When randomness opens new possibilities: Acknowledging the stimulus sampling variability in Experimental Psychology

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Introduction

Stimuli are fixed, respondents are random

Respondents are random

Sampled from a lager population

Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

Stimuli/item are fixed

Taken to be entire population

There is no sampling variability

There is no need to generalize the results because the stimuli are the population

However...

The stimuli can also represent a sample of a larger universe:

Example

Positive vs. **Negative** attributes in Linguistic
There is a universe of positive words
Only a sample of positive attributes (e.g., good, nice) and negative attributes (e.g., bad, evil) are administered
So... **there must be a sampling variability**

Introduction

What if the sampling variability is not acknowledged

Generalizability

The results can be generalized at the respondents level but not at the stimulus level → Results can be generalized if and only if the exact same set of stimuli is used

Generalizability is bounded to the specific set of stimuli used in the experiment

Robustness of the results

Random variability at the stimulus level might inflate the probability of committing Type I errors

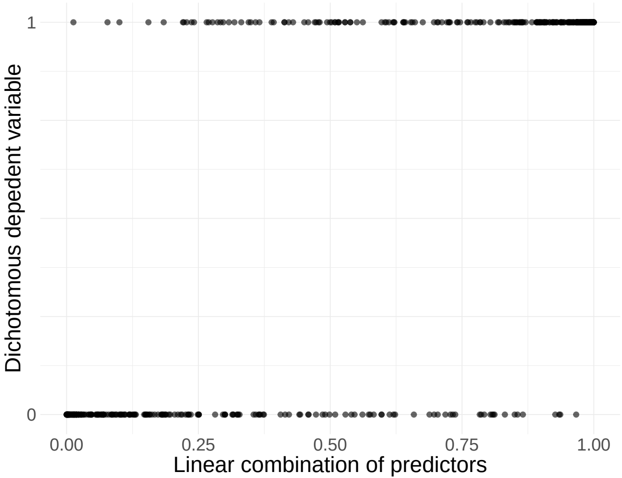
Averaging across stimuli to obtain a person-level score results in biased estimates due to the noise in the data

Loss of information

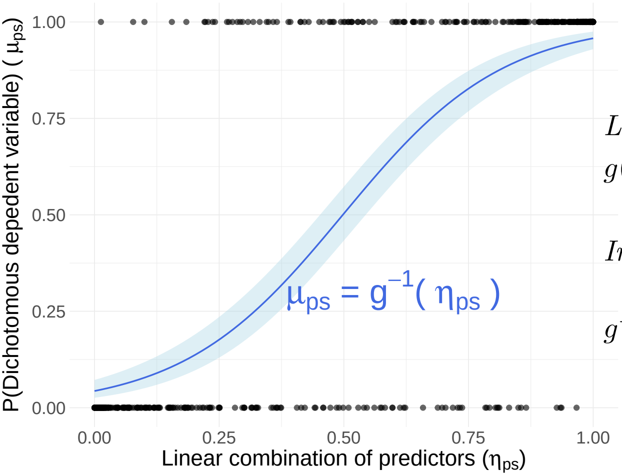
Every stimulus is assumed to be equal → to have the same effect on the observed score

All the variability is not considered as well as all the information that can be obtained from it

Generalized linear model for dichotomous responses



Generalized linear model for dichotomous responses



Logit link function g

$$g(\eta_{ps}) = log\left(\frac{\mu_{ps}}{1-\mu_{ps}}\right)$$

Inverse g^{-1}

$$g^{-1} = \frac{exp(\eta_{ps})}{1+exp(\eta_{ps})}$$

Random effects and random factors

Linear component in a (G)LM:

$$\eta = \beta X, \tag{1}$$

where β indicates the coefficients of the fixed intercept and slope(s), and X is the model-matrix.
Linear components in a (Generalized) Linear Mixed-Effects Model (GLMM):

$$\eta = \beta X \, Zd, \tag{2}$$

where Z is the matrix and d is the vector of the random effects (not parameters!)

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Best Linear Unbiased Predictors

The Rasch model

$$P(x_{ps} = 1|\theta_p, b_s) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

where:
θ_p: ability of respondent p (i.e., latent trait level of respondent p)
b_s: difficulty of stimulus s (i.e., "challenging" power of stimulus s)

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$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

GLM

$$P(x_{ps} = 1) = \frac{\exp(\theta_p + b_s)}{1 + \exp(\theta_p + b_s)}$$

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Random stimuli in Experimental Psychology

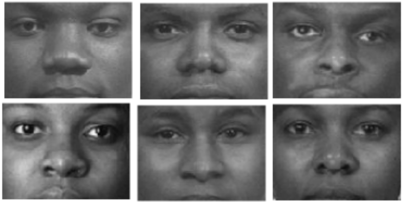
Experiment

The stimuli

12 Object stimuli

White people faces

Black people faces



16 Attribute stimuli

Positive attributes

Good, laughter, pleasure, glory, peace, happy, joy, love

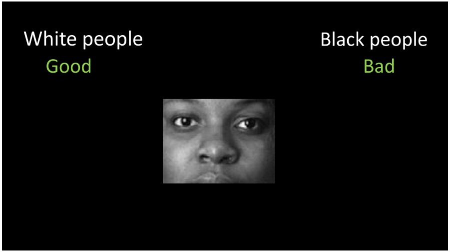
Negative attributes

Evil, bad, horrible, terrible, nasty, pain, failure, hate

The task

Two experimental conditions

White-Good/Black-Bad (WGBB): 60 trials



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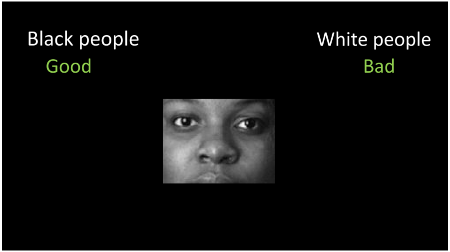

Random stimuli in Experimental Psychology

Models

The expected response y for the observation $i = 1, \ldots, I$ for respondent $p = 1, \ldots, P$ on stimulus $s = 1, \ldots, S$ in condition $c = 1, \ldots, C$:

Model 1:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \alpha_{s[i]} + \varepsilon_i)$$

$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2).$$

Model 2:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i + \varepsilon_i)$$

$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\beta_s \sim \mathcal{MVN}(0, \Sigma_{sc}).$$

Model 3:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{s[i]} + \beta_{p[i]} c_i + \varepsilon_i)$$

$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2),$$
$$\beta_p \sim \mathcal{MVN}(0, \Sigma_{pc}).$$

Accuracy: $\epsilon \sim Logistic(0, \sigma^2)$

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Fixed Effects

Random structure

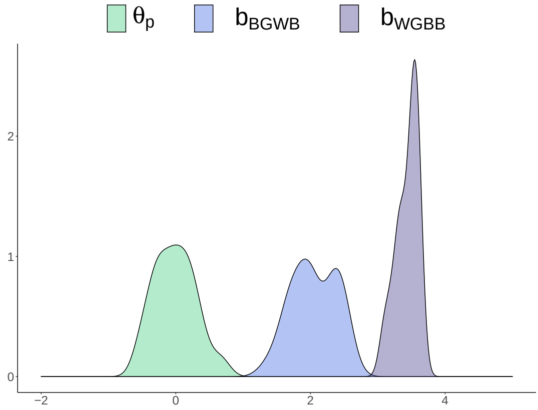
Random stimuli in Experimental Psychology

Results

Model 2 is the least wrong model

Rasch model:

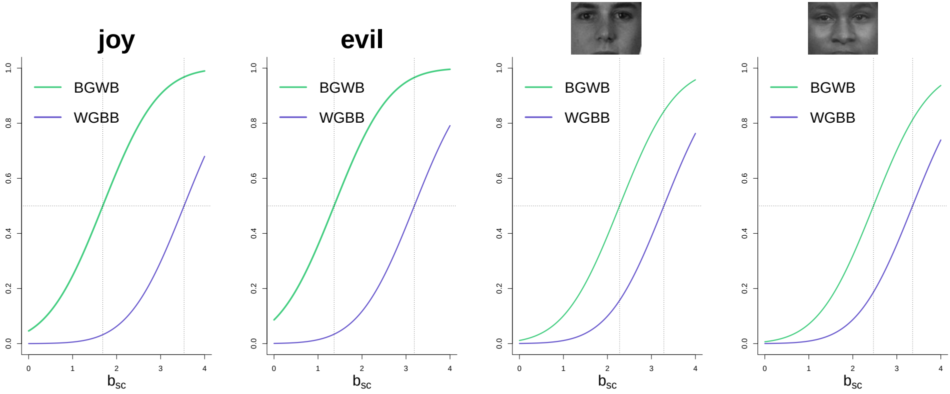
Model 2



Condition–specific easiness

Highly contributing stimuli

Lowly contributing stimuli



- Improve generalizability of the results to other sets of stimuli
- Control for random variance in the data
- Allow for obtaining a Rasch-like parametrization of the data
- Possibility of extending the (linear) model to other dependent variables (e.g., response times)