

It's how you use the items that counts: An intelligent procedure for item selection in Item Response Theory

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Developing, administering and refining measurement instruments in
Social Sciences



1 Aim

2 Item Response Theory and Information Functions

- 2-Parameter Logistic Model
- Item and Test Information Functions

3 Item Selection Procedures

- Item Locating Algorithm – ILA
- Brute Force Procedure – BFP

4 Simulation Study

- Simulation design
- Results
- Conclusions

Item Response Theory (IRT) for the development of Short Test Form (STF):

Typical procedure: Manually inspecting the item characteristics to recreate the desired characteristics of a test

Automated (new) procedure: A priori definition of latent trait levels of interest on which the STF should be focusing the most

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AIM

New automated procedure for item selection in IRT that only requires the definition of the desired characteristics of a test

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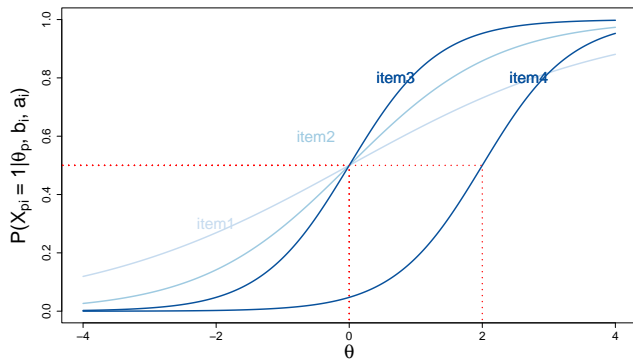
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$$P(x_{pi} = 1 | \theta_p, b_i, a_i) = \frac{\exp[a_i(\theta_p - b_i)]}{1 + \exp[a_i(\theta_p - b_i)]}$$



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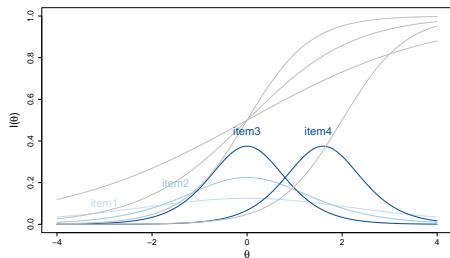
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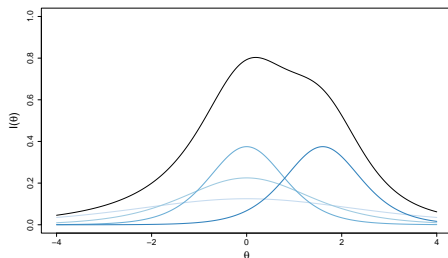
Item Information Function (IIF):

$$I_i(\theta) = a_i^2 P_i(\theta, b_i, a_i)[1 - P_i(\theta, b_i, a_i)]$$



Test Information Function (TIF):

$$I(\theta) = \sum_{i=1}^I I_i(\theta)$$



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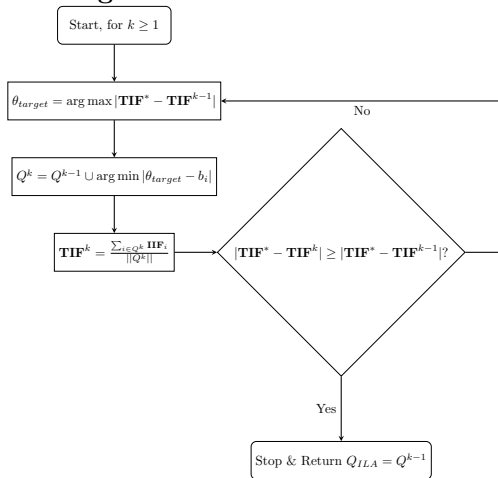
Set up:

N : number of items included in the item bank

Q^k : Set of item indexes selected for inclusion in the STF up to iteration k ($Q^0 = \emptyset$)

\mathbf{TIF}^* : TIF target

$\mathbf{TIF}^0 = (0, 0, \dots, 0)$

ILA Algorithm:

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For each $Q_m \subset Q$ with $Q_m \neq \emptyset$, calculate:

$$\textcircled{1} \quad \mathbf{TIF}^{Q_m} = \frac{\sum_{i \in Q_m} IIF_i}{||Q_m||}$$

$$\textcircled{2} \quad \bar{\Delta}_{\mathbf{TIF}^{Q_m}} = \text{mean}(|\mathbf{TIF}^* - \mathbf{TIF}^{Q_m}|)$$

$$Q_{BFP} = \arg \min_{\emptyset \neq Q_m \subset Q} \bar{\Delta}_{\mathbf{TIF}^{Q_m}}$$

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100 iterations:

- ① Generate an item bank B of $N = 6$ items:
 - Difficulty parameters: $\mathcal{U}(-3, 3)$
 - Discrimination parameters: $\mathcal{U}(.90, 2.0)$
- ② Random item selections of lengths l from B ($M_l = 3.34 \pm 1.13$) + modification parameters $\mathcal{U}(-0.20, 0.20) \rightarrow \mathbf{TIF}^*$
- ③ Considering \mathbf{TIF}^* at Step 2 and item parameters at Step 1:
 - ILA \rightarrow *Forwardly searches*
 - BFP \rightarrow *Systematically tests*

Comparison:

- $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}||$
- Percentile rank of the distance $\mathbf{TIF}_{\text{BFP}} - \mathbf{TIF}_{\text{ILA}}$

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$$57\% \rightarrow \|Q_{BFP}\| - \|Q_{ILA}\| = 0$$

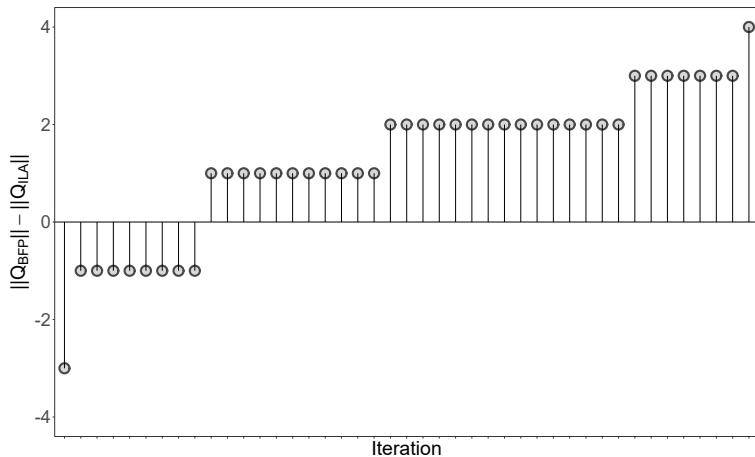
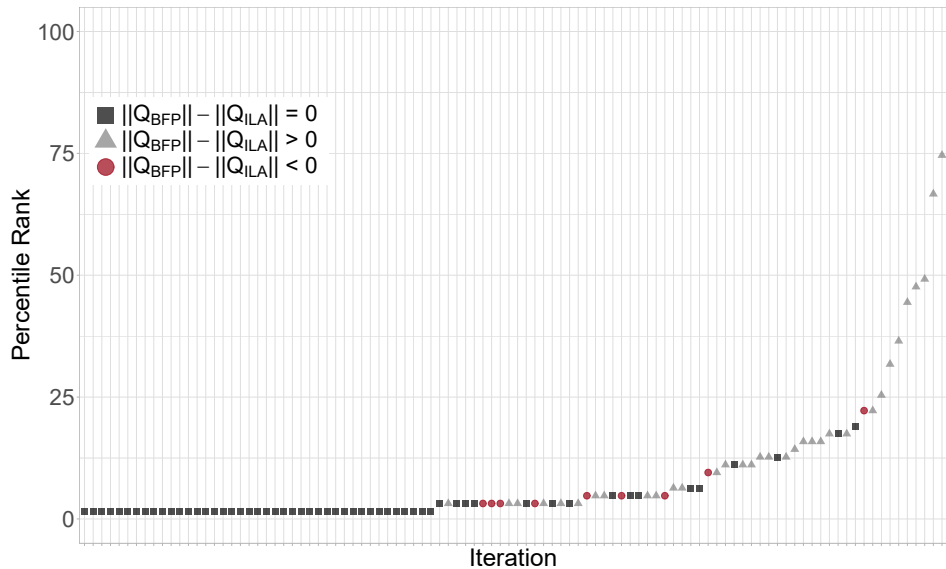
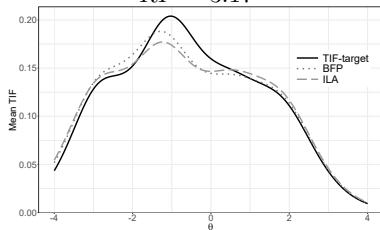


Figure: 43% $\rightarrow \|Q_{BFP}\| - \|Q_{ILA}\| \neq 0$

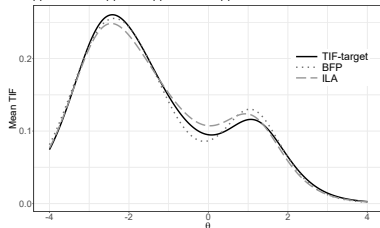


$$\|Q_{\text{BFP}}\| = \|Q_{\text{ILA}}\|, Q_{\text{BFP}} \neq Q_{\text{ILA}},$$

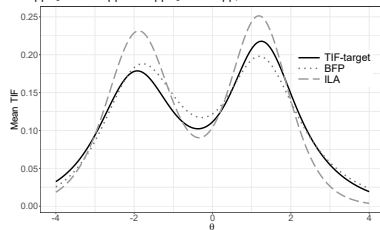
$$RP = 3.17$$



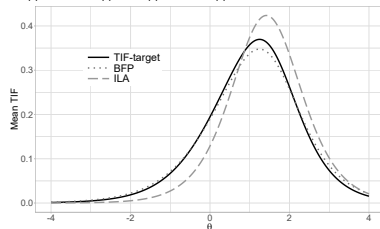
$$\|Q_{\text{BFP}}\| < \|Q_{\text{ILA}}\|, RP = 3.17$$



$$\|Q_{\text{BFP}}\| > \|Q_{\text{ILA}}\|, RP = 4.76$$



$$\|Q_{\text{BFP}}\| > \|Q_{\text{ILA}}\|, RP = 12.70$$



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Pros

- ILA selects items that are able to recreate the desired characteristics of a test (usually)
- It is computationally “Light”

Cons

- ILA grounds its selection on a single θ_{target} → it might select items minimizing the distance on that target but that are useless for the test
- ILA only forwardly searches an item → once it is in, it can't get out
- ILA does not account for the discrimination parameters of the items