Don't say CAT: New Item Response Theory approaches for developing short test forms

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CAT

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Computerized **A**daptive **T**esting

Item Response Theory and short test forms

ADAPTIVE SHORT FORMS: *Ad-hoc* tests for each person $\rightarrow$ The information is maximized for each level of θ (i.e., for each respondent) $\rightarrow$ (**CAT**: Computerized Adaptive Testing)

STATIC SHORT FORMS: Static tests equal for all respondents $\rightarrow$ The information is maximized across θ levels (i.e., across all respondents)

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Different short test forms for each respondent $\rightarrow$ Potential fairness issues in assessments, e.g. for recruitment

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STATIC SHORT FORMS: Static tests equal for all respondents $\rightarrow$ The information is maximized across θ levels (i.e., across all respondents)

Issue

Not being tailored to any θ level of interest $\rightarrow$ Potentially more items are needed to cover a wide range of θs

Aim

New IRT-based procedures for shortening tests

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Equal for all respondents

Aim

New IRT-based procedures for shortening tests

Equal for all respondents

Tailored to specific levels of the latent trait

Item Response Theory

2-PL Model

$$P(x_{pj} = 1|\theta_p, b_j, a_j) = \frac{exp[a_j(\theta_p - b_j)]}{1 + exp[a_j(\theta_p - b_j)]}$$

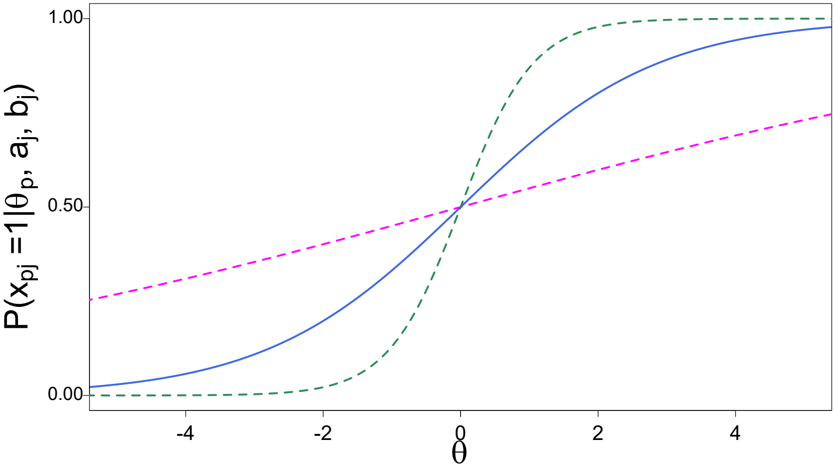
where:

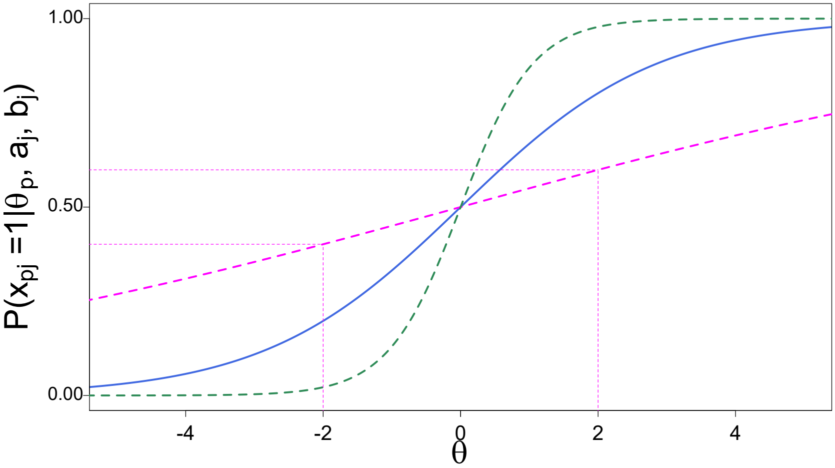
$P(x_{pj} = 1)$: Probability of a correct response to item j by respondent p

θ_p: Ability of respondent p

b_j: Difficulty of item j

a_j: Discrimination of item j


1.00
0.50
0.00
-4
-2
0
2
4
θ
P(x_pj =1|θ_p, a_j, b_j)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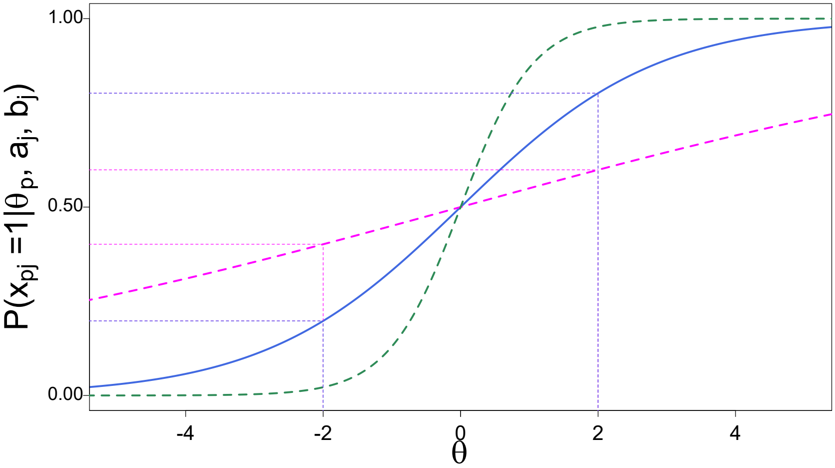
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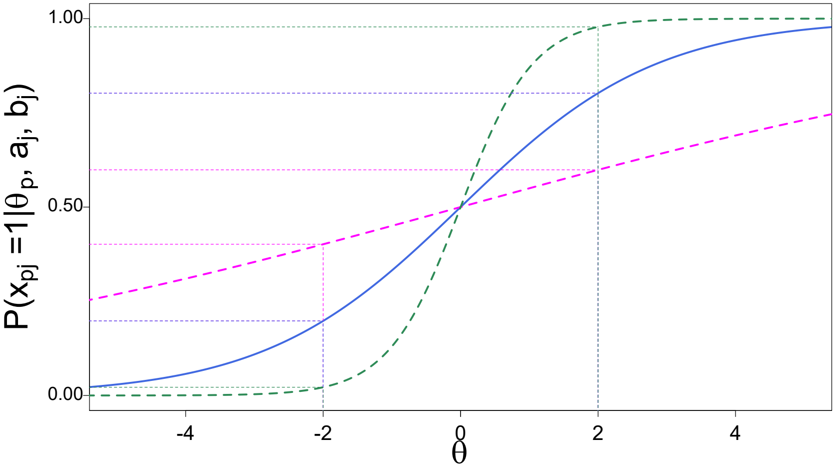
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Information functions

Item Information Function

$$IIF_j = a_j^2[P(\theta)(1 - P(\theta))]$$

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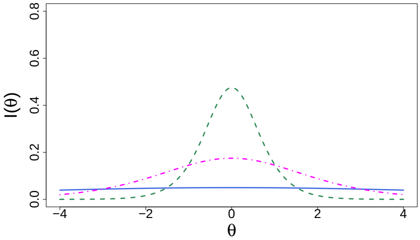

Figure 1: $a = 0.20$, $a = 0.70$, $a = 1.90$, $b = 0$

Information functions

Item Information Function

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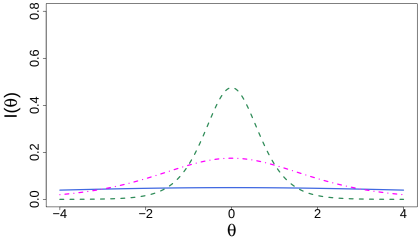
Test Information Function

$$TIF = \sum_{j=1}^{J} IIF_j$$



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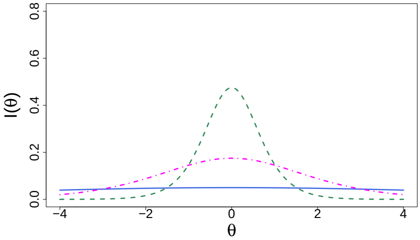

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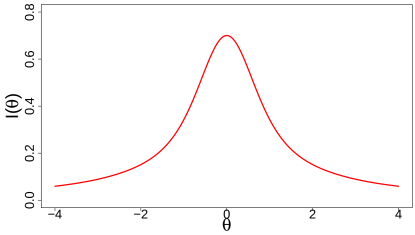

Figure 2: TIF $= IIF_1 + IIF_2 + IIF_3$

Benchmark procedure

Selected items → items with the highest *IIF*s

e.g.: 3-item short form from 10-item full-length test

item	b	a	IIF
1	−0.67	0.71	0.08
2	0.50	1.19	0.15
3	−2.43	0.25	0.01
4	2.12	1.98	0.24
5	1.72	0.39	0.03
6	−2.28	1.62	0.19
7	0.64	0.50	0.05
8	−2.51	1.68	0.19
9	−0.66	0.44	0.04
10	0.72	0.33	0.02

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6	−2.28	1.62	0.19
2	0.50	1.19	0.15
1	−0.67	0.71	0.08
7	0.64	0.50	0.05
9	−0.66	0.44	0.04
5	1.72	0.39	0.03
10	0.72	0.33	0.02
3	−2.43	0.25	0.01

θ-target procedures

Selected items $\rightarrow$ items with highest *IIF*s in respect to θ targets (θ')

e.g.: 3-item short form from 10-item full-length test

	θ'_1	θ'_2	θ'_3
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3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
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Segmenting the latent trait

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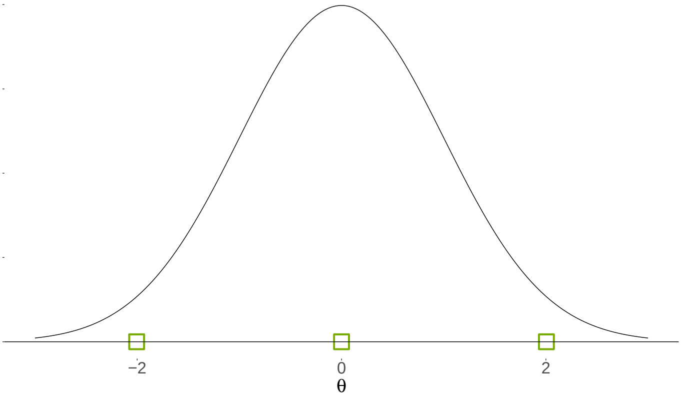

Equal Intervals Procedure

Equal segmentation

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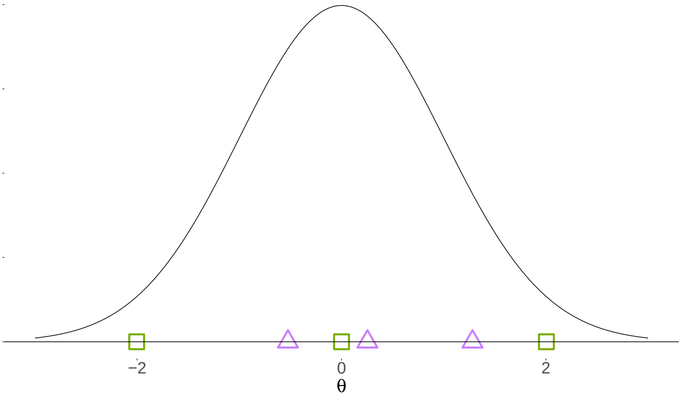

Equal Intervals Procedure
Equal segmentation

Unequal Intervals Procedure
Clustering

Comparison between the item selection procedures:

- **Benchmark procedure (BP)**: The N items with the highest *IIF*s are selected from the full-length test
- **Equal Intervals Procedure (EIP)**: The N items that maximize the information for each θ' obtained by dividing the latent trait into equal intervals are selected
- **Unequal Intervals Procedure (UIP)**: The N items that maximize the information for each θ' obtained by clustering the latent trait are selected
- **Random Procedure (RP)**: N items are randomly selected from the full-length tests

10, 30, 50, 70, 90-item short test forms from a 100-item full-length test

1000 respondents p

1. Normal distribution
   $p \sim \mathcal{N}(0, 1)$
2. Positive skewed distribution
   $p \sim Beta(1, 100)$ (linearly transformed to obtain negative values)
3. Uniform distribution
   $p \sim \mathcal{U}(-3, 3)$

100 items j:

- $b \sim \mathcal{U}(-3, 3)$
- $a \sim \mathcal{U}(0.40, 2)$

An overall look

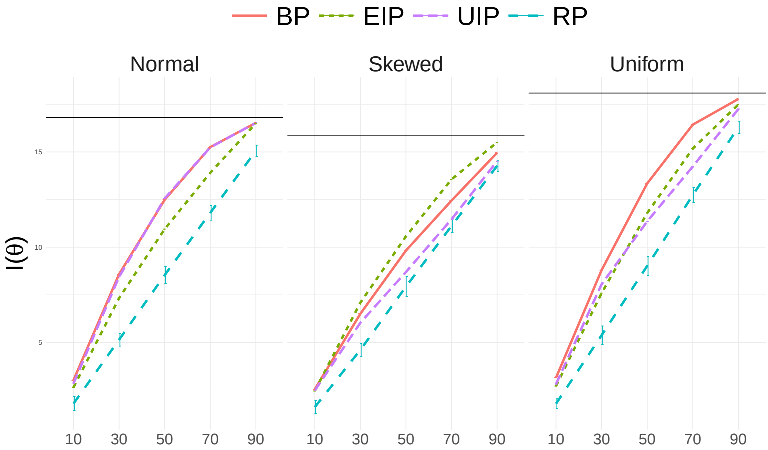

Figure 3: Overall Information of the short test forms

A closer look

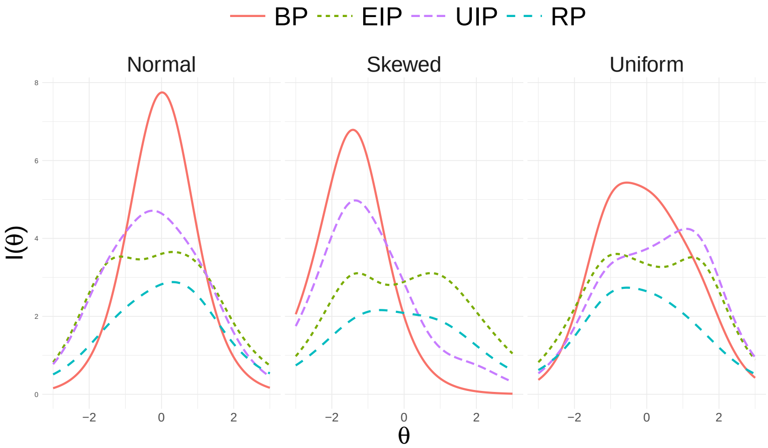

Figure 4: TIF of the 10-item short test form

An even closer look

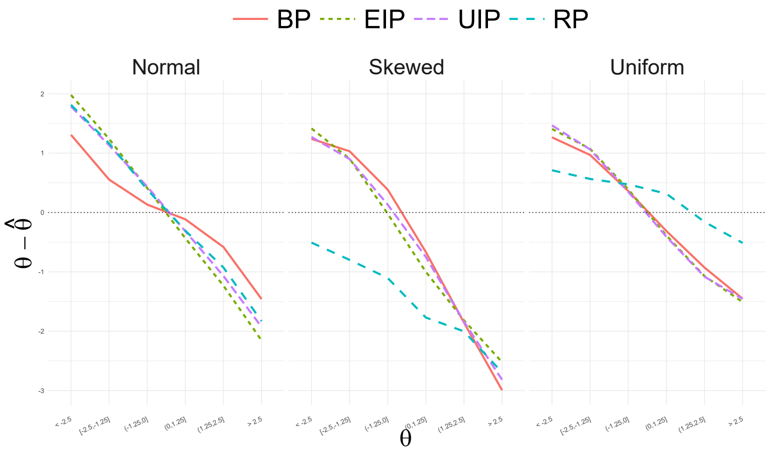

Figure 5: *bias* $= \theta - \hat{\theta}$ of the 10-item short test form

Good!

There's no "one-fits-all" solution

The θ distribution is a key element

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There's no "one-fits-all" solution

The θ distribution is a key element

..but work is still needed

Real life applications are missing

The CAT is missing

Thank you!

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