Chapter 1

Multiple measures: Models specification

In this chapter, a comprehensive modeling approach for multiple implicit measures is illustrated. This modeling framework is obtained by exploiting the flexibility of Linear Mixed effect Models for obtaining Rasch model and log-normal model parameters estimates.

Two levels of model complexity are presented. At a first level, each implicit measure is modeled separately by employing the models presented in Chapter **??**. These models will not be further illustrated here, and only a brief summary of the parameters that can be estimated is provided.

At a second level, the between–measures variability is accounted for either by considering just the within–respondents between–measures variability (Model 2) or the within–respondents between–conditions variability across implicit measures (Model 3).

1.1 Single measures models

The models presented in Chapter **??** for both accuracy and log-time responses can be used for modeling each implicit measure independently from one another. This implies that, even though the variability within each measure is accounted for, the between–measures variability at both the respondents and the stimuli vary can still affect the parameter estimates. Nonetheless, this approach is a valid approach when implicit measures are administered independently from each another. Their data have been analyzed separately mainly for two reasons: (i) to investigate whether the modeling approach for IAT data in Chapter **??** can be extended to SC-IAT data, and (ii) to investigate whether and how these estimates are different from the ones obtained with a more sound approach that can account for the between–measures sources of variability, both within respondents and stimuli.

Irrespective of the implicit measure under investigation or dependent variable (i.e., accuracy or log-time responses), the fixed intercept was set at 0. The effect of the associative condition of each implicit measure was the fixed effect of the models. Since the fixed intercept was set at 0, the fixed effects can be interpreted as either the expected *log-odds* of the probability of a correct response in each associative condition (Accuracy models) or the expected average of the log-time responses in each associative condition (Log-time models).

Model 1 (the Null model) accounts for the between–respondents and between–stimuli variability across associative conditions by specifying them as random intercepts. For each separate implicit measure ($m = 1, \ldots, M$ Implicit measure), overall estimates at respondents' (θ_{pm} and τ_{pm}) and stimuli (b_{sm} and δ_{sm}) levels can be obtained for the Rasch and log-normal models.

The random structure of Model 2 accounts for the between–stimuli and across conditions variability and the within–respondents between–conditions variability by specifying stimuli as random intercepts and respondents' random slopes in the associative condition. For each, overall stimuli estimates (b_{sm} and δ_{sm}) and condition–specific respondents' estimates (θ_{pcm} and τ_{pcm}) are obtained.

Finally, Model 3 results in the estimation of condition–specific stimuli parameters (b_{klm} and δ_{klm}) and overall respondents' parameters (θ_{jlm} and τ_{jlm}) for each implicit measure by considering stimuli random slopes in the associative conditions and respondents' random intercepts across conditions. This model accounts for the within–stimuli between–conditions variability and the between–respondents variability across conditions.

As stated above, by separately analyzing the data obtained from implicit measures that were administered together, the within–respondents between–measures variability, as well as the within–stimuli between–measures one, are neglected. Therefore, the estimates of the models parameters can still be affected by a part of error variance. Moreover, since the estimates for both the respondents and the stimuli are obtained from separate, independent models, they cannot be directly compared between each other.

The models presented in Section 1.2 overcome this issue by considering implicit measures data altogether. The models presented in this Section are identified by superscript "C" (i.e., "Comprehensive").

1.2 Comprehensive models

Data from IAT and SC-IATs are considered and modeled together. In all models, the fixed intercept is set at 0, while the fixed effect varies. Specifically, in the Null and first models, the fixed effect β is the type of implicit measure, while in the third model the fixed effect β is the the effect of the associative condition of each implicit measure.

The only differences concerning the models applied on accuracy or log-time responses concern the dependent variable and the assumption on the distribution of the error terms. GLMMs (Section 1.2.1) are applied on accuracy responses, and the error term ϵ_i is assumed to follow a logistic distribution. These models are identified with a capital "A". LMMs (Section 1.2.2) are applied on log-time responses and the error term ϵ_i is assumed to follow a normal distribution. These models are identified with a capital "T".

In all models, only the between–stimuli across–measures variability was considered. The investigation of the stimuli functioning according to the specific implicit measure would indeed provide interesting information. For instance, the SC-IAT is known to be an easier task than the IAT. By having an information at the stimuli level it would be possible to understand whether the task is made easier by just some of the stimuli. Nonetheless, for specifying the random slopes of the stimuli in each implicit measure, a high within–stimuli between–measures variability is needed, but previous studies (e.g., ?, ?, ?) already highlighted a low within–stimuli between–conditions variability, especially for what concerns their time responses. Moreover, the focus was more oriented on understanding the intra and inter individuals differences in performing different implicit measures. Consequently, multidimensionality of the error variance was allowed only at level of the respondents.

1.2.1 Comprehensive GLMMs

Model A1$^{\mathrm{C}}$ is considered as the Null model:

$$y_i = logit^{-1}(\alpha + \beta_m X_m + \alpha_{p[i]} + \alpha_{s[i]} + \varepsilon_i), \tag{1.1}$$

with

$$\alpha_p \sim \mathcal{N}(0, \sigma^2_{\alpha_p}), \tag{1.2}$$

$$\alpha_s \sim \mathcal{N}(0, \sigma^2_{\alpha_s}), \tag{1.3}$$

where both respondents' and stimuli are specified as random intercepts across both associative conditions and type of implicit measure.

Model A1$^{\mathrm{C}}$ results in overall respondents ability estimates (θ_p^{C}) and overall stimuli easiness estimates (b_s^{C}) of the Rasch model. These estimates inform about the general ability of the respondents to perform the categorization task, as well as the general easiness of the stimuli, across implicit measures. This model should be preferred when both a low within–respondents and between–measures variability and a low within–stimuli between–measures variability are observed. The lack of variability at both levels might already indicate that respondents' ability is not affected by the specific implicit measure or, in other words, that their ability is constant across measures. Similarly, stimuli easiness does not vary across implicit measures.

In Model A2$^{\mathrm{C}}$, between–stimuli variability across implicit measures and within–respondents between–measures variability are accounted for by specifying stimuli random intercepts and respondents' random slopes in the implicit measures:

$$y_i = logit^{-1}(\alpha + \beta_m X_m + \alpha_{k[i]} + \beta_{p[i]} m_i + \varepsilon_i), \tag{1.4}$$

with:

$$\beta_{pm} \sim \mathcal{MVN}(\mathbf{0}, \boldsymbol{\Sigma_{pm}}) \tag{1.5}$$

$$\alpha_s \sim \mathcal{N}(0, \alpha_s^2), \tag{1.6}$$

where $\boldsymbol{\Sigma}_{pm}$ is the variance-covariance matrix of the population of the respondents and it expresses the by-respondents variability according to the implicit measure.

Model A2$^{\text{C}}$ results in overall stimuli easiness estimates across implicit measures (b_s^{C}) and measure–specific respondents' ability estimates (θ_{pm}^{C}). A high within–respondents between–measures variability is needed for this model to be the best fitting one. This variability indicates that respondents' ability performance is affected by the specific implicit measure. The estimates provided by this model can hence inform about how respondents' ability performance has been affected by the type of implicit measures. However, no information on the effect of the associative condition is available.

In both Model A1$^{\text{C}}$ and Model 2$^{\text{C}}$, the fixed effect is the type of measure. Therefore, it provides the expected *log-odds* of the probability of a correct response in each implicit measure. Since these estimates are obtained from the same model, they can be directly compared between each other.

The random structure of Model A3$^{\text{C}}$ accounts for the between–stimuli variability across implicit measures and the within–respondents between–conditions variability. Stimuli random intercepts and respondents' random slopes in each associative condition of each implicit measure are specified:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{s[i]} + \beta_{p[i]} c_i + \varepsilon_i), \tag{1.7}$$

with:

$$\beta_{pc} \sim \mathcal{MVN}(\mathbf{0}, \boldsymbol{\Sigma_{pc}}), \tag{1.8}$$

$$\alpha_s \sim \mathcal{N}(0, \alpha_s^2), \tag{1.9}$$

where $\boldsymbol{\Sigma}_{sc}$ is the variance-covariance matrix of the population of the respondents, expressing the by-respondent adjustment in each associative condition on each implicit measure. Model A3$^{\text{C}}$ results in overall stimuli easiness estimates (b_sC) and condition–specific respondents' ability estimates, for each implicit measure (θ^{C}_{pmc}). Model A3$^{\text{C}}$ requires a high within–respondents between–conditions variability to result as the best fitting model. The high variability between their responses in each condition of each measure already stands for an effect of the associative conditions, and of the implicit measure to which they belong, on respondents' performance. By taking the difference between the condition–specific estimates of each implicit measure, a measure of the bias on respondents' performance due to the associative conditions can be obtained.

1.2.2 Comprehensive LMMs

Models with the same random structures as those presented in Section 1.2.2 are specified for obtaining log-normal estimates from the log-time responses.

Model T1$^{\text{C}}$ accounts for the between–respondents and the between–stimuli variability across implicit measures. As such, it is considered as the Null model:

$$y_i = \alpha + \beta_m X_m + \alpha_{p[i]} + \alpha_{p[i]} + \varepsilon_i, \tag{1.10}$$

with

$$\alpha_p \sim \mathcal{N}(0, \sigma^2_{\alpha_p}), \tag{1.11}$$

$$\alpha_s \sim \mathcal{N}(0, \sigma^2_{\alpha_s}), \tag{1.12}$$

The random structure specification of Model T1$^{\text{C}}$ results in the estimation of overall respondents' speed parameters (τ_p^{C}) and overall stimuli time intensity parameters (δ_s^{C}). Consequently, only general information about respondents' performance and stimuli functioning across implicit measures are available.

Model T2$^{\text{C}}$ accounts for the between–stimuli variability across implicit measures and the within–respondents between–measures variability by specifying stimuli random intercepts and respondents' random slopes in the implicit measures:

$$y_i = \alpha + \beta_m X_m + \alpha_{s[i]} + \beta_{p[i]} m_i + \varepsilon_i, \tag{1.13}$$

with:

$$\beta_{pm} \sim \mathcal{MVN}(\mathbf{0}, \boldsymbol{\Sigma_{pm}}) \tag{1.14}$$

$$\alpha_s \sim \mathcal{N}(0, \alpha^2_s), \tag{1.15}$$

where $\boldsymbol{\Sigma}_{pm}$ represents the variance-covariance of the population of the respondents, expressing their variability due the effect of the implicit measure. Model T2$^{\text{C}}$ results in overall stimuli time intensity estimates across implicit measures (δ_s^{C}) and measure–specific respondents' speed estimates (τ_{pm}^{C}). A high within–respondents between–measures variability is needed for this model to be the best fitting one. This variability indicates that respondents' speed performance is affected by the specific implicit measure. However, it is not possible to rule out the possibility that this variability is due to the effect of the associative conditions.

The fixed effect in both Model T1$^{\text{C}}$ and Model T2$^{\text{C}}$ is the type of implicit measure. Therefore, the expected average log-time in each implicit measure are obtained.

The variability due to the effect of the associative conditions of each implicit measure can be understood with the random structure specification of Model T3$^{\text{C}}$. By specifying the respondents' random slopes in each associative condition of each implicit measure, this model accounts for the within–respondents between–conditions and between–measures variability, as well as the between–stimuli across–measures variability:

$$y_i = \alpha + \beta_c X_c + \alpha_{s[i]} + \beta_{p[i]} c_i + \varepsilon_i, \tag{1.16}$$

with:

$$\beta_{pc} \sim \mathcal{MVN}(\mathbf{0}, \mathbf{\Sigma}_{pc}), \tag{1.17}$$

$$\alpha_s \sim \mathcal{N}(0, \alpha_s^2), \tag{1.18}$$

where $\mathbf{\Sigma}_{pc}$ represents the covariance matrix of the population of the respondents, expressing the variability due to their adjustments to each of the associative conditions in each of the implicit measures. Model T3$^{\text{C}}$ results in overall stimuli time intensity estimates (δ_sC) and condition–specific respondents' speed estimates, for each implicit measure (τ^{C}_{pmc}). Model T3$^{\text{C}}$ should be preferred when a high within–respondents between–conditions variability is observed. The high variability between respondents' responses in each condition of each measure stands for an effect of the associative conditions, and of the implicit measure to which they belong, on their performance. By taking the difference between the condition–specific estimates of each implicit measure, a measure of the bias on respondents' performance due to the associative conditions can be obtained.

A measure of the bias due to the associative conditions of each implicit measure can be obtained from the estimates provided by the Single measures models in Section 1.1 as well. However, as already stated, those estimates are affected by both the within–respondents between–measures variability and the within–stimuli between–measures variability. Conversely, these sources of variability are accounted for in the Comprehensive modeling framework, potentially resulting in more reliable estimates. Moreover, the estimates obtained with the Comprehensive modeling approach are directly comparable between each other because they are derived from the same model.