Pauci sed moni:
An Item Response Theory approach for shortening tests

Ottavia M. Epifania[1,2], Pasquale Anselmi[1] & Egidio Robusto[1]

[1] University of Padua, Padova (IT)

[2]Catholic University of the Sacred Heart, Milano (IT)

ottavia.epifania@unipd.it

Introduction

Item Response Theory (IRT) is the theoretical framework often used for shortening existing tests. IRT models describe the probability of observing a response as a function of the characteristics of respondent p (i.e., the latent trait level θ) and the characteristics of item s. IRT models provide detailed information on how well each item measures a certain θ level (i.e., *item information function*, *IIF*). Two types of short forms can be created by exploiting the *IIF*s:

1. **Adaptive short forms**: *Ad-hoc* tests for each person (i.e., Computerized Adaptive Testing, CAT. The items administered to each respondent vary according to the responses that this respondent gave to the previously administered items) $\rightarrow$ the information is maximized for each level of θ (i.e., each respondent)

   Issue: *Different short test forms for each respondent $\rightarrow$ Unfair assessments in recruitment or admissions tests*
2. **Static short forms**: Static tests equal for all respondents (i.e., only the items from the full-length test that provide the highest information are included in the short form) $\rightarrow$ the information is maximized across θ levels (i.e., across all respondents)

   Issue: *Not being tailored to any θ level of interest $\rightarrow$ Potentially more items are needed to cover a wide range of θs*

Aim

New IRT-based procedures for the development of short test forms combining the advantages of adaptive short test forms (i.e., tailoring the tests to different θ levels) and those of static short forms (i.e., being equal for all respondents).

The new item selection procedures are based on the definition of trait levels of interest (i.e., θ targets, denoted as θ') $\rightarrow$ The items that best assess the trait levels represented by the θ' targets (i.e., optimal items with highest *IIF*s for each θ') are included in the short form.

Item Response Theory and information functions

This illustration is based on the 2-parameter logistic model (2PL) for dichotomous responses:

$$P(x_{ps} = 1|\theta_p, b_s, a_s) = \frac{exp[a_s(\theta_p - b_s)]}{1 + exp[a_s(\theta_p - b_s)]} \quad (1)$$

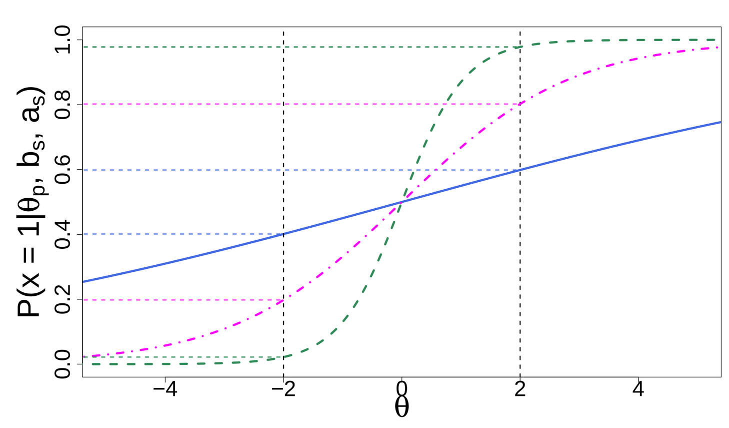
where $P(x_{ps} = 1|\theta_p, b_s, a_s)$ is the probability of respondent p to respond correctly to item s given the ability (θ) of p and difficulty (b) and discrimination (a) of s. The *Item Characteristics Curves* (*ICCs*) of three items with same difficulty but different discriminations are illustrated in Figure 1a.

The *item information function* (*IIF*) informs about the precision with which the item measures the abilities θs. In the 2PL model, the *IIF* is obtained as:

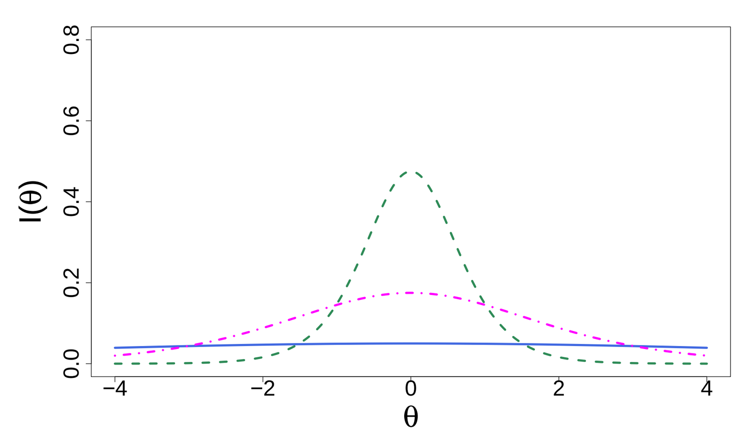
$$IIF = a^2[P(\theta)(1 - P(\theta))], \quad (2)$$

where $P(\theta)$ is the probability of a respondent with a certain θ of responding correctly to an item, and $1 - P(\theta)$ is their probability of responding incorrectly to the same item. The *IIF*s of the items depicted in Figure 1a are illustrated in Figure 1b.

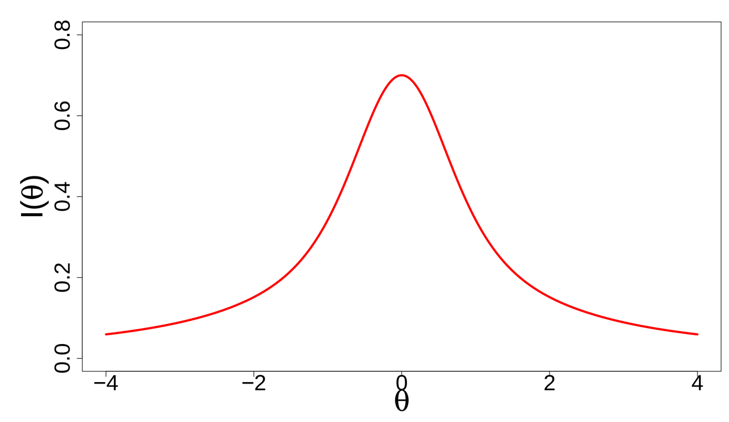
The *test information function* (*TIF*) is obtained by summing the IIFs across items (*test information function*, $TIF = \sum_{s=1}^{S} IIF_s$, Figure 1c).



(a) Item Characteristics Curves (*ICCs*) of items with $b = 0$, and $a = 0.20$, $a = 0.70$, $a = 1.90$



(b) Item Information Functions (*IIFs*) of items with $b = 0$, and $a = 0.20$, $a = 0.70$, $a = 1.90$



(c) Test Information Function (*TIF*) of the test composed of the items in Fig. 1a and Fig. 1b.

Figure 1: 2-PL and information functions

Item Selection Procedures

- **Benchmark**: The N items with the highest *IIF*s are selected from the full-length test to be included in the static short form, where N is the desired length of the short form (Benchmark Procedure, BP).
- **Procedures based on θ'**:
  - **Cluster**: *The latent trait is clustered in N clusters, where n is the number of items to be included in the short form. The centroids of the clusters are the θ' (Unequal Intervals Procedure, UIP).*
  - **Intervals**: *The latent trait is segmented into $N + 1$ intervals. Each interval is defined by $[\theta'_{n-1}; \theta'_n]$. The θ's are obtained by averaging between the lower and upper bound of each interval to avoid that the first and the last θ's correspond to the minimum and maximum θ values (Equal Intervals Procedure, EIP).*
- **Random**: Items are randomly selected from the full-length tests (RP).

Development of a 5-item short form from a 10-item full-length test:

Typical procedure

item	b	a	*IIF*
1	−2.51	1.68	0.10
2	−2.43	0.25	0.02
3	−2.28	1.62	0.13
4	−0.67	0.71	0.11
5	−0.66	0.44	0.05
6	0.50	1.19	0.27
7	0.64	0.50	0.06
8	0.72	0.33	0.03
9	1.72	0.39	0.03
10	2.12	1.98	0.16

θ'-based procedures

	θ_1	θ_2	θ_3	θ_4	θ_5
item	−3.07	−1.54	−0.01	1.53	3.06
1	0.07	0.12	0.12	0.07	0.03
2	0.02	0.11	0.32	0.25	0.06
3	0.02	0.02	0.01	0.01	0.01
4	0.01	0.01	0.06	0.71	0.45
5	0.02	0.03	0.03	0.04	0.03
6	0.45	0.46	0.06	0.01	0.01
7	0.03	0.05	0.06	0.06	0.04
8	0.57	0.38	0.04	0.01	0.01
9	0.04	0.05	0.05	0.04	0.03
10	0.02	0.02	0.03	0.03	0.02

Method

Comparison between the item selection procedures:

- Benchmark procedure (BP)
- Unequal Intervals Procedure (UIP)
- Equal Interval Procedure (EIP)
- Random Procedure (RP)

in the development of 10, 30, 50, 70, 90 items test short forms from a 100-item full-length test (For each short test form, RP randomly selects the items 10 times).

1000 respondents p

Three θ distributions:

1. Normal distribution $p \sim \mathcal{N}(0, 1)$
2. Positive skewed distribution $p \sim Beta(1, 100)$ (linearly transformed to obtain negative values)
3. Uniform distribution $p \sim \mathcal{U}(-3, 3)$

100 items s:

- $b \sim \mathcal{U}(-3, 3)$
- $a \sim \mathcal{U}(0.40, 2)$

Results

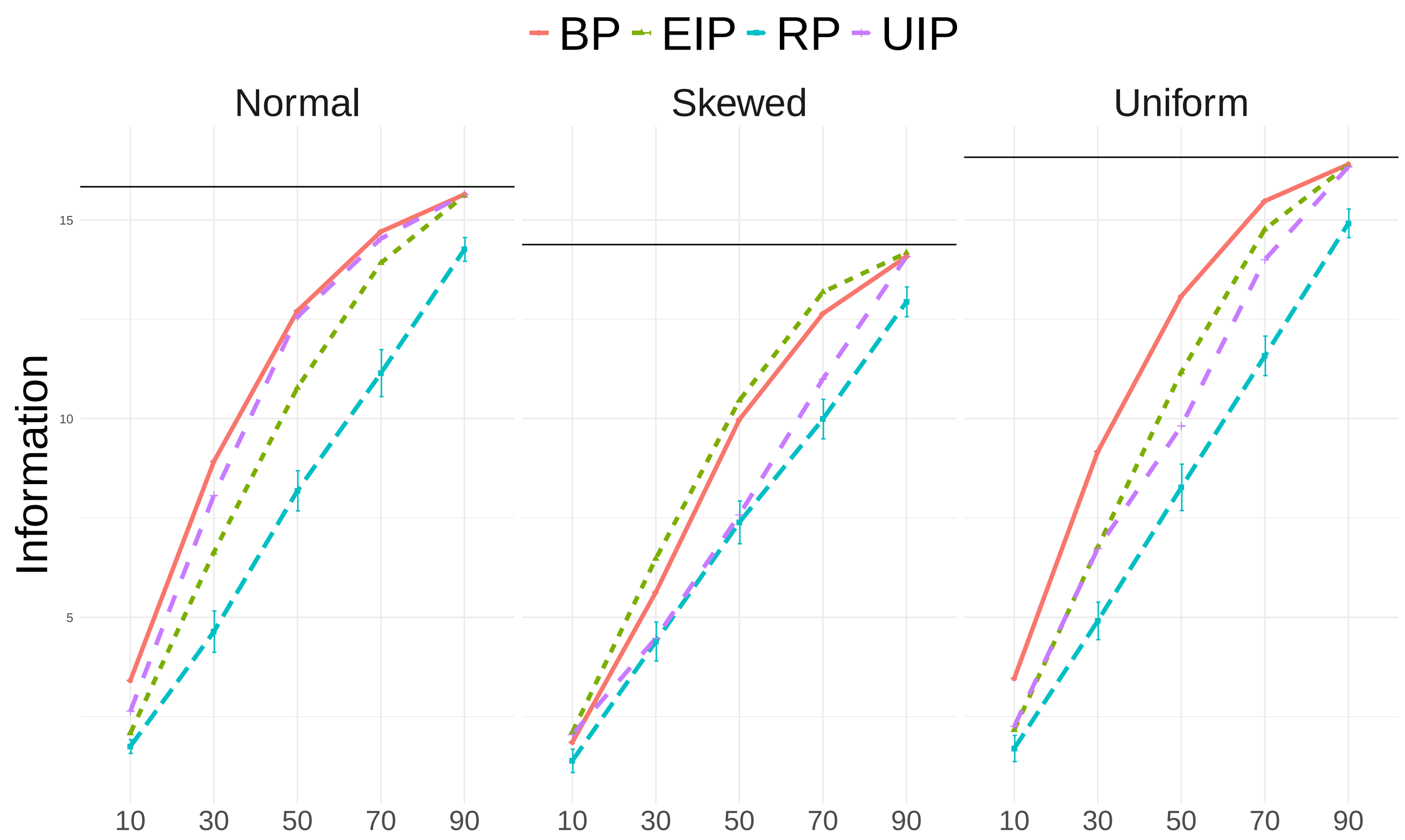


Figure 2: Overall information of the short test forms

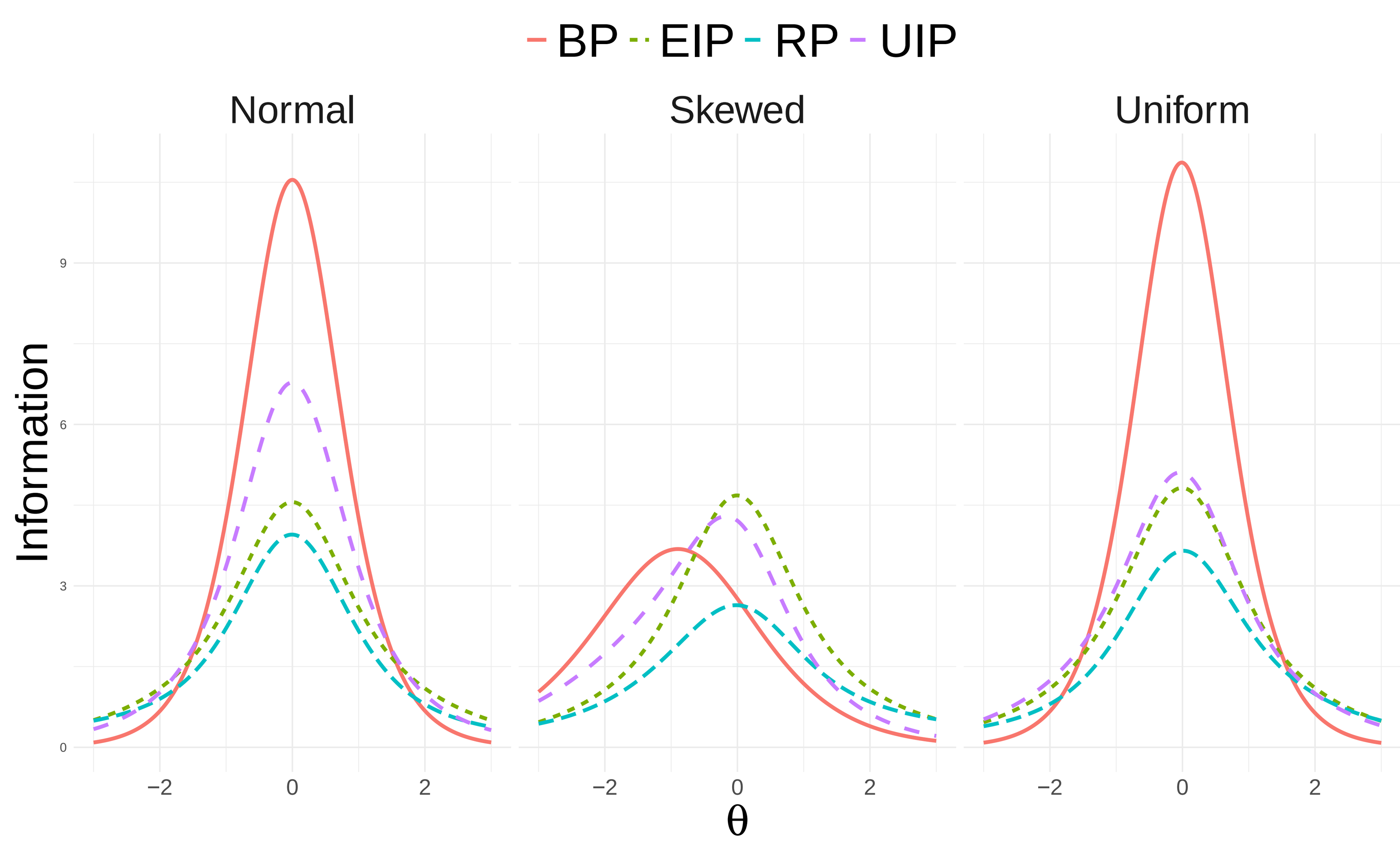


Figure 3: Detailed information of the short test forms

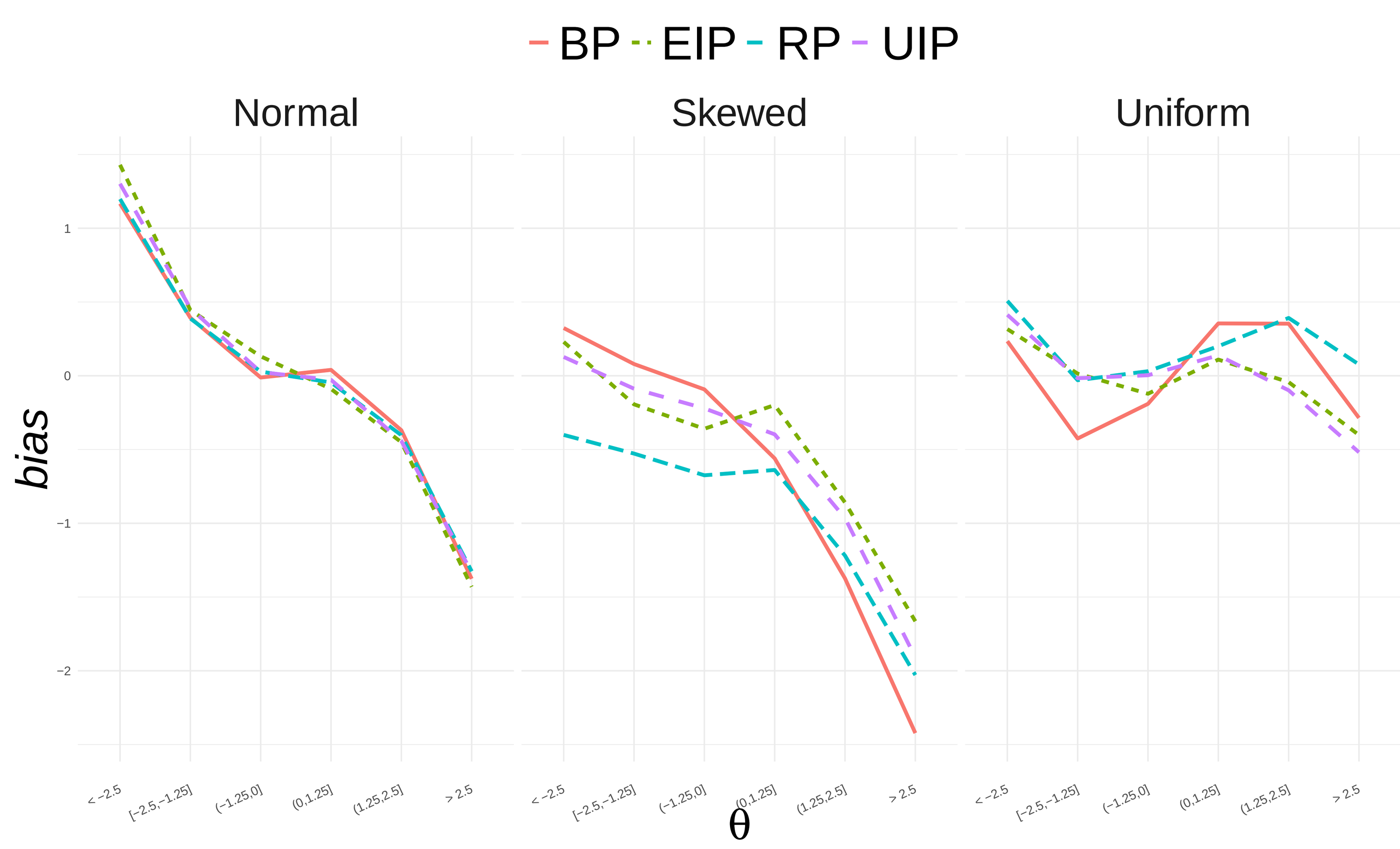


Figure 4: Bias for different group of θ

Discussion

- Different methods for different θ distributions
- Better performance of θ-based procedures on the extreme ends of the distributions
- By considering the θ' in the item selection procedures $\rightarrow$ not the highest information but best coverage of the entire latent trait