$\boxed{9.8}$ Suppose the mean square error is approximated using a running average filter of order $M-1$ as follows.

$$\epsilon(w) \approx \frac{1}{M}\sum_{i=0}^{M-1} e^2(k-i)$$

(a) Find an expression for the gradient vector $\nabla\epsilon(w)$ using this approximation for the mean square error.

(b) Using the steepest descent method and the results from part (a), find a weight update formula.

(c) How many floating point multiplications (FLOPs) are required per iteration to update the weight vector? You can assume that 2μ is computed ahead of time.

(d) Verify that when $M = 1$ the weight update formula reduces to the LMS method.

Solution

(a) Using (9.2.3) and (9.2.4), the partial derviative of this approximation to the mean square error with respect to w_i is

$$\begin{aligned}
\frac{\partial\epsilon(w)}{\partial w_i} &= \left(\frac{\partial}{\partial w_i}\right)\frac{1}{M}\sum_{q=0}^{M-1} e^2(k-q) \\
&= \frac{1}{M}\sum_{q=0}^{M-1} 2e(k-q)\frac{\partial e(k-q)}{\partial w_i} \\
&= \frac{2}{M}\sum_{q=0}^{M-1} e(k-q)\left(\frac{\partial}{\partial w_i}\right)[d(k-q)-y(k-q)] \\
&= \frac{-2}{M}\sum_{q=0}^{M-1} e(k-q)\left(\frac{\partial}{\partial w_i}\right)y(k-q) \\
&= \frac{-2}{M}\sum_{q=0}^{M-1} e(k-q)\left(\frac{\partial}{\partial w_i}\right)w^T\theta(k-q) \\
&= \frac{-2}{M}\sum_{q=0}^{M-1} e(k-q)\theta_i(k-q) \quad , \quad 0 \le i \le m
\end{aligned}$$

Thus the gradient vector is

$$\nabla\epsilon(w) = \frac{-2}{M}\sum_{i=0}^{M-1} e(k-i)\theta(k-i)$$

(b) Using (9.3.3) and the results from part (a), the weight update formula using the the steepest descent method is

$$\begin{aligned} w(k+1) &= w(k) - \mu \nabla \epsilon[w(k)] \\ &= w(k) + \frac{2\mu}{M} \sum_{i=0}^{M-1} e(k-i)\theta(k-i) \end{aligned}$$

(c) Since $\theta(k-i)$ is a vector of length $m+1$, the number of FLOPs per iteration to update the weight vector is

$$n_w = (m+1)(M+1)$$

(d) Starting with the answer to part (b) and setting $M = 1$ yields

$$\begin{aligned} w(k+1) &= w(k) + \frac{2\mu}{M} \sum_{i=0}^{M-1} e(k-i)\theta(k-i) \\ &= w(k) + 2\mu e(k)\theta(k) \ \surd \end{aligned}$$