2.22 Consider a discrete-time system described by the following impulse response.

$$h(k) = [2 - (.5)^k + (.2)^{k-1}]u(k)$$

(a) Find the transfer function $H(z)$.
(b) Find the difference equation.
(c) Sketch the signal flow graph.

Solution

(a) Using Table 2.2.1, the transfer function is

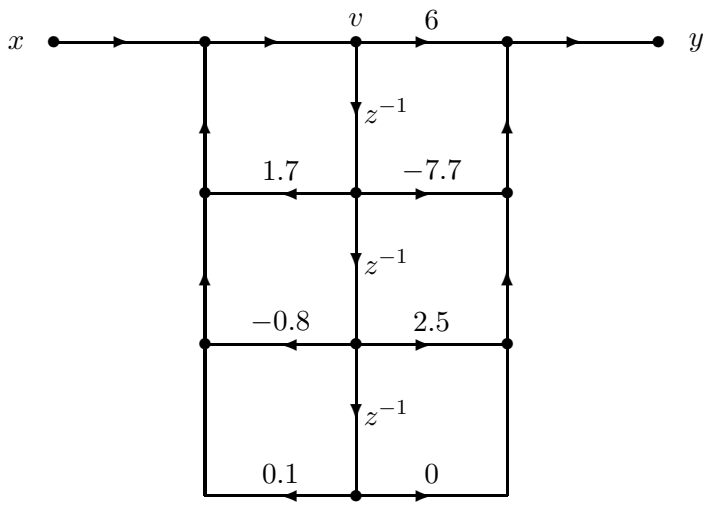
$$\begin{aligned} H(z) &= Z\{h(k)\} \\ &= \frac{2z}{z-1} - \frac{z}{z-0.5} + \frac{5z}{z-0.2} \\ &= \frac{2z(z-0.5)(z-0.2) - z(z-1)(z-0.2) + 5z(z-1)(z-0.5)}{(z-1)(z-0.5)(z-0.2)} \\ &= \frac{z[2(z^2-0.7z+0.1) - (z^2-1.2z+0.2) + 5(z^2-1.5z+0.5)]}{(z-1)(z-0.5)(z-0.2)} \\ &= \frac{z(6z^2-7.7z+2.5)}{(z-1)(z-0.5)(z-0.2)} \end{aligned}$$

(b) Expanding the denominator and converting to negative powers of z yields

$$\begin{aligned} H(z) &= \frac{6z^3-7.7z^2+2.5z}{(z-1)(z^2-0.7z+0.1)} \\ &= \frac{6z^3-7.7z^2+2.5z}{z^3-0.7z^2+0.1z-z^2+0.7z-0.1} \\ &= \frac{6z^3-7.7z^2+2.5z}{z^3-1.7z^2+0.8z-0.1} \\ &= \frac{6-7.7z^{-1}+2.5z^{-2}}{1-1.7z^{-1}+0.8z^{-2}-0.1z^{-3}} \end{aligned}$$

Thus, by inspection, the difference equation is

$$y(k) = 1.7y(k-1) - 0.8y(k-2) + 0.1y(k-3) + 6x(k) - 7.7x(k-1) + 2.5x(k-2)$$



(c) Signal Flow Graph