

QUESTIONNAIRES AND BEYOND: THE RASCH MODEL

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 A_{Bart}  A_{Lisa}

 A_{Bart} **Q1**

$$4 + 5 = ?$$

 d_{q1} **Q2**

$$\frac{3}{2}x^2 + \frac{5}{4}x = ?$$

 d_{q2}  A_{Lisa}


 A_{Bart}
Q1

$$4 + 5 = ?$$

 d_{q1}
Q2

$$\frac{3}{2}x^2 + \frac{5}{4}x = ?$$

 d_{q2}

 A_{Lisa}

$$\frac{A_p}{d_i} \quad (1)$$

$$> 1 \text{ if } A_p > d_i$$

$$< 1 \text{ if } A_p < d_i$$

$$P(X_{pi} = 1) = \frac{\frac{A_p}{d_i}}{1 + \frac{A_p}{d_i}} \quad (2)$$



$$\ln(A_p) = \beta_p$$

$$\ln(d_i) = \delta_i$$

$$P(X_{pi} = 1 | \beta_p, \delta_i) = \frac{\exp(\beta_p - \delta_i)}{1 + \exp(\beta_p - \delta_i)} \quad (3)$$

The intuition



The model



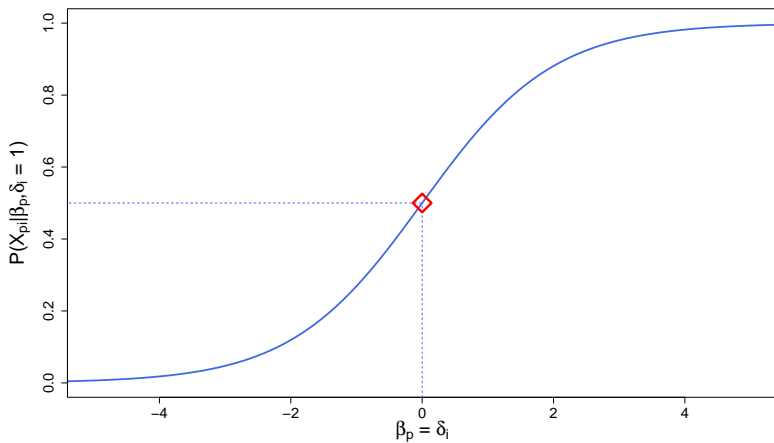
Wait...



Why is it useful?



Closing time



The intuition



The model



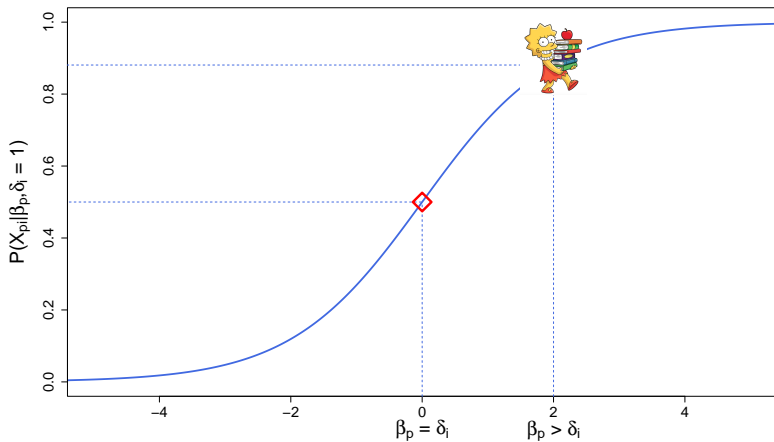
Wait...



Why is it useful?



Closing time



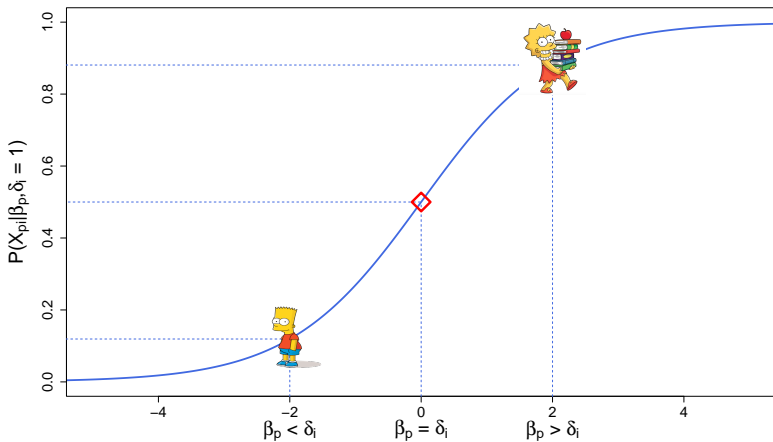
The intuition

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Wait...

Why is it useful?

Closing time



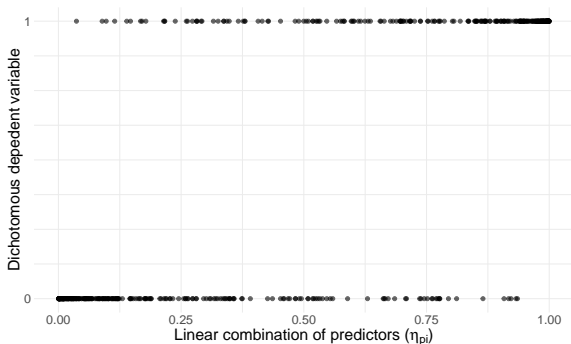
* Eureka moment *



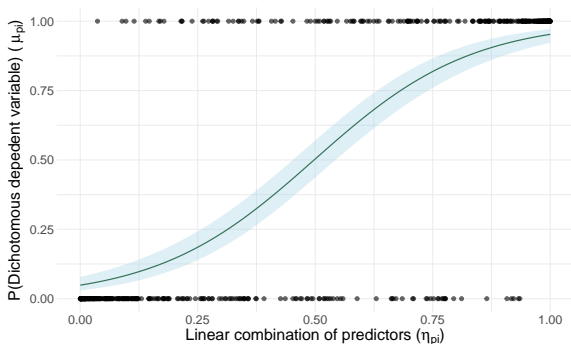
$$P(X_{pi} = 1 | \beta_p, \delta_i) = \frac{\exp(\beta_p - \delta_i)}{1 + \exp(\beta_p - \delta_i)}$$

Generalized Linear Model (GLM)

binomially distributed responses

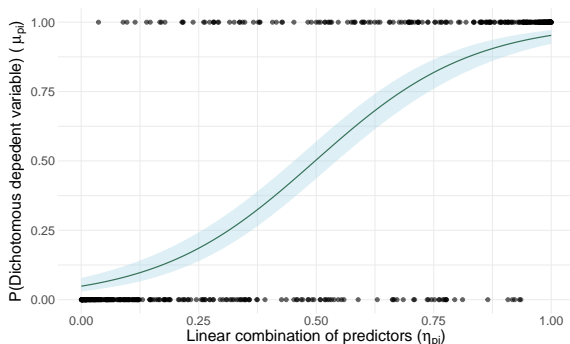


Generalized Linear Model (GLM) binomially distributed responses



$$\mu_{pi} = g(\eta_{pi}) = \log \left(\frac{\mu_{pi}}{1 - \mu_{pi}} \right)$$

Generalized Linear Model (GLM) binomially distributed responses



$$\mu_{pi} = g(\eta_{pi}) = \log \left(\frac{\mu_{pi}}{1 - \mu_{pi}} \right)$$

$$g^{-1} = \frac{\exp(\eta_{pi})}{1 + \exp(\eta_{pi})}$$

Rasch model: Dichotomous responses

Issue

Quite limiting in Psychological Research

(Generalized) Linear Model: “Any” kind of response

e.g.: Response times

log-transformation and log-normal model parametrization

- **Linearity of the scores**

Logarithm transformation → Respondents and items on the same latent trait

- **Comparison invariance**

Respondents can be compared between each other without considering the items....and vice versa!

- **Local independence**

Given the person → The responses to the items are independent

Unidimensionality

- **Linearity of the scores**

Logarithm transformation → Respondents and items on the same latent trait

- **Comparison invariance**

Respondents can be compared between each other without considering the items....and vice versa!

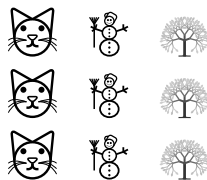
- **Local independence**

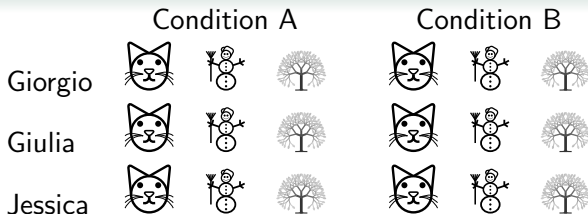
Given the person → The responses to the items are independent

Condition A



Condition B





Local independence

Rasch model

- Can't be applied
- The estimates would make no sense

Generalized Linear Model

- Add the random part (Go Mixed)
- Obtain a Rasch-like parametrization of the data

Think outside of the box!

Yes

Rasch estimates

The sky is the limit

Keep it maximal

But

Rasch-like parametrization

Don't over complicate things

Keep it minimal



Thank you

Questions!



<https://ottaviae.github.io/AIP2022/Rasch/epifaniaRasch.pdf>