# The good, the bad, and the ugly

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**Abstract**

# 1 Bruto

$\theta = \{x \in \mathbb{R} \,|\, \theta_{min} \leq x \leq \theta_{max}\}$: Latent trait levels, where $\theta_{min}$ is the minimum value of $\theta$ and $\theta_{max}$ is its maximum value (non sono sicura della x)

$||\theta||$: cardinality of the latent trait

$\forall Q \in \mathcal{Q} = 2^B \setminus \{\emptyset, B\}$,

1. $\mathbf{TIF}^Q = \frac{\sum_{i \in Q} IIF_i}{||Q||}$
2. $\overline{\Delta}_{\mathbf{TIF}^Q} = mean(|\mathbf{TIF}^* - \mathbf{TIF}^Q|)$ ho dei dubbi su questa notazione. la media è calcolata attraverso i valori di theta. In alternativa a questa notazione: $\overline{\Delta}_{\mathbf{TIF}^Q} = \frac{|\mathbf{TIF}^* - \mathbf{TIF}^Q|}{||\theta||}$

$Q_{bruto} = \arg\min_{Q \in \mathcal{Q}} \overline{\Delta}_{\mathbf{TIF}^Q}$

# 2 Item Locating Algorithm

Siccome la filosofia di ILA e ISA è molto simile sarebbe carino trovare loro un nome comune e poi declinarle nelle loro specificità. ci pensiamo

**Set up:**

$B$: Item bank

$Q^k \subset B$: Set of item indexes selected for inclusion in the STF up to iteration $k$

$\mathbf{TIF}^*$: TIF target

$D$: Item selected at each iteration

$\mathbf{PIF}^k_D = \frac{\sum_{i \in Q^k \cup \{D^k\}} IIF_i}{||Q^k \cup \{D\}||}$, provisional mean tif at iteration $k$, including the item selected in $D^k$

$||Q^k||$: cardinality of $Q^k$ at iteration $k$

At $k = 0$: $\mathbf{PIF}^0_D = (0, 0, \ldots, 0)$, $Q^0 = \emptyset$, $D^0 = \emptyset$,

1. $\theta_{target} := \arg\max |\mathbf{TIF}^* - \mathbf{PIF}^k_D|$
2. $D^k = \arg\min_{i \in B \setminus Q^k} |\theta_{target} - b_i|$
3. $\mathbf{PIF}^k_D = \frac{\sum_{i \in Q^k \cup \{D\}} IIF_i}{||Q^k \cup \{D\}||}$

4. Termination Criterion: $|\mathbf{TIF}^* - \mathbf{PIF}_D^k| < |\mathbf{TIF}^* - \mathbf{TIF}^k|$:

   - TRUE: $k := k+1$, $Q^k = Q^{k-1} \cup \{D\}$, iterates 1-5 (Ho scritto $k-1$ perché siccome ho scritto all'inizio che $k$ si aggiorna e diventa $k+1$ mi sembrava sensato)
   - FALSE: The item in $D$ does not contribute to reduce the distance from the TIF target, hence: $Q_{ISA} = Q^k$

# 3 Item Selecting Algorithm

Same as ILA but based on the Item Information Functions.

Set up same as ILA: $B$: Item bank

$Q^k \subset B$: Set of item indexes selected for inclusion in the STF up to iteration $k$

$\mathbf{TIF}^*$: TIF target

$D$: Item selected at each iteration

$\theta_{target}$: theta target selected at each iteration

$\mathbf{PIF}_D^k = \frac{\sum_{i \in Q^k \cup \{D\}} IIF_i}{||Q^k \cup \{D\}||}$, provisional mean tif at iteration $k$, including the item selected in $D$

$||Q^k \cup \{D\}||$: cardinality of $Q^k \cup \{D\}$ at iteration $k$

At $k = 0$: $\mathbf{PIF}_D^0 = (0, 0, \ldots, 0)$, $Q^0 = \emptyset$, $D^0 = \emptyset$,

1. $\theta_{target} := \arg\max |\mathbf{TIF}^* - \mathbf{PIF}_D^k|$
2. $IIF_{i \in B \setminus Q^k} = a_i^2 P(\theta_{target}, a_i, b_i)[1 - P(\theta_{target}, a_i, b_i)]$
3. $D := \arg\min_{i \in B \setminus Q^k} |\theta_{target} - IIF_i|$
4. $\mathbf{PIF}_{D^k} = \frac{\sum_{i \in Q^k \cup \{D\}} IIF_i}{||Q^k \cup \{D\}||}$
5. Termination Criterion: $|\mathbf{TIF}^* - \mathbf{PIF}_D^k| < |\mathbf{TIF}^* - \mathbf{TIF}^k|$:

   - TRUE: $k := k+1$, $Q^k = Q^{k-1} \cup \{D\}$, iterates 1-5 (Ho scritto $k-1$ perché siccome ho scritto all'inizio che $k$ si aggiorna e diventa $k+1$ mi sembrava sensato)
   - FALSE: The item in $D$ does not contribute to reduce the distance from the TIF target, hence: $Q_{ISA} = Q^k$

# 4 Frank

The setup is like the one of ILA and ISA:

$B$: Item bank

$Q^k$: set of items selected at iteration $k$

$D$: provisional item selected at each iteration

**PIF**: provisional mean tif

At $k = 0$, $\mathbf{PIF}^0 = (0, 0, \ldots, 0)$, $Q^0 = \emptyset$, iterate

1. $A^k = B \setminus Q^k$ (sets of available items at iteration $k$)
2. $\forall i \in A^k$, $\mathbf{PIF}_i^k = \frac{\mathbf{TIF}^k + \mathbf{IIF}_i}{||Q^k|| + 1}$
3. $D = \arg\min_{i \in A^k} |\mathbf{TIF}^* - \mathbf{PIF}_i^k|$
4. Termination criterion: $|\mathbf{TIF}^* - \mathbf{PIF}_D^k| < |\mathbf{TIF}^* - \mathbf{TIF}^{k-1}|$:
   - If true, $k := k + 1$, $Q^k = Q^{k-1} \cup \{D\}$, restart from 1 C'è il meno 1 per la stessa ragione scritta per ILA e ISA
   - If false, stops, $Q_{Frank} = Q^k$