



TECHNISCHE
UNIVERSITÄT
WIEN

165.144 Simulation of condensed matter

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Summary chapter 7

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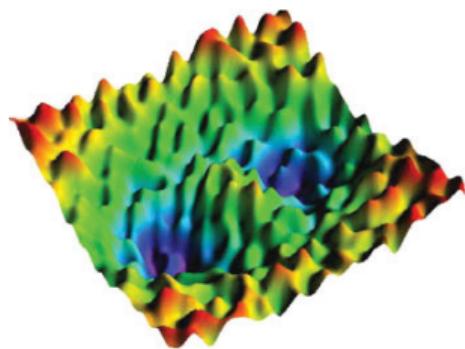
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- Quantum mechanics provides an "ab-initio/first principles" (no prior knowledge or measurements required) means to study small systems
- Force fields can be parametrized using quantum mechanics

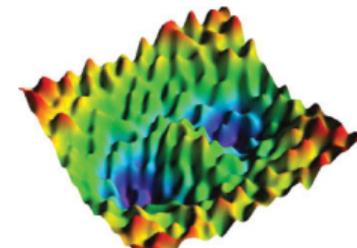


Chapter 8: Machine-learned force fields

Why settle on a functional form?

The potential energy $E_{\text{pot}}(\mathbf{x})$ has many complex features:

- Number of minima exponential on N
- Multiple transition paths
- Features at different length and energy scales



<https://doi.org/10.1515/pac-2014-0212>

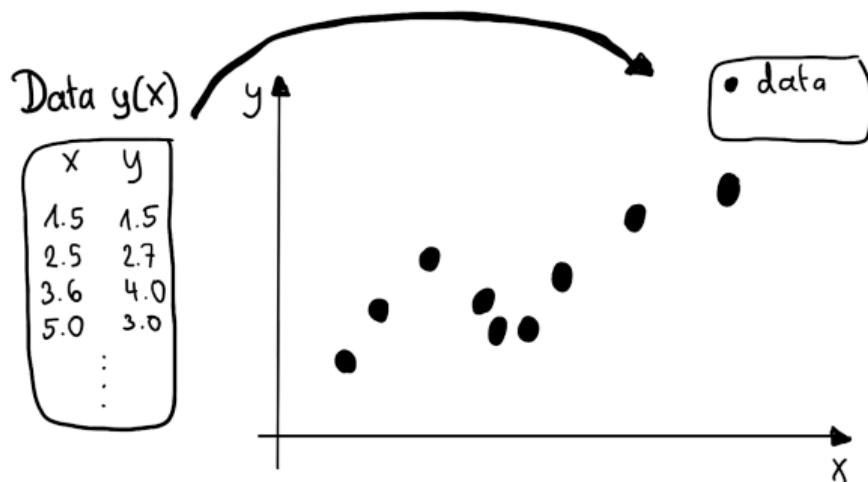
Approaches to approximate the true potential energy function:

- Quantum-mechanical calculations would be very accurate, but are way too expensive
- The classical forcefields from the previous lectures are cheap but only a crude approximation
- Machine learning can approximate a region of the function close to QM accuracy but with much less expensive function evaluations

Machine learning

Regression: Predict the continuous outcome/function value given some new unseen input

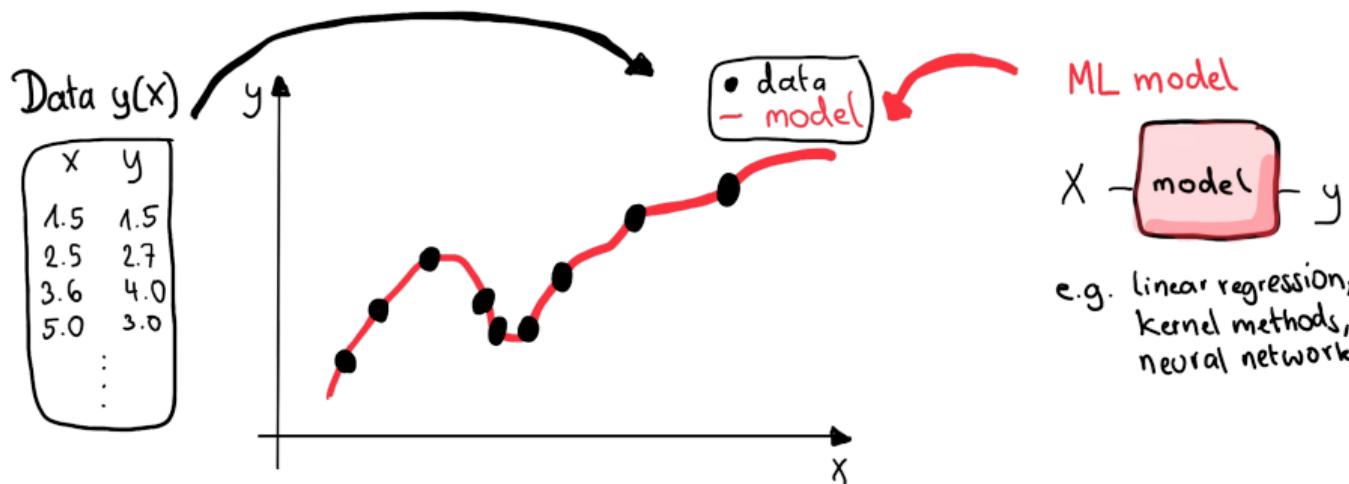
Step 1: Collect data (measurements, calculations)



Machine learning (regression task)

Regression: Predict the continuous outcome/function value given some new unseen input

Step 2: Fit model (find best parameters of function)



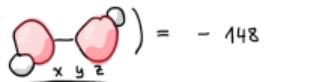
Step 3: Apply model, e.g. $y(2.0)=2.1$

Machine-learning (ML) potentials

Data: $E_{\text{Pot}}(\mathbf{x})$... potential energy as a function of the atomic coordinates from QM calculations for many relevant conformations (+ optionally atomic forces \mathbf{f})

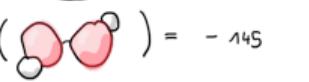
Ground truth $E_{\text{Pot}}(\mathbf{x})$:

$E_{\text{pot}} \left(\begin{array}{c} \text{H} \\ \text{O} \\ \text{H} \end{array} - \begin{array}{c} \text{O} \\ \text{H} \end{array} \right) = -148$



H	0	0	0
O	0.9	0.9	0
O	2.3	0.9	0
H	3.0	4.4	0

$E_{\text{pot}} \left(\begin{array}{c} \text{O} \\ \text{H} \\ \text{H} \end{array} - \begin{array}{c} \text{O} \\ \text{H} \end{array} \right) = -145$



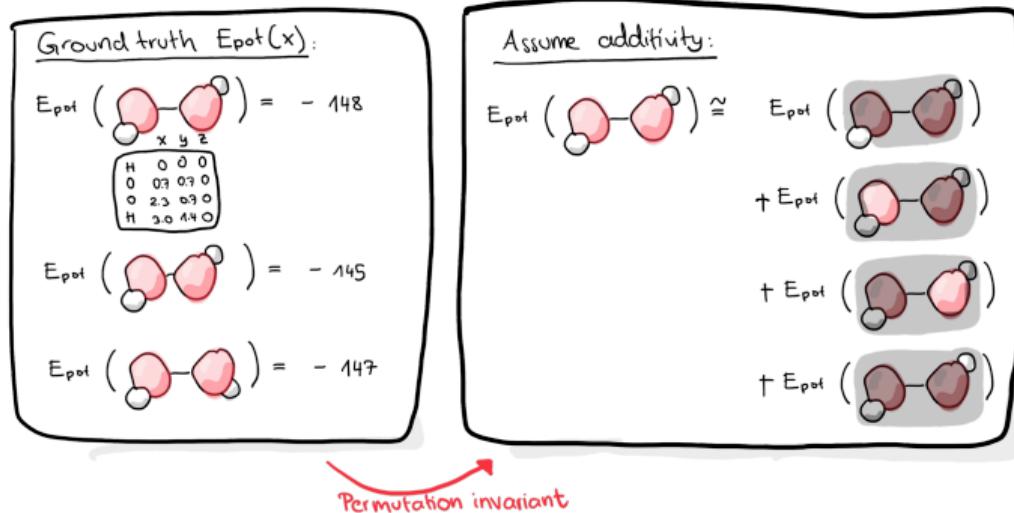
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Problem: Learned potential energy would not be permutation, translation, or rotation invariant!

Machine-learning (ML) potentials

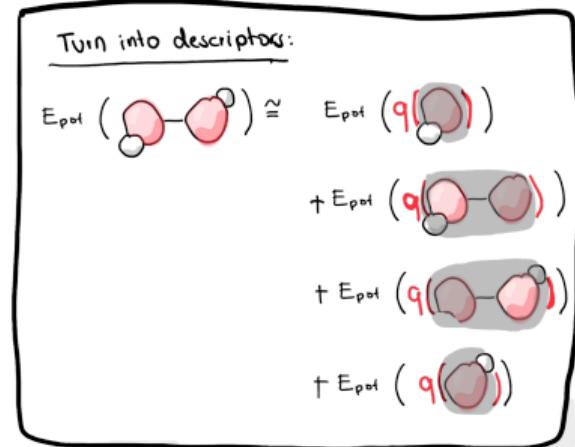
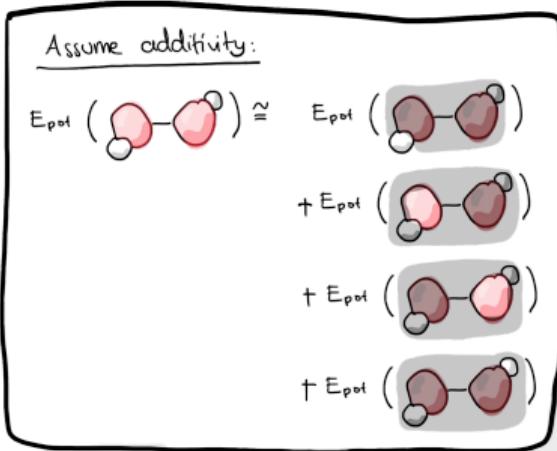
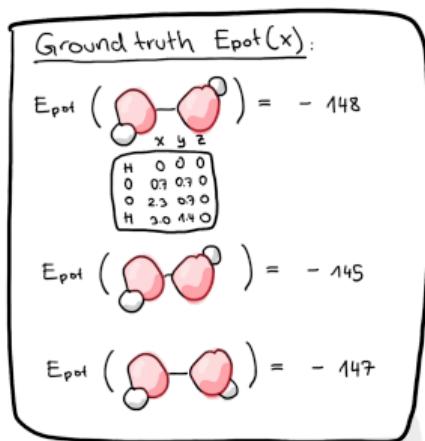
Assumption 1: $E_{\text{Pot}}(\mathbf{x})$ is approximately additive with respect to the atomic contributions $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$



Problem: Learned potential energy would not be translation or rotation invariant!

Machine-learning (ML) potentials

Assumption 2: The atomic contribution $E_{\text{Pot},N}(x)$ can also be learned from $E_{\text{Pot},N}(q_N)$ with q_N being a descriptor representation of the local environment around atom N



Permutation invariant

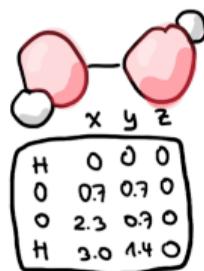
Translation/rotation invariant

Problem: We must choose/design suitable descriptors

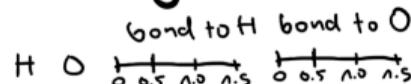
How to design a descriptor

Descriptors must be invariant with respect to translation, and invariant or equivariant with respect to rotation.

Simplest choice: Two-body (atom + distance information), or three-body (atom + distance + angle information)

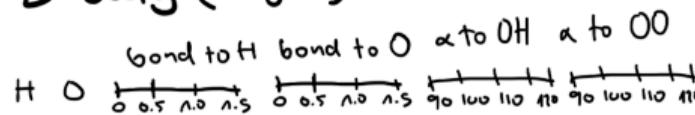


2-body (distances):



1	0	0		0		0		0		1	0
0	1	0		1		0		0		0	1
0	1	0		1		0		0		0	1
1	0	0		0		0		0		1	0

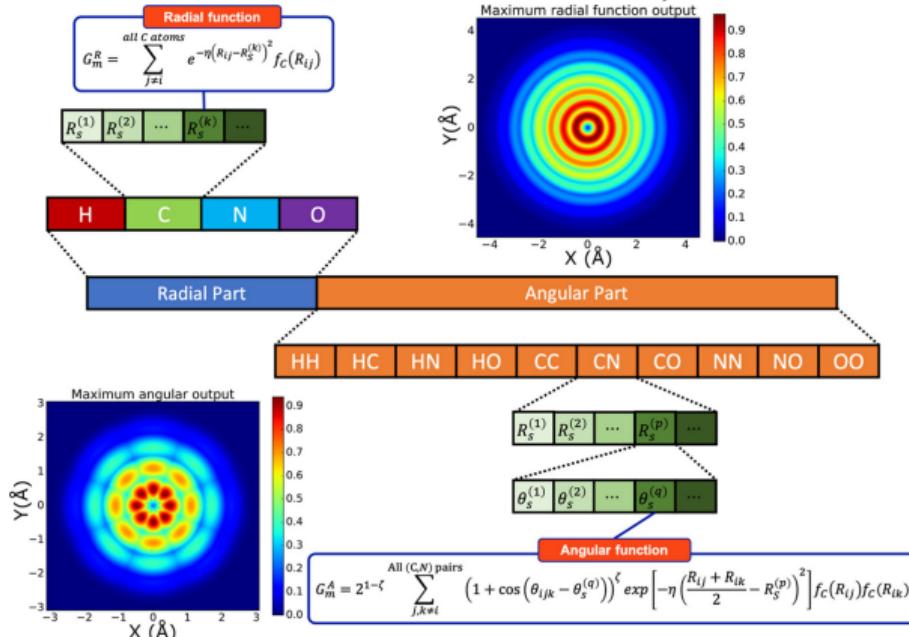
3-body (angles):



1	0	0		0		0		0		1	0
0	1	0		1		0		0		0	0
0	1	0		1		0		0		0	0
0	1	0		1		0		0		0	0
1	0	0		0		0		0		1	0

How to design a descriptor

Usually: Many-body representation (all neighboring atoms up to a cutoff) with a radial part and an angular part (usually based on spherical harmonics):



Descriptor-based architectures

Design of descriptors is still a work in progress, many variants published recently:

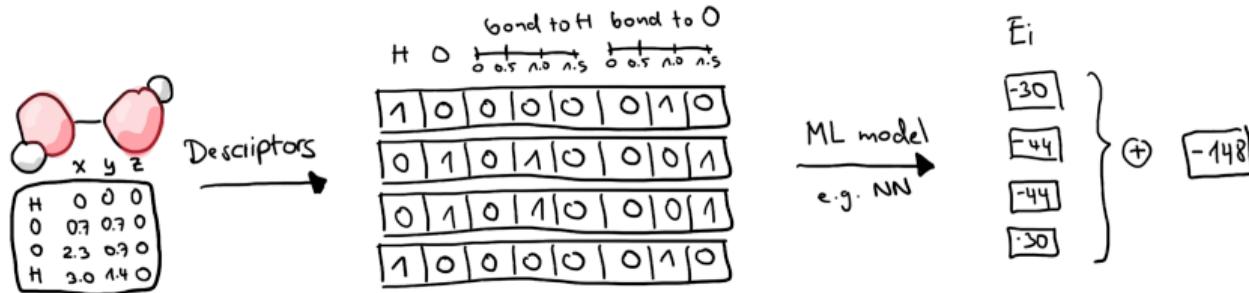
- Atom-centered symmetry functions (ACSFs)
- Smooth overlap of atomic positions (SOAP)
- Eigenvalues of the Coulomb matrix

Exemplar architectures:

- GAP: Gaussian process regression on SOAP descriptors
- NeuralIL: Residual neural network on Bessel descriptors
- ANI: Neural network on ACSFs

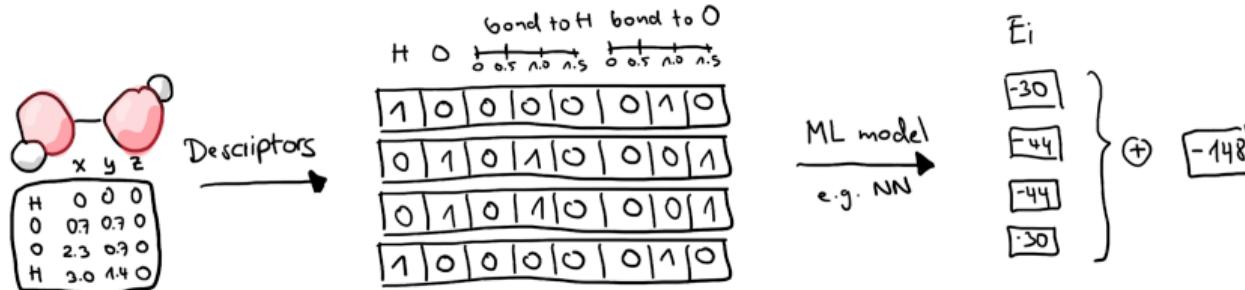
Learned descriptors

So far, we have custom-made the descriptors, relying on expert knowledge which information is important:

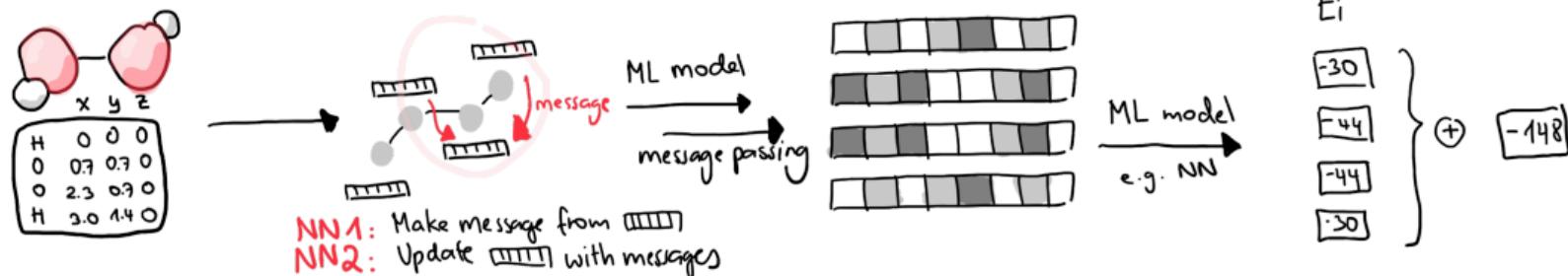


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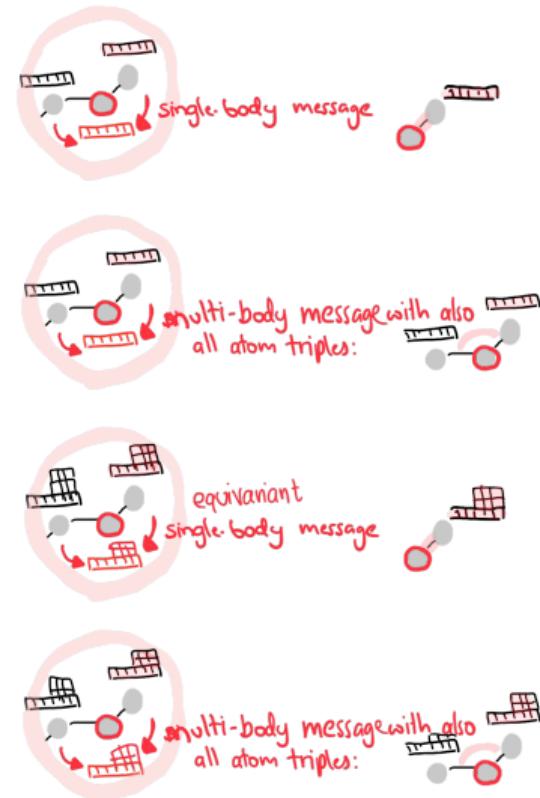
Instead, we can learn the descriptor from the data using message passing (= another model):



Learned descriptors

Evolution of learned descriptors:

- ① 2-Body (atoms I, J) invariant message passing with scalar features: Very basic learned features
- ② Multi-body messages including also triples (atoms I, J, K) or more: Can capture more information
- ③ 2-Body (atoms I, J) equivariant message passing with scalar and vectorial features: Can also capture directional information
- ④ Combination of equivariant message-passing with multi-body messages: Currently one of the best models

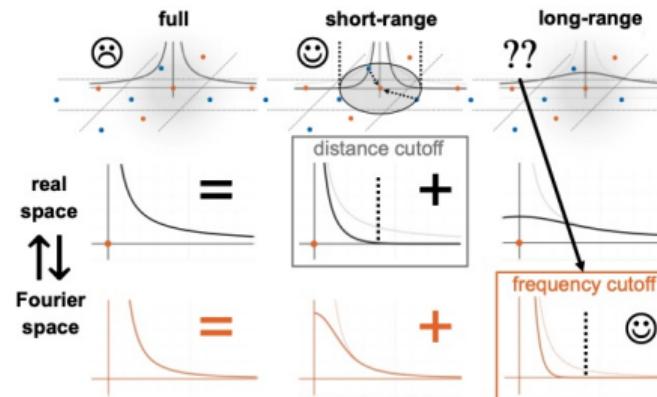


What about the long-range part?

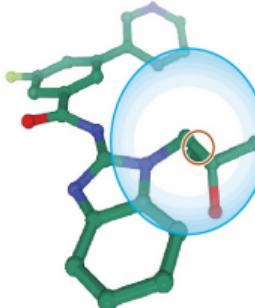
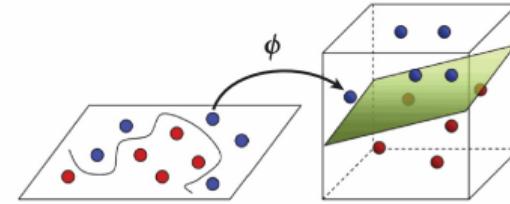
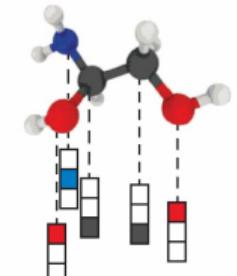
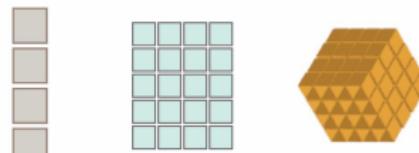
Previous methods only use local descriptors. What about long-range forces though?

No commonly accepted approach yet. Some possibilities:

- Fixed atomic multipoles (charges, dipoles, etc)
- Machine-learning method for the charge density and the derived potential energy
- Project long-range part onto local descriptors
- Ewald-type ML



Overview of machine-learned potentials

Neural network models		Kernel-based models	Others (linear and polynomial models)	
Predefined descriptor				
ANI DeepMD EANN HDNN PIP-NN, FI-NN TensorMol wACSF, SingleNN, ...		GPR/KRR GAP sGDML SNAP ...		ACE ChIMEs MTP PIP ...
Learnable descriptor				



Useful software

-  (scikit-learn.org): Comprehensive, well documented ML kit for Python, covering whole workflow.
-  TensorFlow (tensorflow.org): Google's open source software library for numerical computation using data flow graphs
-  PyTorch (pytorch.org): Meta's competitor to TensorFlow.
-  (github.com/google/jax): Automatic differentiation and JIT compilation for Python and Numpy with the goal of high-performance machine-learning research.

Disadvantages/Challenges

The development of machine-learned force fields is far from finished, and many challenges remain. Maybe you want to contribute?

- No functional form means no safety net - predictions can get arbitrarily different from the ground truth. Predictions can thus even have a wrong sign, i.e. be attractive instead of repulsive
- Generalization ability often poor
- High dependence on training set
- Currently promising results after some finetuning, but not off-the-bench

