Test length doesn't matter, it's how you use the items that counts: An intelligent procedure for item selection in Item Response Theory

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Convegno ASA 2024, Contributed session:
Developing, administering and refining measurement instruments in Social Sciences

Item Response Theory (IRT) for the development of Short Test Form (STF):

Typical procedure: Manually inspecting the item characteristics to recreate the desired characteristics of a test

Automated (new) procedure: A priori definition of latent trait levels of interest on which the STF should be focusing the most

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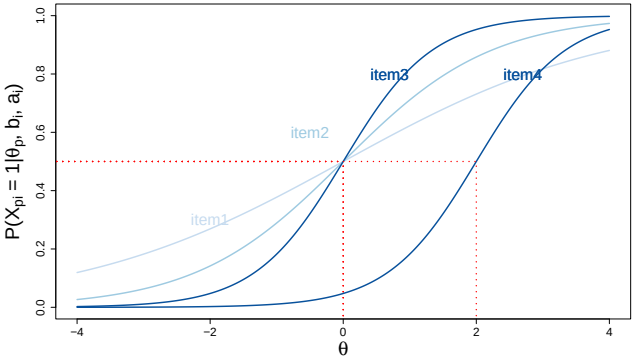
AIM

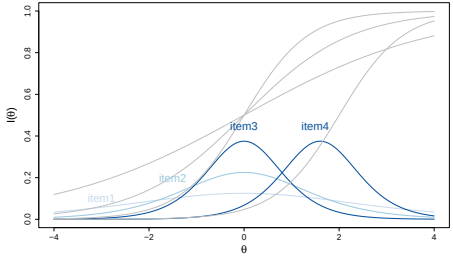
New automated procedure for item selection in IRT that only requires the definition of the desired characteristics of a test

$$P(x_{pi} = 1|\theta_p, b_i, a_i) = \frac{\exp[a_i(\theta_p - b_i)]}{1+\exp[a_i(\theta_p - b_i)]}$$

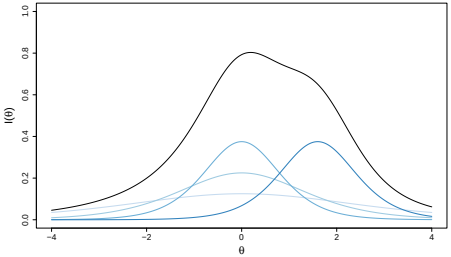


θ_p: Latent trait level of person p
b_i: Location of item i on θ
a_i: Discrimination ability of item i

Item Information Function (IIF):
$I_i(\theta) = a_i^2 P_i(\theta, b_i, a_i)[1 - P_i(\theta, b_i, a_i)]$



Test Information Function (TIF):
$I(\theta) = \sum_{i=1}^{N} I_i(\theta)$



Set up:

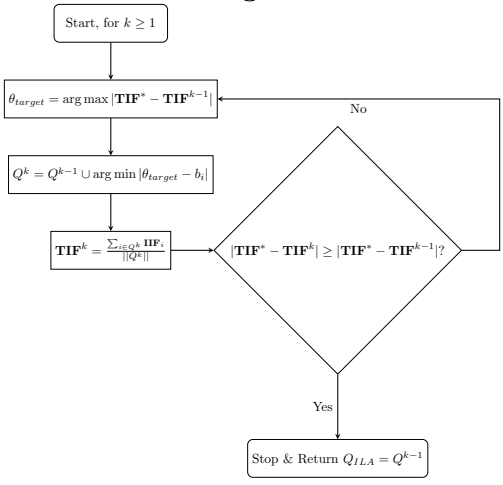
N: number of items included in the item bank

Q^k: Set of item indexes selected for inclusion in the STF up to iteration k ($Q^0 = \emptyset$)

$\mathbf{TIF}^*$: TIF target

$\mathbf{TIF}^0 = (0, 0, \ldots, 0)$

ILA Algorithm:



For each $Q_m \subset Q$ with $Q_m \neq \emptyset$, calculate:

1. $\mathbf{TIF}^{Q_m} = \dfrac{\sum_{i \in Q_m} IIF_i}{||Q_m||}$
2. $\overline{\Delta}_{\mathbf{TIF}^{Q_m}} = mean(|\mathbf{TIF}^{*} - \mathbf{TIF}^{Q_m}|)$

$Q_{BFP} = \arg\min_{\emptyset \neq Q_m \subset Q} \overline{\Delta}_{\mathbf{TIF}^{Q_m}}$

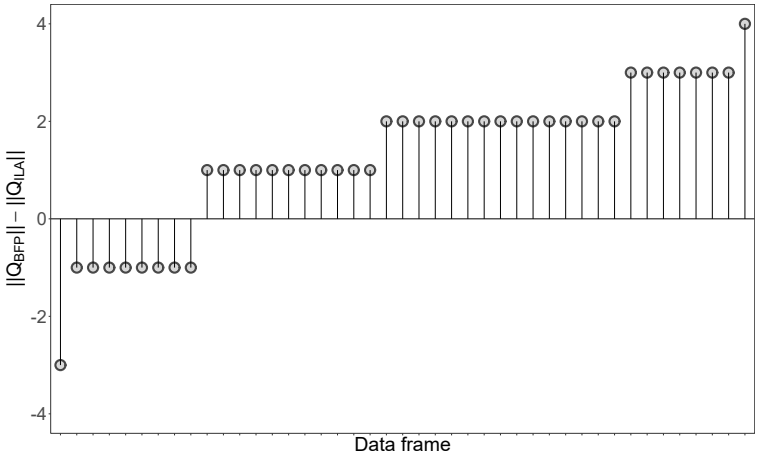
100 data frames:

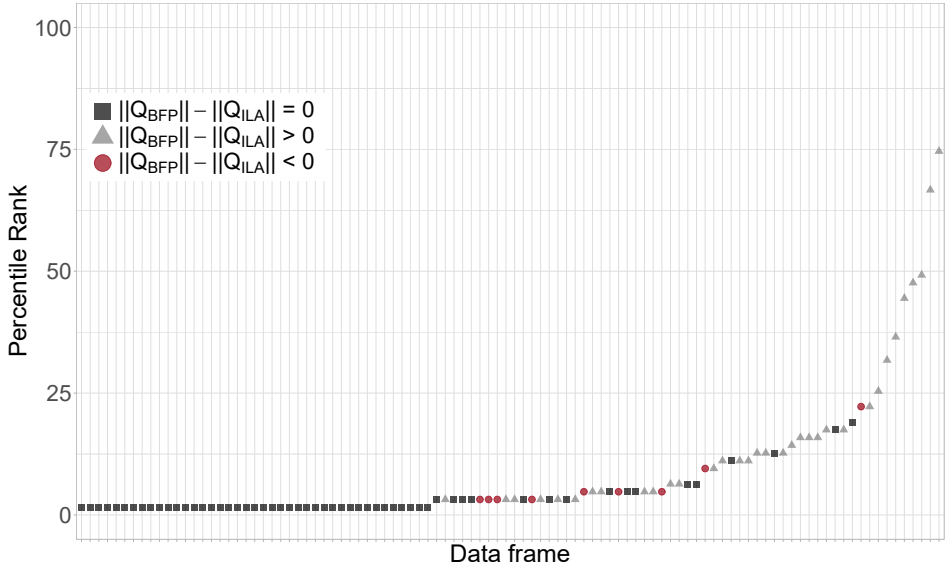
1. Generate an item bank B of $N = 6$ items:
   - Difficulty parameters: $\mathcal{U}(-3, 3)$
   - Discrimination parameters: $\mathcal{U}(.90, 2.0)$
2. Random item selections of lengths l from B ($M_l = 3.34 \pm 1.13$) + modification parameters $\mathcal{U}(-0.20, 0.20) \rightarrow \mathbf{TIF}^*$
3. Considering $\mathbf{TIF}^*$ at Step 2 and item parameters at Step 1:
   - ILA $\rightarrow$ *Forwardly searches*
   - BFP $\rightarrow$ *Systematically tests*

Comparison:

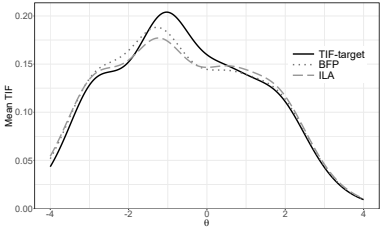
- $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}||$
- Percentile rank of the distance $\mathbf{TIF}_{\text{BFP}} - \mathbf{TIF}_{\text{ILA}}$

$||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}|| = 0$ in 57% of cases

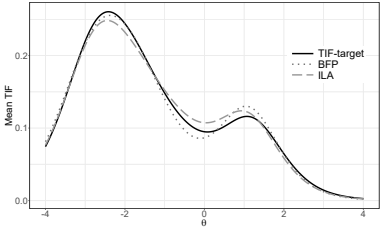



100
75
50
25
0
Percentile Rank
Data frame
||Q_BFP|| − ||Q_ILA|| = 0
||Q_BFP|| − ||Q_ILA|| > 0
||Q_BFP|| − ||Q_ILA|| < 0

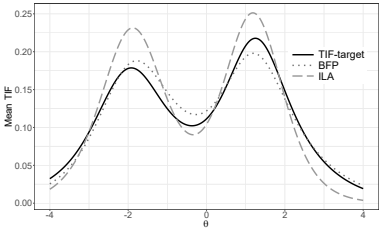
$||Q_{\text{BFP}}|| = ||Q_{\text{ILA}}||$, $Q_{\text{BFP}} \neq Q_{\text{ILA}}$, $RP = 3.17$



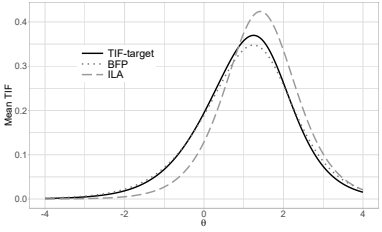
$||Q_{\text{BFP}}|| < ||Q_{\text{ILA}}||$, $RP = 3.17$



$||Q_{\text{BFP}}|| > ||Q_{\text{ILA}}||$, $RP = 4.76$



$||Q_{\text{BFP}}|| > ||Q_{\text{ILA}}||$, $RP = 12.70$



Pros of ILA

- It selects items that are able to recreate the desired characteristics of a test (usually)
- It is computationally "Light"

Cons of ILA

- It grounds its selection on a single θ_{target} at a time $\rightarrow$ it might select items minimizing the distance on that target but that are not very useful for the test
- It only forwardly searches an item $\rightarrow$ once it is in, it can't get out
- It does not account for the discrimination parameters of the items