$\boxed{2.14}$ Consider the following Z-transform. Find $x(k)$.

$$X(z) = \frac{5z^3}{(z^2 - z + .25)(z+1)} \quad , \quad |z| > 1$$

Solution

The factored form of $X(z)$ is

$$X(z) = \frac{5z^3}{(z-0.5)^2(z+1)}$$

Using the residue method, the initial value of $x(k)$ is

$$\begin{aligned} x(0) &= \lim_{z\to\infty} X(z) \\ &= 5 \end{aligned}$$

The residues of $X(z)z^{k-1}$ at the two poles are

$$\begin{aligned} R_1 &= \frac{d}{dz}\{(z-0.5)^2 X(z) z^{k-1}\}|_{z=0.5} \\ &= \frac{d}{dz}\left\{\frac{5z^{k+2}}{z+1}\right\}\bigg|_{z=0.5} \\ &= \frac{(z+1)5(k+2)z^{k+1} - 5z^{k+2}}{(z+1)^2}\bigg|_{z=0.5} \\ &= \frac{7.5(k+2)(0.5)^{k+1} - 5(0.5)^{k+2}}{(1.5)^2} \\ &= \frac{2.5(0.5)^{k+1}[3(k+2)-1]}{2.25} \\ &= \left(\frac{10}{9}\right)(3k+5)(0.5)^{k+1} \end{aligned}$$

$$\begin{aligned} R_2 &= (z+1)X(z)z^{k-1}|_{z=-1} \\ &= \frac{5z^{k+2}}{(z-0.5)^2}\bigg|_{z=-1} \\ &= \frac{5(-1)^{k+2}}{(-1.5)^2} \\ &= \left(\frac{20}{9}\right)(-1)^{k+2} \end{aligned}$$

Thus

$$
\begin{aligned}
x(k) &= x(0)\delta(k) + (R_1 + R_2)u(k-1) \\
&= 5\delta(k) + \left(\frac{10}{9}\right)[(3k+5)(0.5)^{k+1} + 2(-1)^{k+2}]u(k-1) \\
&= \left(\frac{10}{9}\right)[(3k+5)(0.5)^{k+1} + 2(-1)^{k+2}]u(k)
\end{aligned}
$$