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Evaluating the recharge mechanism of the Lower Kuiseb Dune area using mixing cell modeling and residence time data

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Summary A mixing cell approach was extended by a method incorporating mean cell residence times derived from ^{14}C to further constrain and validate the modeling results. This extended approach was used to model the groundwater system of the Lower Kuiseb Dune area in Namibia. The Kuiseb river is a 560 km long ephemeral river that crosses the Namib Desert from east to west. Transmission losses from the riverbed during flood events are an important source of groundwater recharge to the underlying aquifer system. The Lower Kuiseb area is located in a hyper arid region with annual precipitation less than 25 mm/yr. Hydrochemical data from 13 wells in the area were used in the mixing cell model (MCM). End members were identified as sources for the groundwater found in the Lower Kuiseb, including inflow from the crystalline basement plateau north of the Kuiseb as well as floodwater from the Kuiseb river. A conceptual groundwater recharge and flow model was developed, and then inverse mixing cell modeling was carried out using hydrochemical tracers. This approach generally allows for several possible solutions. After completing the inverse modeling, a forward mixing cell model was developed by varying the mean residence time of each cell to fit calculated ^{14}C data to the measured ^{14}C data. This new approach joins previously developed methods solely based on conservative mixing or residence time optimization. Based on the results of the model the fraction of floodwater in different sections of the Lower Kuiseb groundwater systems was calculated, ranging from 61% to 98.2%. In addition to floodwater, groundwater inflow from the crystalline basement north of the Kuiseb was shown to

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contribute to the Lower Kuiseb aquifer, accounting for salinization during periods without flooding.

Introduction

With few exceptions, only limited hydrologic data are available in regions with the highest scarcity of water. Especially in basins with limited data on hydrogeological structure, it is difficult to accurately characterize groundwater flow systems and their recharge sources. Therefore, estimation of available water resources in drylands requires methods that are based on existing data and on information that can be obtained with affordable efforts. Even in desert regions, chemical and isotope data are often available or can be obtained quickly from reconnaissance studies. Mixing cell models (MCM), also known as multi-compartment models, are based mainly on measurements of water chemistry and isotopes at various locations within the system. In addition, they provide an intrinsic integration of time and space for the examination of areas with scarce hydrologic information and for long time periods (Adar, 1996). A method for the combined use of several conservative tracers (stable isotopes, conservative ions) and of age tracers with decay (^{14}C, tritium) is presented in this paper. This approach combines an inverse mixing cell model (MCM) with a forward MCM to derive mean compartmental residence times and uses observed age information (^{14}C) for model validation.

MCM include various types of models that use environmental tracer data. Simpson and Duckstein (1976) and Campana and Simpson (1984) determined groundwater residence time and recharge rate in the Tucson Basin Aquifer using a compartment model with ^{14}C as the only tracer. A mixing cell approach was used to calculate bulk recharge to the Edwards-Aquifer, Texas, a chalk-aquifer, with tritium data (Campana and Mahin, 1985). A similar study with deuterium as a tracer was carried out by Kirk and Campana (1990). Meanwhile Adar and co-workers (Adar, 1984, 1996; Adar and Neuman, 1988; Adar et al., 1988, 1992, 2002; Adar and Sorek, 1989, 1990; Adar et al., 1992; Adar and Külls, 2002; Adar et al., 2002) developed an approach for using several dissolved minerals and isotopes simultaneously in a model based on mass-balance equations for water, stable isotopes and dissolved chemicals with conservative behaviour. MCMs have also been applied to the Aravaipa catchment, Arizona, the Arava Rift Valley, Israel, and in Kalahari Desert, Namibia (Külls, 2000).

Several attempts were made to further develop and validate the MCM methodology. Since the introduction of quadratic programming (Wolfe, 1959) to the hydrological problem of sub-basin streamflow contributions by Woolhiser et al. (1979,1982) different solvers were tested (Adar et al., 2002; Gieske and De Vries, 1990) and evaluated. Appelo and Willemsen (1987) combined a MCM with the geochemical model EQ3/6 for taking into non-conservative behaviour of tracers resulting from phase reactions. Further attempts were made to extend compartment modeling to non-steady conditions (Campana, 2002; Adar et al., 2002). Tezcan (2002) coupled a mixing cell approach with a flow routing procedure to model a non-steady large scale karst aquifer system. Some authors (Harrington et al., 1999; Dahan et al., 2004) coupled MCMs with the numerical groundwater model MODFLOW (McDonald and Harbaugh, 1988) to validate advection-dispersion processes and mass balance. The integration of groundwater residence times into multi-compartment mixing cell models could provide a further improvement of the approach for the analysis of groundwater flow systems in regions with scarce data. This extended approach was applied to the groundwater system of the hyperarid Lower Kuiseb Dune area of Namibia (Fig. 1) for the investigation of the impact of flood recharge on alluvial aquifers and paleochannels.

Study area and data

Study area

The study area is within the Kuiseb catchment, Namibia, south-west Africa (Fig. 1). The Kuiseb is a 560 km long ephemeral river with a catchment size of 14,700 km^2. Only about 9000 km^2 contribute significantly to runoff generation, whereas the rest is a desert plain (Hattle, 1985). The Kuiseb river reaches the Atlantic Ocean near Walvis Bay after crossing the Namib Desert, a coastal desert, from east to west (Jacobson, 1997). The study area in the lower part of the Kuiseb catchment forms a triangular fan with dune covered paleochannel branches. The north-eastern boundary of the study area is the Kuiseb river bed, the southern boundary is formed by a step-fault approximately along the line Anichab-Gobabeb, the western boundary is a north—south line through Rooibank at 14.6500°E. Gobabeb, the eastern boundary (14.9667°E) is about 60 km from the coast (Fig. 1).

Precipitation in the Kuiseb catchment varies from 14 mm/yr at the coast to 360 mm/yr in the headwaters (Kalinki, 1998; Schmidt and Plöthner, 1999). The study area is a hyper-arid region with annual precipitation at Gobabeb less than 25 mm/yr. According to Shanyengana (1997) the precipitation at Gobabeb in the years 1962 to 1996 varied from 2.0 to 115.1 mm/yr. Of the annual precipitation 77% occurs in summer and 23% occurs in winter. The climatic conditions are controlled by the cold Benguela Current, which leads to significant low precipitation in the coastal Namib Desert. The annual mean temperature in the Namib Desert is 18 °C. In the coastal regions fog appears frequently, the mean annual fog drip is 80 mm at Rooibank, 25 km inland, and 31 mm at Gobabeb (Kalinki, 1998).

Runoff events are mainly generated in the eastern headwaters of the catchment and contribute to groundwater recharge by transmission losses. In general, it is assumed that precipitation in the coastal regions (fog drip and rare precipitation events) does not contribute much to total groundwater recharge due to high potential evaporation.

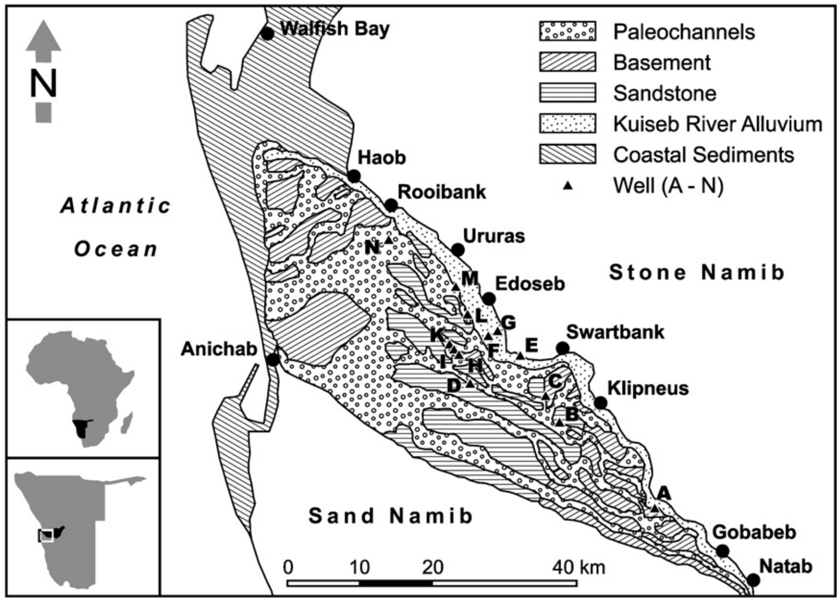

Figure 1 Location of Namibia, the Kuiseb river and map of the study area with wells.

The geology of the Namib Desert consists of Precambrian basement with granite, gneiss and shale. The oldest Tertiary rocks are part of the Tsondab-Sandstone-Formation, which underlies most of the central Namib south of the Kuiseb (AIN, 1999). North of the Kuiseb a flat gravel plain on a crystalline basement is found. The underlying rocks consist of calcareous and gypsic metamorphic bedrock or granite. The dune area south of the Kuiseb river reaches 100 km inland from the Atlantic Ocean and ranges from Lüderitz in South-Namibia nearly to Walvis Bay (Shanyengana, 1997). Several studies have investigated the geology of the Kuiseb Dune area (Geyh, 1994; BGR, 1999). In addition to the Precambrian basement, paleochannels have been identified, which are incised in the Tsondab Sandstone and the basement. The paleochannels are filled with a 40–90 m thick layer of calcareous silty fine sands and covered by up to 100 m of sand dunes. Five paleochannels have been identified that extend between 20 and 65 km in length and 0.5–5 km in width (BGR, 1999). Based on hydrogeological properties, Schmidt and Plöthner (1999) suggest that floodwater of the Kuiseb enters these paleochannels at selective locations along the riverbed. Such potential corridors from the riverbed aquifer to the paleochannels were found between Gobabeb and Rooibank by an aerial geophysical survey of the area (Sengpiel and Siemon, 1995,1997). The transmissivity of the different geological units in the Kuiseb Dune area was determined by pumping tests (Schmidt and Plöthner, 1999) as follows: (1) sediments in the paleochannels 2–6 m^2/d, (2) Tsondab-Sandstone 4 m^2/d; (3) crystalline basement 0.03 m^2/d. The groundwater system of the Lower Kuiseb is an important source of water for the central Namib, and supplies the coastal cities Walvis Bay and Swakopmund, and the local rural population (Schmidt and Plöthner, 1999). One study reports, that the alluvial aquifer of the Kuiseb river consists of a fresh water aquifer that overlies a saltier groundwater layer below (AIN, 1999). The thickness of the aquifer is 3–15 m with a porosity of 35%, and an average width of 150 m (AIN, 1999). The thickness of the alluvium may be as much as 50 m and the depth to groundwater is 5–15 m (DWA, 1987). A conceptual model of the geology is presented in Fig. 2. From upstream of Gobabeb to Rooibank the Kuiseb supplies dense forest vegetation (DWA, 1987).

Data

Hydrochemical and well data (Tables 1 and 2) were provided by DWA (Department of Water Affairs, Namibia) and BGR (Bundesanstalt für Geowissenschaften und Rohstoffe, Federal Institute for Geosciences and Natural Resources, Germany) and were made available by Plöthner (1995). The chemistry of groundwater in the basement beneath the gravel plain north of the Kuiseb river and for Kuiseb floodwater (Table 3) derives from Schmitz (2004). Analyses of ^{14}C are from previous investigations in the Kuiseb area that were interpreted by Geyh (1994). Most data compiled by Plöthner (1995,1998) result from CSIR-studies from 1967 to 1977 (Heaton et al., 1981; Vogel and van Urk, 1975; Vogel et al., 1982). In general, samples were taken from production boreholes once stability of in situ parameters was reached. Sampling depths were compared to lithological information from borehole logs, geophysical data (BGR) and well construction records in the DWA database in order to assign the chemical analysis to a defined aquifer.

Methodology

The first step of an MCM approach is the division of the examined aquifer into different compartments or cells based on chemical and/or isotopic data. Each cell and each source represents a hydrological subunit with a distinct chemical and/or isotopic concentration. The sources and upgradient cells are mixed in the downgradient cell. Within each cell, complete mixing is assumed, which determines the chemical and isotopic composition of the cell water

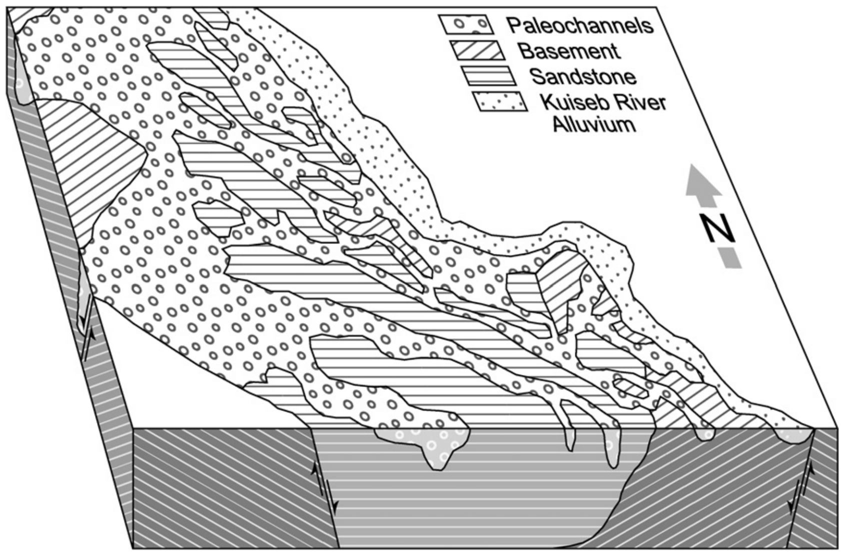

Figure 2 Conceptual geological model of the study area. The paleochannels are incised into Tsondab sandstone and crystalline basement.

Table 1 Data for surface elevation (m.a.s.l), head (m.a.s.l), TDS (total dissolved solids in mg/L, ^{14}C as pmC (percent modern carbon) and pH value used in the model

Well (map)	Well (DWA)*	Cell number	Surface elevation	Head	^{14}C	TDS	pH
A	21898	1	350	311.0	108.3	760	7.9
B	21613	2	312	251.0	78.1	533	7.6
C	21611	3	290	259.4	73.2	772	7.5
D	7898	3	259	187.0	66.0	750	8.2
E	21616	4	254	249.3	94.0	340	8.0
F	20172	4	232	212.9	87.3	325	8.4
G	12808	4	232	225.7	96.5	335	8.4
H	7871	5	224	173.1	70.5	402	7.6
I	7869	6	221	168.1	86.0	327	8.1
K	7892	7	225	166.9	81.0	733	7.2
L	20199	8	214	195.1	100.4	609	8.5
M	20198	9	210	180.7	67.1	348	8.4
N	21610	10	145	126.3	70.9	970	8.2

* Department of water affairs.

(Simpson and Duckstein, 1976; Campana and Simpson, 1984).

The model is based on the availability of measurements of environmental tracers in samples from the aquifer and from potential recharge sources. The following assumptions were made for the aquifer (Adar, 1996): (1) tracers are conservative, that is changes in the concentration of solutes are solely due to mixing (non-conservative tracers can be utilized as long as the rate of change is known), (2) seasonal pulsations of fluxes for each cell can be represented by mean values spanning a time interval in which the hydraulic head may be regarded as constant or as an average of a cyclic process in time, and (3) transport of dissolved constituents is dominated by advection.

All flows entering or leaving the mixing cell system must be identified. Tracer concentrations are assumed to be constant in the system or the changes are periodic. This study used the mixing cell model for steady-state flow and the Mixing-Cell-Input-Generator (MIG) that was developed by Adar and Külls (2002).

Modeling equations

Two different mass-balance equations exist, one for the water balance (1) and one for the mass balance of dissolved chemicals and isotopes (2) that account for each cell of the model domain.

$$Q_n - W_n + \sum_{i=1}^{In} q_{in} - \sum_{j=1}^{jn} q_{nj} = 0 \qquad (1)$$

I_n and J_n denote the number of sources and/or compartments from which flow enters the n-th compartment, and leaves it; q_{in} und q_{nj} denote the fluxes from the i-th source or compartment into the n-th one, and from n-th one into the j-th one; Q_n and W_n denote the fluid sources and sinks (point injection, pumping etc.). All values are average values of a time period.

$$C_{qnk}Q_n - C_{nk}\left[W_n + \sum_{J=1}^{Jn} q_{nj}\right] + \sum_{i=1}^{In} q_{in}C_{qink} = 0 \quad k = 1,2,\ldots,K \qquad (2)$$

C_{qnk} is the concentration of the tracer k associated with source Q_n. C_{nk} denotes the concentration of the k-th constituent/tracer within cell n. C_{qink} is the concentration of solute k entering cell n together with the flux coming from cell I. A parameter that accounts for any deviation of the flux from mass balance in each cell can be introduced, as in Adar (1996). The mass-balance equations are transformed in matrix and vector notation and solved with a mathematical optimization technique (Wolfe, 1959,1967). Eq. (2) is solved for source flows Q_n and flows between cells q_n. This type of inverse solution of flow systems based on sufficient independent tracers does not require total cell volumes for the calculation of Q_n and q_n as flows are derived solely

Table 2 Hydrochemical data (mg/L) for the same wells as in Table 1

Well (map)	Well (DWA)	cell number	Mg^{2+}	K^{+}	Na^{+}	Ca^{2+}	Cl^{-}	SO_4^{2-}	HCO_3^{-}	Error (%)
A	21898	1	75	27	213	75	278	91	576	1.8
B	21613	2	66	12	108	85	116	58	603	0.9
C	21611	3	86	17	175	65	235	81	586	1.1
D	7898	3	74	13	172	43	214	81	487	0.8
E	21616	4	56	14	33	139	55	34	653	1.3
F	20172	4	42	13	37	113	40	29	547	1.5
G	12808	4	37	12	43	118	48	43	521	1.2
H	7871	5	74	14	61	75	30	75	614	1.1
I	7869	6	49	11	64	35	68	1	404	1.5
K	7892	7	45	13	184	50	216	81	391	0.6
L	20199	8	86	18	140	5	200	1	472	1.6
M	20198	9	21	8	90	3	99	11	169	1.5
N	21610	10	70	23	246	58	286	80	586	1.4

Table 3 Hydrochemical data (mg/L) of the identified sources

Source	Mg^{2+}	K^{+}	Na^{+}	Ca^{2+}	Cl^{-}	SO_4^{2-}
GW-G	80.8	36	495	225	761	405
floodwater	5.5	7.5	13.3	40.3	14.3	11.8
GW-S	62.7	29	354	174	617	259
Saltrivier	108	35	342	262	5065	4391

based on tracer concentrations. Details of the equation's transformation and different solution procedures are given by Adar (1996).

Model extension

To include radioactive decay of a tracer like ^{14}C in the mixing cell approach used in this study, mass-balance Eq. (2) was modified by applying the decay law as an e-function. This approach required two new model parameters, the decay constant λ and the mean residence time t of water in each cell as in (3):

$$Q_n C_{qnk} \exp(-\lambda_k t_n) - W_n C_{nk} + \sum_{i=1}^{In} q_i n C_{qink} \exp(-\lambda_k t_n) - \sum_{j=1}^{jn} q_n j C_{nk} = 0 \tag{3}$$

exp represents the exponential function, λ_k denotes the decay constant of the tracer k ($\lambda = 0$ if the tracer is not decaying) and t_n is the time the tracer decays in cell n which is the mean residence time of the water in cell n.

A two-step modeling procedure was developed. In the first step, Eq. (2) was solved inversely (Adar and Külls, 2002), providing a set of solutions for inflows to each cell. For each set of solutions water- and tracer-mass-balance errors were derived. These results were then used in a second step to calculate the concentrations of ^{14}C by a forward mixing cell model based on Eq. (3). The mean residence time in the individual cells t_n was varied to fit the calculated concentrations of the decay tracer (^{14}C) to measured sample values.

Selection of environmental tracers

Radiocarbon decay was calculated using the exponential law with the decay constant $\lambda_k = 1.2097 \times 10^{-4}$ 1/yr, corresponding of a half-life of 5730 years. The ^{14}C activity in groundwater at time t is the result of the initial activity in the water and subsequent radioactive decay (Geyh and Schleicher, 1990); however estimation of the initial activity is not always straightforward. Activities are expressed as percent of the standard 'modern carbon' (pmC). Anthropogenic effects have altered the natural regime. Combustion of fossil fuel has diluted the atmosphere with ^{14}C-free CO_2 and nuclear weapons testing and nuclear power plants have released additional artificial ^{14}C into the atmosphere. An additional 'reservoir effect' was taken into account for the correction of initial ^{14}C activities (Geyh, 1995). Based on an evaluation of previous samples taken in the Kuiseb (Vogel, 1970; Vogel and van Urk, 1975), it can be shown that due to the aridity of the environment and as a result of major indirect recharge components the reservoir effect is limited and initial activity close to 100 pmC. In hydrogeological systems where groundwater is isolated from input of modern carbon sources containing ^{14}C, radioactive decay of ^{14}C may be used for an estimation of mean residence times. In two samples the impact of anthropogenic bomb ^{14}C results in measured activities above 100 pmC. For the use of inorganic ^{14}C in groundwater end member analyses, different concentrations of DIC need to be taken into account (see also Phillips and Koch, 2002; Robins et al., 2002; Koch and Phillips, 2002). This is because a mixing of Water A (100 pmC and a DIC concentration of 100 mg/l) with Water B (80 pmC, DIC: 50 mg/l) will not lead to a concentration of 90 pmC but to a concentration of 93.3 pmC. In this study the correction was not relevant because all waters within the system showed a similar DIC concentration.

Mixing cell calculations were carried out based on the assumption of conservative behaviour of hydrochemical tracers. In order to assure the validity of the concept, prior to the selection of tracers, the conservative behaviour of potential tracers in the specific environment of the Lower Kuiseb groundwater was evaluated. Due to the mineral phase assemblage, ion exchange and sorption were generally low in the study area. Only tracers that were not affected by significant mineral phase dissolution and precipitation resulting from evaporation and mixing were considered. The evaluation of conservative behaviour was performed using thermodynamic phase equilibrium modeling techniques (PHREEQC, Parkhurst and Appelo, 1999). Chloride, sulphate, potassium, sodium, and magnesium were found to suffice these criteria.

Application to the Lower Kuiseb Dune area

The extended mixing cell approach was used to model the groundwater system of the Kuiseb Dune area, Namibia. Thirteen wells (Fig. 1) with hydrochemical and ^{14}C data in the model area were used. In the samples, chemical analyses of nitrate (NO_3^-), magnesium (Mg^{2+}), potassium (K^+), silicate (SiO_2), sodium (Na^+), calcium (Ca^{2+}), chloride (Cl^-), fluoride (F^-), sulphate (SO_4^{2-}), TDS (Total Dissolved Solids), alkalinity and pH-value were available. Cluster analysis was performed with the statistical programme PAST (Hammer et al., 2001) for different combinations of dissolved matter. The Ward method was used in the cluster analysis (Ward, 1963). The cluster analysis resulted in the identification of two different major water types, representing an upper and lower aquifer. The identified water types correspond to the geological formation of the area. While the upper aquifer (alluvium/paleochannels) is composed of fresher groundwater strongly influenced by flood recharge, the lower aquifer (basement rock) contains additional saline end members.

Based on the cluster analysis, geological investigations, geophysical surveys and water level readings a conceptual flow model for the Kuiseb delta was developed and the model domain was separated into ten mixing cells (Fig. 3). Cell boundaries are formed by hydrogeological units and further subdivisions representing different mixing conditions within a given aquifer. Four different sources of water entering the groundwater system were identified. Three groundwater samples were used as end member to account for saline inflows from the northern boundary stemming from a saline spring located in the basement rock (Saltrevier) and the fractured aquifer that underlies the gravel plain. The samples from the fractured aquifer were taken in Gobabeb (GW-G) and Swartbank (GW-S). The fourth source is the water of the Kuiseb river (Fig. 3). The hydrochemical data are shown in Table 3 and are based on the data of Schmitz (2004). Samples from two flood events were used to calculate average chemical concentrations of floods in the Kuiseb river.

Based on the water levels and the position of the paleochannels, potential inflows into the different cells from upstream cells and inflow from the identified sources were assumed. Then the different groundwater compartments were analysed in terms of impact of flood recharge, inflow from saline groundwater and flow connections between alluvial aquifer, paleochannels and different basement rock types (crystalline basement or Tsondab sandstone).

After the model was initiated with the first estimation of the flow system and the classification into ten different cells, inverse modeling of each cell was carried out based on the approach of Adar and Külls (2002). In this approach, both the tracers and the potential sources were varied for each cell. As a result, different possible proportions of the different end members were obtained for each cell. To reduce the number of possible solutions, the error of the water balance and the mass balance of the different tracers was considered. The mixing cell modeling was carried out by including ^{14}C values to validate the results from inverse

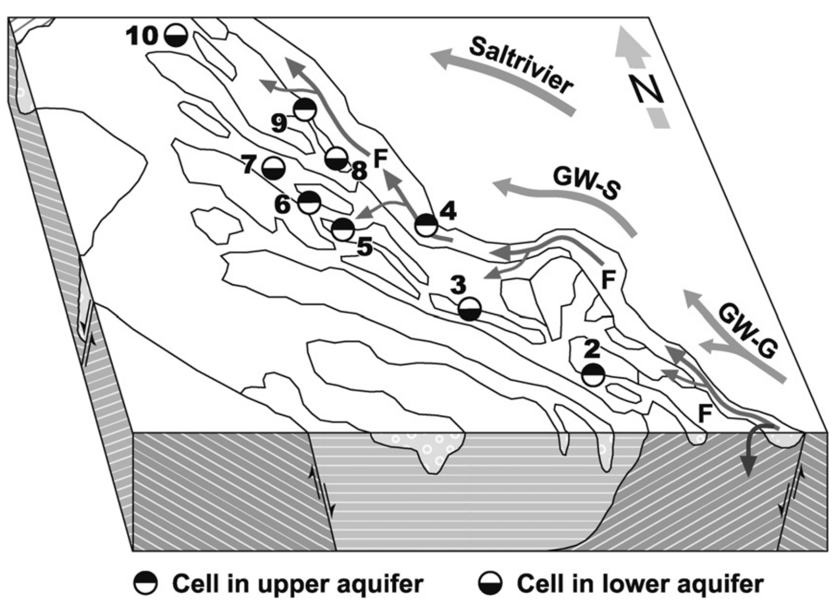

Figure 3 Hydrogeological map of the model area in the Kuiseb Dune area (based on BGR, 1995) including mixing cells and sources of the conceptual flow model (F, Saltrevier, GW-G and GW-S). The arrows denots the assumed area of the appearance of the sources GW-S and GW-G.

modeling of the hydrochemical tracers only. For the floodwater a ^{14}C-value of 100 pmC was used, corresponding to the present atmospheric concentration. The ^{14}C-concentration of the groundwater inflow from the northern crystalline basement was assigned based on samples taken by Vogel (1975). The measured ^{14}C-value of 33.8 pmC was compared to a residence time estimation based on annual groundwater recharge, recharge area and porosity. For the Saltrivier water, a value of <1 pmC was used. The calculated mean cell residence times were iterated and optimized to fit the measured ^{14}C-values in each cell. Implausible solutions were rejected, reducing the number of alternatives for each cell.

The floodwater fraction of the different cells was calculated based on the different model results for each cell of the MCM. A mean value was derived with deviations. If a given cell was affected by an upstream cell, the mean value of the upstream cell (with its deviation) was used to calculate the floodwater fraction. From the different results the mean value with its maximum positive and negative deviations was used to calculate the floodwater fraction for the downstream cells.

Sensitivity analysis

The model parameters are the concentrations of the different tracers. The sensitivity analysis was performed with three different tracers by varying a single tracer while the others were kept constant. The chemical concentration of each tracer was varied by 1% steps until ±10% and then by ±20%, ±30% and ±50% from the measured concentration. These concentrations were then used to run the MCM.

Results

Inverse mixing cell model

In this section the inverse modeling was carried out based on the setup described in Section 'Application to the Lower Kuiseb Dune area'. The description of modeling results follows the geological location of the cells. At first the results for the alluvial aquifer are described (cell 4 and 9), then the results for the paleochannels (cell 2, 5 and 6) and finally the results for the cells located in the basement are reported.

Wells E, G and F represent groundwater from the Kuiseb alluvial aquifer (cell 4) with water levels of 249, 226 and 213 m.a.s.l. Different model runs indicated that floodwater is by far the dominant source of recharge to this compartment of the alluvial aquifer (94—95.2%). Still, inflow of saline groundwater from the crystalline basement north of the Kuiseb (Table 3, GW-S) was found to contribute about 4.8—6.0% of water. This provides an explanation for observed salinization of water supplies in this section of the Kuiseb alluvial aquifer during extended periods without flood events. Calculations were done with three potential sources (floodwater, GW-S and water from Saltrivier) and with different tracer combinations ($Na^+/Cl^-/K^+$, Cl^-/K^+, Cl^-). Among different potential sources of saline groundwater GW-S could be identified. The end-member 'Saltrivier' had no detectable influence. Water balance deviation was less than 11.6%, tracer balance deviation was about −9% for sodium and between 0% and 6.3% for chloride. Potassium had significantly higher deviations.

The next compartment downstream in the alluvial aquifer (cell 9, well M, groundwater level 181 m.a.s.l.) again is characterized by a dominant floodwater fraction (97.6—98.1%). Inflows of saline groundwater from GW-S and Saltrivier as well as of a direct contribution from the upstream compartment were minor. Calculations were carried out with 14 different model configurations, only three of them led to acceptable deviations without water and tracer mass balance error (0%). All other solutions showed significant deficits in water balance (>15%). Deviations in tracer mass balance mostly exceeded the error tolerance (30%) for at least with one constituent.

For the compartments in the paleochannel system (cells 2, 5 and 6), solutions from inverse hydrochemical mixing indicate a significant influence of the end member floodwater. The most upstream compartment in the paleochannel system (cell 2, well B, 251 m.a.s.l., 61 m depth to water table) contains 76.6—86.5% of floodwater while the inflow of saline groundwater (GW-G) amounts to 11.3—15.4%. The very saline groundwater from Saltrivier did not have any detectable influence. The solution of the water and ion mass balance equations also allows for an inflow from the upstream basement compartment (cell 1) between 6.7% and 12.1%. The water mass balance error did not exceed 4%, and the tracer mass balance deviation was less than 20% for any tracer. For the paleochannel compartment cell 5 (well H, groundwater level at 173 m.a.s.l.) a single solution with two dominant inflows, 53.2% floodwater and 46.8% lateral subsurface inflow from the alluvial channel compartment (cell 4), showed acceptable deviations in water balance and tracer mass balances. For cell 6 (well I, water table at 168 m.a.s.l.) 16 feasible solutions pointed to inflows from the end members floodwater, GW-S, Saltrivier and indicated lateral subsurface inflow from the alluvial channel compartment (cell 4), from the upstream paleochannel compartment (cell 5) and from compartments representing groundwater in the basement aquifer (cells 3 and 8). Six model runs that fully satisfied water and tracer mass balances (0% error) all identified floodwater as major source (39—65.2%). Differing additional sources were calculated using K^+ and Cl^- as tracers. The inflows from saline groundwater (GW-S and Saltrivier) consistently had a small impact (below 8.5% and 0.9%, respectively). Deviations in water-balance were also small for the remaining runs (−12.7% to 11.9%). The deviations in the mass balance varied between −10.3% and 14.3% for K^+, −13.5% and −0.1% for Na^+ with one exception (−22.3%) and −3.2% and 10% for Cl^-.

Furthermore, the groundwater flow system within the crystalline basement (cells 1, 3, 8, 10) south of the Kuiseb river and the interaction with the alluvial channel compartment could be investigated. In the most upstream basement compartment (cell 1) of the study area (well A, mean water level at 311 m.a.s.l.) flood recharge and an inflow of saline groundwater (GW-G, see Table 2) were identified as major inflows. The inflow of recharge from floods was calculated to range from 62.2% to 62.6% and the fraction of inflow from the north at Gobabeb (GW-G) ranged from 37.4% to 37.8% depending on the choice of tracers for mass balance calculations. Using sodium and chloride led to a deviation in mass balance of −0.2%. Model runs including potassium as an additional tracer yielded comparable results but resulted in higher deviations in the specific potassium ion mass balance.

Inverse mixing analysis indicates that groundwater composition in the basement compartment downstream (cell 3, wells C and D, groundwater table at 260 and 187 m.a.s.l.) is more complex. The end member floodwater contributes with 35.9% to 65.2%, but there are also model runs indicating no floodwater in cell 3. Saline groundwater inflows are varying between 36.5% and 16.6% or 0% (GW-S) and between 0% and 2.3% for water from Saltrivier. In addition, several solutions indicate inflows from other adjacent cells in the basement, e.g. between 5.9% and 62.7% from the upstream compartment in the basement (cell 1) and 72.6% and 82.6% from an adjacent paleochannel compartment (cell 2). Some model runs indicate also no inflow from these cells. Different sets of sources and inflows were used and run with two (K^+, Cl^-) and with three tracers (Na^+, K^+, Cl^-). Depending on model set-up and tracer selection, eleven different solutions were identified for cell 3. The deviations of the water balance ranged from −11.7% to 1.4%. Deviations in tracer mass balances were lowest for chloride. The modeling allowed for multiple solutions, an identification of the flow system required further constraints on mean residence time.

Downstream compartments in the basement located closer to the alluvial channel (cell 8, well L, 195 m.a.s.l.) indicate increased contributions from the recent alluvial aquifer. The best solution (lowest deviation in water balance) was found for an inflow of 71.5% water from the alluvial channel (cell 4). However, saline groundwater still contributed 28.5% (GW-S). The deviation in water balance was 1% and −1.3%, −7.7% and 6.0% in the tracer mass balances for K^+, Na^+ and Cl^-, respectively. It is noted that the paleochannels south of this compartment (cell 5, 6 and 7) did not contribute due to lower hydraulic potential. In some solutions additional contributions of floodwater and saline groundwater (Saltrivier) end members as well as inflow from the upstream basement compartment (cell 3) were detected. The inclusion or exclusion of certain end members (e.g., Saltrivier, GW-S) predetermined the type of contributions from additional inflows. In every model setup, where GW-S was a potential inflow to cell 8, it had a contribution between 19.6% and 32.4%. Further model runs showed that the inclusion of inflow from the alluvial aquifer (cell 4) excluded the source floodwater (except for one case with minor floodwater inflow of 2.3%).

The hydraulically lowest basement compartment (cell 10, well N, water table 126 m.a.s.l.) again showed complex and variable solutions. Saline groundwater (GW-S) contributes with 19.4% to 47.6%, while the alternative saline source (Saltrivier) was rejected. The contribution of floodwater as a source and of inflow from the upstream basement compartment (cell 8) could not be discerned and replaced each other in different solutions. Some model runs indicate additional contributions from other compartments (cell 7 or 9).

Six solutions were found within the accepted range of deviation for water and tracer mass balance. In all six cases deviation in water balance was between 3.2% and 5.8%, tracer mass balance pointed to potassium and sodium deficiencies and chloride excess in cell 10.

While inverse multi-compartment mixing analysis provided distinct solutions for most of the compartments in the alluvial aquifer and in the paleochannel system, the composition of groundwater in the basement aquifer beneath the dune sea south of the Kuiseb river is complex. Due to this complexity the hydrochemical composition could not be derived from known end-members for cell 7 (well K, water level at 167 m.a.s.l.). Solute balances and the water-balance did not close within acceptable limits. The lithology of the study area (mainly alluvial sand, Tsondab sandstone, granite basement rock) and thermodynamic phase equilibrium analysis do not suggest a major influence of geochemical evolution. Therefore, the difficulty in matching hydrochemical composition with known sources may point to a missing source end-member. It is also noted that for compartments with a complex source and inflow situation, a strictly pre-defined conceptual model excluded feasible solutions while a less specific conceptual model favoured multi-finality. In such cases various solutions can only be discriminated based on additional criteria such as residence time.

Sensitivity analysis

A sensitivity analysis of the hydrochemical end member analysis showed only a small change of calculated sources when concentrations of tracers were modified. Varying tracer concentration in floodwater source by up to ±10% caused a maximum change of −0.3% compared to the original floodwater contribution of 84.6%. Uncertainties in tracer concentrations result in linearly correlated deviations of computed flow rates for most tracers. The absolute change of floodwater fraction is more sensitive to variations of tracer concentrations in groundwater than in floodwater because tracer concentrations are higher in groundwater. For the same reason, chloride showed the highest sensitivity and potassium the lowest. The change of the floodwater fraction in cell 2 is −1.5% for a decrease of 10% in groundwater chloride concentration and +1.3% for a 10% increase. The change in water balance error is −0.37% when chloride concentration is lowered by 10% and increased linearly to −0.51% when the concentration was increased by 10%.

Validation of the results by forward mixing cell modeling with ^{14}C

Based on different solutions of the inverse mixing cell model, modeling with ^{14}C was carried out following the above-described stepwise approach. A mean residence time for each cell was calculated according to this approach. The results of the ^{14}C modeling were used to validate the results of Section 'Inverse mixing cell model'. The number of solutions could be narrowed down for the different cells. The validated results are shown in Figs. 4 and 5. The ^{14}C modeling of cell 1 and 8 led to no results for the residence time, because the ^{14}C activity exceeds 100 pmC in those two cells.

The average ^{14}C-activity in the Kuiseb alluvial channel compartment (cell 4) is 92.6 pmC, groundwater is recent. The results obtained by mixing cell analysis were confirmed by ^{14}C-MCM. The calculated mean residence time in cell 4

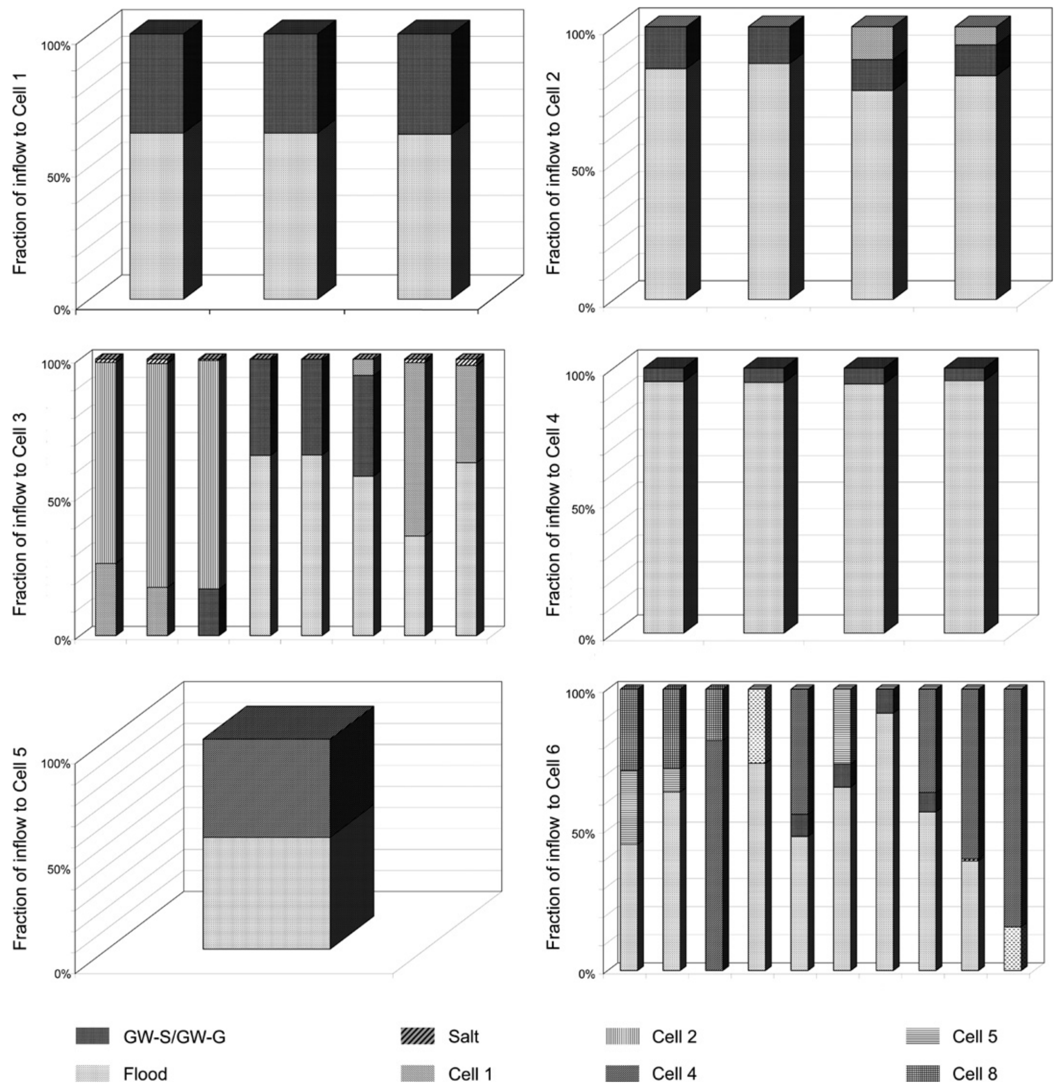

Figure 4 Results of the Mixing Cell Model (cell 1 to cell 6). The bars indicate the percentage of inflow from different cells and sources.

varied between 300 and 369 yr with an average of 345 yr and a standard deviation of 26 yr. The next downstream compartment in the alluvial channel (cell 9) has a much lower measured ^{14}C-activity of 67.1 pmC pointing to significantly higher mean residence times. Three possible sets of solutions found by inverse hydro-chemical mixing cell analysis all showed very similar mean residence times (3145 yr, 3152 yr and 3157 yr). In terms of the interpretation of the flow system the result is significant as cell 9 is located directly at the alluvial aquifer and hydraulic estimations would suggest much lower mean residence times (BGR, 1999). Hence, hydrochemical mixing cell analysis and ^{14}C both provide consistent evidence, that the alluvial may consist of sections divided by basement ridges.

For all the three paleochannel compartments (cell 2, 5 and 6) the solutions of the mixing cell analysis could be validated. The most upstream paleochannel compartment (cell 2) has a measured ^{14}C activity of 78.1 pmC. All solutions obtained from hydrochemical mixing analysis were plausible and resulted in calculated mean residence times of 1154—1490 yr, with a mean value of 1343 yr and a standard deviation of 159 yr. Similar results were obtained for the paleochannel compartment downstream (well 7871, cell 5, with 70.5 pmC). The calculated mean residence time was 2598 yr. Other inflow alternatives with higher deviations in water- and/or tracer mass-balance showed similar mean residence times. For the most downstream paleochannel compartment (cell 6) some hydrochemically valid solutions did not agree with the measured ^{14}C activity of 86 pmC. Eight of 18 solutions of the inverse MCM were rejected because they could not reproduce the measured ^{14}C values. The remaining possible solutions had mean residence times between 69 and 809 yr with an average of 516 yr and a standard deviation of 263 yr. Without the outlier of 69 yr the minimum value is 287 yr. All three modelled paleochannels have relatively short mean residence times, indicating active or recent recharge.

For the compartments assigned to the basement aquifer, all characterized by complex flow contributions (cell 3, 10), the analysis of ^{14}C data helped to constrain the set of hydrochemically possible solutions. Only some of the flow connections identified for cell 3 could also reproduce

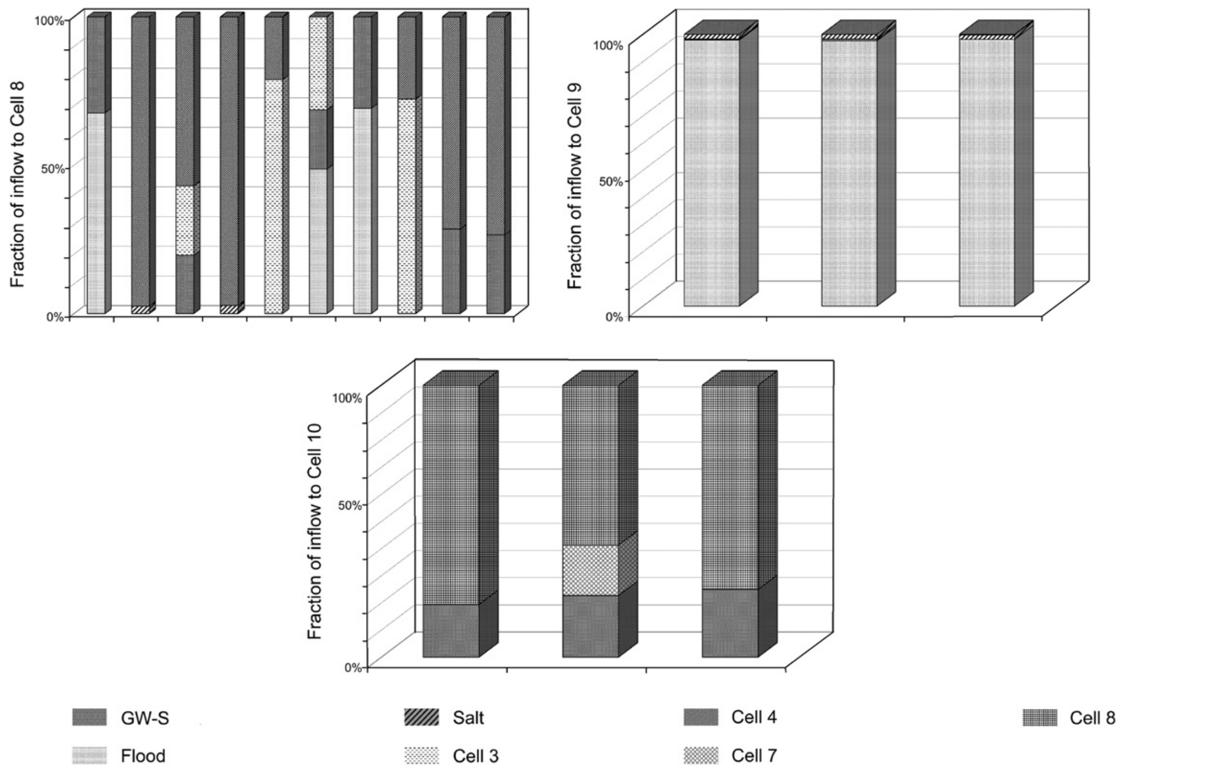

Figure 5 Results of the Mixing Cell Model (cell 8 to cell 10).

measured ^{14}C values, corresponding to a mean residence time of up to 3305 yr. For the lowermost basement compartment (cell 10), the inverse MCM led to six possible inflow alternatives, which had a measured ^{14}C-concentration of 70.9 pmC. The validation of the results led to a rejection of all model runs including floodwater recharge as a source, because mean residence times became negative in these cases. The remaining solutions correspond to a relatively narrow range of mean residence times of 1169, 1384 and 1737 yr. These solutions showed that groundwater in the basement contains a significant fraction of saline groundwater (around 25%) and receives lateral subsurface inflow from upstream basement compartments (cell 8, ~60% to 80%). In summary, the groundwater in the base rock into which paleochannels were incised is still connected with adjacent paleochannels. Again, patterns of increasing mean residence times with increasing distance from the alluvial channel and from paleochannels are not evident.

Discussion

In summary, the mixing cell model provided detailed information on the interactions between the alluvial aquifer, the paleochannel system and the basement aquifers (Tsondab sandstone and crystalline basement) into which the paleochannels are incised. It was possible to calculate the overall floodwater fraction of each compartment (Table 4, Fig. 6). Two types of groundwater were identified and the results indicate that floodwater is the main recharge source in the area but that the groundwaters from the aquifer beneath the Namib desert gravel supplies a amount of recharge that should not be underestimated in its relevance (Table 4).

The modeling allow for different sets of inflows to each cell, which produced a variety of MCM solutions. The variety of existing solutions results from uncertainties about the conceptual hydrogeological model. The use of ^{14}C as an additional validation tool helped to reduce the number of possible solutions. In some cases, forward modeling allowed for the rejection of inflow alternatives calculated by the inverse hydrochemical model, because invalid solutions for calculated mean residence could be rejected. Some samples (cells 1 and 8) that were affected by bomb ^{14}C and exceeding 100 pmC could not be validated. For the other compartments, the inclusion of age indicators (here ^{14}C) introduced an independent constraint that helped reduce model uncertainty. Still, the study showed that equifinality

Table 4 Minimum, mean and maximum floodwater fraction of the different cells

Cell	Percentage of calculated contribution from floodwater			Well
	Minimum	Mean	Maximum	
No.1		62.4		A
No.2	83.6	85.2	86.5	B
No.3	61.3	71.2	84.5	C, D
No.4	94.0	94.8	95.2	E, F, G
No.5	97.2	97.5	97.8	H
No.6	89.5	92.7	97.7	I
No.7				K
No.8	67.6	75.2	92.4	L
No.9	98.1	98.2	98.3	M
No.10	50.8	61.0	74.5	N

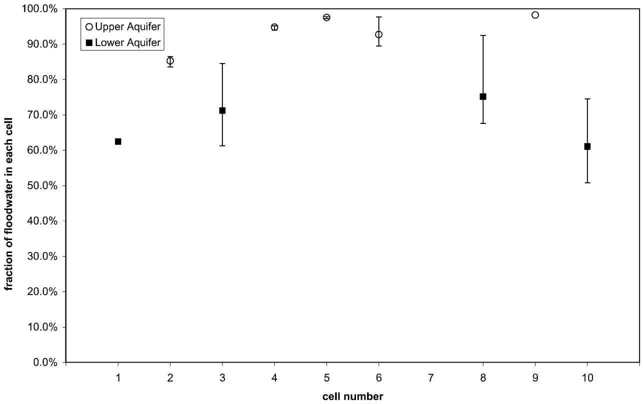

Figure 6 Fraction of floodwater in different cells of the Lower Kuiseb groundwater system. The upper (alluvium, paleochannels) and the lower aquifer (basement) are shown separately.

can be an issue in mixing cell modeling, resulting in more then one possible solution of the flow system. It may be caused by linear combinations of water types representing potential complex inflows and existing uncertainty or similarity in the composition of end-members.

For cells located in the recent alluvial channel (4, 9), it was found that groundwater has the same dominant source but that residence time varies significantly (Fig. 7). Floodwater was found to constitute 94.8—97.9% of groundwater in the alluvial channel, with the remaining percentage from different sources of saline groundwater from the fractured basement aquifers. The detection of an inflow of saline groundwater explains salinization trends during long periods without flooding. The data on residence times suggest the existence of compartments, likely by basement ridges beneath the alluvium. The compartments (e.g., cells 4 and 9, see Fig. 5) have at most weak flow connections, and likely no connections. Groundwater level monitoring data (Dahan et al., 2008) indicating a stabilization in the decline of water level recession in the alluvial channel is also in agreement with the assumption of compartments in the Kuiseb alluvial aquifer. Further geophysical investigations may help to delineate these compartments and improve the management of pumping wells.

It was found that groundwater in the paleochannel system in the vicinity of the alluvial aquifer and for which groundwater samples exist, still contains a high percentage of floodwater fraction (85.2—97.5%). However, the results indicate additional recharge and subsurface flow mechanism. Groundwater composition in the most upstream paleochannel compartment (cell 2) resembles the composition of groundwater in the alluvial aquifer, characterized by floodwater recharge and saline groundwater. In two out of four solutions that are valid based on ^{14}C activities, minor inflows from the fractured basement aquifer are identified (6.7—12.1%). Within the paleochannel system the hydrochemical composition is getting increasingly complex, with sources and inflows from floodwater, saltwater from the fractured basement aquifer and from upstream cells (4 and 5). In the investigated area a flow connection between the alluvial aquifer (cell 4) and paleochannels (cell 5 and 6) could be clearly detected, confirming the assumptions of BGR (1999). These results only apply to the investigated area in the vicinity of the alluvial aquifer.

Finally, five compartments that were all allocated to a lower aquifer tier by cluster analysis, represent groundwater found in the basement rock into which paleochannels are incised. Results obtained from mixing cell analysis also provided insight into recharge and subsurface flow interactions of the lower aquifer tier with the Kuiseb alluvial aquifer and with paleochannels. All compartments are characterized by a typical floodwater fraction of 61.0—75.2%, differing significantly from both the alluvial channel and the paleochannel compartments. The specific composition

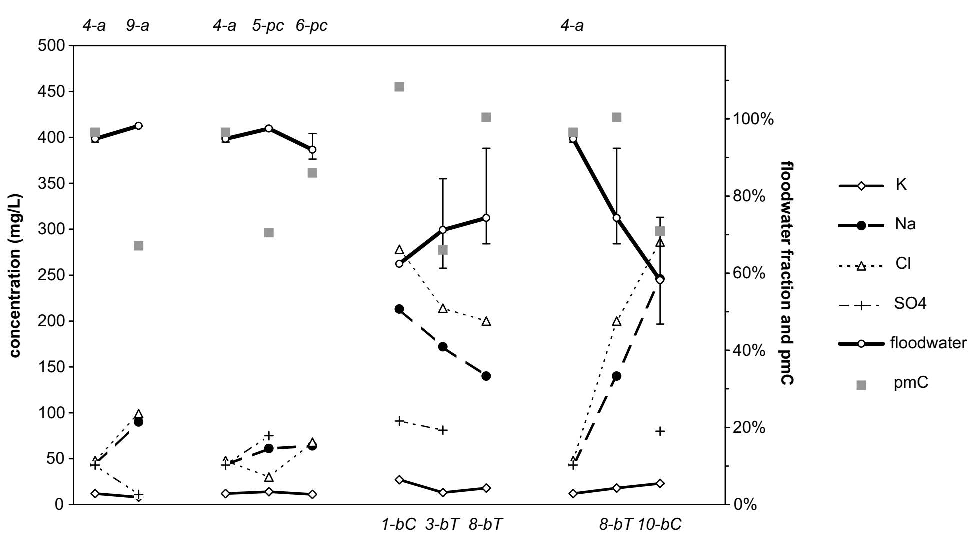

Figure 7 Profiles along flow paths within the alluvial (a) aquifer (cell 4—9) with no or a minor flow connection, from the alluvium into the paleochannels (pc) represented by the cells 4, 5, 6 and into the basement (cell 1, 3, 8 and 10) with hydrochemistry, ^{14}C and floodwater fraction. The basement consists of crystalline basement (bC) and Tsondab sandstone (bT). The profiles derive from mixing of different water types not from hydrochemical evolution. The different values can be seen in the Tables 1, 2 and 4.

depends on the location within the flow system. In the most upstream compartment (cell 1) the floodwater source is mixed with 36.6–37.8% of the salt-water end member GW-G. In sections where paleochannels are incised into Tsondab sandstone, the paleochannel groundwater can be traced into the lower aquifer tier (cell 3). Some compartments assigned to the lower aquifer indicated additional internal flow connections (between cell 1 and 3; cell 3 and 8; cell 8 and 10, see Figs. 4 and 5).

The results confirm that floodwater recharge is the dominant source of groundwater in the study area. The hydrochemically and hydrogeologically based distinction of an upper and lower aquifer tier leads to consistent profiles of floodwater fraction in each aquifer tier along flowpaths.

Conclusion

Use of an inverse mixing cell model (Adar et al., 1988; Adar and Sorek, 1989,1990; Adar and Külls, 2002) together with a newly developed ^{14}C validation procedure proved to be a useful method to derive a conceptual flow system for the hyper arid Kuiseb Dune area of Namibia. In this study, the fractions of flood recharge and of inflow of saline groundwater from the basement north of the Kuiseb were calculated for different aquifer compartments (cells) of the study area. The MCM indicates two distinctly different groundwater types, a fresh groundwater type with higher floodwater proportions found in the alluvium and in the paleochannel system and a saltier groundwater type mainly found in the Tsondab and crystalline basement rock. Also, the less frequent occurrence of flood events in the western-most part of the system was evident in a lower proportion of floodwater in that area. A detectable inflow from the northern boundary (basement rock plain) contributes to the groundwater system of the Lower Kuiseb Dune area.

This study demonstrated the usefulness of ^{14}C data as a validation tool for inverse MCMs through forward mixing cell modeling, using the calculated mean residence times as additional proof of possible inflow alternatives. Other dating tracers could also be used as validation tools, with only small changes to this method. As such, the model could also be used for groundwater systems with smaller residence times through use of tracers such as CFCs, SF_6, ^{85}Kr, ^{39}Ar, or ^{3}H and for systems with longer residence times by using ^{36}Cl.

Acknowledgments

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