C++ midterm 1082

Problem 1

- Given the coordinates of a triangle in xy plane, A(10, 3), B(-4, 4), and C(2,-9). Ask the user to input the coordinates of a point P. Check if this point is inside the triangle, on the border, or outside the triangle ΔABC. You can use outer products of vectors to calculate the area of a triangle.

$$\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$
$$= \frac{1}{2}\left|(xb - xa)(yc - ya) - (xc - xa)(yb - ya)\right|$$

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Calculate ΔABC

Calculate ΔPAB

If ΔPAB is not equal to zero

Calculate ΔPAC

If ΔPAC is not equal to zero

Calculate ΔPBC

If ΔPBC is not equal to zero

If ΔABC equals to the sum of ΔPAB+ΔPAC+ΔPBC

The point is inside ΔABC (1)

Else

The point is not inside ΔABC (3)

Else

The point is on the line between BC (2)

Else

The point is on the line between AC (2)

Else

The point is on the line between AB (2)

This algorithm has a bug!! For example, if ΔPAC=0, it still could be outside the triangle.

Problem 1

- Useful tips:
  - Use a simpler case for testing
  - Floating point arithmetic

Problem 2

- Find the largest distance between two adjacent prime numbers in the range of [2,10000]. Hint: we know that 2 is a prime number. Declare a variable to save the previous prime number (oldprime, for example) and set it equal to 2. Declare a variable to store the maximum distance (maxdist, for example) and set it to be zero at the beginning. When you find the next prime number, find the distance to the previous prime number. If it is larger than replace maxdist by this new distance.

Set maxdist equal to 1
Set oldprime equal to 2
Use a for loop to run between 3 and 10000
    Check if the number is a prime number
    If yes
        Calculate the distance to the previous prime number
        If the new distance is larger than maxdist
            replace maxidist by the new distance.
        Replace oldprime by the new prime number
Output the maxdist

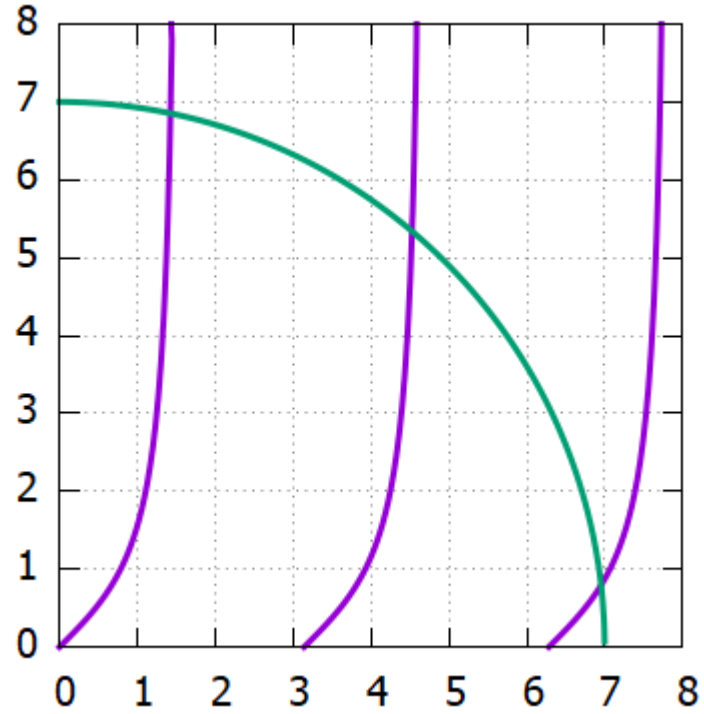
Problem 2

- Useful tips:
  - Double for-loop
  - To check the possible factors, one only needs to search the range from 2 to $\sqrt{i}$
  - Use a flag to signal if a number is a prime number.

- What if you are asked to output the adjacent prime numbers that give the largest distance?

Problem 3

- Find all the roots for $\tan x = \sqrt{r^2 - x^2}$, for a given r.
  - The number of roots depends on r.
  - If $n\pi \leq r < (n+1)\pi$, there are n+1 roots, one in each interval of π.
  - Steps
    - Find n
    - Use a loop to search for root in each interval
    - Within the loop, use the bisection method with the initial interval $[(k-1)\pi, k\pi]$ for $k = 1,2,..,n$.



Find n (take the integer of r/pi)

Use a for loop for $i = 1\ to\ \ n + 1$ to find root in each interval $[(i-1)\pi, i\pi]$

$a = (i-1)\pi, b = i\pi$ *(b=r for the last interval),* $fa = f(a), fb = f(b)$

Use a while loop (the interval is larger than 2* tol)

Take the midpoint $c = (a+b)/2$ and compute $fc = f(c)$

if($fc * fa < 0$)

b=c

else

a=c

fa=fc

output $(a+b)/2$

Problem 3

- Useful tips:
  - Problematic with $\tan n\pi$
  - $\sqrt{r^2 - x^2}$ is defined only for $x \in [0, r]$

Additional problem

- A twin prime is a prime number that is either 2 less or 2 more than another prime number—for example, either member of the twin prime pair (41, 43). In other words, a twin prime is a prime that has a prime gap of two.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Additional problem

- Find the number of twin prime number pairs between 2 and 10000.

- Find the number of twin prime number pairs between n and m.

- Find the number of twin prime numbers between n and m.
- Print out all twin prime numbers between n and m.