$\boxed{6.4}$ Consider the problem of using the windowing method to design a lowpass filter to meet the following specifications.

$$
\begin{aligned}
(f_s, F_p, F_s) &= (100, 30, 50) \text{ Hz} \\
(A_p, A_s) &= (0.02, 50) \text{ dB}
\end{aligned}
$$

(a) Which types of windows can be used to satisfy these design specifications?

(b) For each of the windows in part (a), find the minimum order of filter m that will satisfy the design specifications.

(c) Assuming an ideal piecewise-constant amplitude response is used, find an appropriate value for the cutoff frequency F_c.

Solution

(a) From Table 6.2.3, the only windows which satisfy the passband ripple and stopband attenuation specifications are the Hamming and the Blackman windows.

(b) The normalized transition bandwidth required is

$$
\begin{aligned}
\hat{B} &= \frac{|F_s - F_p|}{f_s} \\
&= \frac{|50 - 30|}{100} \\
&= 0.2
\end{aligned}
$$

For the Hamming window, the normalized transition bandwidth is $\hat{B} = 3.3/m$. Thus $3.3/m = 0.2$ or

$$
\begin{aligned}
m &= \text{ceil}\left(\frac{3.3}{0.2}\right) \\
&= \text{ceil}(16.5) \\
&= 17
\end{aligned}
$$

For the Blackman window, the normalized transition bandwidth is $\hat{B} = 5.5/m$. Thus $5.5/m = 0.2$ or

$$
\begin{aligned}
m &= \text{ceil}\left(\frac{5.5}{0.2}\right) \\
&= \text{ceil}(27.5) \\
&= 28
\end{aligned}
$$

Thus the Blackman window requires a higher order filter to meet the transition bandwidth specification, but it has superior passband ripple and stopband attenuation.

(c) The ideal cutoff frequency should be placed in the middle of the transition band. Thus

$$\begin{aligned} F_c &= \frac{F_p + F_s}{2} \\ &= 40 \text{ Hz} \end{aligned}$$