Quantum Computing Lecture 1

Maths for Quantum Mechanics
- Set Theory
- Complex numbers
- Linear Algebra *

Basic Quantum Mechanics

Do Quantum Computing!

Set Theory

Sets are collections of things

$\{a, b, c\}$ $\{cat, dog\}$

$\{1, 2, 5\}$

$\{1, 2, 5, 5\} = \{1, 2, 5\}$

$\{1, 2, 3\} \neq \{1, 2\}$

$\{3, 2, 1\} = \{1, 2, 3\}$

Combining sets

$\{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\}$
union

$\{a, b, c\} \cup \{d\} = \{a, b, c, d\}$

$\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$

$\{1, 2, 3\} \cap \{2, 4, 6\} = \{2\}$
intersection

Subsets

Is one set inside another?

$\{1, 2\} \subseteq \{1, 2, 3\}$

$\{1, 2\}$ is a subset of $\{1, 2, 3\}$

$\{a, b\} \subseteq \{a, b, c\}$

$\{a, b, d\} \not\subseteq \{a, b, c\}$

Size of sets / cardinality

$|\{1, 2, 3\}| = 3$

$|\{a, b\}| = 2$

Membership

Is an element in a set?

Is 1 in $\{1, 2, 3\}$? Yes

$1 \in \{1, 2, 3\}$
belongs to

$4 \notin \{1, 2, 3\}$

Common / Useful Sets to Know

Empty set $\emptyset$ (no items)

Integers $\mathbb{Z}$ $\{\ldots -1, 0, 1, \ldots\}$

Natural numbers $\mathbb{N}$ $\{1, 2, 3 \ldots\}$

Real Numbers $\mathbb{R}$ $\pi, e, 1.73, \frac{4}{5}$

Complex Number $\mathbb{C}$ $\{a + bi\}$

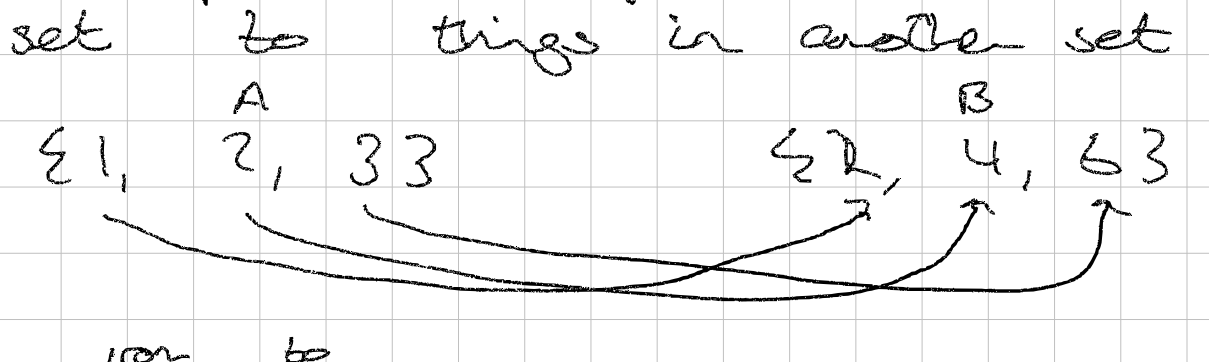
Pure Imaginary $i\mathbb{Z}$

Extra Topics

Russel's Pardox / Barber's paradox
ZFC

Maps / Functions

A map relates objects in one set to things in another set



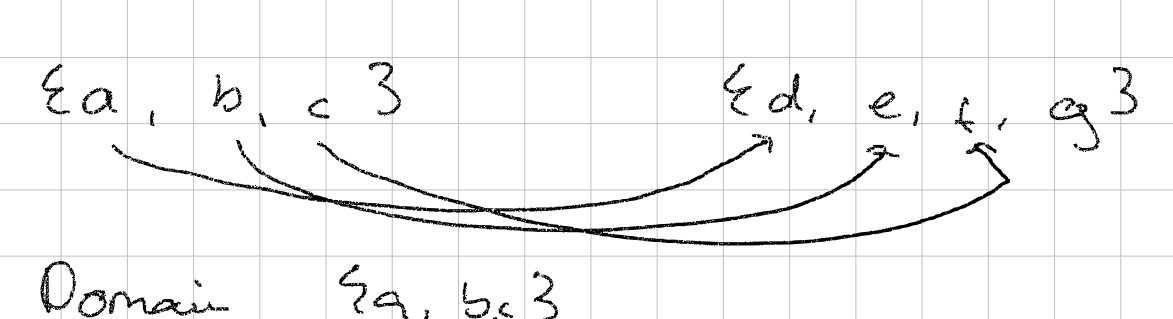
from to
$f: A \to B$

$f(x) = 2x$

Domains / Ranges

$f: A \to B$ A domain
B codomain

Range all the values f takes



Domain $\{a, b, c\}$

Codomain $\{d, e, f, g\}$

Range $\{d, e, f\}$

$g: \mathbb{R} \to \mathbb{R} \subseteq \mathbb{C}$ $f: \mathbb{R} \to \mathbb{R}$
$g(x) = x^2$ $f'(x)$

Injective / Surjective Maps

Injective maps are ones where every input has unique output

$f(x) = x + 3$

$f(4) = 7$

$g(x) = x^2$

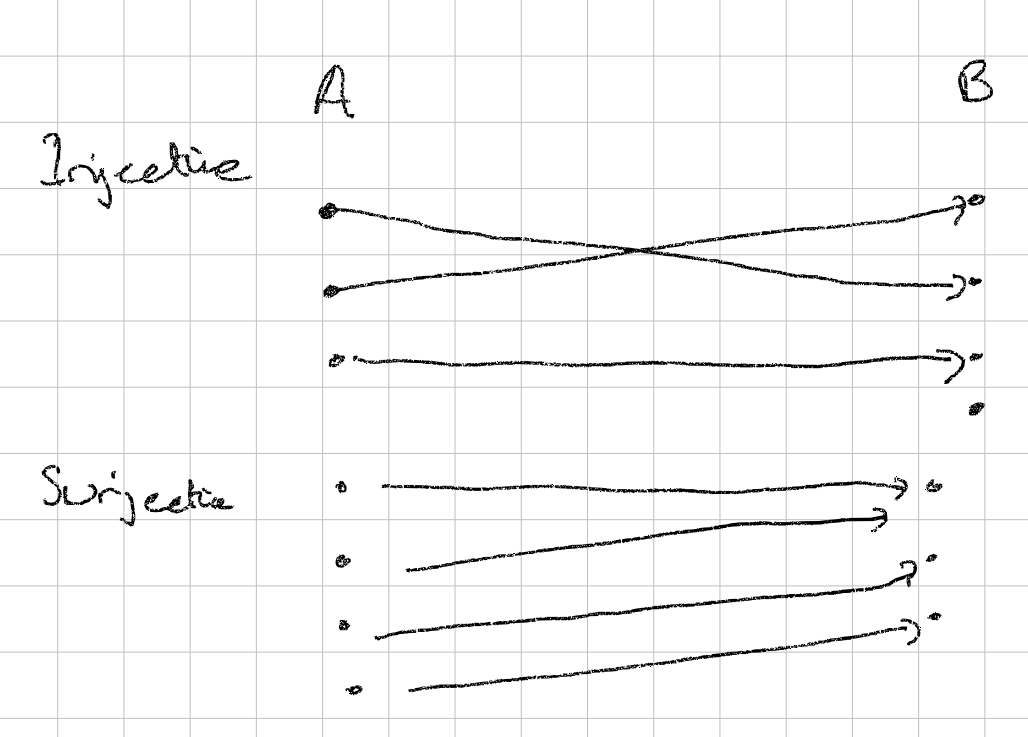
$g(1) = g(-1) = 1$

Surjective maps are maps that cover codomain

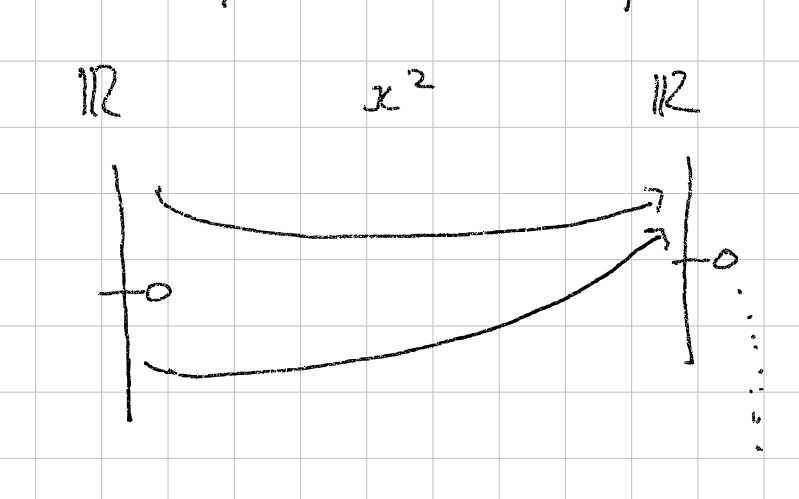
$f: A \to B$

$f(x) = x - 2$

$g(x) = x^2$ $-1, -2$



Bijective : Injective + Surjective



Binary Operations

A B C

$f(x, y) = z$

$f: A \times B \to C$

$A = \{1, 2\}$ $B = \{3, 4\}$ $C = \mathbb{Z}$

(1, 3)	(1, 4)	(2, 3)	(2, 4)
↓	↓	↓	↓
4	5	5	6

$f(1, 4) = 5$

Complex Numbers

$x^2 = -1$ $x \in \mathbb{R}$

No solutions

$i = \sqrt{-1}$

$i^2 = -1$

$(3i)^2 = 3^2 i^2 = 9 \cdot -1 = -9$

xi $x \in \mathbb{R}$

$3i + 4i = 7i$

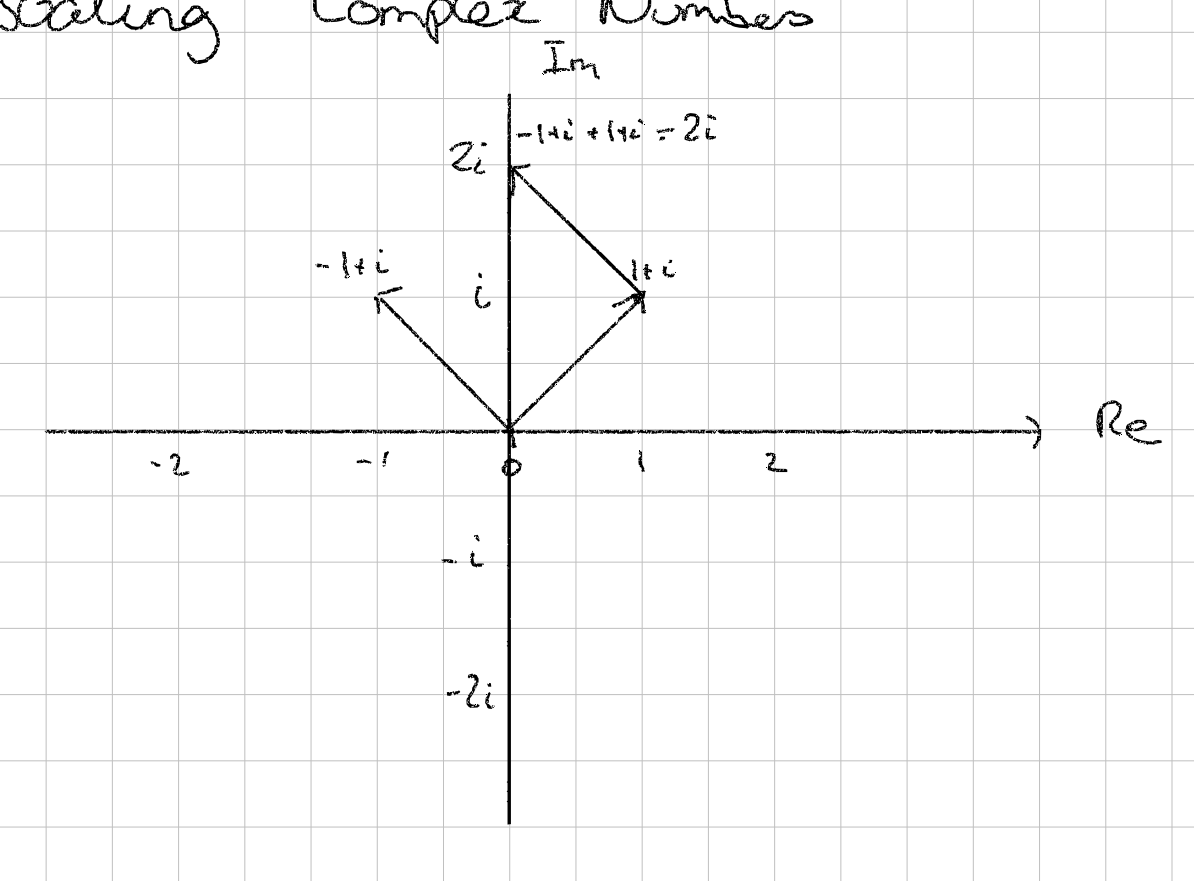
$4 + 2i$

$a + bi$ ← "Complex numbers"

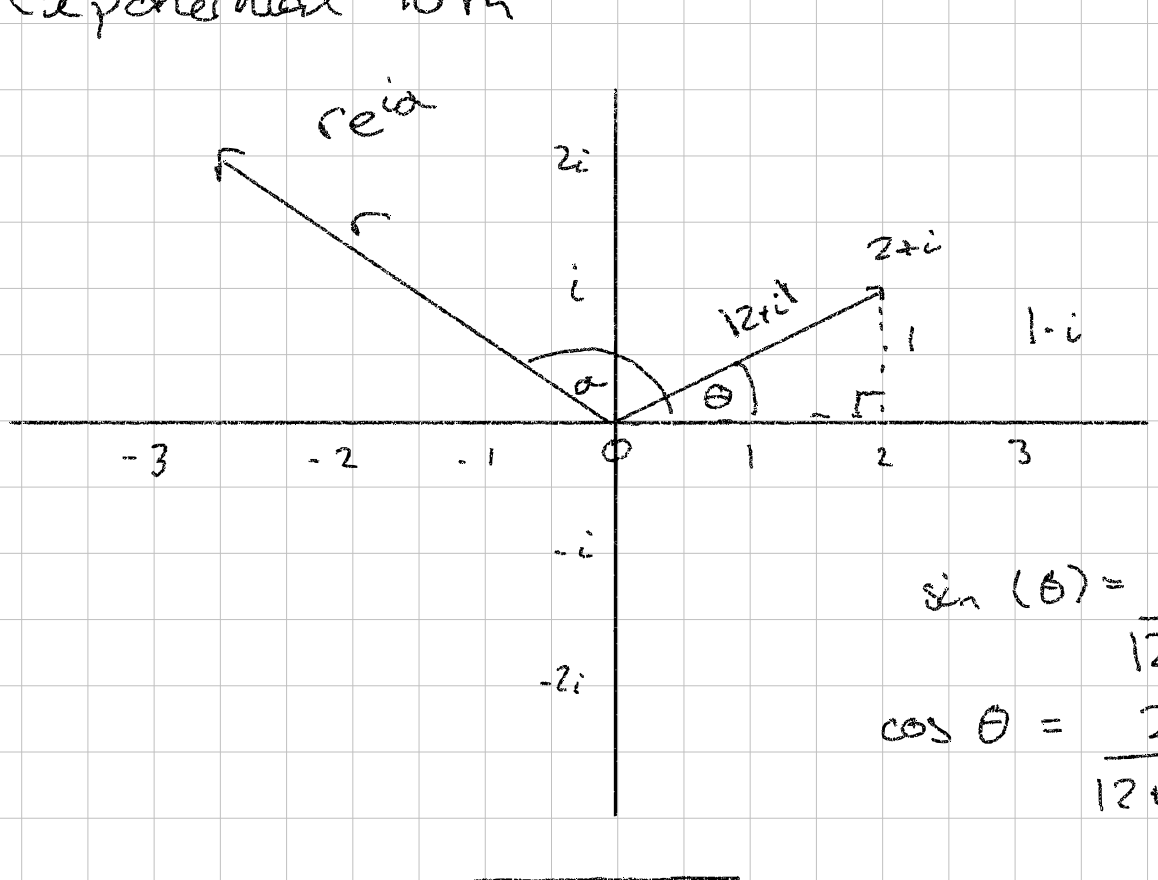
$(4 + 2i) + (7 + 3i) = 11 + 5i$

$$(4 + 2i)(7 + 3i) = 28 + 14i + 12i + 6i^2$$
$$= 28 - 6 + 26i$$
$$= 22 + 26i$$

Visualising Complex Numbers



Exponential Form



$\sin(\theta) = \frac{1}{|2+i|}$

$\cos\theta = \frac{2}{|2+i|}$

$|2+i| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\sin\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\sqrt{5}\sin\theta = 1$

$\sqrt{5}\cos\theta = 2$

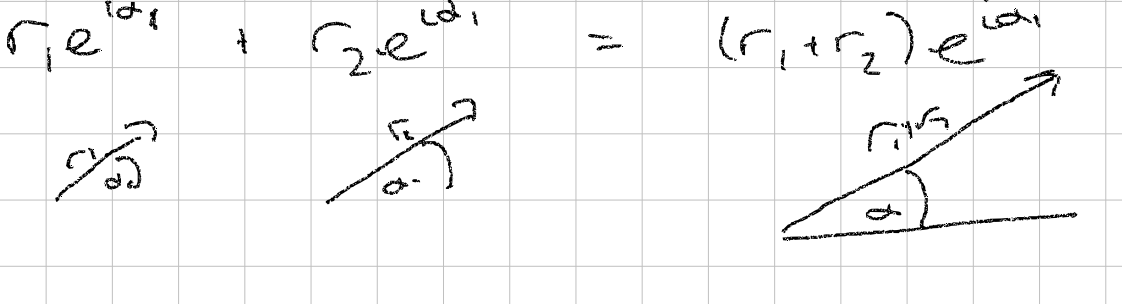
$$2 + i = \sqrt{5}\cos(\theta) + \sqrt{5}\sin(\theta)\, i$$
$$= \sqrt{5}(\cos\theta + i\sin\theta)$$
$$= \sqrt{5}e^{i\theta}$$

Polar coords

Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$

$r_1 e^{i\alpha_1} r_2 e^{i\alpha_2} = r_1 r_2 e^{i(\alpha_1 + \alpha_2)}$

$r_1 e^{i\alpha_1} + r_2 e^{i\alpha_1} = (r_1 + r_2) e^{i\alpha_1}$



Next time

- Linear Algebra Vectors + Matrices