It's how you use the items that counts: An intelligent procedure for item selection in Item Response Theory

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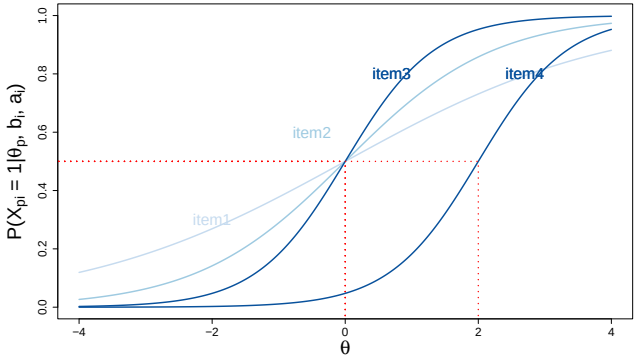
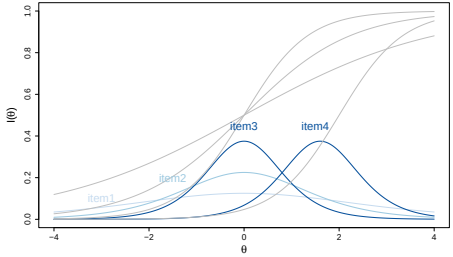
New Item Response Theory-based algorithm for the development of informative short test forms

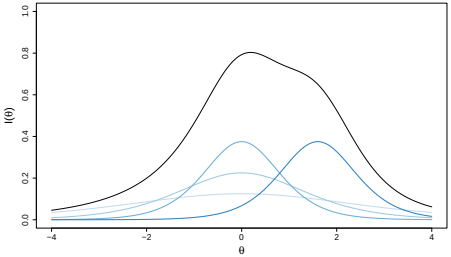
$$P(x_{pi} = 1|\theta_p, b_i, a_i) = \frac{\exp[a_i(\theta_p - b_i)]}{1+\exp[a_i(\theta_p - b_i)]}$$



Item Information Function (IIF):
$I_i(\theta) = a_i^2 P_i(\theta, b_i, a_i)[1 - P_i(\theta, b_i, a_i)]$



Test Information Function (TIF):
$I(\theta) = \sum_{i=1}^{I} I_i(\theta)$



Q^k: vector of the item indexes selected for inclusion in the STF up to iteration k ($Q^0 = \emptyset$)
TIF^*: TIF target
$TIF^0 = 0$
Termination criterion: $|TIF^* - TIF^k| \leq |TIF^* - TIF^{k-1}|$
Iterate:

1. $\theta_{target} = \arg\max |TIF^* - TIF^{k-1}|$
2. $Q^k = Q^{k-1} \cup \arg\min_{i \in \{1,\ldots,N\} \setminus Q^{k-1}} |\theta_{target} - b_i|$
3. $TIF^k = \frac{\sum_{i \in Q^k} IIF_i}{||Q^k||}$
4. $k = k + 1$

N: Number of items in the item bank
$L = N - 1$: Maximum length of the STFs that can developed from the item bank
$\binom{N}{l}$: number of STFs resulting from the combination of the $l = \{1, \dots L\}$ items
The algorithm iterates the following steps until the termination criterion is reached:

1. $\overline{TIF}$ for each STF of length l
2. $\Delta_{TIF} = TIF^* - \overline{TIF}$
3. $\overline{\Delta}_{TIF}$

The best STF is the one with the lowest value of $\overline{\Delta}_{TIF}$, that is the one that presents the lowest absolute distance from the TIF target.

100 iterations:

1. Generate an item bank of $N = 6$ items:
   - Difficulty parameters: $\mathcal{U}(-3, 3)$
   - Discrimination parameters: $\mathcal{U}(.90, 2.0)$
2. Generate TIF^* by randomly selecting items from the item bank (Mean number of items $= 3.34 \pm 1.13$). The parameters of the selected items are modified according to values drawn from uniform distributions $\mathcal{U}(-0.20, 0.20)$.
3. Considering the TIF^* at Step 2 and the item parameters at Step 1:
   - ILA $\rightarrow$ Forwardly searches for the best item selection to recover the TIF^*
   - BFP $\rightarrow$ tries every possible item combination to find the STF best able to recover TIF^* $N = 6$ items, $L = 5$ and $\binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} = 62$ STFs are developed and compared.

- $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}||$
- Percentile rank of the distance between the STF selected by BFP and that selected by ILA

57% $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}|| = 0 \rightarrow$ 72% same item selection

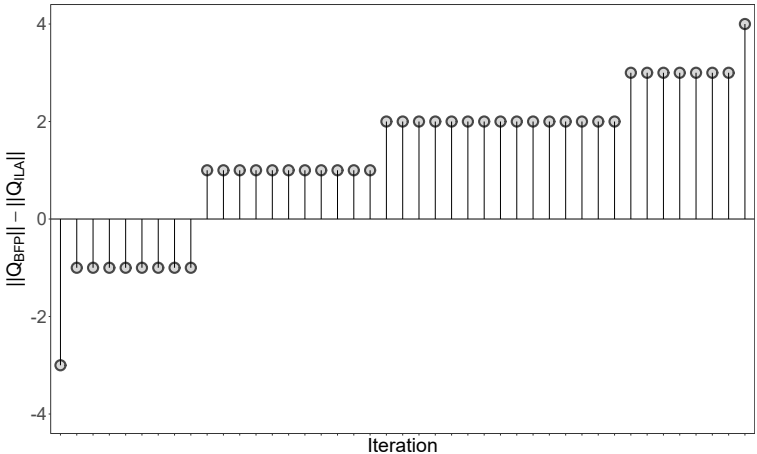
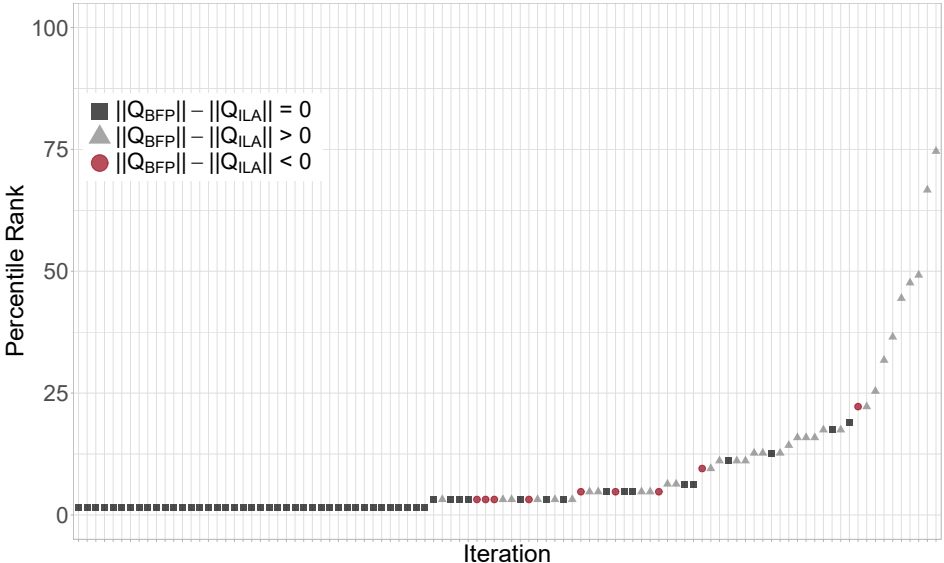
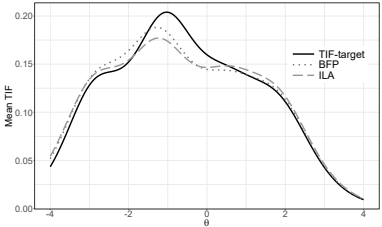


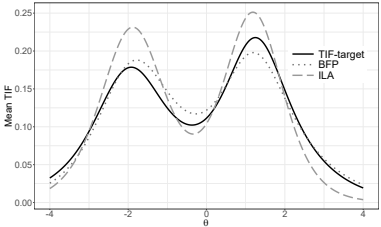
Figure: 43% $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}|| \neq 0$


100
75
50
25
0
Percentile Rank
Iteration
$||Q_{BFP}|| - ||Q_{ILA}|| = 0$
$||Q_{BFP}|| - ||Q_{ILA}|| > 0$
$||Q_{BFP}|| - ||Q_{ILA}|| < 0$

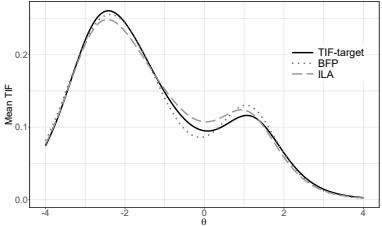
$||Q_{\text{BFP}}|| = ||Q_{\text{ILA}}||$, $Q_{\text{BFP}} \neq Q_{\text{ILA}}$



$||Q_{\text{BFP}}|| > ||Q_{\text{ILA}}||$



$||Q_{\text{BFP}}|| < ||Q_{\text{ILA}}||$



$||Q_{\text{BFP}}|| < ||Q_{\text{ILA}}||$