

Test length doesn't matter, it's how you use the items that counts: An intelligent procedure for item selection in Item Response Theory

Ottavia M. Epifania^{1,2}, Pasquale Anselmi³, Egidio Robusto³

¹ Psychology and Cognitive Science Department, University of Trento, Italy

² Psicostat, University of Padova, Italy

³ Department of Philosophy, Sociology, Education, and Applied Psychology, University of Padova, Italy

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Item Response Theory (IRT) for the development of Short Test Form (STF):

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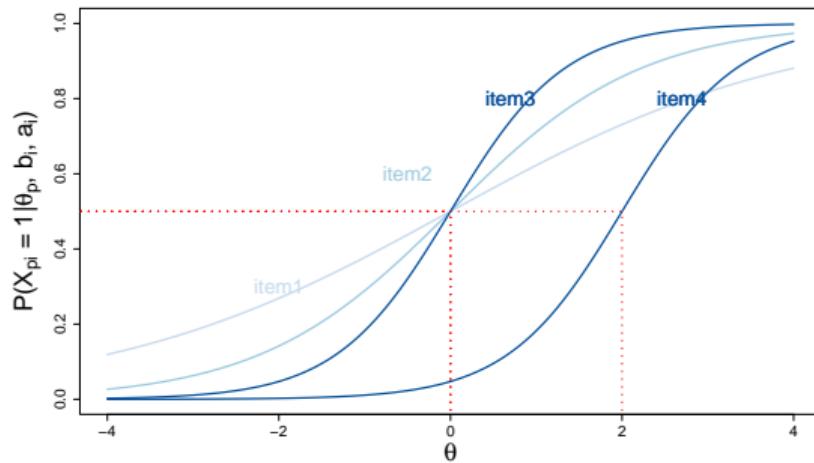
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AIM

New automated procedure for item selection in IRT that only requires the definition of the desired characteristics of a test

2-Parameter Logistic Model

$$P(X_{pi} = 1 | \theta_p, b_i, a_i) = \frac{\exp[a_i(\theta_p - b_i)]}{1 + \exp[a_i(\theta_p - b_i)]}$$



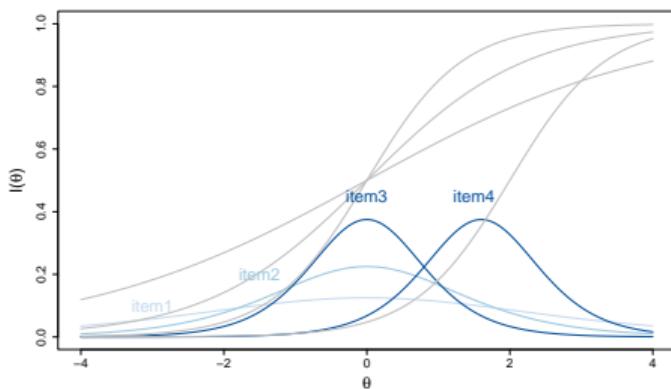
θ_p : Latent trait level of person p

b_i : Location of item i on θ

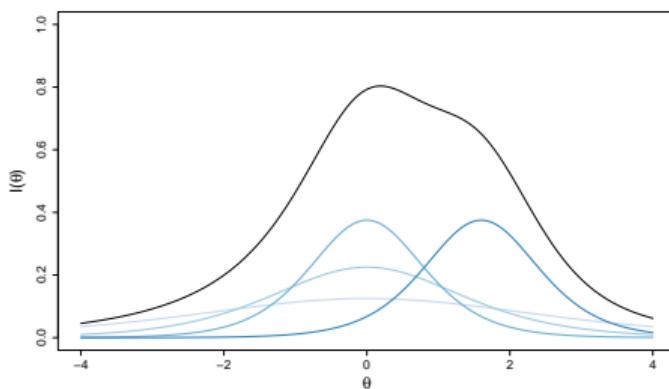
a_i : Discrimination ability of item i

Item and Test Information Functions

Item Information Function (IIF):
 $I_i(\theta) = a_i^2 P_i(\theta, b_i, a_i)[1 - P_i(\theta, b_i, a_i)]$



Test Information Function (TIF):
 $I(\theta) = \sum_{i=1}^N I_i(\theta)$

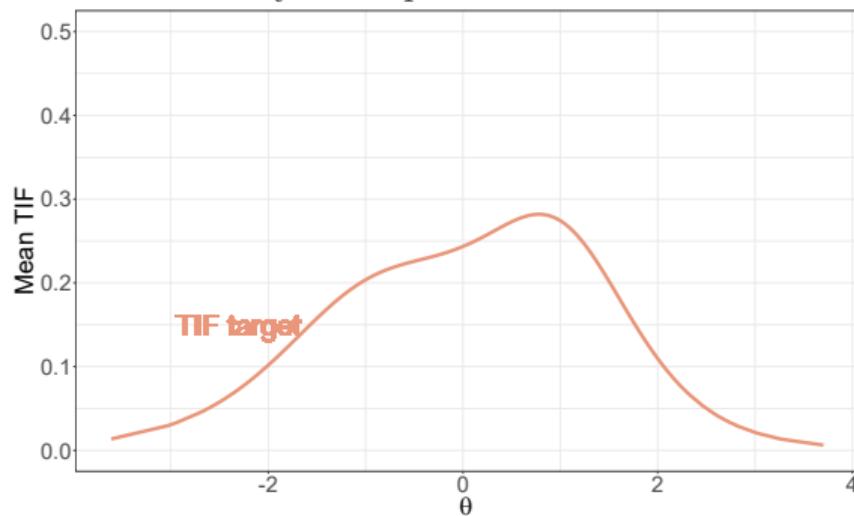


Item Locating Algorithm – ILA

B : Set of items of the item bank , items $i = \{1, 2, \dots, ||B||\}$

TIF*: TIF target describing the desired characteristic of a test

$Q_{ILA}^k \subset B$: Set of items selected by ILA up to iteration k



$$k = 0, Q^0 = \emptyset,$$

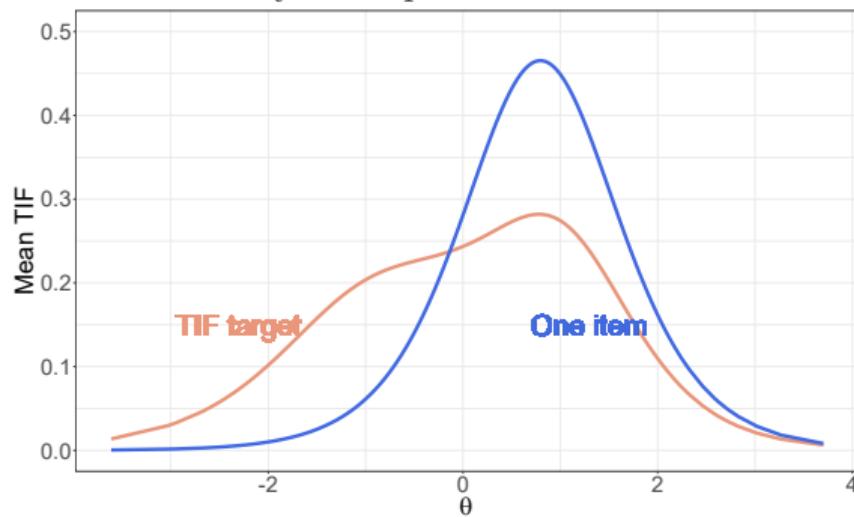
$$\theta_{target} := \arg \max |\mathbf{TIF}^* - \mathbf{TIF}^0|, \text{ where } \mathbf{TIF}^0 = (0, 0, \dots, 0)$$

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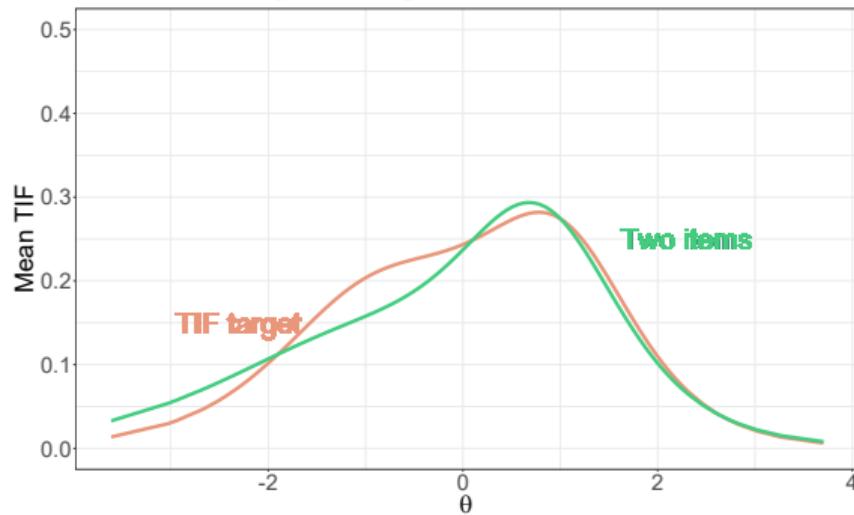
$$\begin{aligned} k &= 1, Q^1 = Q^0 \cup \arg \min_{i \in B \setminus Q^0} |\theta_{target} - b_i|, ||Q^1|| = 1 \\ |\mathbf{TIF}^* - \mathbf{TIF}^1| &\geq |\mathbf{TIF}^* - \mathbf{TIF}^0| \rightarrow \text{false}, k = 2 \\ \theta_{target} &:= \arg \max |\mathbf{TIF}^* - \mathbf{TIF}^1| \end{aligned}$$

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$$Q^2 = Q^1 \cup \arg \min_{i \in B \setminus Q^1} |\theta_{target} - b_i|, ||Q^2|| = 2$$

$|\text{TIF}^* - \text{TIF}^2| \geq |\text{TIF}^* - \text{TIF}^1| \rightarrow$
false, $k = 3$

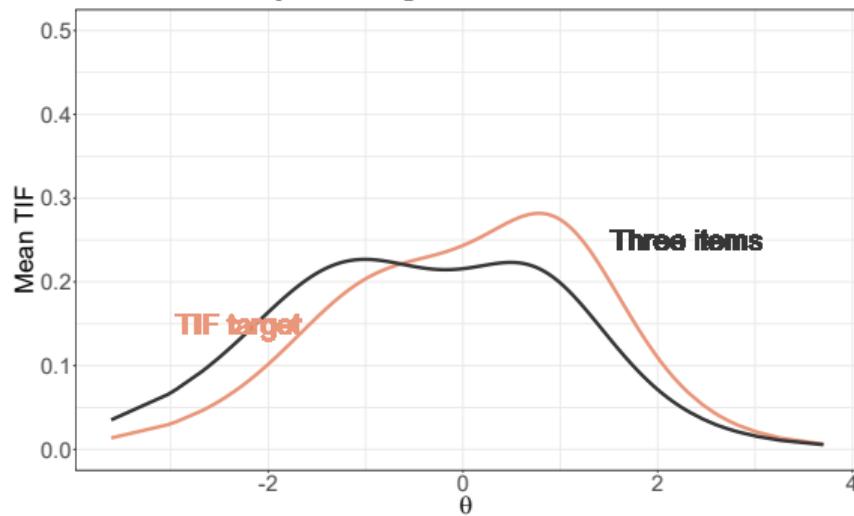
$$\theta_{target} := \arg \max |\text{TIF}^* - \text{TIF}^2|$$

Item Locating Algorithm – ILA

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TIF^* : TIF target describing the desired characteristic of a test

$Q_{\text{ILA}}^k \subset B$: Set of items selected by ILA up to iteration k



$$Q^3 = Q^2 \cup \arg \min_{i \in B \setminus Q^2} |\theta_{target} - b_i|, ||Q^3|| = 3$$

$$|\text{TIF}^* - \text{TIF}^3| \geq |\text{TIF}^* - \text{TIF}^2| \rightarrow \text{true} \rightarrow \text{end}, Q_{\text{ILA}} = Q^2$$

Brute Force Procedure – BFP

Item bank B

$Q_m \subset B$ item combinations of different lengths $l = 1, 2, \dots, N - 1$

Total number of item combinations $2^{\|B\|-2}$

$$\bar{\Delta}_{\text{TIF}^{Q_m}} = \text{mean}\left(\left|\text{TIF}^* - \frac{\sum_{i \in Q_m} IIF_i}{\|Q_m\|}\right|\right)$$

$$Q_{BFP} = \arg \min_{\emptyset \neq Q_m \subset Q} \bar{\Delta}_{\text{TIF}^{Q_m}}$$

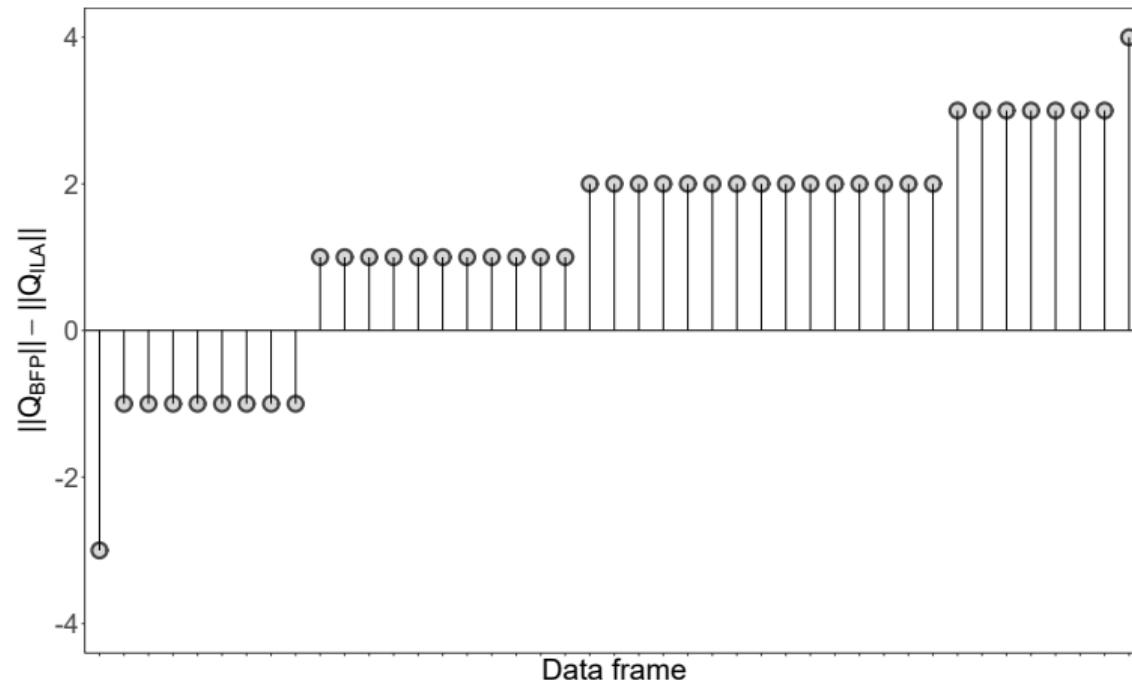
100 data frames:

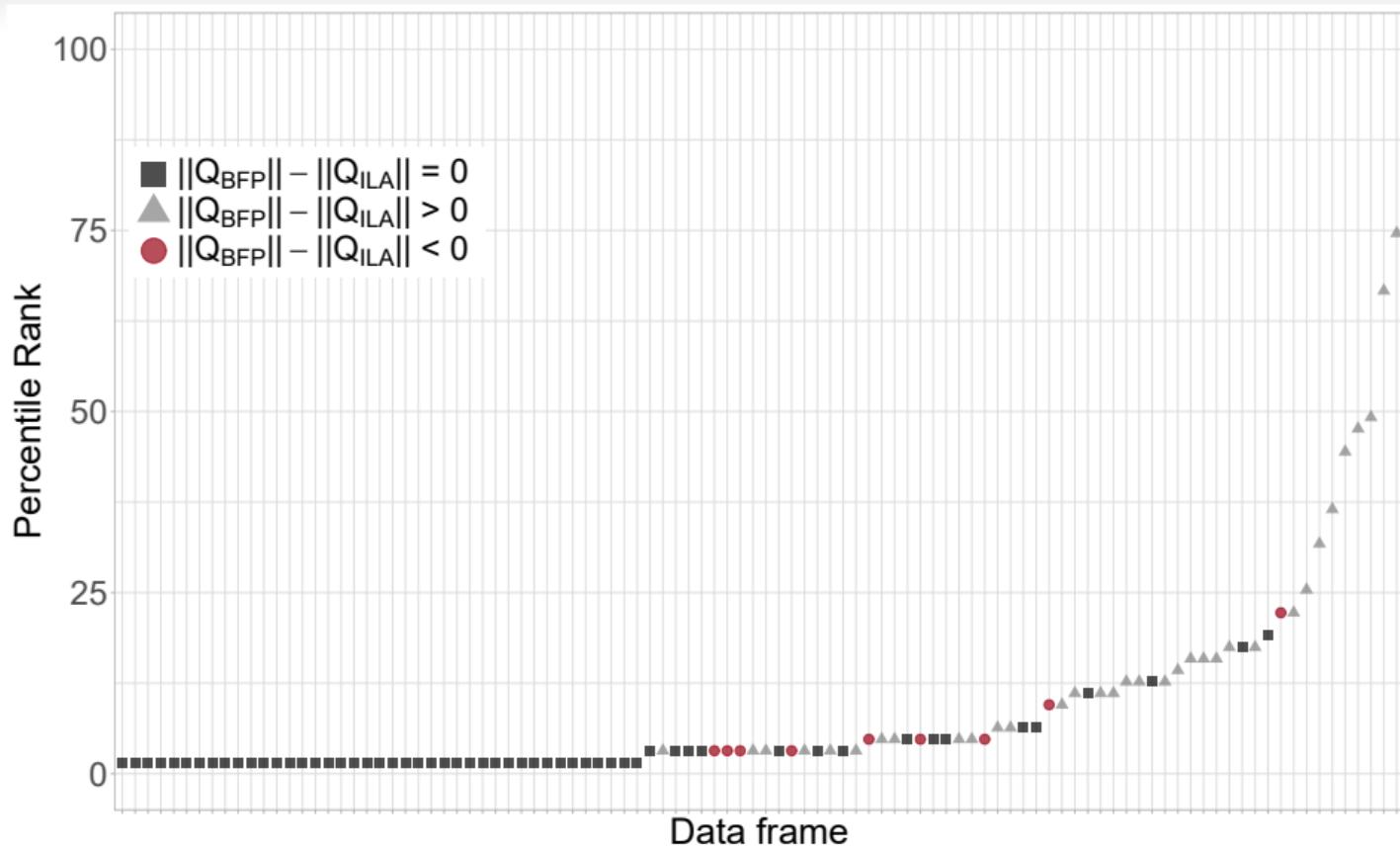
- ① Generate an item bank B of $N = 6$ items:
 - Difficulty parameters: $\mathcal{U}(-3, 3)$
 - Discrimination parameters: $\mathcal{U}(.90, 2.0)$
- ② Random item selections of lengths l from B ($M_l = 3.34 \pm 1.13$) + modification parameters $\mathcal{U}(-0.20, 0.20) \rightarrow \mathbf{TIF}^*$
- ③ Considering \mathbf{TIF}^* at Step 2 and item parameters at Step 1:
 - ILA \rightarrow *Forwardly searches*
 - BFP \rightarrow *Systematically tests*

Comparison:

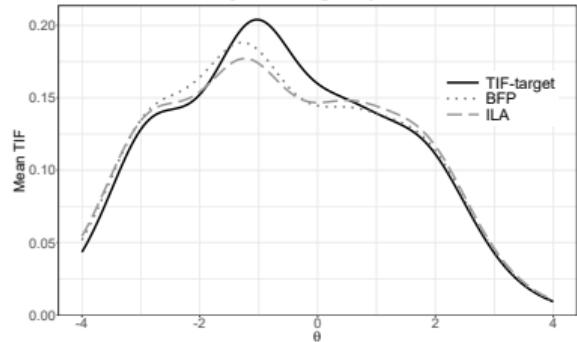
- $\|Q_{\text{BFP}}\| - \|Q_{\text{ILA}}\|$
- Percentile rank (RP) of the distance $\mathbf{TIF}_{\text{BFP}} - \mathbf{TIF}_{\text{ILA}}$

$\|Q_{BFP}\| - \|Q_{ILA}\| = 0$ in 57% of cases

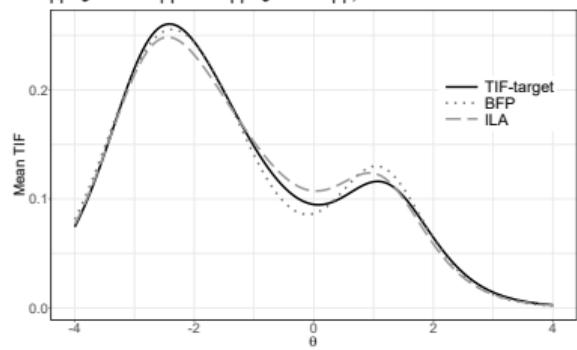




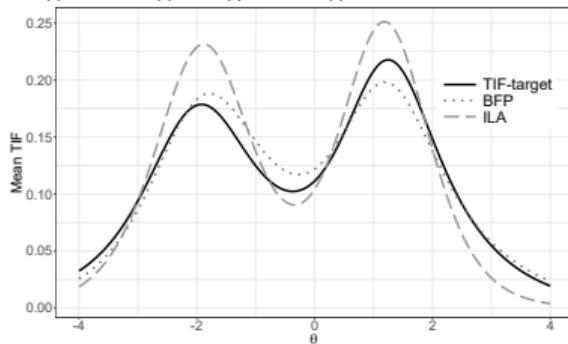
$\|Q_{BFP}\| = \|Q_{ILA}\|, Q_{BFP} \neq Q_{ILA},$
 $RP = 3.17$



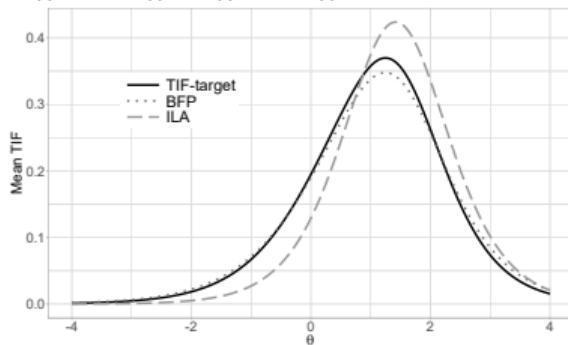
$\|Q_{BFP}\| < \|Q_{ILA}\|, RP = 3.17$



$\|Q_{BFP}\| > \|Q_{ILA}\|, RP = 4.76$



$\|Q_{BFP}\| > \|Q_{ILA}\|, RP = 12.70$



Pros of ILA

- It selects items that are able to recreate the desired characteristics of a test (usually)
- It is computationally “Light”

Cons of ILA

- It grounds its selection on a single θ_{target} at a time → it might select items minimizing the distance on that target but that are not very useful for the test
- It only forwardly searches an item → once it is in, it can't get out
- It does not account for the discrimination parameters of the items