$\boxed{6.10}$ Consider a type 3 linear-phase FIR filter of order $m = 2r$. Find a simplified expression for the amplitude response $A_r(f)$ similar to (6.4.7), but for a type 3 linear-phase FIR filter.

Solution

Starting from (6.4.5), the frequency response of $H(z)$ can be expressed as follows where $\theta = 2\pi fT$.

$$H(f) \quad = \quad \exp(-jr\theta)\sum_{i=0}^{m} b_i \exp[-j(i-r)\theta]$$

For a type 3 filter of order m, the odd symmetry constraint is $b_{m-i} = -b_i$. Since m is even for a type 3 filter, the middle or rth term can be separated out. Euler's identity from Appendix 4 then can be used to combine the remaining pairs of terms as follows.

$$\begin{aligned}
H(f) &= \exp(-jr\theta)\{b_r + \sum_{i=0}^{r-1} b_i \exp[-j(i-r)\theta] + b_{m-i}\exp[-j(m-i-r)\theta]\} \\
&= \exp(-jr\theta)\sum_{i=0}^{r-1} b_i\{\exp[-j(i-r)\theta] - \exp[-j(m-i-r)\theta]\} \\
&= \exp(-jr\theta)\sum_{i=0}^{r-1} b_i\{\exp[-j(i-r)\theta] - \exp[j(i-r-m+2r)\theta]\} \\
&= \exp(-jr\theta)\sum_{i=0}^{r-1} b_i\{\exp[-j(i-r)\theta] - \exp[j(i-r)\theta]\} \\
&= -j2\exp(-jr\theta)\sum_{i=0}^{r-1} b_i \sin[(i-r)\theta)] \\
&= j\exp(-jr\theta)A_r(f)
\end{aligned}$$

Here $b_r = 0$ due to the odd symmetry. Recall that $\theta = 2\pi fT$. Thus the amplitude response for a type 3 linear-phase filter is

$$A_r(f) \quad = \quad -2\sum_{i=0}^{r-1} b_i \sin[2\pi(i-r)fT]$$