When randomness opens new possibilities: Acknowledging the stimulus sampling variability in Experimental Psychology

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Introduction

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Stimuli are fixed, respondents are random

Respondents are random factors

Sampled from a larger population

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Need for acknowledging the sampling variability

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Results can be generalized to other respondents belonging to the same population

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Stimuli/items are fixed factors

Taken to be entire population

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There is no sampling variability

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Stimuli/items are fixed factors

Taken to be entire population

There is no sampling variability

There is no need to generalize the results because the stimuli are the population

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Stimuli are fixed, respondents are random

However...

The stimuli can also represent a sample of a larger universe

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Processing speed of positive and negative attributes

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There is a universe of **positive attributes** as well as an universe of
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Only samples of **positive attributes** (e.g., good, nice, ...) and
negative attributes (e.g., bad, evil, ...) are administered

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So... **there must be a sampling variability!**

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What if the sampling variability is not acknowledged

Generalizability

Generalizability is bounded to the specific set of stimuli used in the experiment

Results can be generalized if and only if the exact same set of stimuli is used

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Robustness of the results

Random variability at the stimulus level might inflate the probability of committing Type I errors

Averaging across stimuli to obtain person-level scores results in biased estimates due to the noise in the data

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Loss of information

All the variability is not considered as well as all the information that can be obtained from it
Every stimulus is assumed to be equally informative

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What if the sampling variability is not acknowledged

This contribution

Focus on the loss of information...the other side of the coin

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This contribution

Focus on the loss of information...the other side of the coin

The information at the stimulus level that can be retrieved from the accuracy responses (correct vs. incorrect) from a typical experiment where the response times are usually employed for scoring the data

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This contribution

Focus on the loss of information...the other side of the coin

The information at the stimulus level that can be retrieved from the accuracy responses (correct vs. incorrect) from a typical experiment where the response times are usually employed for scoring the data

It can actually help in disentangling what is known to be a shortcoming of the score usually employed for analyzing the data of this experiment

Random effects for random factors

Random effects and random factors

Linear combination of predictors in a Linear Model:

$$\eta = X\beta,$$

where β indicates the coefficients of the fixed intercept and slope(s), and X is the model-matrix.

Random effects and random factors

Linear combination of predictors in a Linear Model:

$$\eta = X\beta,$$

where β indicates the coefficients of the fixed intercept and slope(s), and X is the model-matrix.

Linear combination of predictors in a Linear Mixed-Effects Model (LMM):

$$\eta = X\beta + Zd,$$

where Z is the matrix and d is the vector of the random effects (not parameters!)

Best Linear Unbiased Predictors
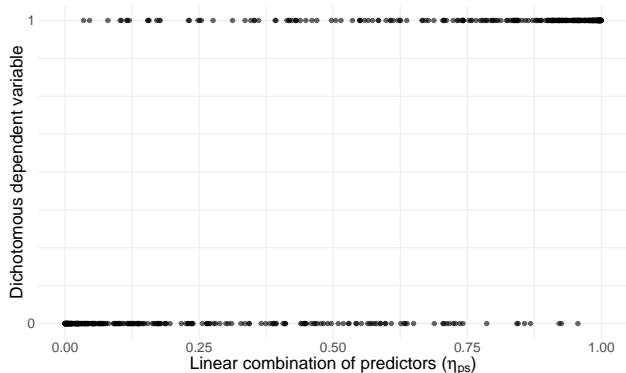
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Generalized linear model (GLM) for dichotomous responses

Generalized linear model (GLM) for dichotomous responses
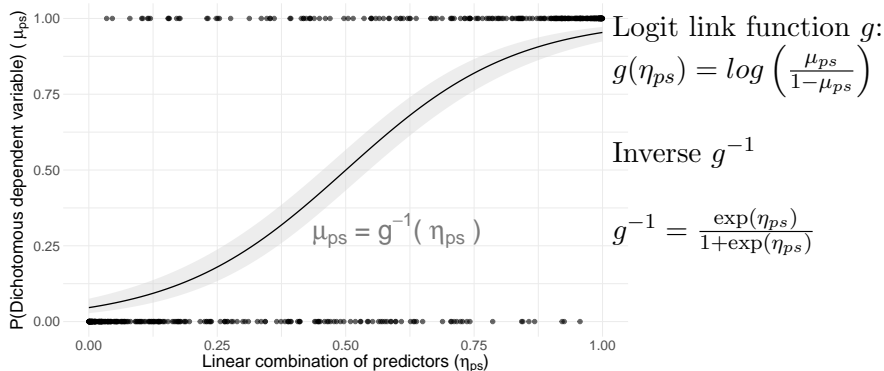
Logit link function g:

$$g(\eta_{ps}) = log\left(\frac{\mu_{ps}}{1-\mu_{ps}}\right)$$

Inverse g^{-1}

$$g^{-1} = \frac{\exp(\eta_{ps})}{1+\exp(\eta_{ps})}$$

The Rasch model

$$P(x_{ps} = 1|\theta_p, b_s) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

where:

θ_p: ability of respondent p (i.e., latent trait level of respondent p)

b_s: difficulty of stimulus s (i.e., "challenging" power of stimulus s)

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Standard

$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

GLM

$$P(x_{ps} = 1) = \frac{\exp(\theta_p + b_s)}{1 + \exp(\theta_p + b_s)}$$

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Experiment with the Implicit Association Test

12 Object stimuli

White people faces

Black people faces
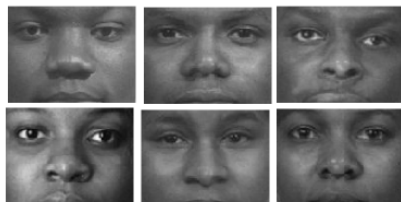
16 Attribute stimuli

Positive attributes

Negative attributes

Good, laughter, pleasure, glory, peace, happy, joy, love

Evil, bad, horrible, terrible, nasty, pain, failure, hate

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Experiment with the Implicit Association Test

Two experimental conditions

White-Good/Black-Bad
(WGBB): 60 trials
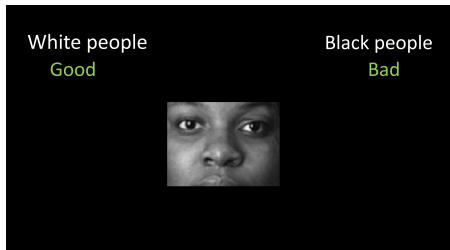
Black-Good/White-Bad
(BGWB): 60 trials

$$D = \frac{M_{\text{BGWB}} - M_{\text{WGBB}}}{s_{\text{BGWB, WGBB}}}$$

Participants: 62 (F = 48.39%, Age = 24.92 ± 2.11 years)

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Models

The expected response y for the observation $i = 1, \ldots, I$ for respondent $p = 1, \ldots, P$ on stimulus $s = 1, \ldots, S$ in condition $c = 1, \ldots, C$:

Model 1:
$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \alpha_{s[i]})$$
$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2).$$

Model 2:
$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i)$$
$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\beta_s \sim \mathcal{MVN}(0, \Sigma_{sc}).$$

Model 3:
$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{s[i]} + \beta_{p[i]} c_i)$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2),$$
$$\beta_p \sim \mathcal{MVN}(0, \Sigma_{pc}).$$

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Model 3:

$$y_i = logit^{-1}(\alpha_{p[i]} + \alpha_{s[i]}) \vdash \quad \alpha + \beta_c X_c$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2),$$
$$\beta_p \sim \mathcal{MVN}(0, \Sigma_{pc}).$$

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$$\alpha_{p[i]} + \beta_{s[i]} c_i)$$

$$\alpha + \beta_c X_c$$

Model 2 is the least wrong model

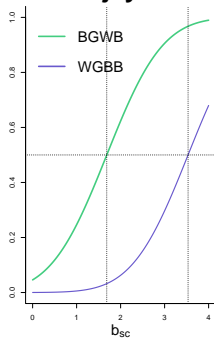$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i)$$

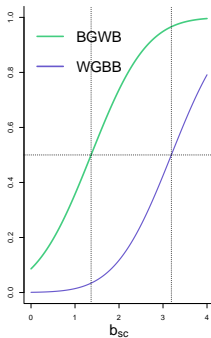Introduction   Random effects for random factors   **Random stimuli in Experimental Psychology**   Discussion
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Results
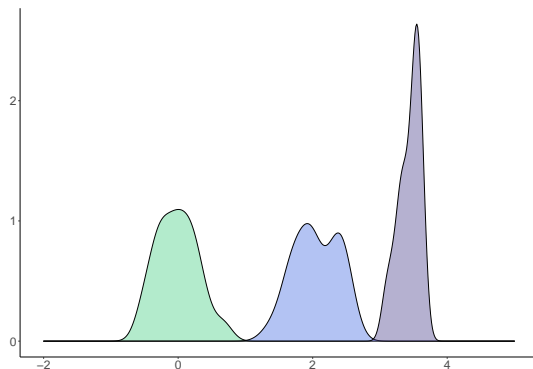
Condition–specific easiness

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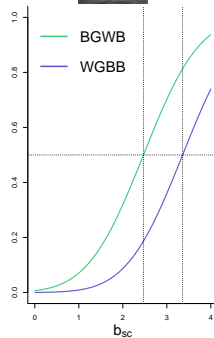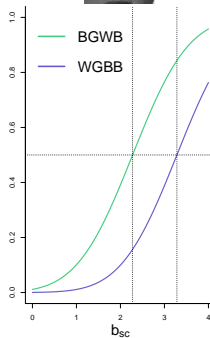
Discussion
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- Acknowledge and gather the information at the stimulus level

- Improve generalizability of the results to other sets of stimuli

- Control for random variance in the data

- Allow for obtaining a Rasch-like parametrization of the data

Thank you!

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