1.10 Consider the following bandlimited signal.

$$x_a(t) = \sin(4\pi t)[1 + \cos^2(2\pi t)]$$

(a) Using the trigonometric identities in Appendix 4, find the maximum frequency present in $x_a(t)$.

(b) For what range of values for the sampling interval T can this signal be reconstructed from its samples?

Solution

(a) From Appendix 4

$$\begin{aligned} x_a(t) &= \sin(4\pi t) + \sin(4\pi t)\cos^2(2\pi t) \\ &= \sin(4\pi t) + 0.5\sin(4\pi t)[1 + \cos(4\pi t)] \\ &= \sin(4\pi t) + 0.5\sin(4\pi t) + 0.5\sin(4\pi t)\cos(4\pi t) \\ &= \sin(4\pi t) + 0.5\sin(4\pi t) + 0.25\sin(8\pi t) \end{aligned}$$

Thus the highest frequency present in $x_a(t)$ is $f_0 = 4$ Hz.

(b) From Theorem 1.3.1, we needed $f_s > 8$ Hz. Thus

$$0 < T < 0.125 \text{ sec}$$