$\boxed{7.3}$ Consider the problem of designing a sampling rate decimator with a decimation factor of $M = 8$.

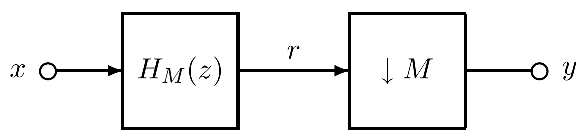
(a) Sketch a block diagram of the sampling rate decimator.

(b) Find the required frequency response of the ideal anti-aliasing digital filter. Here f_s is the sampling rate of $x(k)$.

(c) Using Table 6.2.1 and Table 6.2.2, design an anti-aliasing filter of order $m = 40$ using the windowing method with a Hanning window.

(d) Find the difference equation for the sampling rate decimator.

Solution



Sampling Rate Decimator Block Diagram

(b) From (7.2.3), the ideal cutoff frequency is

$$\begin{aligned} F_M &= \frac{f_s}{2M} \\ &= \frac{f_s}{16} \end{aligned}$$

The required frequency response for the ideal anti-aliasing digital filter is then

$$H_M(f) = \begin{cases} 1 \ , & 0 \le |f| < f_s/16 \\ 0 \ , & f_s/16 \le |f| \le f_s/2 \end{cases}$$

(c) Using Table 6.2.1 and Table 6.2.2 with $m = 40$, $p = m/2$, and the Hanning window, the FIR filter coefficients are

$$\begin{aligned} b_i &= w(i)h(i) \\ &= 0.5\left[1 - \cos\left(\frac{\pi i}{0.5m}\right)\right]\frac{\sin[2\pi(i-p)F_M T]}{\pi(i-p)} \\ &= 0.5\left[1 - \cos\left(\frac{\pi i}{20}\right)\right]\frac{\sin[2\pi(i-20)/16]}{\pi(i-20)} \qquad , \qquad i \ne 20 \end{aligned}$$

The middle term is

$$\begin{aligned} b_{20} &= w(p)h(p) \\ &= 0.5[1 - \cos(\pi)]2F_M T \\ &= 2/16 \\ &= 0.125 \end{aligned}$$

(d) From (7.2.4) the decimator difference equation is

$$\begin{aligned} y(k) &= \sum_{i=0}^{m} b_i x(Mk - i) \\ &= \sum_{i=0}^{40} b_i x(20k - i) \end{aligned}$$