

DON'T SAY CAT: NEW ITEM RESPONSE THEORY APPROACHES FOR DEVELOPING SHORT TEST FORMS

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1 Introduction

2 Item Response Theory and information functions

3 IRT procedures for shortening tests

Benchmark procedure

Procedures based on θ targets

4 Simulation study

5 Some final remarks

CAT



CAT



Computerized Adaptive Testing

Item Response Theory and short test forms

ADAPTIVE SHORT FORMS: *Ad-hoc* tests for each person → The information is maximized for each level of θ (i.e., for each respondent) → (**CAT**: Computerized Adaptive Testing)

STATIC SHORT FORMS: Static tests equal for all respondents → The information is maximized across θ levels (i.e., across all respondents)

Item Response Theory and short test forms

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Issue

Different short test forms for each respondent → Potential fairness issues in assessments, e.g. for recruitment

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STATIC SHORT FORMS: Static tests equal for all respondents → The information is maximized across θ levels (i.e., across all respondents)

Issue

Not being tailored to any θ level of interest → Potentially more items are needed to cover a wide range of θ s

Aim

New IRT-based procedures for shortening tests

Aim

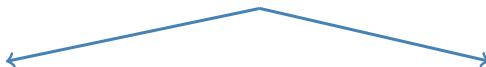
New IRT-based procedures for shortening tests



Equal for all respondents

Aim

New IRT-based procedures for shortening tests



Equal for all respondents

Tailored to specific levels of
the latent trait

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Item Response Theory

2-PL Model

$$P(x_{ps} = 1 | \theta_p, b_j, a_j) = \frac{\exp[a_j(\theta_p - b_j)]}{1 + \exp[a_j(\theta_p - b_j)]}$$

where:

$P(x_{ps} = 1)$: Probability of a correct response to item s by respondent p

θ_p : Ability of respondent p

b_j : Difficulty of item s

a_j : Discrimination of item s

Information functions

Item Information Function

$$IIF_j = a_j^2 [P(\theta)(1 - P(\theta))]$$

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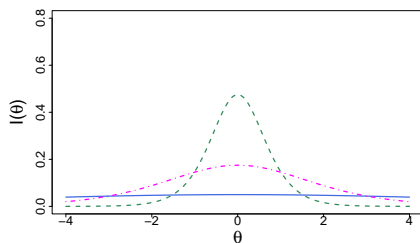


Figure 1: $a = 0.20$, $a = 0.70$, $a = 1.90$,
 $b = 0$

Information functions

Item Information Function

$$IIF_j = a_j^2 [P(\theta)(1 - P(\theta))]$$

Test Information Function

$$TIF = \sum_{s=1}^S IIF_j$$

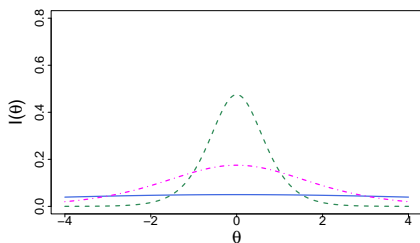


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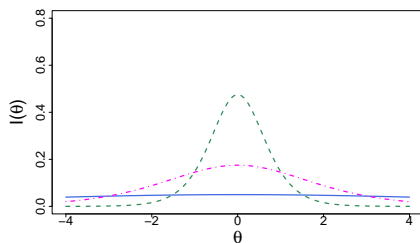


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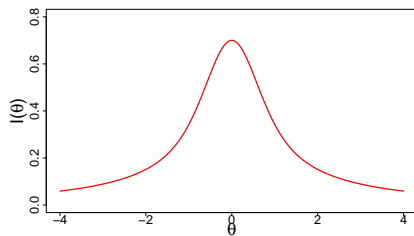


Figure 2: $TIF = IIF_1 + IIF_2 + IIF_3$

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Benchmark procedure

Selected items → items with the highest *IIFs*

e.g.: 3-item short form from 10-item full-length test

Benchmark procedure

Selected items → items with the highest *IIFs*

e.g.: 3-item short form from 10-item full-length test

item	<i>b</i>	<i>a</i>	<i>IIF</i>
1	−0.67	0.71	0.08
2	0.50	1.19	0.15
3	−2.43	0.25	0.01
4	2.12	1.98	0.24
5	1.72	0.39	0.03
6	−2.28	1.62	0.19
7	0.64	0.50	0.05
8	−2.51	1.68	0.19
9	−0.66	0.44	0.04
10	0.72	0.33	0.02

Benchmark procedure

Selected items \rightarrow items with the highest $IIFs$

e.g.: 3-item short form from 10-item full-length test

item	b	a	IIF
4	2.12	1.98	0.24
8	-2.51	1.68	0.19
6	-2.28	1.62	0.19
2	0.50	1.19	0.15
1	-0.67	0.71	0.08
7	0.64	0.50	0.05
9	-0.66	0.44	0.04
5	1.72	0.39	0.03
10	0.72	0.33	0.02
3	-2.43	0.25	0.01

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- └ Short form procedures
- └ Procedures based on θ targets

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θ -target procedures

Selected items \rightarrow items with highest $IIFs$ in respect to θ targets (θ')

e.g.: 3-item short form from 10-item full-length test

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2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
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Don't say CAT

└ Short form procedures

└ Procedures based on θ targets

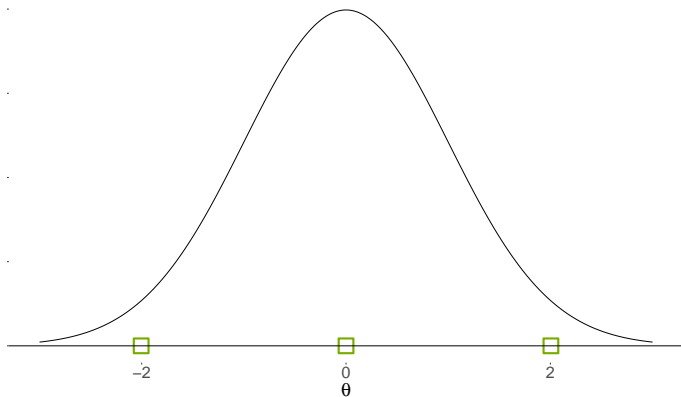
Segmenting the latent trait

Don't say CAT

└ Short form procedures

└ Procedures based on θ targets

Segmenting the latent trait



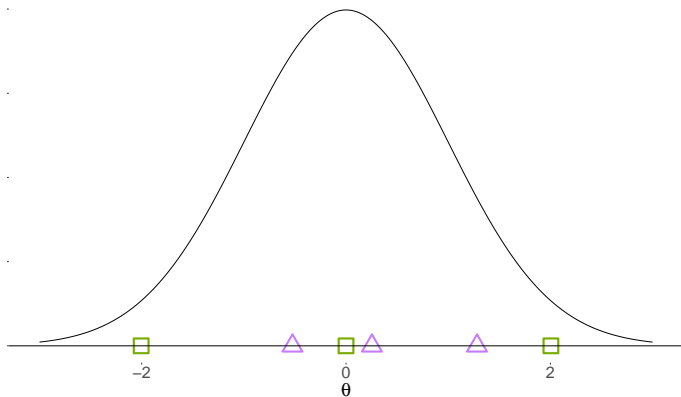
Equal Intervals Procedure
Equal segmentation

Don't say CAT

└ Short form procedures

└ Procedures based on θ targets

Segmenting the latent trait



Equal Intervals Procedure
Equal segmentation

Unequal Intervals Procedure
Clustering

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Comparison between the item selection procedures:

- **Benchmark procedure (BP)**: The N items with the highest $IIFs$ are selected from the full-length test
- **Equal Intervals Procedure (EIP)**: The N items that maximize the information for each θ' obtained by dividing the latent trait into equal intervals are selected
- **Unequal Intervals Procedure (UIP)**: The N items that maximize the information for each θ' obtained by clustering the latent trait are selected
- **Random Procedure (RP)**: N items are randomly selected from the full-length tests

10, 30, 50, 70-item short test forms from a 100-item full-length test

1000 respondents p

- ① Normal distribution
 $p \sim \mathcal{N}(0, 1)$
- ② Positive skewed distribution
 $p \sim \text{Beta}(1, 100)$ (linearly transformed to obtain negative values)
- ③ Uniform distribution
 $p \sim \mathcal{U}(-3, 3)$

100 items s :

- $b \sim \mathcal{U}(-3, 3)$
- $a \sim \mathcal{U}(0.40, 2)$

An overall look

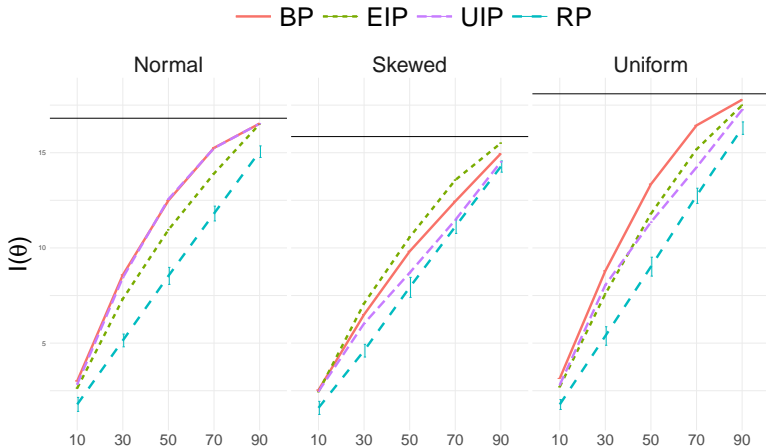


Figure 3: Overall Information of the short test forms

A closer look

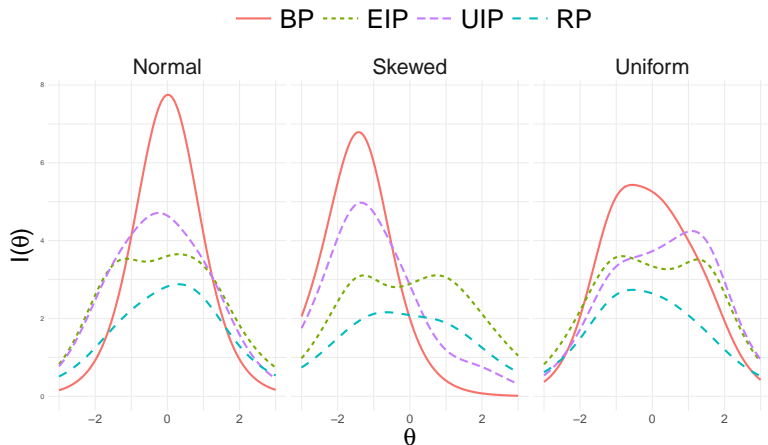


Figure 4: TIF of the 10-item short test form

An even closer look

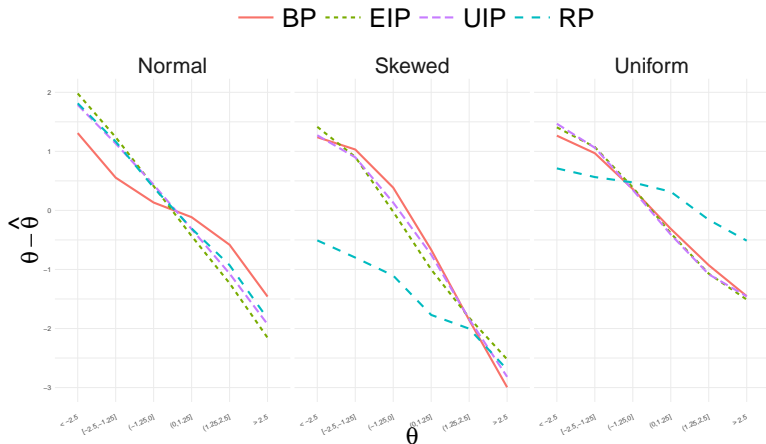


Figure 5: $\text{bias} = \theta - \hat{\theta}$ of the 10-item short test form

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Good!

There's no “one-fits-all” solution

The θ distribution is a key element

Good!

There's no “one-fits-all” solution

The θ distribution is a key element

..but work is still needed

Real life applications are missing

Thank you!

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