A Linear Mixed-Effects Models approach to obtain Rasch-like estimates of accuracies and response times from fully-crossed design data Rasch 2.0

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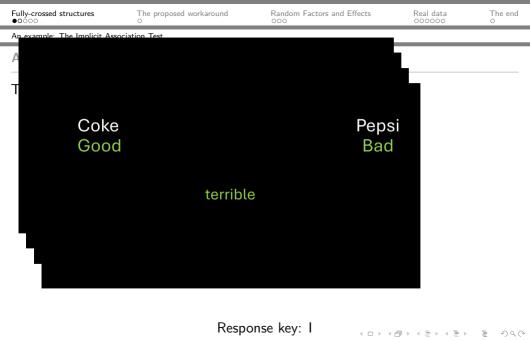
September, 12, 2025

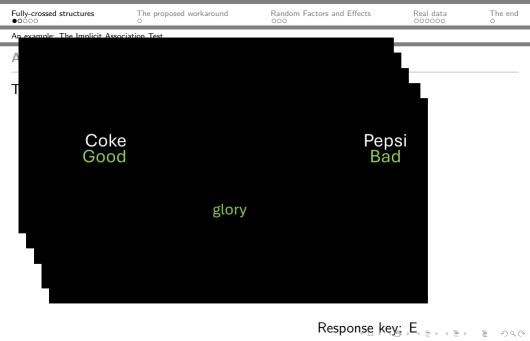
The end

Real data

The end

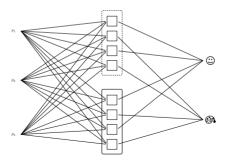






- The "natural" one (so called compatible condition)
 - I love Coke and its easier to associate these stimuli to positive attributes
- The "innatural" one (so incompatible condition)
 - I love Coke and its harder to associate these stimuli to negative stimuli

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Scoring

Person-level scores

$$s_p = \frac{\bar{X}_{p, \text{comp}} - \bar{X}_{p, \text{inc}}}{sd_{\text{pooled}}}$$

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Advantages

Ease of computation Ease of interpretation Scoring

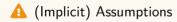
Person-level scores

$$s_p = \frac{\bar{X}_{p, \text{comp}} - \bar{X}_{p, \text{inc}}}{sd_{\text{pooled}}}$$



Advantages

Ease of computation Ease of interpretation



- 2 All stimuli have the same impact (fixed effects)

The issue

A long tradition

Respondents are random factors

Sampled from a larger population

Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

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A long tradition

i Respondents are random factors

Sampled from a larger population Need for acknowledging the sampling variability

Results can be generalized to other respondents belonging to the same population

i Stimuli/items are fixed factors

Taken to be entire population

There is no sampling variability

There is no need to generalize the results because the stimuli are the population

- Generalization of the results is impaired
- Error variance everywhere, left free to bias everything
- The information at the stimulus level is lost.

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Linear Mixed Effects Models

Rasch model

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Linear Mixed Effects Models

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Rasch model



Rasch-like parametrization estimated with Linear Mixed Effects Models

Statistics meets Psychomeerics

i Rasch

$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

GLM (inverse function)

$$P(x_{ps}=1) = \frac{\exp(\theta_p \,+\, b_s)}{1 + \exp(\theta_p \,+\, b_s)}$$

Statistics meets Psychomeerics

i Rasch

$$P(x_{ps}=1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

$$E(t_{ps}) = \delta_s - \tau_p$$

$$P(x_{ps}=1) = \frac{\exp(\theta_p \,+\, b_s)}{1 + \exp(\theta_p \,+\, b_s)}$$

$$E(t_{ps}) = \delta_s + \tau_p + \varepsilon$$

Random Factors and Effects

Real data

The end

The proposed workaround

Fully-crossed structures

Needs to be extended:

In a I M:

 $d\!:$ Random effects associated to the random factors in Z ... Not model parameters! Best Linear Unbiased Predictors

 $n = \mathbf{X}\beta + \mathbf{Z}d$

 Γ : Parameters estimated for the random factors in the model (variances and covariances)

Random structures

i Models

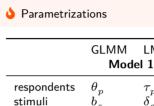
Model 1

 $y = \beta_c X_c + \alpha_{p[i]} + \alpha_{s[i]}$

$$y = \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i$$

Model 3

$$y = \beta_c X_c + \beta_{p[i]} c_i + \alpha_{s[i]}$$



respondents

stimuli b_{sc} δ_{sc}

 $\begin{array}{ccc} & & \textbf{Model 3} \\ \text{respondents} & \theta_{pc} & \tau_{pc} \\ \text{stimuli} & b_s & \delta_s \end{array}$

 $p=1,\ldots,P$: Respondent, $s=1,\ldots,S$: Stimulus, $c\in\{0,1\}$ Associative condition, i Trial

I MM

Model 2

Find the useful model via model comparison: AIC and BIC

The lower the value, the better the model

! AIC, BIC, and model complexity:

Total number of parameters: β and Γ NOT the levels in d

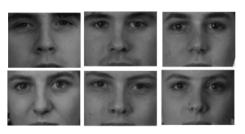
Model 2 and Model 3: Same complexity, different focus

The chosen model is the least wrong model given the considered models

12 Object stimuli

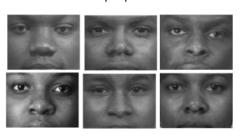
Fully-crossed structures

White people faces



Black people faces

The end



16 Attribute stimuli

Positive attributes

Good, laughter, pleasure, glory, peace, happy, joy, love

Negative attributes

Evil, bad, horrible, terrible, nasty, pain, failure, hate

Best Fitting Models

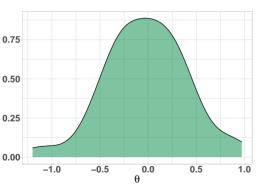
GLMMs Model~2 θ_p b_{WGBB} and b_{BGWB}

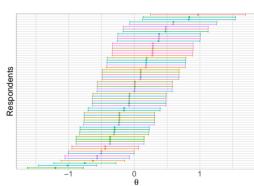
The IAT effect is mostly due to variations in the *stimuli functioning* between conditions, while the performance of the respondents seems unaltered

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Rasch-like estimates

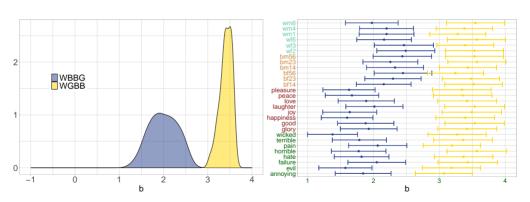
 θ_p





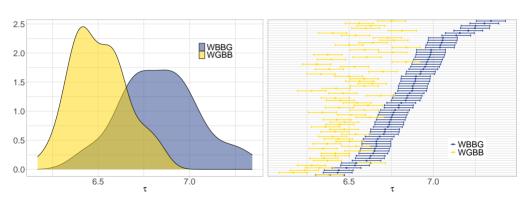
Rasch-like estimates

$b_{\rm WGBB}$ and $b_{\rm WGBB}$



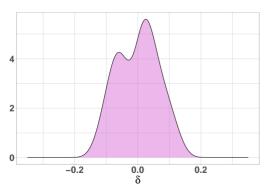
Log-normal estimates

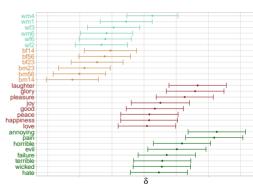
$\tau_{\rm WGBB}$ and $\tau_{\rm BGWB}$



Log-normal estimates

 δ_s





- The best model depends on the other models... sometimes useful, never right
- The sky is the limit... but do not over complicate things

HOWEVER

• Time and accuracy are independent from one another, pretty bold assumption

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Psychological Methods

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Fully-crossed structures

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A Guided Tutorial on Linear Mixed-Effects Models for the Analysis of Accuracies and Response Times in Experiments With Fully Crossed Design

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