

# CUT IT SHORT: A NEW ITEM RESPONSE THEORY-BASED APPROACH FOR SHORTENING TESTS

Ottavia M. Epifania<sup>1,2</sup>, Pasquale Anselmi<sup>1</sup>, Egidio Robusto<sup>1</sup>

`ottavia.epifania@unipd.it`

<sup>1</sup>University of Padova

<sup>2</sup>Catholic University of the Sacred Heart

September 7<sup>th</sup>, Bologna (IT)

Statistics, Technology and Data Science for Economic and Social  
Development, 2023



- 1 **Introduction**
- 2 Item Response Theory and information functions
- 3 IRT procedures for shortening tests
  - Benchmark procedure
  - Procedures based on  $\theta$  targets
- 4 Practical Implications
- 5 Some final remarks

## Many items/questions in a questionnaire

### Good

High assessment precision

High information/reliability

### But

Respondent's fatigue

Response quality might be compromised

## European Social Surveys

Cross-national survey carried on every two years since 2001

Assessment of attitudes, beliefs, and behavior patterns of diverse populations in different countries. Main focus → change/stability of:

- Living conditions
- Social structure
- Public opinion

Round 10:

Socio-demographic information

+

Well being, social exclusion, human values

## European Social Surveys

Cross-national survey carried on every two years since 2001

Assessment of attitudes, beliefs, and behavior patterns of diverse populations in different countries. Main focus → change/stability of:

- Living conditions
- Social structure
- Public opinion

Round 10:

Socio-demographic information

+

Well being, social exclusion, human values

## A viable solution

A short test form (STF) with few items but high reliability

## A viable solution

A short test form (STF) with few items but high reliability

The information at the item level is crucial → Each item taps on a specific location of the latent trait

## A viable solution

A short test form (STF) with few items but high reliability

The information at the item level is crucial → Each item taps on a specific location of the latent trait

## Item Response Theory



## 1 Introduction

## 2 Item Response Theory and information functions

## 3 IRT procedures for shortening tests

- Benchmark procedure
- Procedures based on  $\theta$  targets

## 4 Practical Implications

## 5 Some final remarks

# Item Response Theory

## 2-PL Model

$$P(x_{pj} = 1 | \theta_p, b_j, a_j) = \frac{\exp[a_j(\theta_p - b_j)]}{1 + \exp[a_j(\theta_p - b_j)]}$$

where:

$P(x_{pj} = 1)$ : Probability of endorsing item  $j$  by respondent  $p$

$\theta_p$ : Ability of respondent  $p$

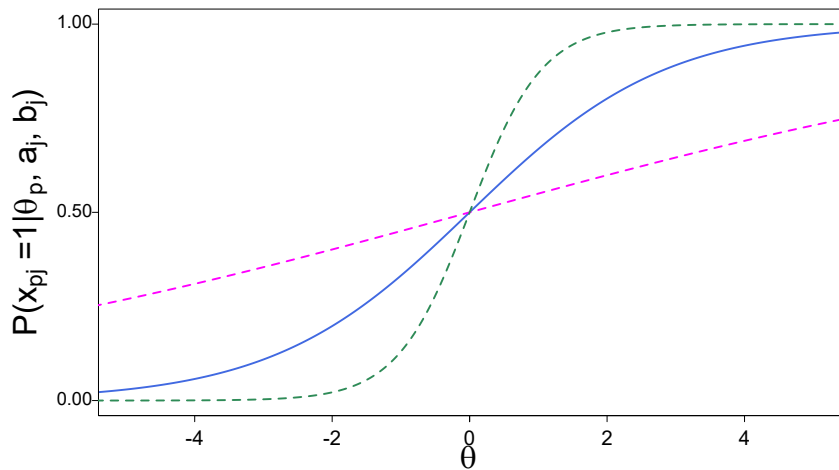
$b_j$ : Difficulty (location on the latent trait) of item  $j$

$a_j$ : Discrimination of item  $j$

Cut it short

└ IRT and Information Functions

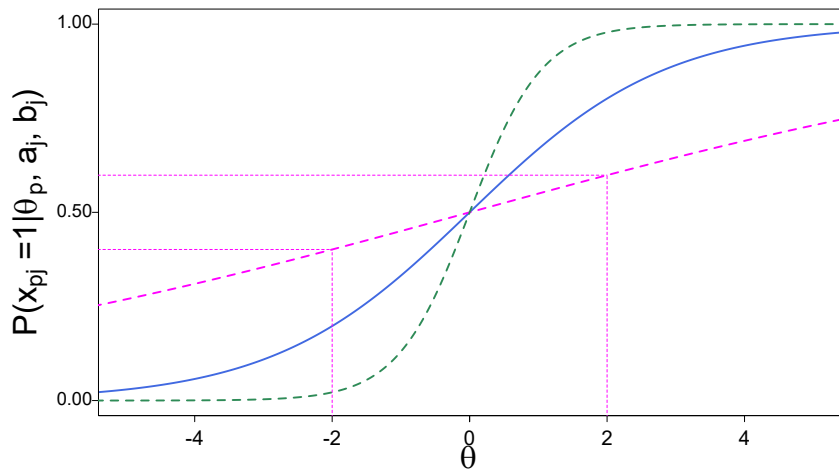
└ 2-PL Model



Cut it short

└ IRT and Information Functions

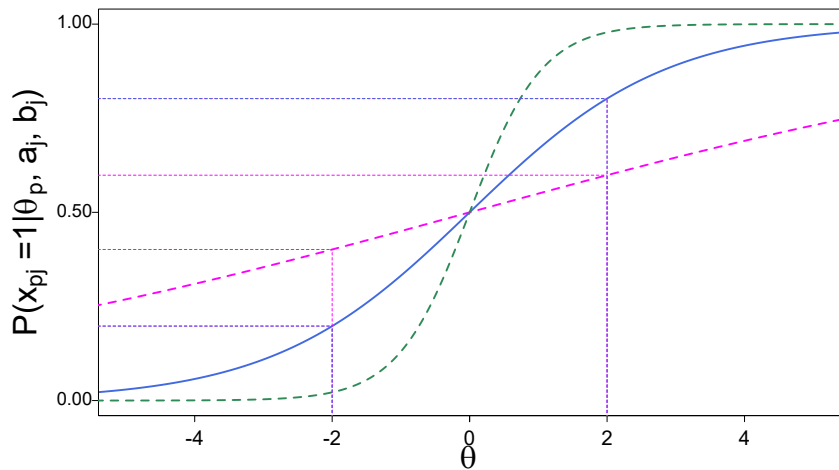
└ 2-PL Model



Cut it short

└ IRT and Information Functions

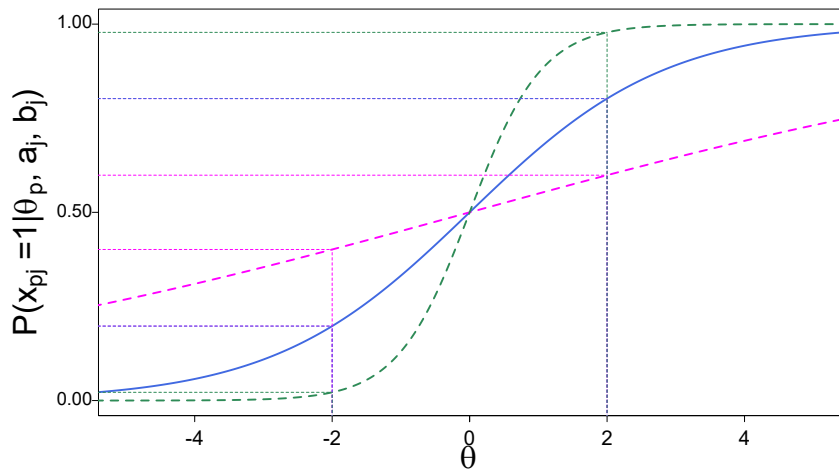
└ 2-PL Model



Cut it short

└ IRT and Information Functions

└ 2-PL Model



## Information functions

Item Information Function

$$IIF_j = a_j^2 [P(\theta)(1 - P(\theta))]$$

## Information functions

### Item Information Function

$$IIF_j = a_j^2 [P(\theta)(1 - P(\theta))]$$

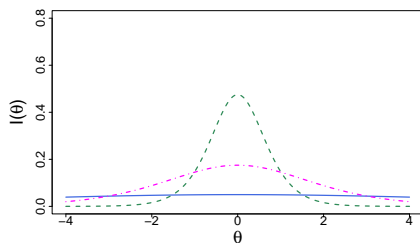


Figure 1:  $a = 0.20$ ,  $a = 0.70$ ,  $a = 1.90$ ,  
 $b = 0$



## Information functions

Item Information Function

$$IIF_j = a_j^2 [P(\theta)(1 - P(\theta))]$$

Test Information Function

$$TIF = \sum_{j=1}^J IIF_j$$

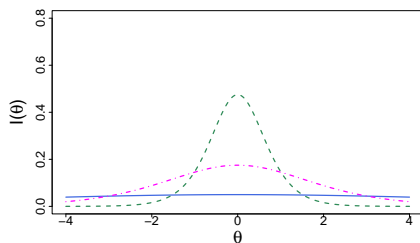


Figure 1:  $a = 0.20$ ,  $a = 0.70$ ,  $a = 1.90$ ,  
 $b = 0$

## Information functions

Item Information Function

$$IIF_j = a_j^2 [P(\theta)(1 - P(\theta))]$$

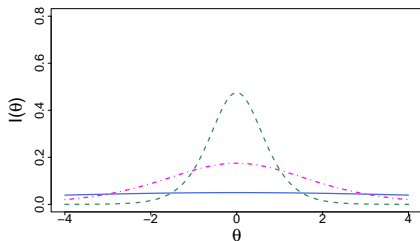


Figure 1:  $a = 0.20$ ,  $a = 0.70$ ,  $a = 1.90$ ,  
 $b = 0$

Test Information Function

$$TIF = \sum_{j=1}^J IIF_j$$

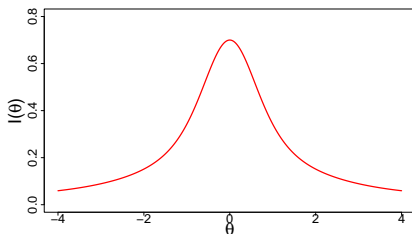


Figure 2:  $TIF = IIF_1 + IIF_2 + IIF_3$

- 1 Introduction
- 2 Item Response Theory and information functions
- 3 IRT procedures for shortening tests**
  - Benchmark procedure
  - Procedures based on  $\theta$  targets
- 4 Practical Implications
- 5 Some final remarks

## 1 Introduction

## 2 Item Response Theory and information functions

## 3 IRT procedures for shortening tests

- Benchmark procedure
- Procedures based on  $\theta$  targets

## 4 Practical Implications

## 5 Some final remarks

## Benchmark procedure

Selected items → items with the highest *IIFs*

*e.g.: 3-item short form from 10-item full-length test*

item	<i>b</i>	<i>a</i>	<i>IIF</i>
1	−0.67	0.71	0.08
2	0.50	1.19	0.15
3	−2.43	0.25	0.01
4	2.12	1.98	0.24
5	1.72	0.39	0.03
6	−2.28	1.62	0.19
7	0.64	0.50	0.05
8	−2.51	1.68	0.19
9	−0.66	0.44	0.04
10	0.72	0.33	0.02

## Benchmark procedure

Selected items → items with the highest *IIFs*

*e.g.: 3-item short form from 10-item full-length test*

item	<i>b</i>	<i>a</i>	<i>IIF</i>
4	2.12	1.98	0.24
8	−2.51	1.68	0.19
6	−2.28	1.62	0.19
2	0.50	1.19	0.15
1	−0.67	0.71	0.08
7	0.64	0.50	0.05
9	−0.66	0.44	0.04
5	1.72	0.39	0.03
10	0.72	0.33	0.02
3	−2.43	0.25	0.01

- └ Short form procedures
- └ Procedures based on  $\theta$  targets

## 1 Introduction

## 2 Item Response Theory and information functions

## 3 IRT procedures for shortening tests

- Benchmark procedure
- Procedures based on  $\theta$  targets

## 4 Practical Implications

## 5 Some final remarks

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

	$\theta'_1$	$\theta'_2$	$\theta'_3$
item	−2.67	0.01	2.67
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			



## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

	$\theta'_1$	$\theta'_2$	$\theta'_3$
item	-2.67	0.01	2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

item	$\theta'_1$ -2.67	$\theta'_2$ 0.01	$\theta'_3$ 2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

item	$\theta'_1$ -2.67	$\theta'_2$ 0.01	$\theta'_3$ 2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

item	$\theta'_1$ -2.67	$\theta'_2$ 0.01	$\theta'_3$ 2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

item	$\theta'_1$ -2.67	$\theta'_2$ 0.01	$\theta'_3$ 2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

item	$\theta'_1$ -2.67	$\theta'_2$ 0.01	$\theta'_3$ 2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

## $\theta$ -target procedures

Selected items  $\rightarrow$  items with highest  $IIFs$  in respect to  $\theta$  targets ( $\theta'$ )

*e.g.: 3-item short form from 10-item full-length test*

item	$\theta'_1$ -2.67	$\theta'_2$ 0.01	$\theta'_3$ 2.67
1	0.04	0.12	0.08
2	0.09	0.33	0.03
3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
7	0.05	0.06	0.03
8	0.01	0.04	0.69
9	0.03	0.05	0.04
10	0.02	0.03	0.02

- 1 Introduction
- 2 Item Response Theory and information functions
- 3 IRT procedures for shortening tests
  - Benchmark procedure
  - Procedures based on  $\theta$  targets
- 4 Practical Implications**
- 5 Some final remarks



## An overall look

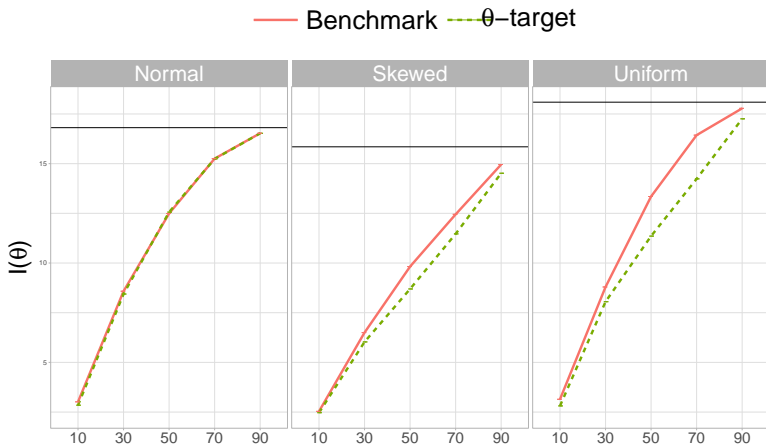


Figure 3: Overall Information of the short test forms

## A closer look

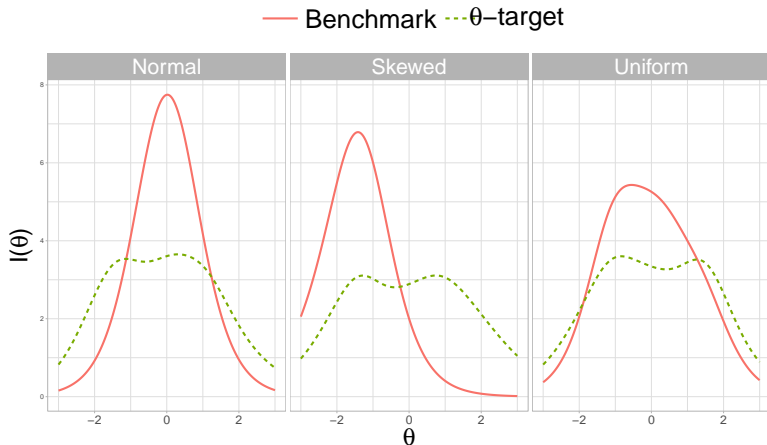


Figure 4: TIF of the 10-item short test form

- 1 Introduction
- 2 Item Response Theory and information functions
- 3 IRT procedures for shortening tests
  - Benchmark procedure
  - Procedures based on  $\theta$  targets
- 4 Practical Implications
- 5 Some final remarks**

- Item response theory provides a valid framework for shortening tests without losing information and reliability
- Targeting vs. ordering: There is no “one-fits-all” solution
- In the future → Which is the ideal number of item?

Thank you!

ottavia.epifania@unipd.it

