta_2)_i^j)^{(s+1)}-((\theta_2)_i^j)^{(s)}\right|<\varepsilon.$$

6th step. Adequacy verification of the task.

5. Conclusion

A mathematical model and an effective numerical algorithm were developed for a comprehensive study of the geofiltration process and for computer experiments to change ground and pressure waters, as well as salt concentration.

Since the process is characterized by a system of nonlinear differential equations and the corresponding initial and boundary conditions, an iterative linearization method was used. Using the developed effective numerical algorithm, the application of the kinetic approach in predicting the movement and changes in the level of wastewater can be used to develop recommendations before implementing decision-making technologies to solve control problems in predicting changes in the concentration of salt in water. As a result of studying the patterns of movement of wastewater flows, computer experiments can be carried out using an algorithm to determine changes in the distance and distribution rate of wastewater with water-soluble chemical and active properties in soil layers. Models and algorithms can be used to predict the process of water infiltration and changes in water levels in water supply zones, many technological processes that occur during the migration of salts, as well as qualitative and quantitative analysis of processes in hydrogeology. The created mathematical and numerical apparatus can significantly reduce the volume of full-scale research and minimize experimental work that requires expensive resources in the process of computing experiments on a computer.

References

[1] Akramov A 1989 *Artificial formation and replenishment of freshwater channel lenses* (Fan: Tashkent)
[2] N Ravshanov et al 2020 *J. Phys.: Conf. Ser.* **1441** 012163
[3] Bogaenko V, Marchenko O and Samoilenko T 2014 *Control Syst. Comput.* **4** 33
[4] Guo Q, Huang J, Zhou Z and Wang J 2019 *Geofluids* **2019** 2316271
[5] A V Malkov et al 2019 *IOP Conf. Ser.: Earth Environ. Sci.* 272 022026
[6] Das P, Begam S and Singh M 2017 *J. Hydrol. Hydromechanics.* **65** 192

IOP Conf. Series: Materials Science and Engineering **896** (2020) 012047 doi:10.1088/1757-899X/896/1/012047

[7] Vázquez-Baez V, Rubio-Arellano A, Garc-A-Toral D and Rodr-Guez Mora I 2019 *Mathematical Problems in Engineering* **2019** 1613726
[8] Zhao M 2011 Communications in Computer and Information Science *Advances in Information Technology and Education. Communications in Computer and Information Science* **201** (Springer: Berlin)
[9] Ravshanov N and Daliev S 2019 *Inf. Technol. Model. Control.* **116** 116
[10] N Ravshanov et al 2020 *J. Phys.: Conf. Ser.* **1210** 012118
[11] Ravshanov N, Zagrebina S and Daliev S 2019 *Probl. Comput. Appl. Math.* **4** 12
[12] Usmanov R et al *Sci. World - Int. Sci. J.* **1** 104
[13] Huang M and Tian Y 2015 *Proc. of Int. Conf. on Sustainable Energy and Environmental Engineering* (Atlantis Press)
[14] Bulavatsky V and Skopetskiy V 2008 *Cybern. Syst. Anal.* **6** 59
[15] Vlasyuk A, Tsvetkova T, Falat P, Klos-Witkowska A and Warwas K 2017 *Proc. of IEEE 9th International Conference on Intelligent Data Acquisition and Advanced Computing Systems* (Bucharest)
[16] Abutaliev F 1972 *Methods of mathematical modeling of hydrogeological processes* (Moscow)
[17] Ravshanov N, Islamov Y and Khurramov I 2018 *Probl. Comput. Appl. Math.* **3** 17
[18] Sharipov D, Aynakulov S and Khafizov O 2019 *E3S Web of Conferences* (Tashkent)
[19] Sharipov D, Muradov F and Akhmedov D 2019 *Applied Mathematics E-Notes* **19** 575
[20] Ravshanov N, Abdullayev Z and Shafiev T 2019 *IEEE International Conference on Information Science and Communications Technologies* (Tashkent)