$\boxed{5.14}$ Consider the following IIR filter.

$$H(z) = \frac{10(z^2-4)(z^2+0.25)}{(z^2+0.64)(z^2-0.16)}$$

(a) Find $H_{\min}(z)$, the minimum-phase version of $H(z)$.

(b) Sketch the poles and zeros of $H_{\min}(z)$.

(c) Find an allpass filter $H_{\text{all}}(z)$ such that $H(z) = H_{\text{all}}(z)H_{\min}(z)$.

(d) Sketch the poles and zeros of $H_{\text{all}}(z)$.

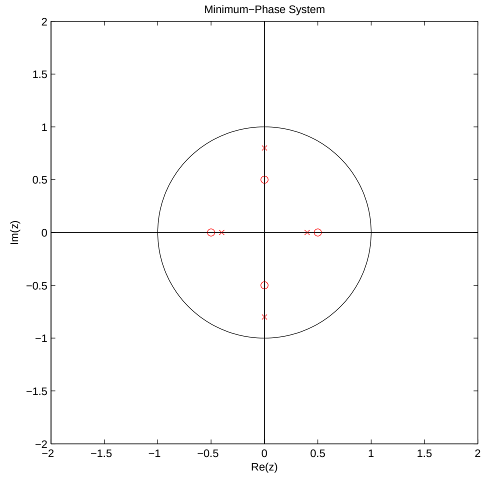
Solution

(a) The factored numerator polynomial is

$$b(z) = 10(z-2)(z+2)(z-j0.5)(z+j0.5)$$

Thus there are two zeros outside the unit circle at $z_{1,2} = \pm 2$. Using (5.4.5), we replace each of these zeros by its reciprocal and multiply by the negative of the zero. This yields

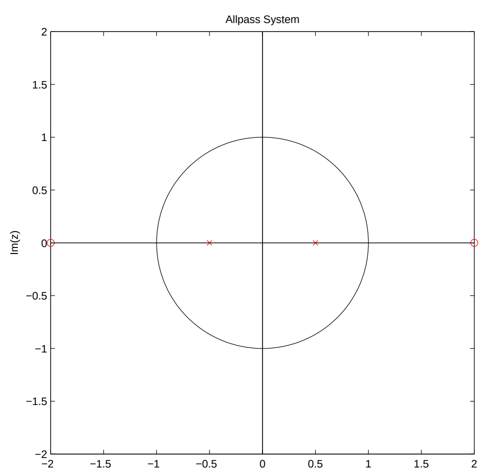
$$\begin{aligned}
H_{\min}(z) &= \frac{z_1 z_2 (z-1/z_1)(z-1/z_2)H(z)}{(z-z_1)(z-z_2)} \\
&= \frac{-4(z-0.5)(z+0.5)H(z)}{(z-2)(z+2)} \\
&= \frac{-4(z^2-0.25)10(z^2+0.25)}{(z^2+0.64)(z^2-0.16)} \\
&= \frac{-40(z^2-0.25)(z^2+0.25)}{(z^2+0.64)(z^2-0.16)}
\end{aligned}$$



(b) **Pole-Zero Plot**

(c) Since $H(z) = H_{\text{all}}(z)H_{\text{min}}(z)$, we can solve for the allpass factor as follows.

$$\begin{aligned}
H_{\text{all}}(z) &= H(z)H_{\text{min}}^{-1}(z) \\
&= \frac{10(z^2-4)(z^2+0.25)}{(z^2+0.64)(z^2-0.16)}\left[\frac{-40(z^2-0.25)(z^2+0.25)}{(z^2+0.64)(z^2-0.16)}\right]^{-1} \\
&= \frac{10(z^2-4)(z^2+0.25)}{(z^2+0.64)(z^2-0.16)}\left[\frac{(z^2+0.64)(z^2-0.16)}{-40(z^2-0.25)(z^2+0.25)}\right] \\
&= \frac{-0.25(z^2-4)}{z^2-0.25}
\end{aligned}$$



(d) **Pole-Zero Plot**