3.7 Consider the following discrete-time signal.

$$x = [1, 2, 1, 0]^T$$

(a) Find $X(i) = \text{DFT}\{x(k)\}$.
(b) Compute and sketch the magnitude spectrum $A(i)$.
(c) Compute and sketch the phase spectrum $\phi(i)$.
(d) Compute and sketch the power density spectrum $S_N(i)$.

Solution

(a) Here $W_4 = \exp(-j2\pi/4) = -j$. Using Definition 3.3.1

$$\begin{aligned}
X(0) &= \sum_{k=0}^{3} x(k) \\
&= 1 + 2 + 1 \\
&= 4 \\
X(1) &= \sum_{k=0}^{3} x(k) W_4^k \\
&= 1 + 2(-j) + 1(-1) \\
&= -j2 \\
X(2) &= \sum_{k=0}^{3} x(k) (W_4^2)^k \\
&= 1 + 2(-1) + 1(1) \\
&= 0 \\
X(3) &= \sum_{k=0}^{3} x(k) (W_4^3)^k \\
&= 1 + 2(j) + 1(-1) \\
&= j2
\end{aligned}$$

Thus the DFT of $x(k)$ is

$$X = [4, -j2, 0, j2]^T$$

(b) The magnitude spectrum of $x(k)$ is

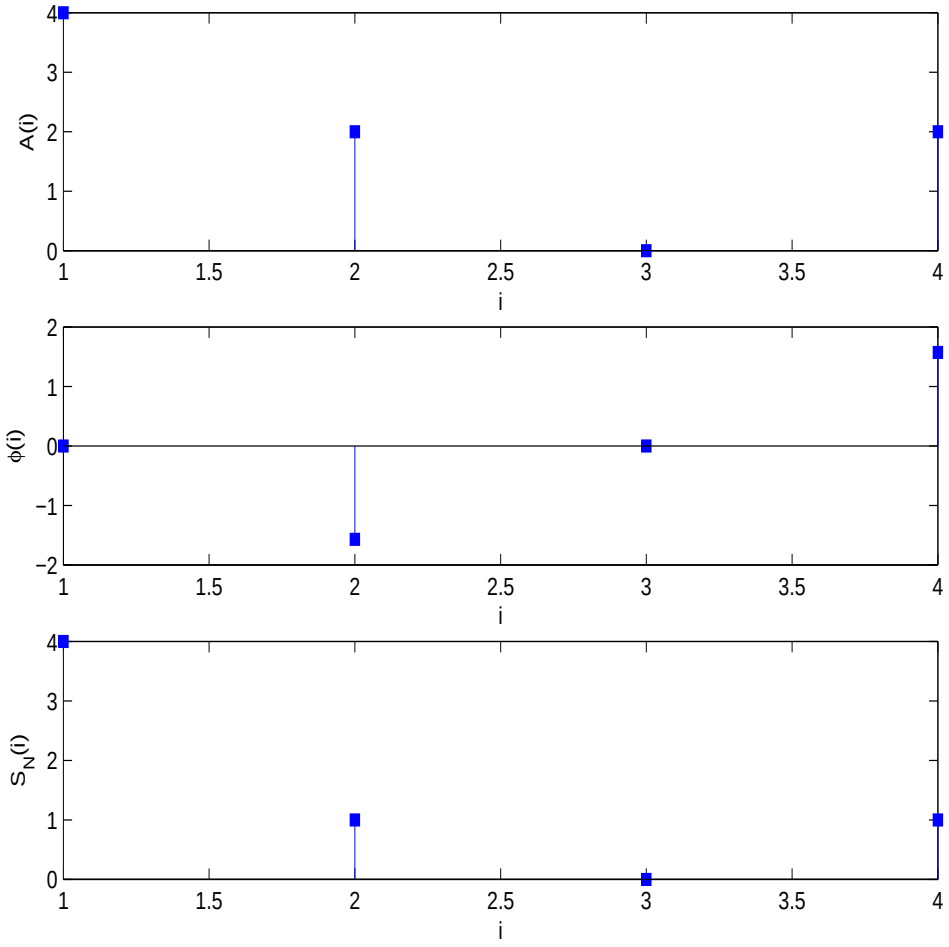
$$\begin{aligned} A &= |X| \\ &= [4, 2, 0, 2]^T \end{aligned}$$

(c) The phase spectrum of $x(k)$ is

$$\begin{aligned} A &= \angle X \\ &= [0, -\pi/2, 0, \pi/2]^T \end{aligned}$$

(d) The power density spectrum of $x(k)$ is

$$\begin{aligned} S_N &= |X|^2/4 \\ &= [4, 1, 0, 1]^T \end{aligned}$$



Magnitude, Phase, and Power Density Spectra of $x = [1, 2, 1, 0]^T$.