Introduction to Complex Analysis Notes

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1 Week 1

1.1 Lecture 2

- Modulus of $z = x + iy$ is $|z| = \sqrt{x^2 + y^2}$
- Multiplication of complex numbers is defined as: $(x + iy) * (u + iv) = (xu - yv) + i(xv + yu)$
- Multiplication of complex numbers is associative (brackets don't matter), commutive (order doesn't matter) and distributive
- Division is defined as $\frac{z}{w} = \frac{x+iy}{u+iv} = \frac{xu+yv}{u^2+v^2} + i\frac{yu-xv}{u^2+v^2}$
- The complex conjugate of z is $\overline{z} = x - iy$
- Triangle inequality: $|z + w| \leq |z| + |w|$
- Fundamental Theorem of Algebra: If $a_0, a_1, ...,$ an are complex numbers with $a_n \neq 0$, then the polynomial $p(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_0$ has n roots $z_1, z_2, ... z_n$ in $\mathbb{C}$. It can be factored as $p(z) = a_n(z - z_1)(z - z_2)...(z - z_n)$

1.2 Lecture 3

- $z = x + iy = r(cos\theta + isin\theta)$
- $r = |z|$
- The principal argument of z, called Arg z, is the value of θ for which $-\pi < \theta \leq \pi$
- arg z = $\{$Arg z $+2\pi k : k = 0, \pm 1, \pm 2, ...\} z \neq 0$
- $e^{i\theta} = cos\theta + isin\theta$
- z = $re^{i\theta}$
- $\overline{e^{i\theta}} = e^{-i\theta}$
- $arg(\overline{z}) = -argz$
- $arg(z + w) = arg(z) + arg(w)$
- De Moivre's Formula: $(cos\theta + isin\theta)^n = cos(n\theta) + isin(n\theta)$

1.3 Lecture 4

- Let w be a complex number. An nth root of w is a complex number z such that $z^n = w$
- If $w \neq 0$ then there are n distinct roots
- Let $z^n = w$ then $z = (re^{i\theta})^n = r^n e^{in\theta} = w = \rho e^{i\phi}$
- This implies $r^n = \rho \Rightarrow r = \sqrt[n]{\rho}$
- Also $e^{in\theta} = e^{i\phi} \Rightarrow cos(n\theta) + isin(n\theta) = cos(\rho) + isin(\rho) = cos(\rho + 2k\pi) + isin(\rho + 2k\pi)$ since cos/sin are periodic with period $2\pi \Rightarrow \theta = \frac{\rho}{n} + \frac{2k\pi}{n}$ If $k \geq n$ then it starts repeating as you're adding more than 2π

1.4 Lecture 5

- $B_r(z_0)$ is a disk centered at z_0 of radius r defined as $\{z \in \mathbb{C} : |z - z_0| < r\}$
- $K_r(z_0)$ is a circle centered at z_0 of radius r defined as $\{z \in \mathbb{C} : |z - z_0| = r\}$
- Let $E \subset \mathbb{C}$ z is an interior point of E if there exists an $r > 0$ such that $B_r(z) \subset E$
- Let $E \subset \mathbb{C}$ b is a boundary point of E if for all $r > 0$, $B_r(b)$ contains a point inside E and a point outside E
- The boundary of the set $E \subset C$, ∂E, is the set of all boundary points of E.
- A set is open if all its elements are interior points
- A set is closed if it contains all its boundary points
- The closure of a set is $\overline{E} = E \cup \partial E$
- The interior of set E° is the set of all interior points of E
- Two sets, X, Y are seperated if there exists two disjoint ($U \cap V = \emptyset$) open sets, U, V with $X \subset U$ and $Y \subset V$
- A set W is connected if it is impossible to find two separated non-empty sets whose union equals W
- Let G be an open set in $\mathbb{C}$. Then G is connected if and only if any two points in G can be joined in G by successive line segments.
- A set A in C is bounded if there exists a number $r > 0$ such that $A \subset B_r(0)$. If no such R exists then A is called unbounded.

2 Week 2

2.1 Lecture 1

- z-plane is the domain, w-plane is the range, you can graph them seperately
- $f^n(z)$ is the nth iterate of f - applying f n times

2.2 Lecture 2

- A sequence $\{s_n\}$ of complex numbers converges to $s \in \mathbb{C}$ if for every $\epsilon > 0$ there exists an index $N \geq 1$ such that $|s_n - s| < \epsilon$ for all $n \geq N$