7.10 Consider the problem of designing an interpolator with $f_s = 12$ Hz and a rate conversion factor $L = 3$.

(a) What is the sampling rate of the output signal?

(b) Sketch the desired magnitude response of the ideal anti-imaging filter $H_L(z)$.

(c) Suppose the anti-imaging filter is a windowed filter of order $m = 20$ using the Hanning window. Use Table 6.2.1 and Table 6.2.2 to find the impulse response, $h_L(k)$.

(d) Suppose a polyphase realization is used. How many polyphase filters are needed, and what is the order of the each polyphase filter?

(e) Sketch a polyphase filter realization of the interpolator.

Solution

(a) The sampling rate of the output signal is

$$\begin{aligned} f_S &= Lf_s \\ &= 36 \text{ Hz} \end{aligned}$$

(b) From (7.2.10), the ideal cutoff frequency is

$$\begin{aligned} F_L &= \frac{f_s}{2L} \\ &= 2 \text{ Hz} \end{aligned}$$

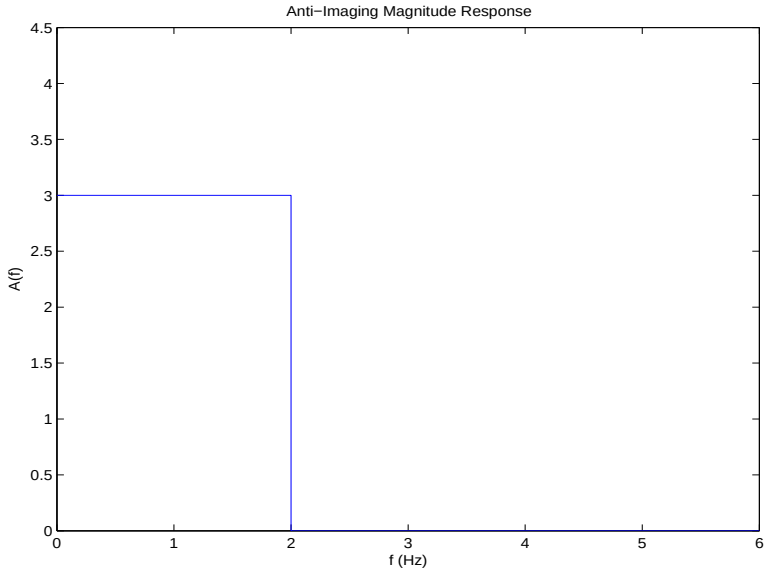
The required frequency response for the ideal anti-aliasing digital filter is then

$$H_L(f) = \begin{cases} 3 & , \quad 0 \le |f| < 2 \\ 0 & , \quad 2 \le |f| \le 6 \end{cases}$$

(c) Using Table 6.2.1 and Table 6.2.2 with $m = 20$, $p = m/2$, and the Hanning window, the FIR filter coefficients are

$$\begin{aligned} h_L(k) &= w(k)h(k) \\ &= 0.5\left[1 - \cos\left(\frac{\pi i}{0.5m}\right)\right]\frac{\sin[2\pi(i-p)F_LT]}{\pi(i-p)} \\ &= 0.5\left[1 - \cos\left(\frac{\pi i}{10}\right)\right]\frac{\sin[2\pi(i-10)/6]}{\pi(i-10)} \quad , \quad k \neq 10 \end{aligned}$$

The middle term is

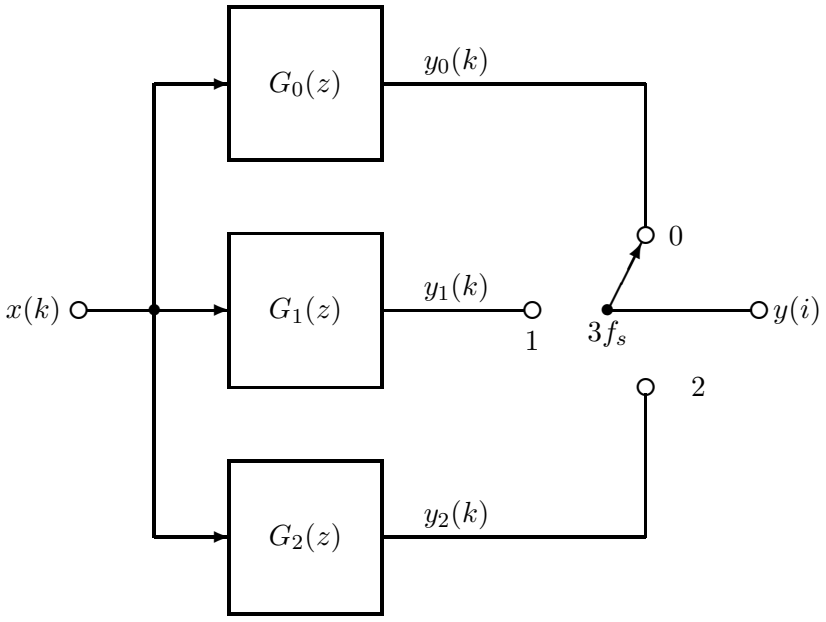


Anti-Imaging Filter Magnitude Resposne

$$\begin{aligned} h_L(10) &= w(p)h(p) \\ &= 0.5[1-\cos(\pi)]2F_LT \\ &= \frac{2}{6} \\ &= 0.333 \end{aligned}$$

(d) From (7.4.9) we need $(p+1) = Mp$ where M is the number of polyphase filters, and p is the order of each polyphase filter. Thus $21 = Mp$ with

$$\begin{aligned} M &= 3 \text{ filters} \\ p &= 7 \text{ filter order} \end{aligned}$$



(e) Polyphase Realization of Interpolator