

# L'importanza di essere significante: Un esempio basato sul test della Torre di Londra

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La psicometria tra oggi e domani:  
Sfide e nuovi orizzonti

20 Giugno 2024



The ratio between the measures of  $a$  and  $b$  is constant and independent of the measurement unit:

$$\frac{\varphi(a)}{\varphi(b)} = \frac{\varphi'(a)}{\varphi'(b)},$$

where  $\varphi$  and  $\varphi'$  are two different scales of measurement of the same variable<sup>1</sup>.

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<sup>1</sup> Strictly referring to extensive physical measures

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## Meaningful comparisons

The comparison between  $a$  and  $b$  is meaningful if it is invariant under all the unit transformations.

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## Meaningful comparisons 2.0

Given that there is a difference between  $a$  and  $b$ , is this difference significant (or not) regardless of the scales of measurement?

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## Admissible and non-admissible transformations

$$\varphi(P) = [0, 1, 2, 3]$$

$$\varphi'(P) = [0, 2, 4, 10]$$

$$\varepsilon(P) = [0, 2, 2, 3]$$

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|            | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ | $q_8$ | $q_9$ |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\varphi$  |       |       |       |       |       |       |       |       |       |
| Joe        | 0     | 1     | 2     | 2     | 2     | 3     | 3     | 3     | 3     |
| Jane       | 0     | 2     | 2     | 2     | 3     | 3     | 3     | 3     | 3     |
| Max        | 0     | 1     | 0     | 2     | 3     | 3     | 3     | 3     | 3     |
| $\varphi'$ |       |       |       |       |       |       |       |       |       |
| Joe        | 0     | 2     | 4     | 4     | 4     | 10    | 10    | 10    | 10    |
| Jane       | 0     | 4     | 4     | 4     | 10    | 10    | 10    | 10    | 10    |
| Max        | 0     | 2     | 0     | 4     | 10    | 10    | 10    | 10    | 10    |
| $\epsilon$ |       |       |       |       |       |       |       |       |       |
| Joe        | 0     | 2     | 2     | 2     | 2     | 3     | 3     | 3     | 3     |
| Jane       | 0     | 2     | 2     | 2     | 3     | 3     | 3     | 3     | 3     |
| Max        | 0     | 2     | 0     | 2     | 3     | 3     | 3     | 3     | 3     |

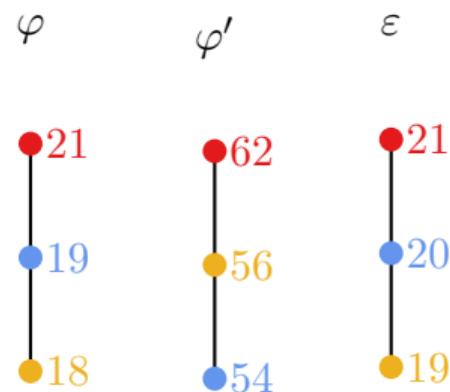
## Admissible and non-admissible transformations

$$\varphi(P) = [0, 1, 2, 3]$$

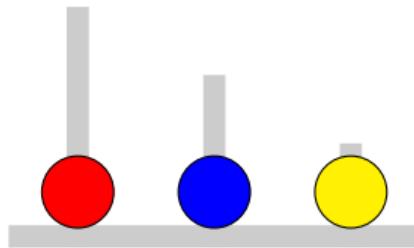
$$\varphi'(P) = [0, 2, 4, 10]$$

$$\varepsilon(P) = [0, 2, 2, 3]$$

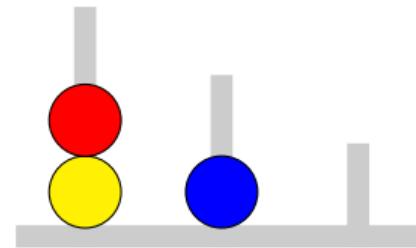
|               | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ | $q_8$ | $q_9$ |
|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\varphi$     |       |       |       |       |       |       |       |       |       |
| Joe           | 0     | 1     | 2     | 2     | 2     | 3     | 3     | 3     | 3     |
| Jane          | 0     | 2     | 2     | 2     | 3     | 3     | 3     | 3     | 3     |
| Max           | 0     | 1     | 0     | 2     | 3     | 3     | 3     | 3     | 3     |
| $\varphi'$    |       |       |       |       |       |       |       |       |       |
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| Max           | 0     | 2     | 0     | 4     | 10    | 10    | 10    | 10    | 10    |
| $\varepsilon$ |       |       |       |       |       |       |       |       |       |
| Joe           | 0     | 2     | 2     | 2     | 2     | 3     | 3     | 3     | 3     |
| Jane          | 0     | 2     | 2     | 2     | 3     | 3     | 3     | 3     | 3     |
| Max           | 0     | 2     | 0     | 2     | 3     | 3     | 3     | 3     | 3     |



## The Tower of London Test (ToL Test)

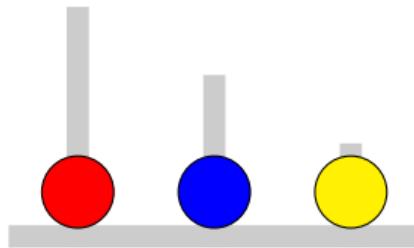


Starting configuration

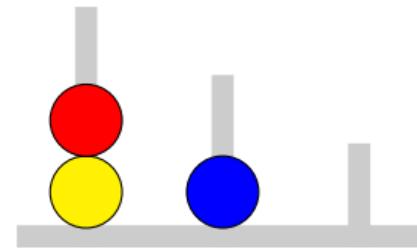


Goal configuration

## The Tower of London Test (ToL Test)



Starting configuration



Goal configuration

| Problem Example | Minimum moves | Alternative paths |
|-----------------|---------------|-------------------|
| 1               | 2             | 1                 |
| 2               | 2             | 1                 |
| 3               | 3             | 2                 |
| 4               | 3             | 1                 |
| 5               | 4             | 2                 |
| 6               | 4             | 1                 |
| 7               | 4             | 1                 |
| 8               | 4             | 1                 |
| 9               | 5             | 2                 |
| 10              | 5             | 1                 |
| 11              | 5             | 1                 |
| 12              | 5             | 2                 |

## Attempt-based SMs

| Scoring system | First attempt | Second attempt | Third attempt | Fourth on | Total sum score |
|----------------|---------------|----------------|---------------|-----------|-----------------|
| KR             | 3             | 2              | 1             | 0         | 0 – 36          |
| SH1            | 1             |                | 0             |           | 0 – 12          |

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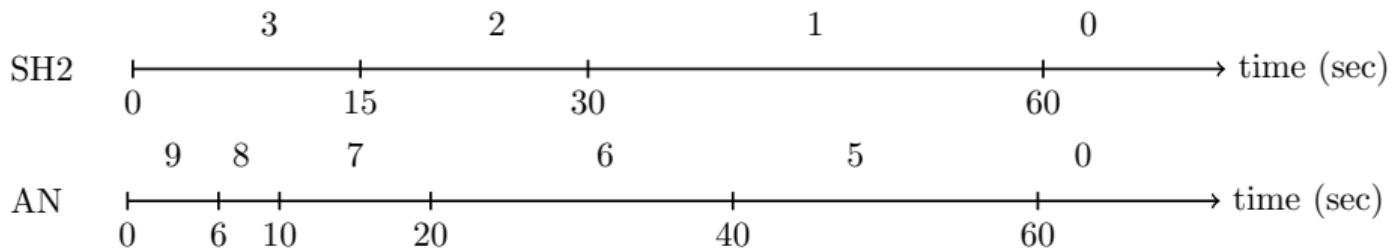
| Scoring system | First attempt | Second attempt | Third attempt | Fourth on | Total sum score |
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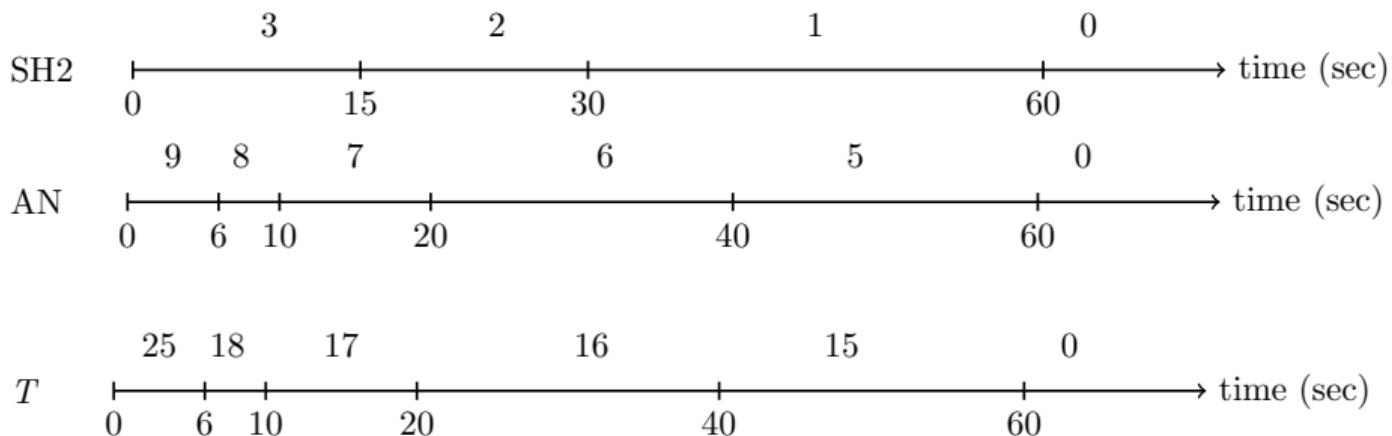
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## Latency-based SMS

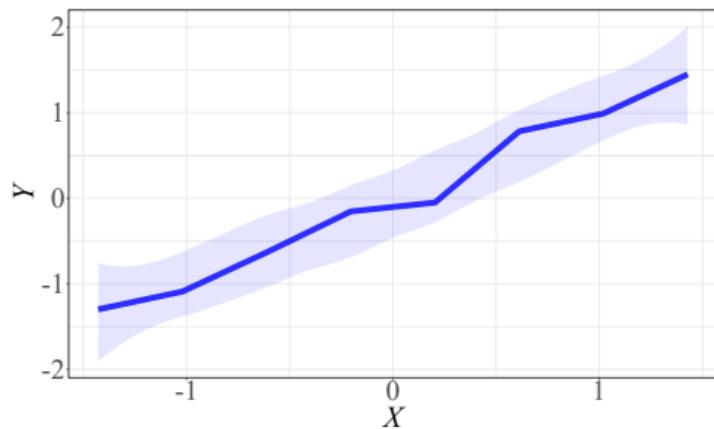


## Latency-based SMS



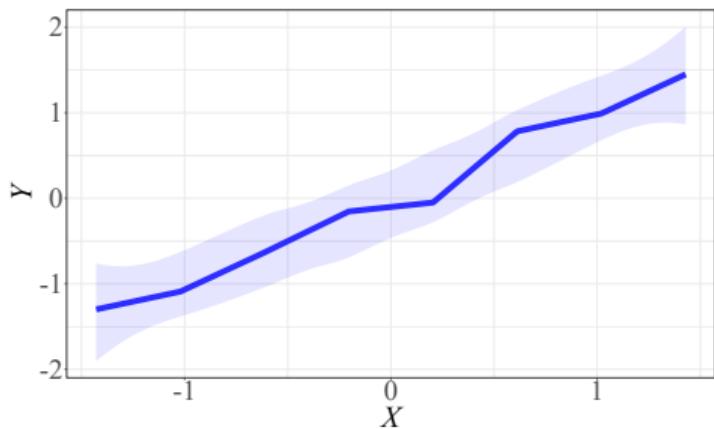
Methods: Individual differences

## Monotonic relation

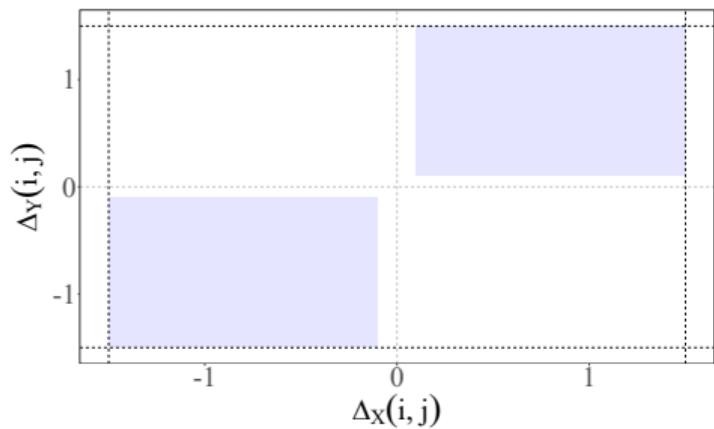


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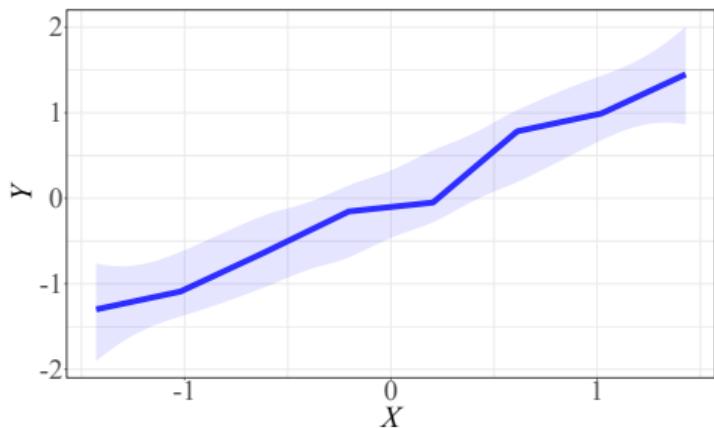


## Distances and inversions

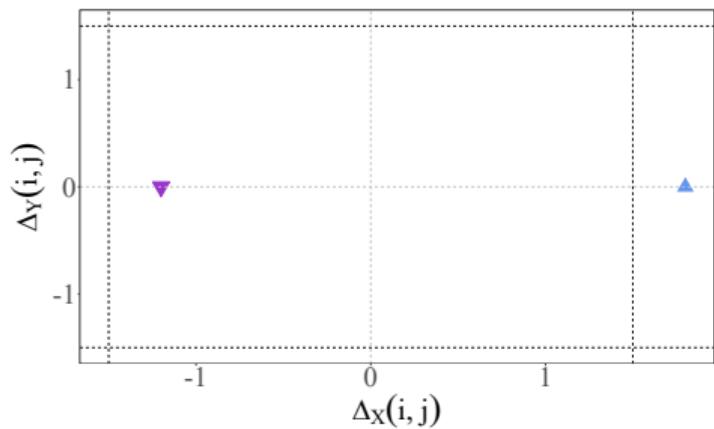


Methods: Individual differences

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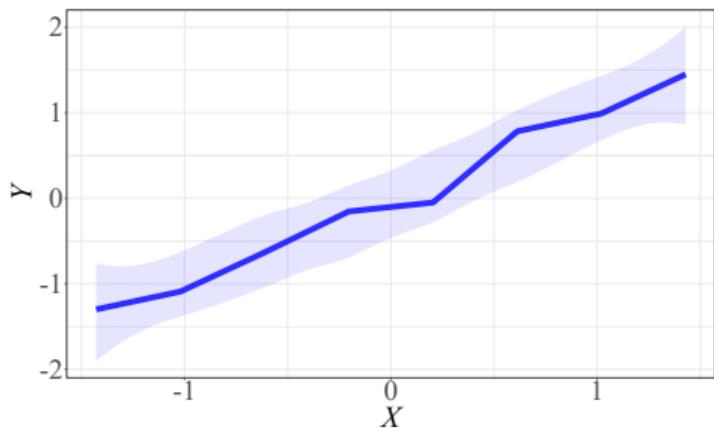


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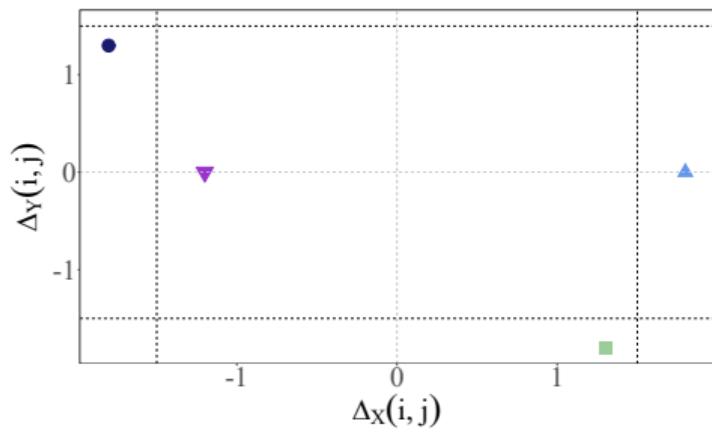


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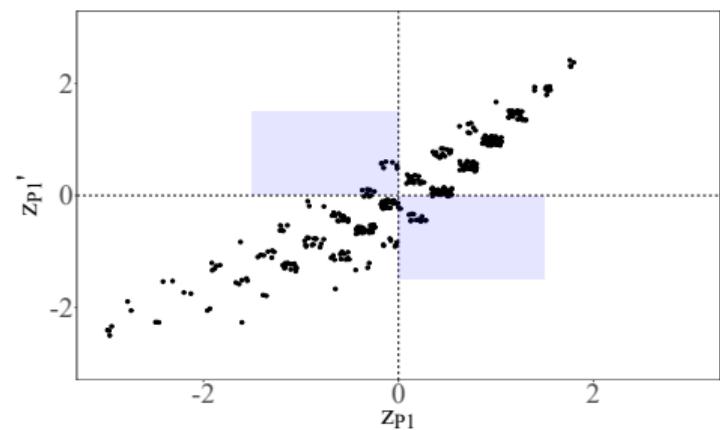
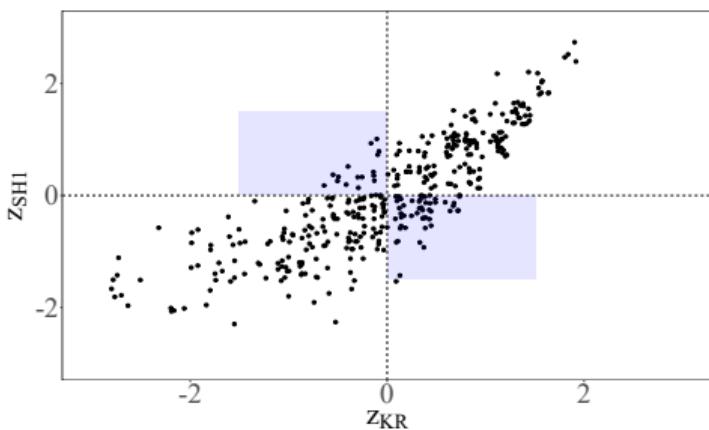


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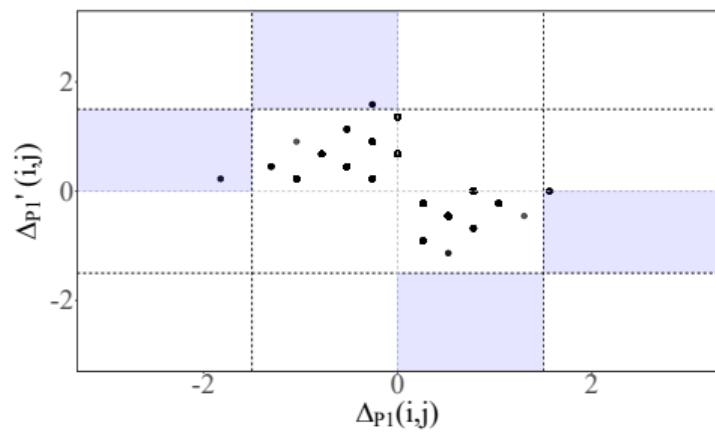
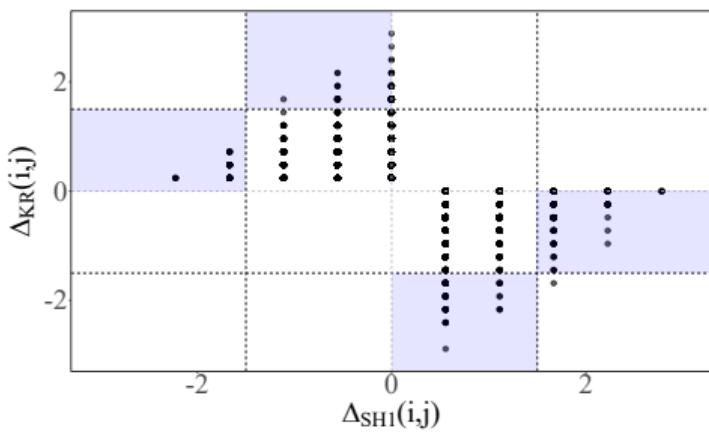
Results: Individual differences

# Attempt-based SM: Monotonic relation



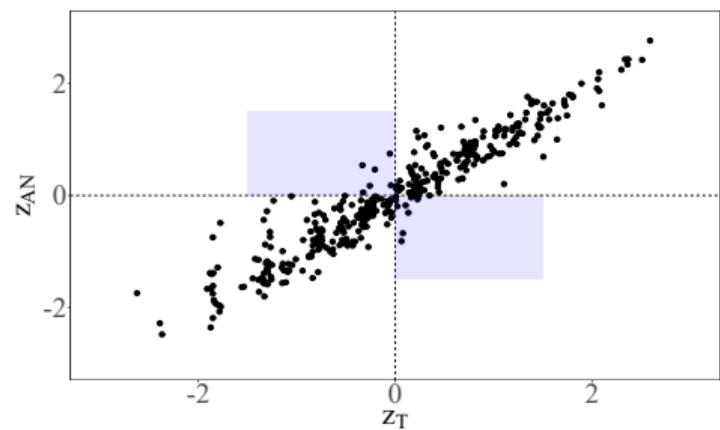
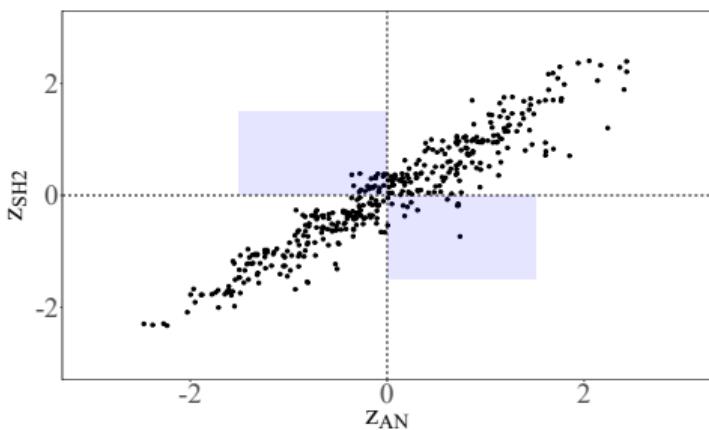
Results: Individual differences

# Attempt-based SM: Differences and distances



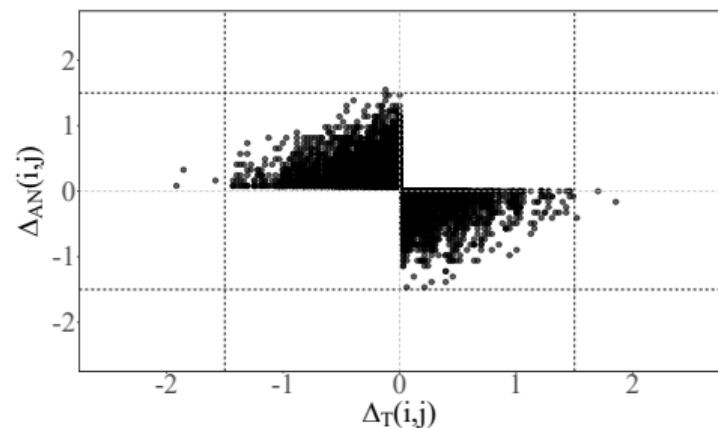
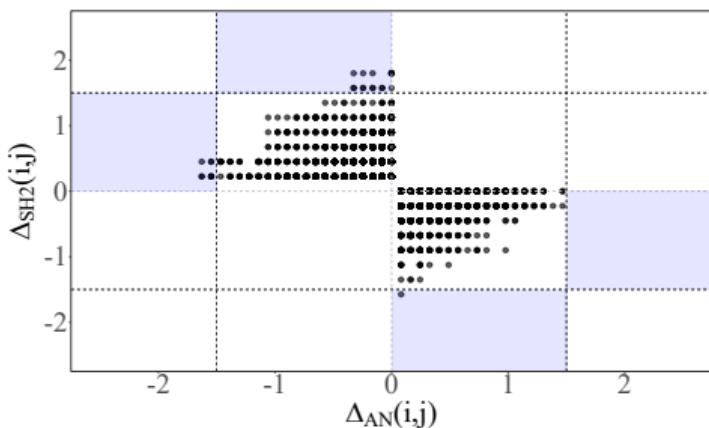
Results: Individual differences

# Latency-based SM: Monotonic relation



Results: Individual differences

# Latency-based SM: Differences and distances



## Methods: Group differences

$$H_0: \mu_{g1} - \mu_{g2} = 0$$

$$H_1: \mu_{g1} - \mu_{g2} \neq 0$$

*t*-test on the standardized scores considering different grouping variables:

| Grouping variable       | $n_1$ | $n_2$ |
|-------------------------|-------|-------|
| Gender                  | 199   | 196   |
| Administration order    | 202   | 193   |
| Administration modality | 211   | 184   |
| Schooling years         | 171   | 224   |

Results: Group differences

# Attempt-based SM

|               | KR       | SH1      | P1       | P1'      |
|---------------|----------|----------|----------|----------|
|               | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> |
| Gender        | 1.84     | 2.11*    | 1.69     | 2.03*    |
|               | 0.19     | 0.21     | 0.17     | 0.20     |
| Test order    | -0.15    | 0.80     | -0.48    | 0.28     |
|               | -0.01    | 0.08     | -0.05    | 0.03     |
| Adm. Modality | -2.85**  | -1.93    | -2.69**  | -2.35*   |
|               | -0.29    | -0.19    | -0.27    | -0.24    |
| Schooling     | 3.95***  | 3.56***  | 3.82***  | 3.85***  |
|               | 0.39     | 0.36     | 0.38     | 0.39     |

Results: Group differences

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Results: Group differences

# Latency-based SM

|            | SH2      | AN       | T        |
|------------|----------|----------|----------|
|            | <i>d</i> | <i>d</i> | <i>d</i> |
| Gender     | 1.64     | 1.88     | 2.10*    |
|            | 0.17     | 0.19     | 0.21     |
| Test order | 0.37     | 0.99     | 0.95     |
|            | 0.04     | 0.10     | 0.10     |
| Adm. Order | -2.90**  | -2.33*   | -2.84**  |
|            | -0.29    | -0.23    | -0.29    |
| Schooling  | 5.52***  | 5.32***  | 5.13***  |
|            | 0.56     | 0.54     | 0.52     |

Results: Group differences

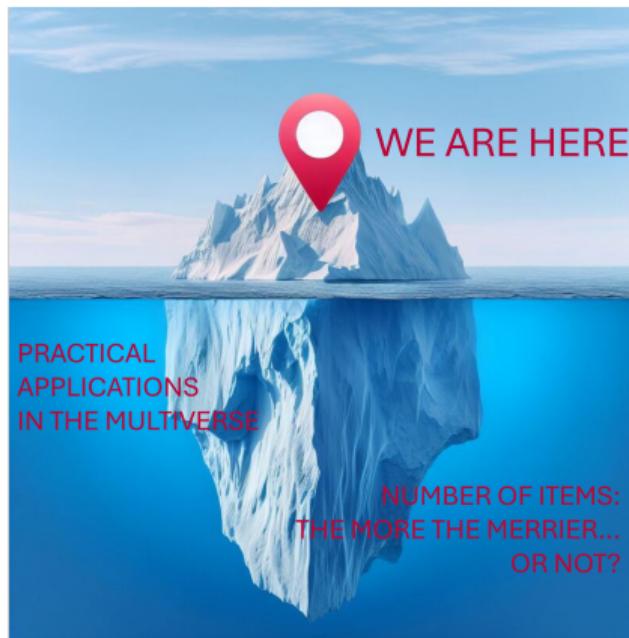
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# Are we sure sum scores are a good idea...?

PSYCHOMETRIKA—VOL. 89, NO. 1, 84–117  
MARCH 2024  
<https://doi.org/10.1007/s11336-024-09964-7>



RECOGNIZE THE VALUE OF THE SUM SCORE, PSYCHOMETRICS' GREATEST  
ACCOMPLISHMENT

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Sum scores of ordinal data bring to a multiverse of contrasting results



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Increasing the number of items does not solve the issue.... it worsens it!

Meaningfulness of psychological measures and reproducibility are interlaced

Research founded by the project “Computerized, Adaptive and Personalized Assessment of Executive Functions and Fluid Intelligence” (PRIN 2020, Prot. 20209WKCLL, P.I. Prof. Luca Stefanutti)



Sum scores of ordinal data bring to a multiverse of contrasting results

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### Bright side:

Sum scores of truly dichotomous data (i.e., true vs. false, correct vs. incorrect) are meaningful

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