

# Test length doesn't matter, it's how you use the items that counts: An intelligent procedure for item selection in Item Response Theory

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Social Sciences



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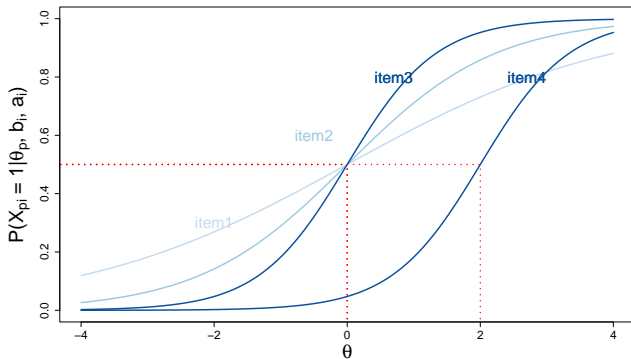
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## AIM

New automated procedure for item selection in IRT that only requires the definition of the desired characteristics of a test

$$P(x_{pi} = 1 | \theta_p, b_i, a_i) = \frac{\exp[a_i(\theta_p - b_i)]}{1 + \exp[a_i(\theta_p - b_i)]}$$



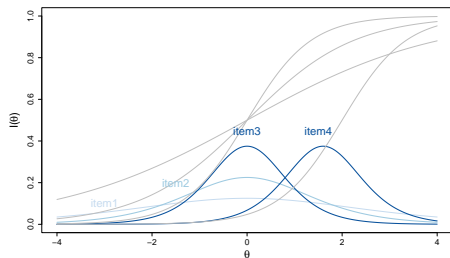
$\theta_p$ : Latent trait level of person  $p$

$b_i$ : Location of item  $i$  on  $\theta$

$a_i$ : Discrimination ability of item  $i$

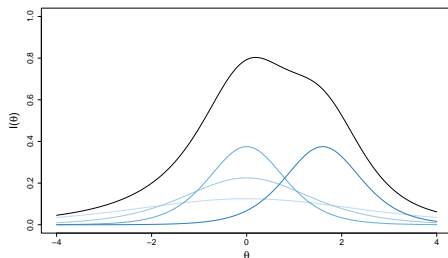
Item Information Function (IIF):

$$I_i(\theta) = a_i^2 P_i(\theta, b_i, a_i)[1 - P_i(\theta, b_i, a_i)]$$



Test Information Function (TIF):

$$I(\theta) = \sum_{i=1}^N I_i(\theta)$$



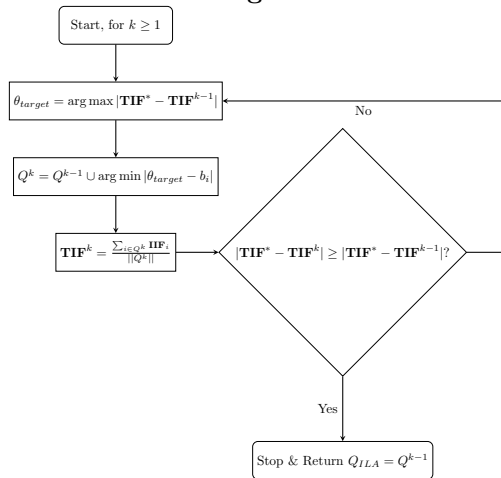
**Set up:**

$N$ : number of items included in the item bank

$Q^k$ : Set of item indexes selected for inclusion in the STF up to iteration  $k$  ( $Q^0 = \emptyset$ )

$\mathbf{TIF}^*$ : TIF target

$\mathbf{TIF}^0 = (0, 0, \dots, 0)$

**ILA Algorithm:**



For each  $Q_m \subset Q$  with  $Q_m \neq \emptyset$ , calculate:

$$\textcircled{1} \quad \mathbf{TIF}^{Q_m} = \frac{\sum_{i \in Q_m} IIF_i}{||Q_m||}$$

$$\textcircled{2} \quad \overline{\Delta}_{\mathbf{TIF}^{Q_m}} = \text{mean}(|\mathbf{TIF}^* - \mathbf{TIF}^{Q_m}|)$$

$$Q_{BFP} = \arg \min_{\emptyset \neq Q_m \subset Q} \overline{\Delta}_{\mathbf{TIF}^{Q_m}}$$

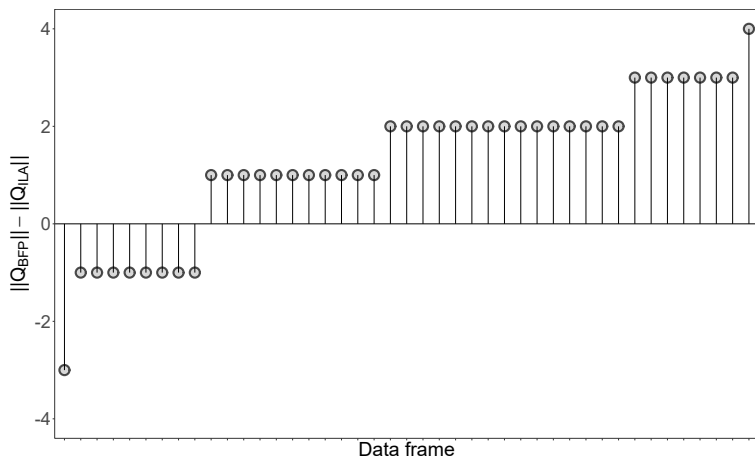
## 100 data frames:

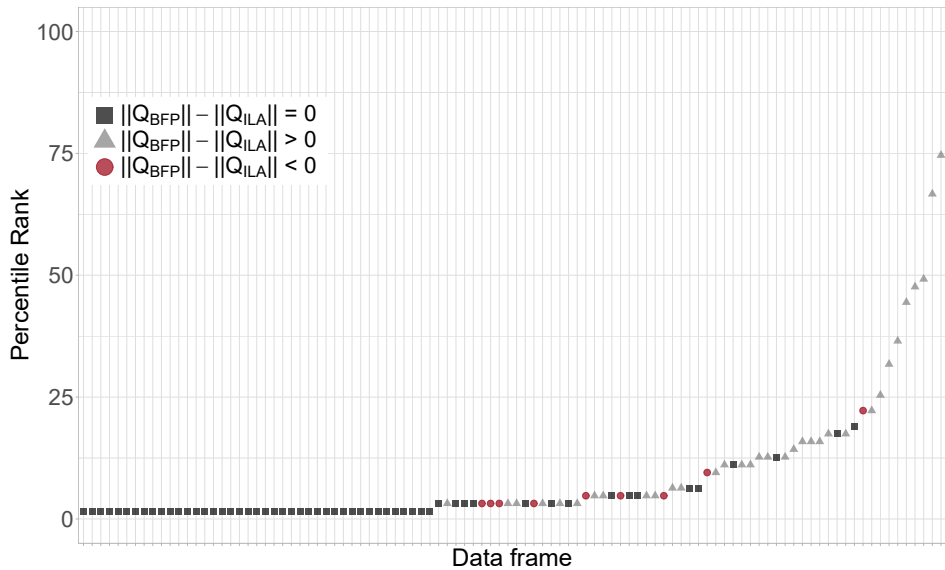
- ① Generate an item bank  $B$  of  $N = 6$  items:
  - Difficulty parameters:  $\mathcal{U}(-3, 3)$
  - Discrimination parameters:  $\mathcal{U}(.90, 2.0)$
- ② Random item selections of lengths  $l$  from  $B$  ( $M_l = 3.34 \pm 1.13$ ) + modification parameters  $\mathcal{U}(-0.20, 0.20) \rightarrow \mathbf{TIF}^*$
- ③ Considering  $\mathbf{TIF}^*$  at Step 2 and item parameters at Step 1:
  - ILA  $\rightarrow$  *Forwardly searches*
  - BFP  $\rightarrow$  *Systematically tests*

## Comparison:

- $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}||$
- Percentile rank of the distance  $\mathbf{TIF}_{\text{BFP}} - \mathbf{TIF}_{\text{ILA}}$

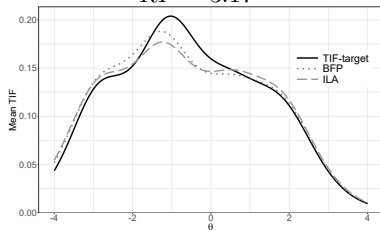
$\|Q_{BFP}\| - \|Q_{ILA}\| = 0$  in 57% of cases



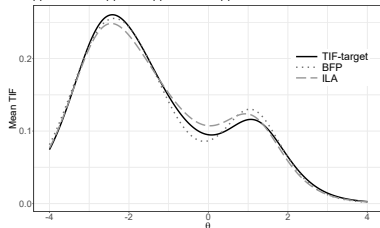


$$\|Q_{\text{BFP}}\| = \|Q_{\text{ILA}}\|, Q_{\text{BFP}} \neq Q_{\text{ILA}},$$

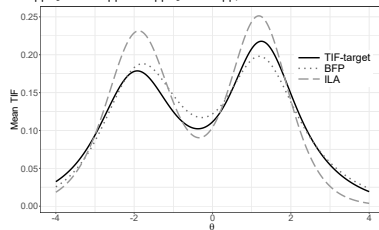
$$RP = 3.17$$



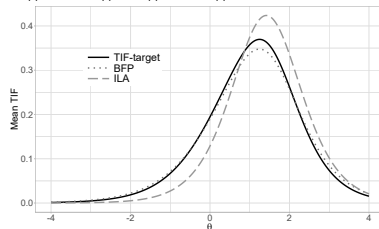
$$\|Q_{\text{BFP}}\| < \|Q_{\text{ILA}}\|, RP = 3.17$$



$$\|Q_{\text{BFP}}\| > \|Q_{\text{ILA}}\|, RP = 4.76$$



$$\|Q_{\text{BFP}}\| > \|Q_{\text{ILA}}\|, RP = 12.70$$



## Pros of ILA

- It selects items that are able to recreate the desired characteristics of a test (usually)
- It is computationally “Light”

## Cons of ILA

- It grounds its selection on a single  $\theta_{target}$  at a time  $\rightarrow$  it might select items minimizing the distance on that target but that are not very useful for the test
- It only forwardly searches an item  $\rightarrow$  once it is in, it can't get out
- It does not account for the discrimination parameters of the items