$\boxed{8.10}$ Consider the problem of designing a lowpass analog Butterworth filter to meet the following specifications.

$$[F_p, F_s, \delta_p, \delta_s] = [300, 500, 0.1, 0.05]$$

(a) Find the minimum filter order n.

(b) For what cutoff frequency F_c is the passband specification exactly met?

(c) For what cutoff frequency F_c is the stopband specification exactly met?

(d) Find a cutoff frequency F_c for which $H_a(s)$ exceeds both the passband and the stopband specification.

Solution

(a) From (8.3.4a), the selectivity factor is

$$\begin{aligned} r &= \frac{F_p}{F_s} \\ &= \frac{300}{500} \\ &= 0.6 \end{aligned}$$

Similarly, from (8.3.4b), the discrimination factor is

$$\begin{aligned} d &= \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}} \\ &= \sqrt{\frac{(0.9)^{-2} - 1}{(0.05)^{-2} - 1}} \\ &= 0.0242 \end{aligned}$$

Next, from (8.4.8), the required filter order is

$$\begin{aligned} n &= \text{ceil}\left[\frac{\ln(d)}{\ln(r)}\right] \\ &= \text{ceil}\left[\frac{\ln(0.0242)}{\ln(0.6)}\right] \\ &= \text{ceil}(7.2813) \\ &= 8 \end{aligned}$$

(b) From (8.4.9), the passband specification will be met exactly using the following cutoff frequency.

$$
\begin{aligned}
F_{cp} &= \frac{F_p}{[(1-\delta_p)^{-2}-1]^{1/(2n)}} \\
&= \frac{300}{[(0.9)^{-2}-1]^{1/(16)}} \\
&= 328.477 \text{ Hz}
\end{aligned}
$$

(c) From (8.4.10), the stopband specification will be met exactly using the following cutoff frequency.

$$
\begin{aligned}
F_{cs} &= \frac{F_s}{[\delta_s^{-2}-1]^{1/(2n)}} \\
&= \frac{500}{[(0.05)^{-2}-1]^{1/(16)}} \\
&= 343.8188 \text{ Hz}
\end{aligned}
$$

(d) Any cutoff frequency in the range $F_{cp} < F_c < F_{cs}$ will exceed both the passband and the stopband specification. For example,

$$
\begin{aligned}
F_c &= \frac{F_{cp}+F_{cs}}{2} \\
&= 336.1698 \text{ Hz}
\end{aligned}
$$