$\boxed{9.16}$ Consider the following periodic input and desired output that form the input-output specification for a pseudo-filter. Suppose $f_i = i f_s/(2N)$ for $0 \le i < N$. Find the cross-correlation vector $p$ for this input and desired output.

$$
\begin{aligned}
x(k) &= \sum_{i=0}^{N-1} C_i \cos(2\pi f_i kT) \\
d(k) &= \sum_{i=0}^{N-1} A_i C_i \cos(2\pi f_i kT + \phi_i)
\end{aligned}
$$

## Solution

Using (9.2.13) and the trigonometric identitites from Appendix 4, we have

$$
\begin{aligned}
p_i &= r_{dx}(i) \\
&= E[d(k)x(k-i)] \\
&= E\left[\sum_{q=0}^{N-1} A_q C_q \cos(2\pi f_q kT + \phi_q) \sum_{r=0}^{N-1} C_r \cos(2\pi f_r [k-i]T)\right] \\
&= \sum_{q=0}^{N-1} \sum_{r=0}^{N-1} A_q C_q C_r E[\cos(2\pi f_q kT + \phi_q)\cos(2\pi f_r [k-i]T)] \\
&= 0.5 \sum_{q=0}^{N-1} \sum_{r=0}^{N-1} A_q C_q C_r E[\cos(2\pi \{f_q k + f_r [k-i]\}T + \phi_q) + \cos(2\pi \{f_q k - f_r [k-i]\}T + \phi_q)] \\
&= 0.5 \sum_{q=0}^{N-1} \sum_{r=0}^{N-1} A_q C_q C_r E[\cos(2\pi \{f_q k - f_r [k-i]\}T + \phi_q)] \\
&= 0.5 \sum_{q=0}^{N-1} A_q C_q^2 E[\cos(2\pi f_q iT + \phi_q)] \\
&= 0.5 \sum_{q=0}^{N-1} A_q C_q^2 \cos(2\pi f_q iT + \phi_q) \quad , \quad 0 \le i \le m
\end{aligned}
$$