$\boxed{4.10}$ Suppose $x(k)$ is as follows.

$$x = [5, 7, -2, 4, 8, 6, 1]^T$$

(a) Construct a 3-point signal $y(k)$ such that $r_{xy}(k)$ reaches its peak positive value at $k = 3$ and $|y(0)| = 1$.

(b) Construct a 4-point signal $y(k)$ such that $r_{xy}(k)$ reaches its peak negative value at $k = 4$ and $|y(0)| = 1$.

Solution

(a) Recall that the cross-correlation $r_{xy}(k)$ measures the degree which $y(k)$ is similar to a subsignal of $x(k)$. In order for $r_{xy}(k)$ to reach its maximum positive value at $k = 3$, we must have maximum positive correlation starting at $k = 3$. Thus for some positive constant α it is necessary that

$$\begin{aligned} y &= \alpha[x(3), x(4), x(5)]^T \\ &= \alpha[4, 8, 6]^T \end{aligned}$$

The constraint, $|y(0)| = 1$, implies that the positive scale factor must be $\alpha = 1/4$. Thus

$$y = [1, 2, 1.5]^T$$

(b) In order for $r_{xy}(k)$ to reach its maximum negative value at $k = 4$, we must have maximum negative correlation starting at $k = 4$. Thus for some positive constant α we need

$$\begin{aligned} y &= -\alpha[x(2), x(3), x(4), x(5)]^T \\ &= \alpha[2, -4, -8, -6]^T \end{aligned}$$

The constraint, $|y(0)| = 1$, implies that the positive scale factor must be $\alpha = 1/2$. Thus

$$y = [1, -2, -4, -3]^T$$

The answers to (a) and (b) can be verified using the FDSP toolbox function *f_corr*.