When randomness opens new possibilities: Acknowledging the stimulus sampling variability in Experimental Psychology

Ottavia M. Epifania[1,2,3], Pasquale Anselmi [1], Egidio Robusto [1]

[1] University of Padova, Padova (IT)
[2] Psicostat Group, University of Padova, Padova (IT)
[3] Università Cattolica del Sacro Cuore, Milano (IT)

Computational and Methodological Statistics
Berlin, Germany

December, 16th 2023

Introduction

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○●○○○   ○○○○   ○○○○○○   ○○○

Stimuli are fixed, respondents are random

Respondents are random

Sampled from a larger population

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○●○○○   ○○○○   ○○○○○○   ○○○

Stimuli are fixed, respondents are random

Respondents are random

Sampled from a larger population

Need for acknowledging the sampling variability

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○●○○○   ○○○○   ○○○○○○   ○○○

Stimuli are fixed, respondents are random

Respondents are random

Sampled from a larger population

Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

Introduction Random effects for random factors Random stimuli in Experimental Psychology Discussion
○●○○○ ○○○○ ○○○○○○ ○○○
Stimuli are fixed, respondents are random

Respondents are random

Sampled from a larger population

Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

Stimuli/items are fixed

Taken to be entire population

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○●○○○   ○○○○   ○○○○○○   ○○○

Stimuli are fixed, respondents are random

Respondents are random

Sampled from a larger population

Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

Stimuli/items are fixed

Taken to be entire population

There is no sampling variability

Respondents are random

Sampled from a larger population

Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

Stimuli/items are fixed

Taken to be entire population

There is no sampling variability

There is no need to generalize the results because the stimuli are the population

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○●○○   ○○○○   ○○○○○○   ○○○
Stimuli are fixed, respondents are random

However...

The stimuli can also represent a sample of a larger universe

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○●○○   ○○○○   ○○○○○○   ○○○
Stimuli are fixed, respondents are random

However...

The stimuli can also represent a sample of a larger universe

Processing speed of positive and negative attributes

Introduction Random effects for random factors Random stimuli in Experimental Psychology Discussion
○○●○○   ○○○○   ○○○○○○   ○○○
Stimuli are fixed, respondents are random

However...

The stimuli can also represent a sample of a larger universe

Processing speed of positive and negative attributes

There is a universe of **positive attributes** as well as an universe of **negative attributes**

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○●○○   ○○○○   ○○○○○○   ○○○

Stimuli are fixed, respondents are random

However...

The stimuli can also represent a sample of a larger universe

Processing speed of positive and negative attributes

There is a universe of **positive attributes** as well as an universe of **negative attributes**

Only samples of **positive attributes** (e.g., good, nice, ...) and **negative attributes** (e.g., bad, evil, ...) are administered

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○●○○   ○○○○   ○○○○○○   ○○○

Stimuli are fixed, respondents are random

However...

The stimuli can also represent a sample of a larger universe

Processing speed of positive and negative attributes

There is a universe of **positive attributes** as well as an universe of **negative attributes**

Only samples of **positive attributes** (e.g., good, nice, . . .) and **negative attributes** (e.g., bad, evil, . . .) are administered

So... **there must be a sampling variability!**

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○○●○   ○○○○   ○○○○○○   ○○○

What if the sampling variability is not acknowledged

Generalizability

Generalizability is bounded to the specific set of stimuli used in the experiment

Results can be generalized if and only if the exact same set of stimuli is used

Introduction Random effects for random factors Random stimuli in Experimental Psychology Discussion
○○○●○ ○○○○ ○○○○○○ ○○○

What if the sampling variability is not acknowledged

Generalizability

Generalizability is bounded to the specific set of stimuli used in the experiment

Results can be generalized if and only if the exact same set of stimuli is used

Robustness of the results

Random variability at the stimulus level might inflate the probability of committing Type I errors

Averaging across stimuli to obtain person-level scores results in biased estimates due to the noise in the data

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○○●○   ○○○○   ○○○○○○   ○○○
What if the sampling variability is not acknowledged

Generalizability

Generalizability is bounded to the specific set of stimuli used in the experiment

Results can be generalized if and only if the exact same set of stimuli is used

Robustness of the results

Random variability at the stimulus level might inflate the probability of committing Type I errors

Averaging across stimuli to obtain person-level scores results in biased estimates due to the noise in the data

Loss of information

All the variability is not considered as well as all the information that can be obtained from it

Every stimulus is assumed to be equally informative

Introduction Random effects for random factors Random stimuli in Experimental Psychology Discussion
○○○○●● ○○○○ ○○○○○○ ○○○

What if the sampling variability is not acknowledged

This contribution

Focus on the loss of information...the other side of the coin

Introduction Random effects for random factors Random stimuli in Experimental Psychology Discussion
○○○○●○ ○○○○ ○○○○○○ ○○○
What if the sampling variability is not acknowledged

This contribution

Focus on the loss of information...the other side of the coin

The information at the stimulus level that can be retrieved from the accuracy responses (correct vs. incorrect) from a typical experiment where the response times are usually employed for scoring the data

Introduction   Random effects for random factors   Random stimuli in Experimental Psychology   Discussion
○○○○●○   ○○○○   ○○○○○○   ○○○
What if the sampling variability is not acknowledged

This contribution

Focus on the loss of information...the other side of the coin

The information at the stimulus level that can be retrieved from the accuracy responses (correct vs. incorrect) from a typical experiment where the response times are usually employed for scoring the data

It can actually help in disentangling what is known to be a shortcoming of the score usually employed for analyzing the data of this experiment

Introduction
○○○○○

Random effects for random factors
●○○○

Random stimuli in Experimental Psychology
○○○○○○

Discussion
○○○

Random effects for random factors

Introduction
00000

Random effects for random factors
0●00

Random stimuli in Experimental Psychology
000000

Discussion
000

Random effects and random factors

Linear combination of predictors in a Linear Model:

$$\eta = X\beta,$$

where β indicates the coefficients of the fixed intercept and slope(s), and X is the model-matrix.

Random effects and random factors

Linear combination of predictors in a Linear Model:

$$\eta = X\beta,$$

where β indicates the coefficients of the fixed intercept and slope(s), and X is the model-matrix.
Linear combination of predictors in a Linear Mixed-Effects Model (LMM):

$$\eta = X\beta + Zd,$$

where Z is the matrix and d is the vector of the random effects (not parameters!)

Best Linear Unbiased Predictors
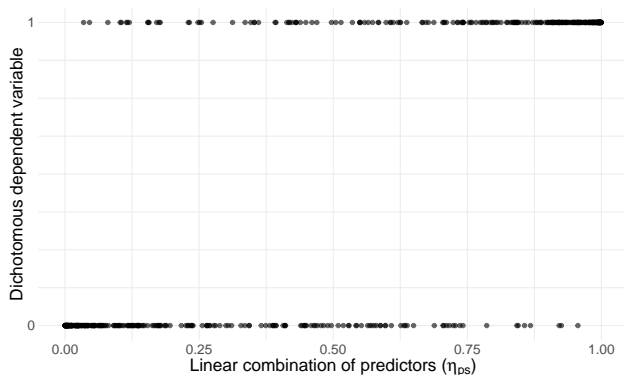
Introduction
00000

Random effects for random factors
0000

Random stimuli in Experimental Psychology
000000

Discussion
000

Generalized linear model (GLM) for dichotomous responses

Introduction
00000

Random effects for random factors
00●0

Random stimuli in Experimental Psychology
000000

Discussion
000

Generalized linear model (GLM) for dichotomous responses

Logit link function g:

$$g(\eta_{ps}) = log\left(\frac{\mu_{ps}}{1-\mu_{ps}}\right)$$

Inverse g^{-1}

$$g^{-1} = \frac{\exp(\eta_{ps})}{1+\exp(\eta_{ps})}$$
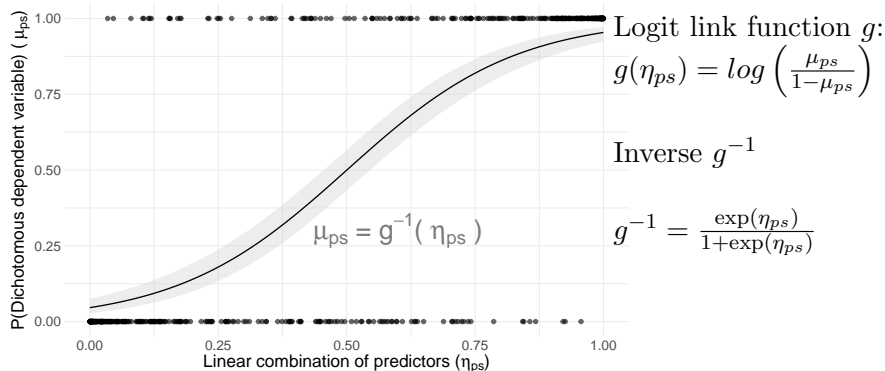
The Rasch model

$$P(x_{ps} = 1 | \theta_p, b_s) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

where:

θ_p: ability of respondent p (i.e., latent trait level of respondent p)

b_s: difficulty of stimulus s (i.e., "challenging" power of stimulus s)

The Rasch model

$$P(x_{ps} = 1|\theta_p, b_s) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

where:

θ_p: ability of respondent p (i.e., latent trait level of respondent p)

b_s: difficulty of stimulus s (i.e., "challenging" power of stimulus s)

Standard

$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

GLM

$$P(x_{ps} = 1) = \frac{\exp(\theta_p + b_s)}{1 + \exp(\theta_p + b_s)}$$

The Rasch model

$$P(x_{ps} = 1|\theta_p, b_s) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

where:

θ_p: ability of respondent p (i.e., latent trait level of respondent p)

b_s: difficulty of stimulus s (i.e., "challenging" power of stimulus s)

Standard	GLM

$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)} \qquad P(x_{ps} = 1) = \frac{\exp(\theta_p + b_s)}{1 + \exp(\theta_p + b_s)}$$

Random stimuli in Experimental Psychology

Introduction Random effects for random factors **Random stimuli in Experimental Psychology** Discussion
○○○○○   ○○○○   ○●○○○○   ○○○
Experiment with the Implicit Association Test

12 Object stimuli

White people faces

Black people faces
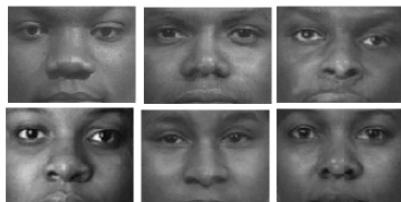
16 Attribute stimuli

Positive attributes

Good, laughter, pleasure, glory, peace, happy, joy, love

Negative attributes

Evil, bad, horrible, terrible, nasty, pain, failure, hate

Introduction Random effects for random factors **Random stimuli in Experimental Psychology** Discussion
○○○○○   ○○○○   ○○●○○○   ○○○
Experiment with the Implicit Association Test

Two experimental conditions

White-Good/Black-Bad
(WGBB): 60 trials
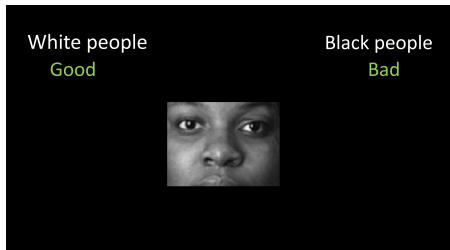
Black-Good/White-Bad
(BGWB): 60 trials

$$D = \frac{M_{\mathrm{BGWB}} - M_{\mathrm{WGBB}}}{s_{\mathrm{BGWB,\ WGBB}}}$$

Participants: 62 (F = 48.39%, Age = 24.92 ± 2.11 years)

Introduction   Random effects for random factors   **Random stimuli in Experimental Psychology**   Discussion
00000   0000   000●00   000

Models

The expected response y for the observation $i = 1, \ldots, I$ for respondent $p = 1, \ldots, P$ on stimulus $s = 1, \ldots, S$ in condition $c = 1, \ldots, C$:

Model 1:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \alpha_{s[i]})$$

$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2).$$

Model 2:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i)$$

$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\beta_s \sim \mathcal{MVN}(0, \Sigma_{sc}).$$

Model 3:

$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{s[i]} + \beta_{p[i]} c_i)$$

$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2),$$
$$\beta_p \sim \mathcal{MVN}(0, \Sigma_{pc}).$$

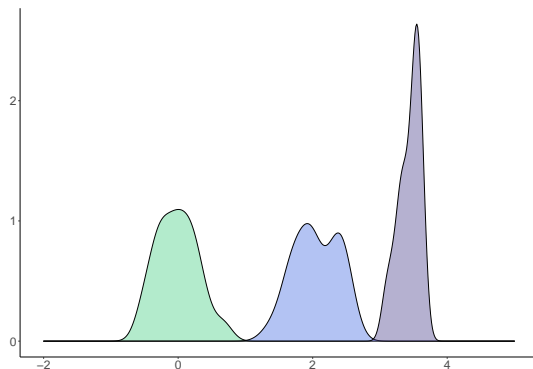The expected response y for the observation $i = 1, \ldots, I$ for respondent $p = 1, \ldots, P$ on stimulus $s = 1, \ldots, S$ in condition $c = 1, \ldots, C$:

Model 1:
$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \alpha_{s[i]})$$
$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2).$$

Model 2:
$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i)$$
$$\alpha_p \sim \mathcal{N}(0, \sigma_p^2),$$
$$\beta_s \sim \mathcal{MVN}(0, \Sigma_{sc}).$$

Model 3:
$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{s[i]} + \beta_{p[i]} c_i)$$
$$\alpha_s \sim \mathcal{N}(0, \sigma_s^2),$$
$$\beta_p \sim \mathcal{MVN}(0, \Sigma_{pc}).$$

Fixed Effects   Random structure

Introduction   Random effects for random factors   **Random stimuli in Experimental Psychology**   Discussion
ooooo   oooo   ooooo●o   ooo
Results

Model 2 is the least wrong model

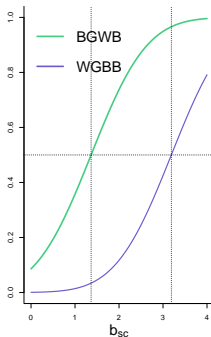$$y_i = logit^{-1}(\alpha + \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i)$$

Condition–specific easiness

Introduction
○○○○○

Random effects for random factors
○○○○

Random stimuli in Experimental Psychology
○○○○○○
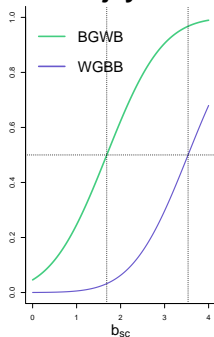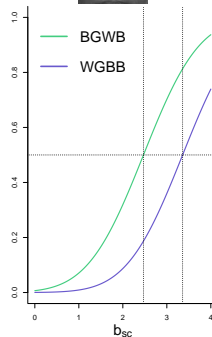
Discussion
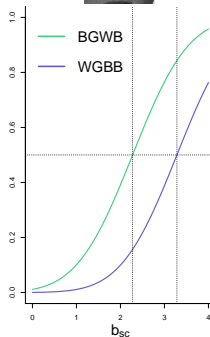●○○

Discussion

- Acknowledge and gather the information at the stimulus level

- Improve generalizability of the results to other sets of stimuli

- Control for random variance in the data

- Allow for obtaining a Rasch-like parametrization of the data

Thank you!

ottavia.epifania@unipd.it

https://psicostat.dpss.psy.unipd.it/