A Linear Mixed-Effects Models approach to obtain Rasch-like estimates of accuracies and response times from fully-crossed design data

Rasch 2.0

Ottavia M. Epifania
University of Trento
ottavia.epifania@unitn.it

AIP 2025, Torino

September, 12, 2025

Fully-crossed structures

An example: The Implicit Association Test

An example: The Implicit Association Test

Coke
Good

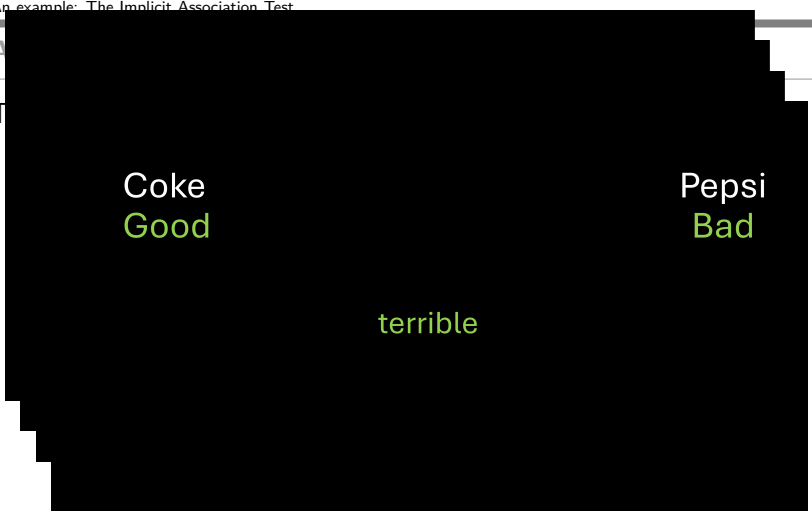
Pepsi
Bad

Response key: E

An example: The Implicit Association Test

Response key: I

An example: The Implicit Association Test



Response key: I

An example: The Implicit Association Test

Response key: E

Sample of stimuli

Two conditions:

1. The "natural" one (so called *compatible* condition)

   I love Coke and its easier to associate these stimuli to positive attributes

2. The "innatural" one (so *incompatible* condition)

   I love Coke and its harder to associate these stimuli to negative stimuli

Person-level scores

$$s_p = \frac{\bar{X}_{p,\text{comp}} - \bar{X}_{p,\text{inc}}}{sd_{\text{pooled}}}$$

Person-level scores

$$s_p = \frac{\bar{X}_{p,\text{comp}} - \bar{X}_{p,\text{inc}}}{sd_{\text{pooled}}}$$

Advantages

Ease of computation
Ease of interpretation

Person-level scores

$$s_p = \frac{\bar{X}_{p,\text{comp}} - \bar{X}_{p,\text{inc}}}{sd_{\text{pooled}}}$$

Advantages

Ease of computation
Ease of interpretation

(Implicit) Assumptions

1. Being slow (less accurate) in one condition = being fast (or more accurate) in the opposite one: 0 means absence of bias
2. All stimuli have the same impact (fixed effects)

A long tradition

Respondents are random factors

Sampled from a larger population
Need for acknowledging the sampling variability
Results can be generalized to other respondents belonging to the same population

A long tradition

Respondents are random factors

Sampled from a larger population
Need for acknowledging the sampling variability
Results can be generalized to other respondents belonging to the same population

Stimuli/items are fixed factors

Taken to be entire population
There is no sampling variability
There is no need to generalize the results because the stimuli are the population

With long lasting consequences

- Generalization of the results is impaired
- Error variance everywhere, left free to bias everything
- The information at the stimulus level is lost

With long lasting consequences

- Generalization of the results is impaired
- Error variance everywhere, left free to bias everything
- The information at the stimulus level is lost

Σ

Linear Mixed Effects Models

ψ

Rasch model

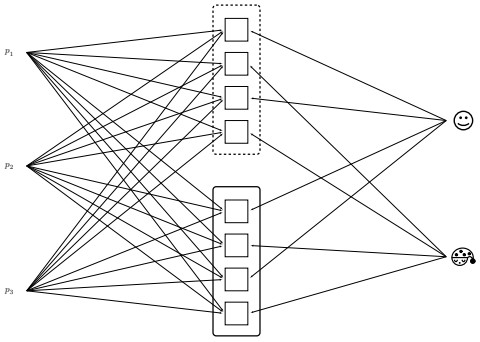
With long lasting consequences

- Generalization of the results is impaired
- Error variance everywhere, left free to bias everything
- The information at the stimulus level is lost

Σ Linear Mixed Effects Models

ψ Rasch model

Rasch-like parametrization estimated with Linear Mixed Effects Models



Sample-level differences:
Compatible and incompatible can be defined *a priori*
(SNARC effect)

Individual differences:
Compatible and incompatible are defined within each respondent
(Implicit Association Test)

The proposed workaround

Rasch

$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_z)}{1 + \exp(\theta_p - b_z)}$$

Log-normal

$$E(t_{ps}|\tau_p, \delta_s) = \delta_s - \tau_p + \varepsilon$$

GLM (inverse function)

$$P(x_{ps} = 1) = \frac{\exp(\theta_p + b_s)}{1 + \exp(\theta_p + b_s)}$$

LM (identity function)

$$E(t_{ps}|\tau_p, \delta_s) = \delta_s + \tau_p + \varepsilon$$

Random Factors and Effects

In a LM:

$$\eta = \mathbf{X}\beta$$

$\mathbf{X}$: Model Matrix

β: Coefficients

Needs to be extended:

$$\eta = \mathbf{X}\beta + Zd$$

d: Random effects associated to the random factors in Z ... Not model parameters! *Best Linear Unbiased Predictors*

Γ: Parameters estimated for the random factors in the model (variances and covariances)

Models

Model 1

$$y = \beta_c X_c + \alpha_p[i] + \alpha_s[i]$$

Model 2

$$y = \beta_c X_c + \alpha_p[i] + \beta_s[i] c_i$$

Model 3

$$y = \beta_c X_c + \beta_p[i] c_i + \alpha_s[i]$$

Parametrizations

	GLMM	LMM
	Model 1	
respondents	θ_p	τ_p
stimuli	b_s	δ_s
	Model 2	
respondents	θ_p	τ_p
stimuli	b_{sc}	δ_{sc}
	Model 3	
respondents	θ_{pc}	τ_{pc}
stimuli	b_s	δ_s

$p = 1, \ldots, P$: Respondent, $s = 1, \ldots, S$: Stimulus, $c \in \{0, 1\}$ Associative condition, i Trial

Find the useful model via model comparison: AIC and BIC

The lower the value, the better the model

> **!** AIC, BIC, and model complexity:
>
> Total number of parameters: β and Γ
> *NOT* the levels in d

Model 2 and Model 3: Same complexity, different focus

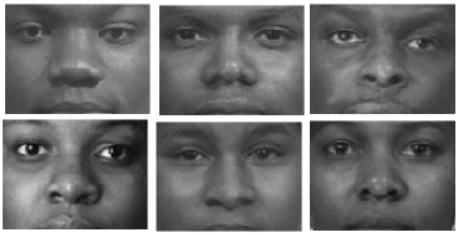
The chosen model is the least wrong model *given the considered models*

Real data

12 Object stimuli

White people faces

Black people faces



16 Attribute stimuli

Positive attributes

Good, laughter, pleasure, glory, peace, happy, joy, love

Negative attributes

Evil, bad, horrible, terrible, nasty, pain, failure, hate

Best Fitting Models

GLMMs

Model 2

θ_p

b_{WGBB} and b_{BGWB}

The IAT effect is mostly due to variations in the *stimuli functioning* between conditions, while the performance of the respondents seems unaltered

LMMs

Model 3

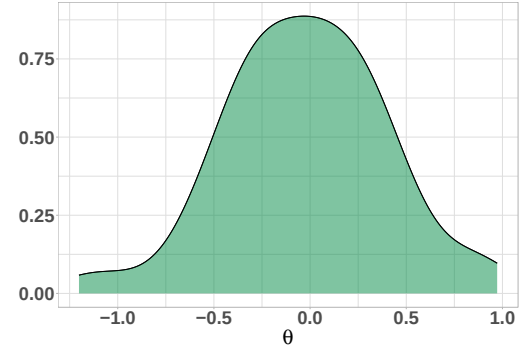
τ_{WGBB} and τ_{BGWB}

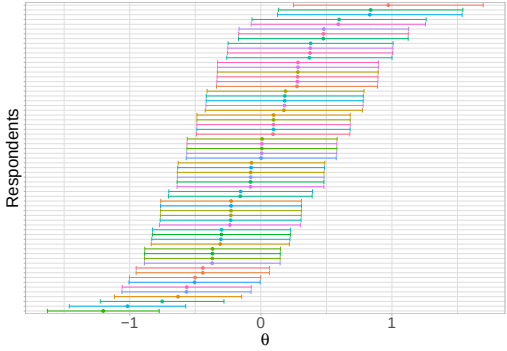
δ_s

The IAT effect is mostly due to variations in the *performance of the respondents* between conditions, while the functioning of the stimuli appears not affected

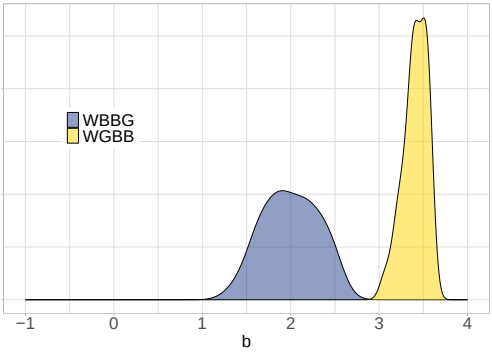
Rasch-like estimates

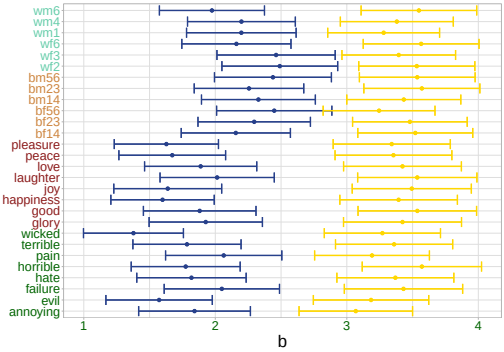
θ_p



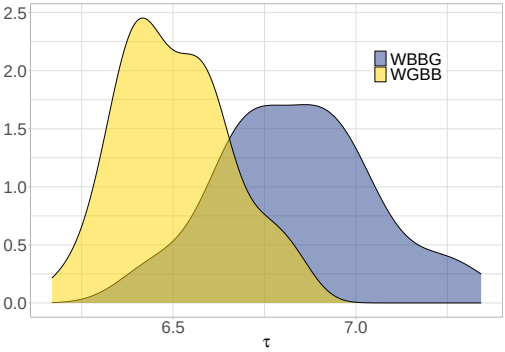


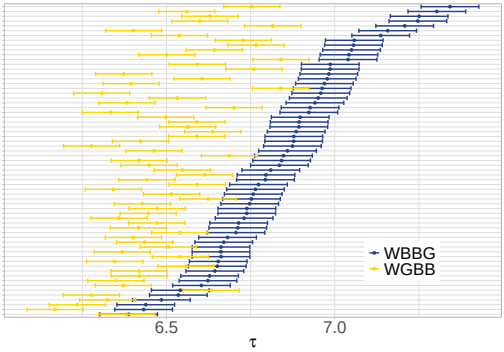
b_{WGBB} and b_{WGBB}





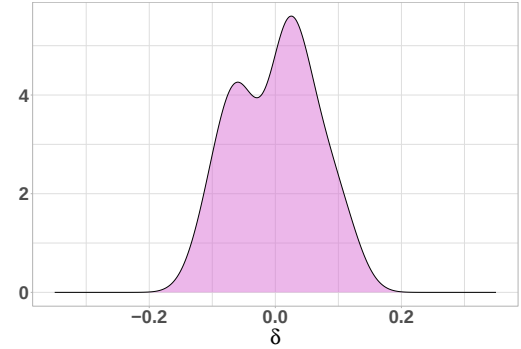
τ_{WGBB} and τ_{BGWB}

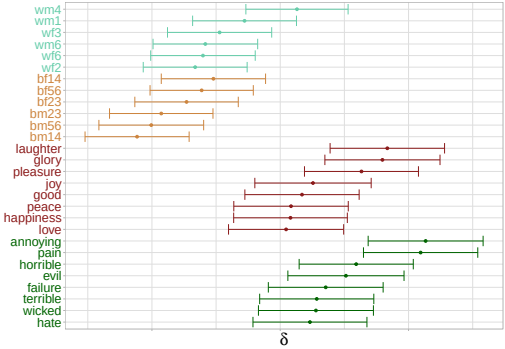




Log-normal estimates

δ_s





The end

- The best model depends on the other models... sometimes useful, never right
- The sky is the limit... but do not over complicate things

- The best model depends on the other models... sometimes useful, never right
- The sky is the limit... but do not over complicate things

HOWEVER

- Time and accuracy are independent from one another, pretty bold assumption

- The best model depends on the other models... sometimes useful, never right
- The sky is the limit... but do not over complicate things

HOWEVER

- Time and accuracy are independent from one another, pretty bold assumption

Psychological Methods

ISSN: 1082-989X
https://doi.org/10.1037/met0000708

A Guided Tutorial on Linear Mixed-Effects Models for the Analysis of Accuracies and Response Times in Experiments With Fully Crossed Design

Ottavia M. Epifania, Pasquale Anselmi, and Egidio Robusto
Department of Philosophy, Sociology, Education and Applied Psychology, University of Padova

https://doi.org/10.1037/met0000708