Quantum Computing Lecture 5

Recap

Qubits $|0\rangle$, $|1\rangle$, $a|0\rangle + b|1\rangle$

Measurement Collapse $|0\rangle$ or $|1\rangle$

Multiple Qubit $|00\rangle + |01\rangle$

Entanglement

Product state

$$a|0_A 0_B\rangle + b|0_A 1_B\rangle + c|1_A 0_B\rangle + d|1_A 1_B\rangle$$

$$\frac{|0_A 0_B\rangle + |0_A 1_B\rangle}{\sqrt{2}} = |0_A\rangle\left(\frac{|0_B\rangle + |1_B\rangle}{\sqrt{2}}\right)$$

An entangled state is anything that is not a product state

$$|0_A\rangle\left(\frac{|0_B\rangle + |1_B\rangle}{\sqrt{2}}\right)$$

Measure 1st qubit $|0\rangle$ 100% time

Measure 2nd qubit $|0\rangle$ 50% time

$|1\rangle$ 50% time

$$\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$$ entangled

Measure 1st qubit $|0_A\rangle$ 50% time
$|1_A\rangle$ 50% time

$|0_A 0_B\rangle$ collapse $|0_B\rangle$ 100%

$|1_A 1_B\rangle$ so was $|1_B\rangle$ 100%

Alice Bob

Earth Alpha Centuri

$$\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$$

Alice measures $\rightarrow |0_A 0_B\rangle$

Super dense coding

Alice and Bob meet up

Create 2 qubit state $\frac{|0_A 0_B\rangle + |1_A 1_B\rangle}{\sqrt{2}}$

Then they go seperate ways each with one qubit

Alice wants to send a message to Bob

She wants to send 2 bit message: 00, 01, 10, 11

She can do this by sending Bob 1 qubit

What she wants to send to Bob	Transformation
00	Do nothing / $\mathbb{1}$
01	Apply X gate to her qubit
10	Z gate
11	iY gate

Send her qubit to Bob

Bob recieves the qubit

Bob has 2 qubit

Start a CNOT gate

Apply a H gate to Alice's qubit

Then when he measures both he'll get Alice's messages

Worked Example

Alice wants to send 01

$$\frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle + |1_A 1_B\rangle\right)$$

X gate / NOT gate $|0\rangle \rightarrow |1\rangle$
$|1\rangle \rightarrow |0\rangle$

$$\frac{1}{\sqrt{2}}\left(|1_A 0_B\rangle + |0_A 1_B\rangle\right)$$

Sends to Bob

CNOT Controlled NOT

If Alice's qubit is $|0\rangle$ do nothing
is $|1\rangle$ flip Bobs

$$\frac{1}{\sqrt{2}}\left(|1_A 1_B\rangle + |0_A 1_B\rangle\right)$$

Apply Hadamard to first qubit

$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $|1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}}\left(\left(\frac{|0_A\rangle - |1_A\rangle}{\sqrt{2}}\right)|1_B\rangle + \left(\frac{|0_A\rangle + |1_A\rangle}{\sqrt{2}}\right)|1_B\rangle\right)$$

$$= \frac{1}{2}\left[|0_A 1_B\rangle - \cancel{|1_A 1_B\rangle} + |0_A 1_B\rangle + \cancel{|1_A 1_B\rangle}\right]$$

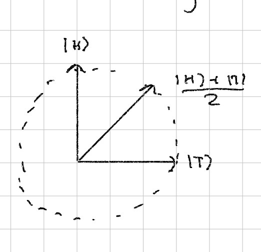
$$= \frac{2|0_A 1_B\rangle}{2} = |0_A 1_B\rangle$$

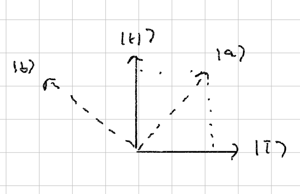
Bob measures 0 1 100%

Why does this work?

Orthonormal states can always be distinguished

In QM working in a vector space





2 states orthogonal always tell the difference

True in higher dimensions

2 qubits form a 4D vector space spanned by

$|00\rangle$, $|01\rangle$, [redacted] $|10\rangle$, $|11\rangle$

Change of basis

B_1 $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$, B_2 $\frac{|01\rangle + |10\rangle}{\sqrt{2}}$

B_3 $\frac{|00\rangle - |11\rangle}{2}$, B_4 $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$

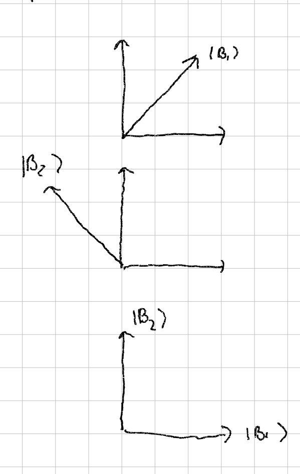
Any 2 qubit state in terms of $B_1 \dots B_4$

B_1

Act on the first qubit $B_1 \rightarrow B_1$
$B_1 \rightarrow B_2$
Send it it too to Bob $B_1 \rightarrow B_3$
$B_1 \rightarrow B_4$

Bob does same thing every time (changes basis back)

Bob measures



No Cloning

One of the reasons QC is hard

If I give you a qubit in an arbitary state e.g.

$$a|0\rangle + b|1\rangle$$

You can't copy it

Fault tolerant QC is really hard!

Shor's Algorithm

Factorise numbers efficiently

Any number can be written as a product of primes

For large numbers this is very hard

$21 = 3 \times 7$

Cryptography uses this fact

RSA uses this

Shor's algorithm does this efficiently

Further reading / resources

- Quantum Computing and Information by Nielsen and Chuang
- Quantum Computing by Mermin
- IBM Qiskit

Special Relativity

- Tells you can't send information faster than the speed of light

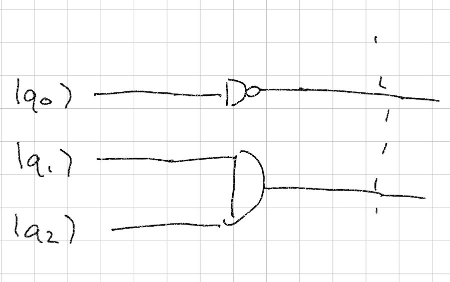
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$|00\rangle$ or $|11\rangle$ $|00\rangle$ or $|11\rangle$

Alice Bob

2 electrons $|\uparrow\rangle$ $|\downarrow\rangle$

$$\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$$



Trapped ion quantum computer