Cut it short: A new item response theory-based approach for shortening tests

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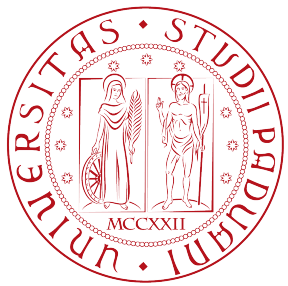


The struggle

Rule of thumb → more questions more reliability
Drawback → too many questions, too many questionnaire → assessment precision might be compromised by the length of the assessment
Esempio che mi manda Andrea per l'indagine dei giovani, tantissimi quetsionari, indagine lunghissima

The Ideal Solution

A short test form (STF) as informative as the full-length test

Focus on the item

The information at the item level is crucial $\rightarrow$ each item taps on a specific location of the latent trait

Item Response Theory

Item Response Theory

2-PL Model

$$P(x_{pj} = 1|\theta_p, b_j, a_j) = \frac{exp[a_j(\theta_p - b_j)]}{1 + exp[a_j(\theta_p - b_j)]}$$

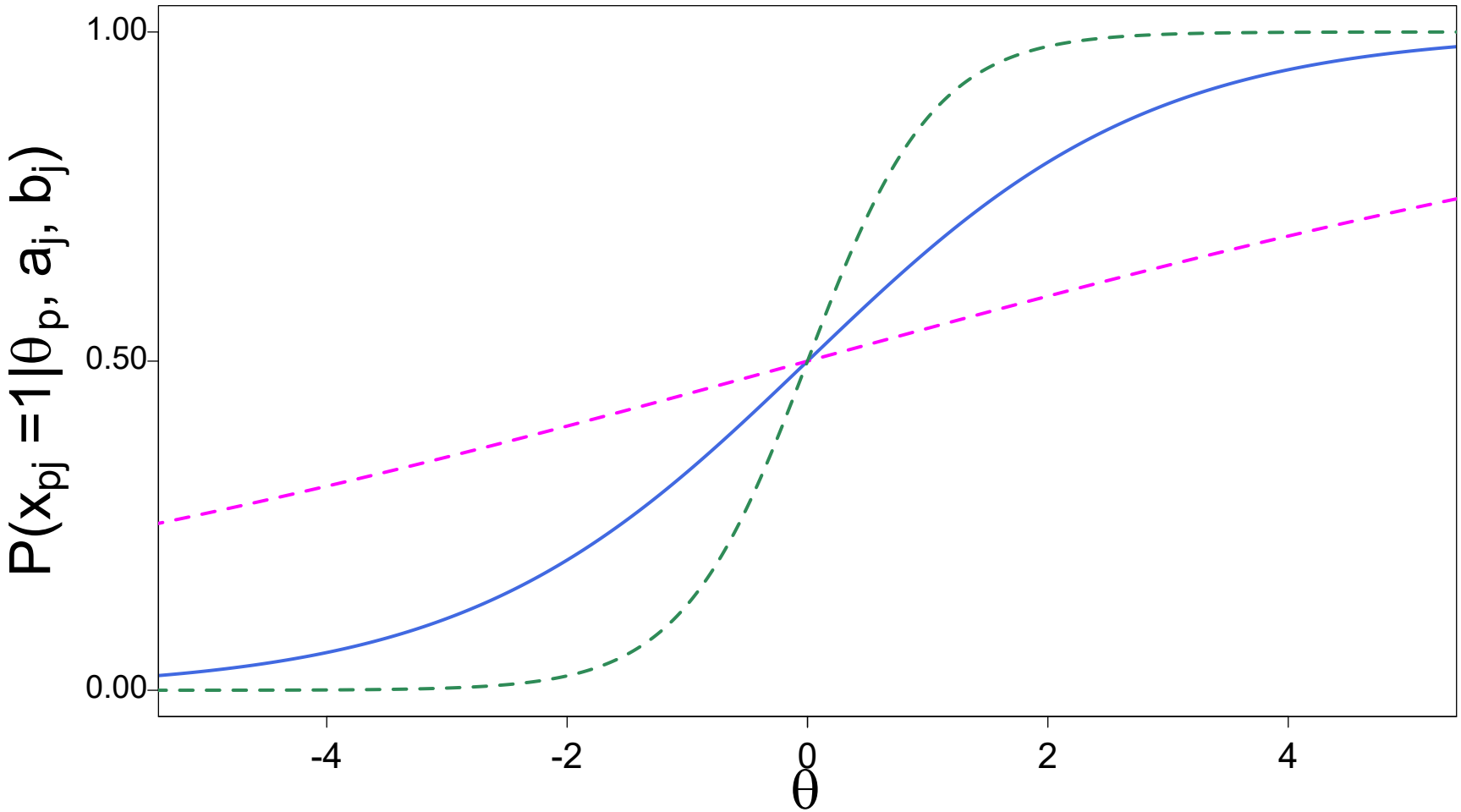
where:

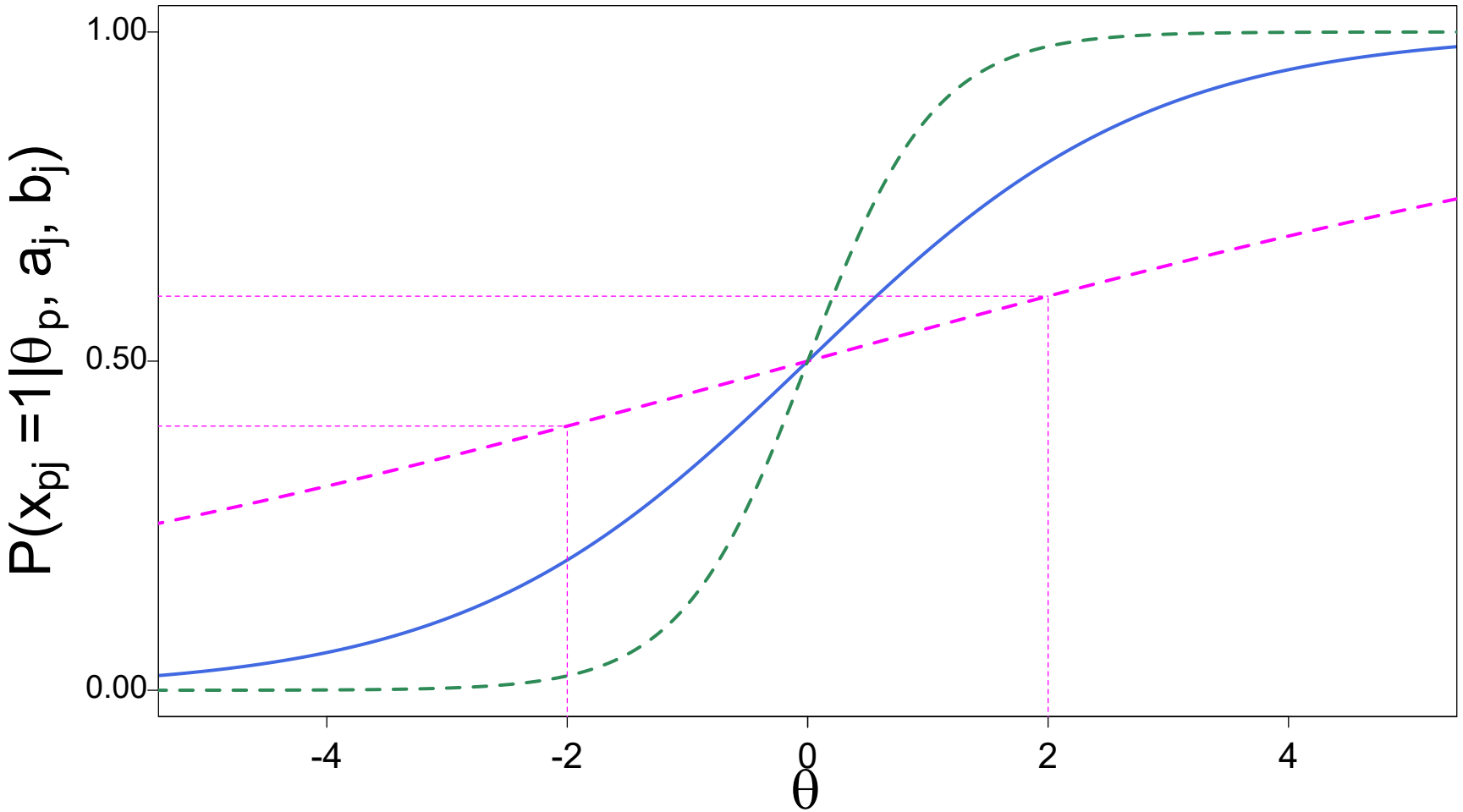
$P(x_{pj} = 1)$: Probability of a correct response to item j by respondent p

θ_p: Ability of respondent p

b_j: Difficulty of item j

a_j: Discrimination of item j


1.00
0.50
0.00
$P(x_{pj} = 1|\theta_p, a_j, b_j)$
-4
-2
0
2
4
θ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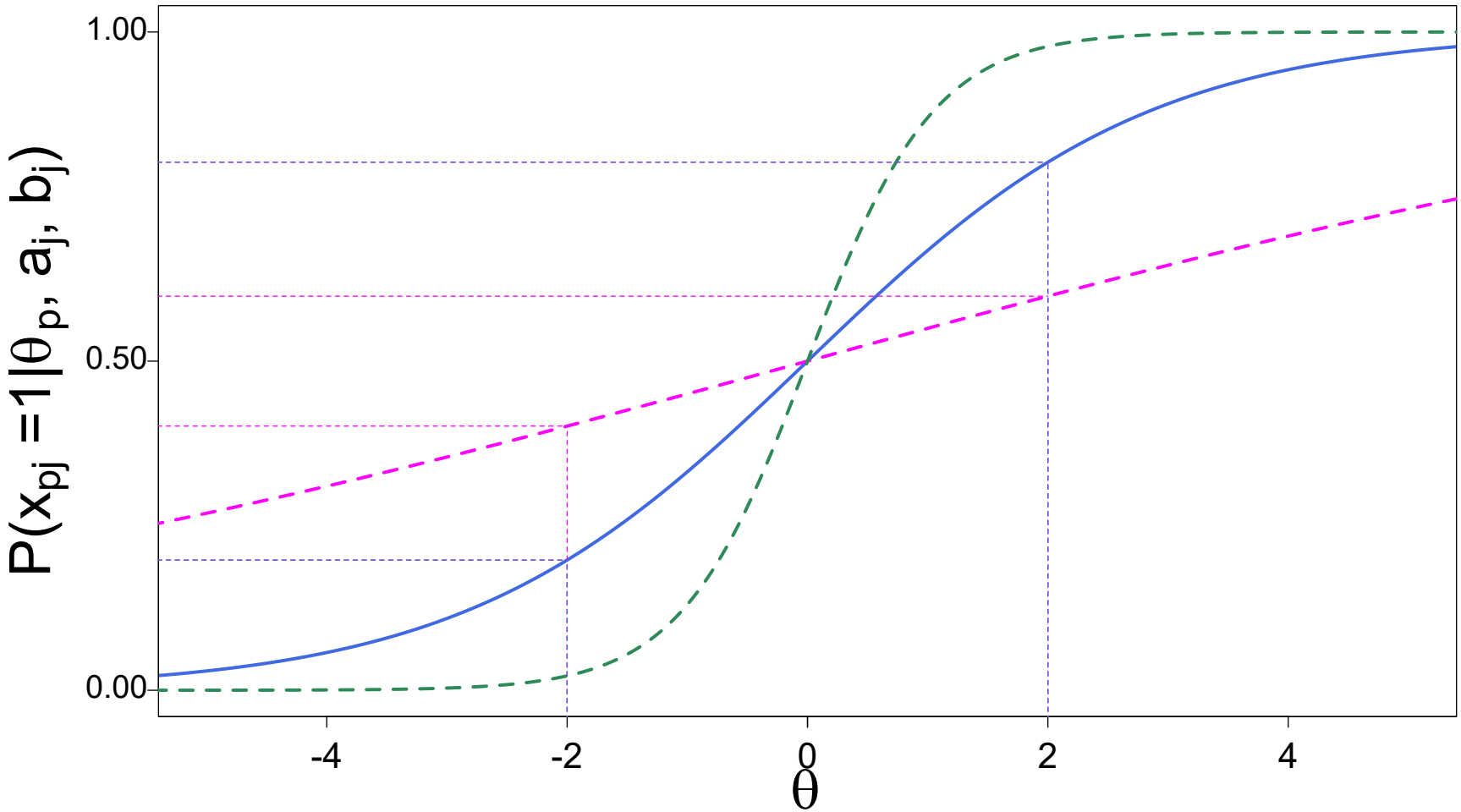
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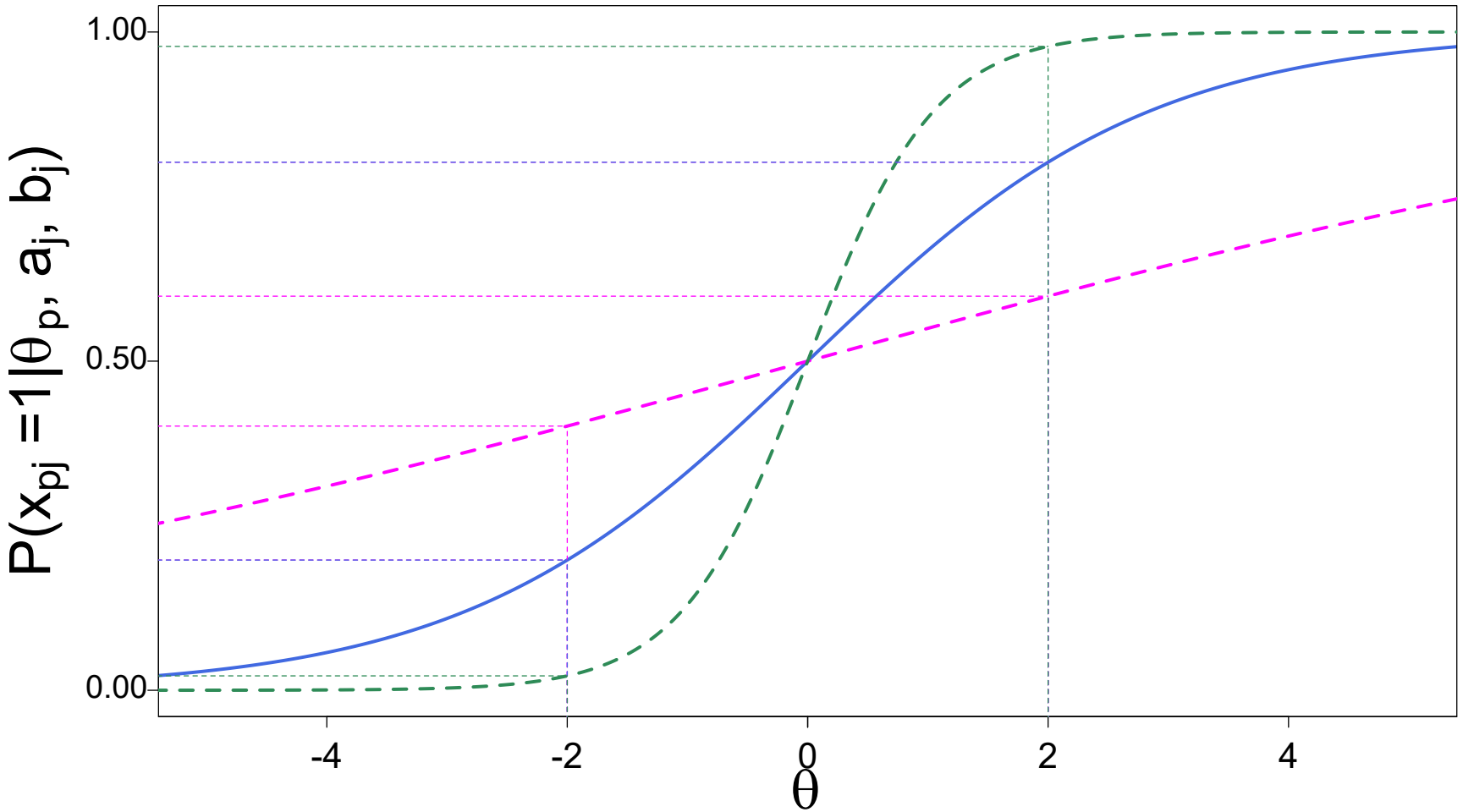
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Information functions

Item Information Function

$$IIF_j = a_j^2[P(\theta)(1 - P(\theta))]$$

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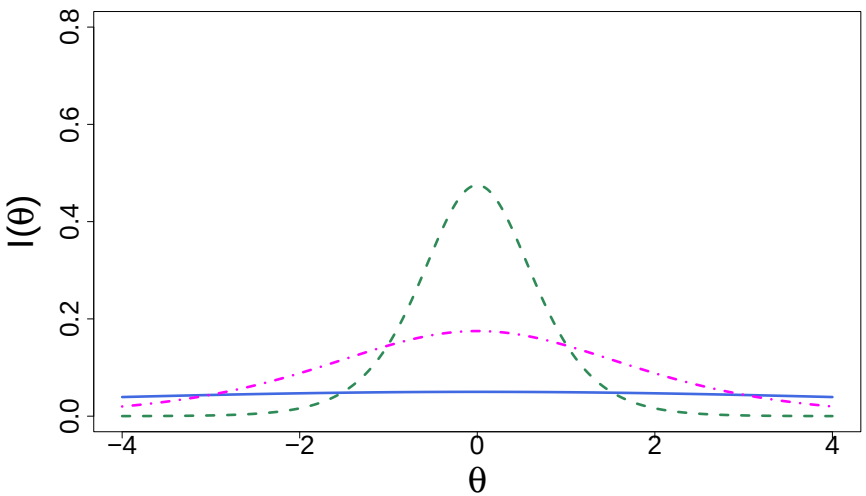

Figure 1: $a = 0.20$, $a = 0.70$, $a = 1.90$, $b = 0$

Information functions

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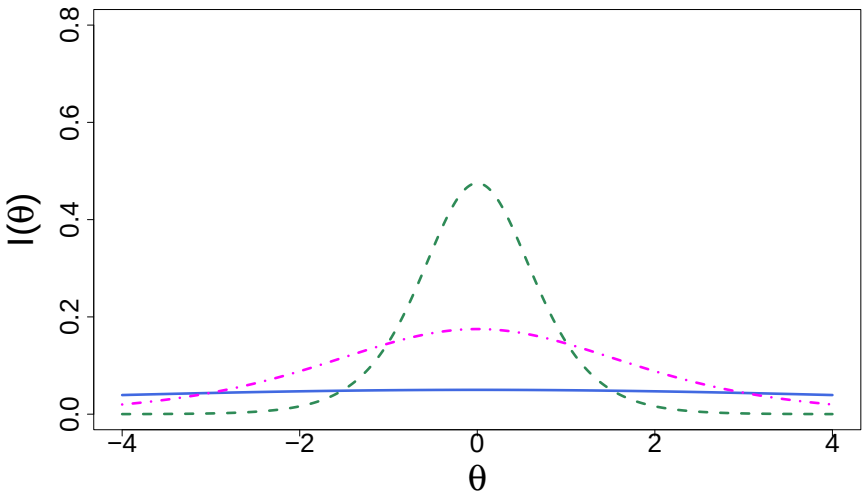
Test Information Function

$$TIF = \sum_{j=1}^{J} IIF_j$$



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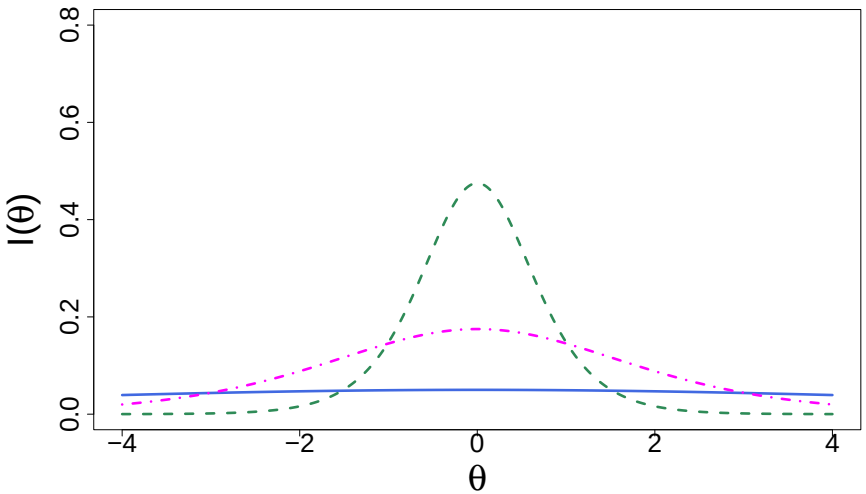

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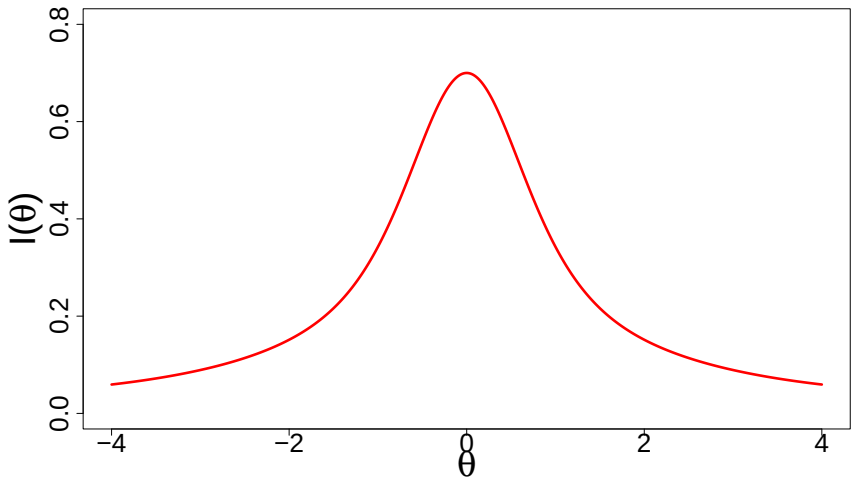

Figure 2: TIF $= IIF_1 + IIF_2 + IIF_3$

Benchmark procedure

Selected items $\rightarrow$ items with the highest *IIF*s

e.g.: 3-item short form from 10-item full-length test

item	b	a	*IIF*
1	-0.67	0.71	0.08
2	0.50	1.19	0.15
3	-2.43	0.25	0.01
4	2.12	1.98	0.24
5	1.72	0.39	0.03
6	-2.28	1.62	0.19
7	0.64	0.50	0.05
8	-2.51	1.68	0.19
9	-0.66	0.44	0.04
10	0.72	0.33	0.02

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3	−2.43	0.25	0.01

θ-target procedures

Selected items $\rightarrow$ items with highest *IIF*s in respect to θ targets (θ')

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3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
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An overall look

Figure 3: Overall Information of the short test forms

A closer look

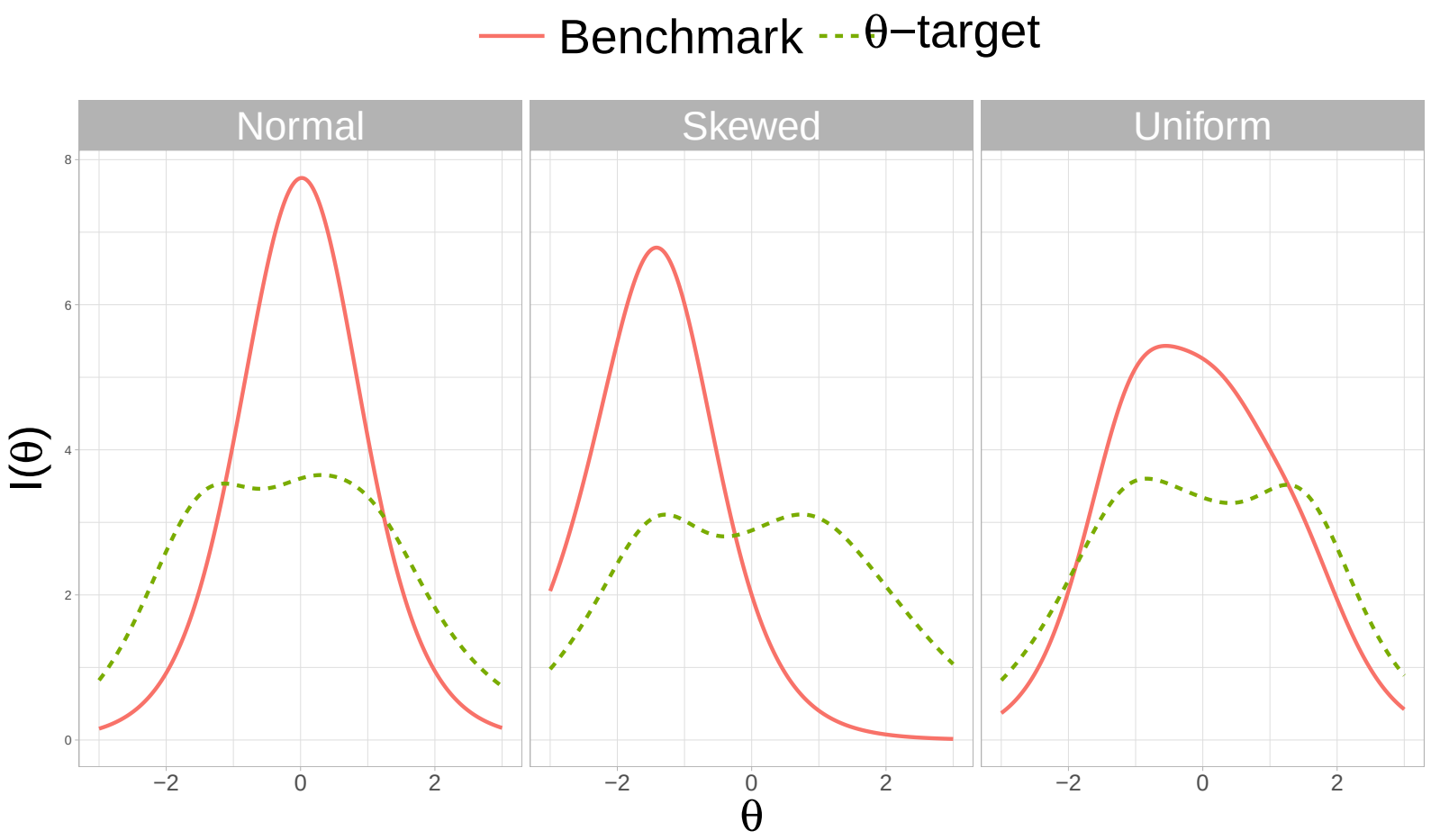

Figure 4: TIF of the 10-item short test form

An even closer look

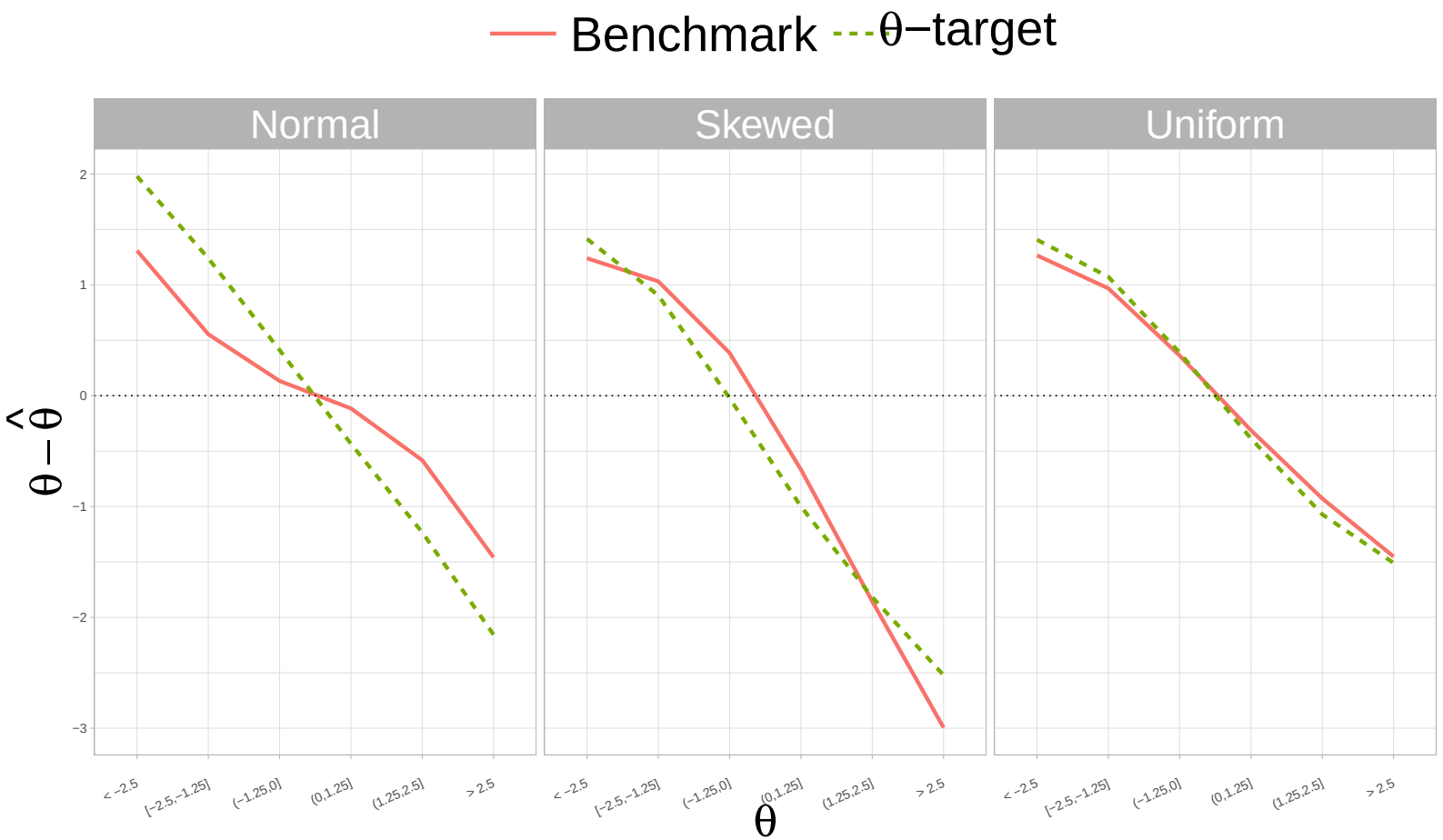

Figure 5: $bias = \theta - \hat{\theta}$ of the 10-item short test form

Good!

There's no "one-fits-all" solution

The θ distribution is a key element

Good!

There's no "one-fits-all" solution

The θ distribution is a key element

..but work is still needed

Real life applications are missing

The CAT is missing

Thank you!

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