

4.5 Suppose $h(k)$ and $x(k)$ are both of length $L = 2048$.

- (a) Find the number of real FLOPs for a fast linear convolution of $h(k)$ with $x(k)$.
- (b) Find the number of real FLOPs for a direct linear convolution of $h(k)$ with $x(k)$.
- (c) Express the answer to (a) as a percentage of the answer to (b).

Solution

- (a) Using (4.3.7) with $L = 2048$, the number of real FLOPs for fast convolution is

$$\begin{aligned}n_{\text{fast}} &= 12L \log_2(2L) + 8L + 4 \\&= 12(2048) \log_2(4096) + 8(2048) + 4 \\&= 311300\end{aligned}$$

- (b) Using (4.3.8) with $L = 2048$, the number of real FLOPs for a direct linear convolution is

$$\begin{aligned}n_{\text{dir}} &= L^2 \\&= (2048)^2 \\&= 4194304\end{aligned}$$

- (c) The ratio of computational efforts when $L = 2048$ is

$$\begin{aligned}\alpha &= \frac{100n_{\text{fast}}}{n_{\text{dir}}} \\&= \frac{100(311300)}{4194304} \\&= 7.422 \%\end{aligned}$$