A Linear Mixed-Effects Models approach to obtain Rasch-like estimates of accuracies and response times from fully-crossed design data

Rasch 2.0

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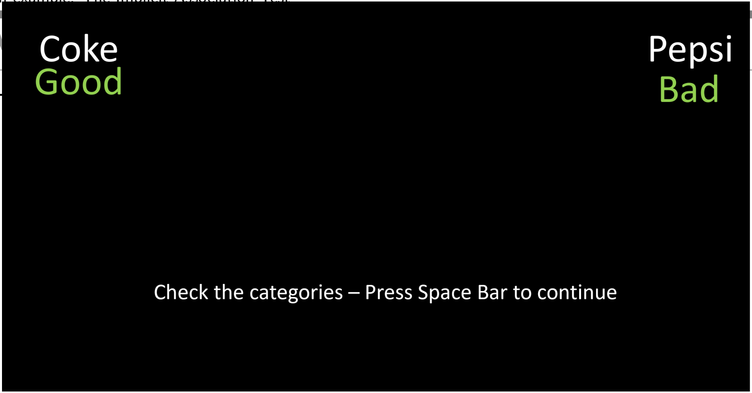
An example: The Implicit Association Test

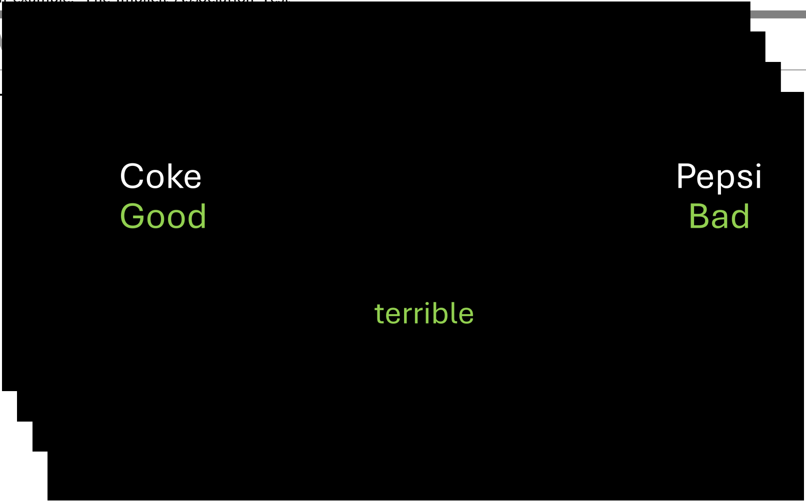


Response key: E

An example: The Implicit Association Test

Response key: I

An example: The Implicit Association Test



Response key: I

An example: The Implicit Association Test

Response key: E

1. The “natural” one (so called *compatible* condition)

   I love Coke and its easier to associate these stimuli to positive attributes

2. The “innatural” one (so *incompatible* condition)

   I love Coke and its harder to associate these stimuli to negative stimuli

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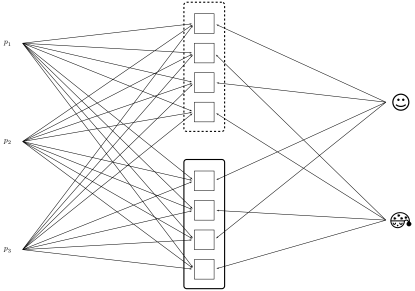

Person-level scores

$$s_p = \frac{\bar{X}_{p,\text{comp}} - \bar{X}_{p,\text{inc}}}{sd_{\text{pooled}}}$$

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Ease of computation
Ease of interpretation

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(Implicit) Assumptions

1. Being slow (less accurate) in one condition = being fast (or more accurate) in the opposite one: 0 means absence of bias
2. All stimuli have the same impact (fixed effects)

A long tradition

Respondents are random factors

Sampled from a larger population
Need for acknowledging the sampling variability
Results can be generalized to other respondents belonging to the same population

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Respondents are random factors

Sampled from a larger population
Need for acknowledging the sampling variability
Results can be generalized to other respondents belonging to the same population

Stimuli/items are fixed factors

Taken to be entire population
There is no sampling variability
There is no need to generalize the results because the stimuli are the population

- Generalization of the results is impaired
- Error variance everywhere, left free to bias everything
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Σ	ψ
Linear Mixed Effects Models	Rasch model

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$\sum$

Linear Mixed Effects Models

ψ

Rasch model

Rasch-like parametrization estimated with Linear Mixed Effects Models

Rasch

$$P(x_{ps} = 1) = \frac{\exp(\theta_p - b_s)}{1 + \exp(\theta_p - b_s)}$$

GLM (inverse function)

$$P(x_{ps} = 1) = \frac{\exp(\theta_p + b_s)}{1 + \exp(\theta_p + b_s)}$$

Rasch

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Log-normal

$$E(t_{ps})=\delta_s-\tau_p$$

LM (identity function)

$$E(t_{ps})=\delta_s+\tau_p+\varepsilon$$

In a LM:

$$\eta = \mathbf{X}\beta$$

$\mathbf{X}$: Model Matrix

β: Coefficients

Needs to be extended:

$$\eta = \mathbf{X}\beta + Zd$$

d: Random effects associated to the random factors in Z ... Not model parameters! *Best Linear Unbiased Predictors*

Γ: Parameters estimated for the random factors in the model (variances and covariances)

Models

Model 1

$$y = \beta_c X_c + \alpha_{p[i]} + \alpha_{s[i]}$$

Model 2

$$y = \beta_c X_c + \alpha_{p[i]} + \beta_{s[i]} c_i$$

Model 3

$$y = \beta_c X_c + \beta_{p[i]} c_i + \alpha_{s[i]}$$

Parametrizations

	GLMM	LMM
	Model 1	
respondents	θ_p	τ_p
stimuli	b_s	δ_s
	Model 2	
respondents	θ_p	τ_p
stimuli	b_{sc}	δ_{sc}
	Model 3	
respondents	θ_{pc}	τ_{pc}
stimuli	b_s	δ_s

$p = 1, \dots, P$: Respondent, $s = 1, \dots, S$: Stimulus, $c \in \{0, 1\}$ Associative condition, i Trial

Find the useful model via model comparison: AIC and BIC

The lower the value, the better the model

! AIC, BIC, and model complexity:

Total number of parameters: β and Γ
NOT the levels in d

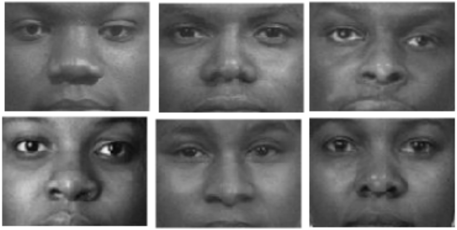
Model 2 and Model 3: Same complexity, different focus

The chosen model is the least wrong model *given the considered models*

12 Object stimuli

White people faces

Black people faces



16 Attribute stimuli

Positive attributes

Good, laughter, pleasure, glory, peace, happy, joy, love

Negative attributes

Evil, bad, horrible, terrible, nasty, pain, failure, hate

Best Fitting Models

GLMMs
Model 2
θ_p
b_{WGBB} and b_{BGWB}

The IAT effect is mostly due to variations in the *stimuli functioning* between conditions, while the performance of the respondents seems unaltered

LMMs
Model 3
τ_{WGBB} and τ_{BGWB}
δ_s

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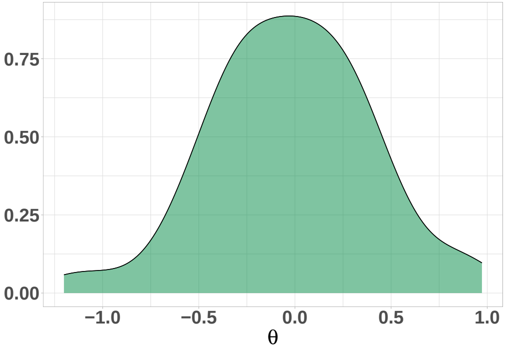
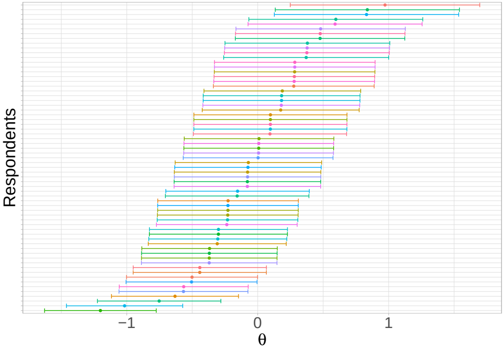
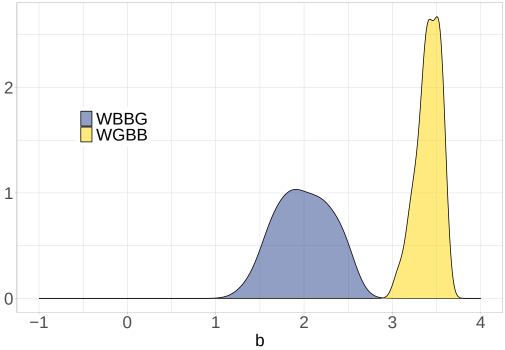
Rasch-like estimates

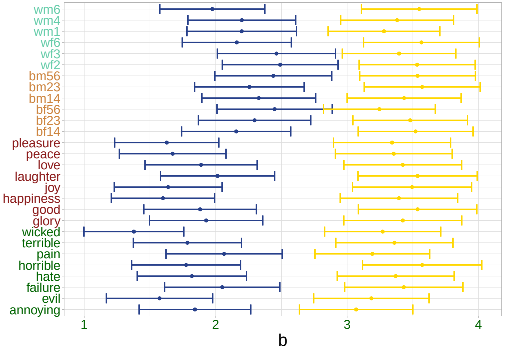
θ_p





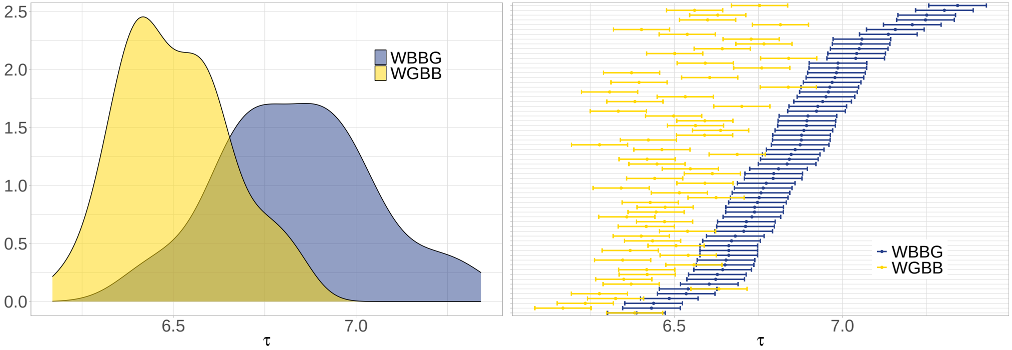
b_{WGBB} and b_{WGBB}





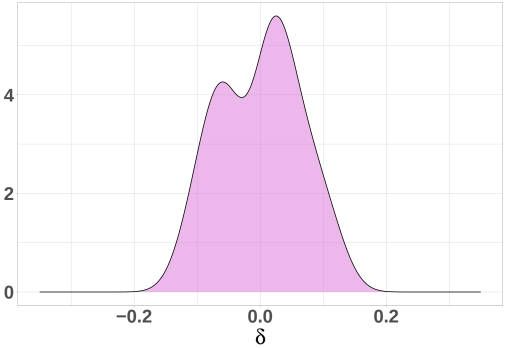
Log-normal estimates

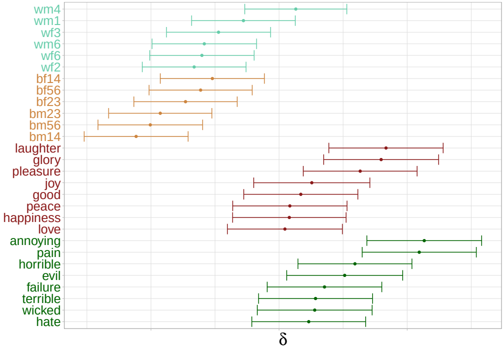
τ_{WGBB} and τ_{BGWB}



Log-normal estimates

δ_s





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A Guided Tutorial on Linear Mixed-Effects Models for the Analysis of Accuracies and Response Times in Experiments With Fully Crossed Design

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