QCD 求和规则（sum rules）

求解 Λ 粒子的质量

目录

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1 Λ 质量（实验测得质量约为 $1115.68MeV/c^2$）

此处使用的方法为 QCD 求和规则（QCD sum rules）

1.1 构造流

$L=0$，$J^p=1/2^+$ 的强子八重态的流

$$J^{\Lambda}(x)=\sqrt{\frac{2}{3}}\epsilon^{abc}[(u^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x))\gamma_5\gamma^\mu d^c(x)-(d^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x))\gamma_5\gamma^\mu u^c(x)] \tag{1}$$

考虑 $J^P=\frac{1}{2}^+$，两点函数以及关联函数

$$\langle 0|\,T[J^\Lambda(x)\bar{J}^\Lambda(0)]\,|0\rangle \tag{2}$$

$$\Pi(p)=\mathrm{i}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}px}\,\langle 0|\,T[J^\Lambda(x)\bar{J}^\Lambda(0)]\,|0\rangle=\Pi_1(q^2)+\not{q}\Pi_2(q^2) \tag{3}$$

考虑含有夸克核胶子凝聚的夸克传播子（仅三项）

$$\begin{aligned}\mathrm{i}S^{ab}&\equiv\langle 0|\,T[q^a(x)\bar{q}^b(0)]\,|0\rangle\\&=\mathrm{i}\frac{\delta^{ab}}{2\pi^2x^4}\not{x}+\frac{\mathrm{i}}{32\pi^2}\frac{\lambda^n_{ab}}{2}g_cG^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\not{x}+\not{x}\sigma^{\mu\nu})-\frac{\delta^{ab}}{12}\langle\bar{q}q\rangle\\&=\frac{\mathrm{i}\delta^{ab}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^n_{ab}}{2}G^{n\ \mu\nu}\epsilon_{\alpha\beta\mu\nu}x^\alpha\gamma^\beta\gamma^5-\frac{\delta^{ab}}{12}\langle\bar{q}q\rangle\end{aligned} \tag{4}$$

对于 s 夸克，多考虑以下两项

$$-\frac{m_s\delta^{ab}}{4\pi^2x^2}+\frac{\mathrm{i}\delta^{ab}m_s\langle\bar{s}s\rangle}{48}\not{x} \tag{5}$$

费曼图如下。

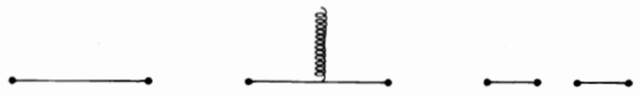

图 1: 共有的传播子部分，公式 4

这里有一个小小的数学技巧:

$$\sigma^{\mu\nu}\not{x}+\not{x}\sigma^{\mu\nu}=-2\epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5x_\rho=2\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\sigma x_\rho$$

图 2: s 夸克额外考虑的传播子部分，公式 5

1.2 两点函数的计算与费曼图

$$
\begin{aligned}
&\langle 0|\,T[J^{\Lambda}(x)\bar{J}^{\Lambda}(0)]\,|0\rangle = \langle 0|\,T[J^{\Lambda}(x)J^{\Lambda\dagger}(0)\gamma^0]\,|0\rangle \\
&= \sqrt{\frac{2}{3}}\epsilon^{abc}[(u^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x))\gamma_5\gamma^\mu d^c(x) - (d^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x))\gamma_5\gamma^\mu u^c(x)] \\
&\quad\times (\sqrt{\frac{2}{3}}\epsilon^{a'b'c'}[(u^{a'\mathrm{T}}(0)\mathcal{C}\gamma_\nu s^{b'}(0))\gamma_5\gamma^\nu d^{c'}(0) - (d^{a'\mathrm{T}}(0)\mathcal{C}\gamma_\nu s^{b'}(0))\gamma_5\gamma^\nu u^{c'}(0)])^\dagger\gamma^0 \\
&= \frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}[(u^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x))\gamma_5\gamma^\mu d^c(x) - (d^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x))\gamma_5\gamma^\mu u^c(x)] \\
&\quad\times [d^{c'\dagger}(0)(\gamma^\nu)^\dagger\gamma_5^\dagger s^{b'\dagger}(0)(\gamma_\nu)^\dagger\mathcal{C}^\dagger(u^{a'\mathrm{T}}(x))^\dagger - u^{c'\dagger}(0)(\gamma^\nu)^\dagger\gamma_5^\dagger s^{b'\dagger}(0)(\gamma_\nu)^\dagger\mathcal{C}^\dagger(d^{a'\mathrm{T}}(0))^\dagger]\gamma^0 \\
&= \frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}[u^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x)\gamma_5\gamma^\mu d^c(x) - d^{a\mathrm{T}}(x)\mathcal{C}\gamma_\mu s^b(x)\gamma_5\gamma^\mu u^c(x)] \\
&\quad\times [\bar{d}^{c'}(0)\gamma^\nu\gamma^0\gamma_5\bar{s}^{b'}(0)\gamma_\nu\mathcal{C}(\bar{u}^{a'}(0))^\mathrm{T} - \bar{u}^{f}(0)\gamma^\nu\gamma^0\gamma_5\bar{s}^{e}(0)\gamma_\nu\mathcal{C}(\bar{d}^{d}(0))^\mathrm{T}]\gamma^0 \\
&= \frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}[u^{a\mathrm{T}}\mathcal{C}\gamma_\mu s^b\gamma_5\gamma^\mu d^c\bar{d}^{c'}\gamma^\nu\gamma^0\gamma_5\bar{s}^{b'}\gamma_\nu\mathcal{C}\bar{u}^{a'\mathrm{T}} \\
&\qquad\quad - u^{a\mathrm{T}}\mathcal{C}\gamma_\mu s^b\gamma_5\gamma^\mu d^c\bar{u}^{c'}\gamma^\nu\gamma^0\gamma_5\bar{s}^{b'}\gamma_\nu\mathcal{C}\bar{d}^{a'\mathrm{T}} \\
&\qquad\quad - d^{a\mathrm{T}}\mathcal{C}\gamma_\mu s^b\gamma_5\gamma^\mu u^c\bar{d}^{c'}\gamma^\nu\gamma^0\gamma_5\bar{s}^{b'}\gamma_\nu\mathcal{C}\bar{u}^{a'\mathrm{T}} \\
&\qquad\quad + d^{a\mathrm{T}}\mathcal{C}\gamma_\mu s^b\gamma_5\gamma^\mu u^c\bar{u}^{c'}\gamma^\nu\gamma^0\gamma_5\bar{s}^{b'}\gamma_\nu\mathcal{C}\bar{d}^{a'\mathrm{T}}]\gamma^0
\end{aligned}
\tag{6}
$$

第 5 步是因为 $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$，$\mathcal{C} = \mathrm{i}\gamma^0\gamma^2, \mathcal{C}^\dagger = -\mathcal{C}$，同时利用 $\gamma^0\gamma^0 = I_{4\times4}$ 简化 $u^{a\mathrm{T}}(x)$，第六步省略了 x 与 0 两点，以 abc、$a'b'c'$ 代表含 x、0 的费米子算符。由于这个流里面的算符都不一样（uds），因此缩并比较简单。

$$
\begin{aligned}
&Equation\ 6 = \\
&\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}[\underset{1}{u}^{a\mathrm{T}}\mathcal{C}\gamma_\mu\underset{2}{s}^b\gamma_5\gamma^\mu\underset{3}{d}^c\underset{4}{\bar{d}}^{c'}\gamma^\nu\gamma^0\gamma_5\underset{5}{\bar{s}}^{b'}\gamma_\nu\mathcal{C}\underset{6}{\bar{u}}^{a'\mathrm{T}} - \underset{1}{u}^{a\mathrm{T}}\mathcal{C}\gamma_\mu\underset{2}{s}^b\gamma_5\gamma^\mu\underset{3}{d}^c\underset{4}{\bar{u}}^{c'}\gamma^\nu\gamma^0\gamma_5\underset{5}{\bar{s}}^{b'}\gamma_\nu\mathcal{C}\underset{6}{\bar{d}}^{a'\mathrm{T}} \\
&- \underset{1}{d}^{a\mathrm{T}}\mathcal{C}\gamma_\mu\underset{2}{s}^b\gamma_5\gamma^\mu\underset{3}{u}^c\underset{4}{\bar{d}}^{c'}\gamma^\nu\gamma^0\gamma_5\underset{5}{\bar{s}}^{b'}\gamma_\nu\mathcal{C}\underset{6}{\bar{u}}^{a'\mathrm{T}} + \underset{1}{d}^{a\mathrm{T}}\mathcal{C}\gamma_\mu\underset{2}{s}^b\gamma_5\gamma^\mu\underset{3}{u}^c\underset{4}{\bar{u}}^{c'}\gamma^\nu\gamma^0\gamma_5\underset{5}{\bar{s}}^{b'}\gamma_\nu\mathcal{C}\underset{6}{\bar{d}}^{a'\mathrm{T}}]\gamma^0
\end{aligned}
$$

费米子算符缩并的时候，只需要计算上述下标数字排列的逆序数，则可以知道该项贡献正负号。例如，使用 Casimir，以数字代表矩阵相乘的下标，相同的数字等价于矩阵指标求和。注意括号内为标量，可以挪动，但是要小心费米子算符满足反对易关系。

$$
(\underset{1}{u}^{a\mathrm{T}}\underset{12}{\mathcal{C}\gamma_\mu}\underset{2}{s}^b)\underset{+33}{\gamma_5\gamma^\mu}\underset{3}{d}^c\underset{4}{\bar{d}}^{c'}\underset{44_+}{\gamma^\nu\gamma^0\gamma_5}(\underset{5}{\bar{s}}^{b'}\underset{56}{\gamma_\nu\mathcal{C}}\underset{6}{\bar{u}}^{a'\mathrm{T}})\underset{+45_+}{\gamma^0}
$$

缩并方式只有 $(16)(25)(34) = \mathrm{i}S^{aa'}\mathrm{i}S^{bb'}\mathrm{i}S^{cc'}$，逆序数 $t(162534)$ 贡献 $(-1)^{t(162534)} = (-)^{0+4+0+2+0} = +1$。因此有

$$
\begin{aligned}
&(\underset{1}{u}^{a\mathrm{T}}\underset{12}{\mathcal{C}\gamma_\mu}\underset{2}{s}^b)\underset{+33}{\gamma_5\gamma^\mu}\underset{3}{d}^c\underset{4}{\bar{d}}^{c'}\underset{44_+}{\gamma^\nu\gamma^0\gamma_5}(\underset{5}{\bar{s}}^{b'}\underset{56}{\gamma_\nu\mathcal{C}}\underset{6}{\bar{u}}^{a'\mathrm{T}})\underset{+45_+}{\gamma^0} \\
&= (-1)^{t(162534)} \times \mathrm{i}\underset{25}{S^{bb'}}\underset{56}{\gamma_\nu\mathcal{C}}(\mathrm{i}\underset{61}{S^{aa'}})^\mathrm{T}\underset{12}{\mathcal{C}\gamma_\mu} \times \underset{+33}{\gamma_5\gamma^\mu}\mathrm{i}\underset{34}{S^{cc'}}\underset{44_+}{\gamma^\nu\gamma^0\gamma_5}\underset{+45_+}{\gamma^0} \\
&= -\mathrm{Tr}[\mathrm{i}S^{bb'}\gamma_\nu\mathcal{C}(\mathrm{i}S^{aa'})^\mathrm{T}\mathcal{C}\gamma_\mu]\gamma_5\gamma^\mu\mathrm{i}S^{cc'}\gamma^\nu\gamma_5
\end{aligned}
\tag{7}
$$

同理可以计算第二、三、四项，得到

$$
\begin{aligned}
&-(\underset{1}{u}^{a\mathrm{T}}\underset{12}{\mathcal{C}\gamma_\mu}\underset{2}{s}^{b})\underset{+33}{\gamma_5\gamma^\mu}\underset{3}{d}^{c}\underset{4}{\bar{u}}^{c'}\underset{44_+}{\gamma^\nu\gamma^0}\gamma_5\underset{5}{\bar{s}}^{b'}\underset{56}{\gamma_\nu\mathcal{C}}\underset{6}{\bar{d}}^{a'\mathrm{T}}\ \underset{+45_+}{\gamma^0}\\
&=-(-)^{t(142536)}\underset{+33}{\gamma_5\gamma^\mu}\mathrm{i}\underset{36}{S}^{ca'}\underset{65}{(\gamma_\nu\mathcal{C})^\mathrm{T}}\underset{52}{(\mathrm{i}S^{bb'})^\mathrm{T}}\underset{21}{(\mathcal{C}\gamma_\mu)^\mathrm{T}}\mathrm{i}\underset{14}{S}^{ac'}\underset{44_+}{\gamma^\nu\gamma^0}\gamma^5\ \underset{+45_+}{\gamma^0}\\
&=-\gamma_5\gamma^\mu\mathrm{i}S^{ca'}(\gamma_\nu\mathcal{C})^\mathrm{T}(\mathrm{i}S^{bb'})^\mathrm{T}(\mathcal{C}\gamma_\mu)^\mathrm{T}\mathrm{i}S^{ac'}\gamma^\nu\gamma^0\gamma^0\gamma^5\\
&=-\gamma_5\gamma^\mu\mathrm{i}S^{ca'}\gamma_\nu\mathcal{C}(\mathrm{i}S^{bb'})^\mathrm{T}\mathcal{C}\gamma_\mu\mathrm{i}S^{ac'}\gamma^\nu\gamma^5
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&-\underset{1}{d}^{a\mathrm{T}}\underset{12}{\mathcal{C}\gamma_\mu}\underset{2}{s}^{b}\underset{+33}{\gamma_5\gamma^\mu}\underset{3}{u}^{c}\underset{4}{\bar{d}}^{c'}\underset{44_+}{\gamma^\nu\gamma^0}\gamma_5\underset{5}{\bar{s}}^{b'}\underset{56}{\gamma_\nu\mathcal{C}}\underset{6}{\bar{u}}^{a'\mathrm{T}}\ \underset{+45_+}{\gamma^0}\\
&=-(-)^{t(142536)}\underset{+33}{\gamma_5\gamma^\mu}\mathrm{i}\underset{36}{S}^{ca'}\underset{65}{(\gamma_\nu\mathcal{C})^\mathrm{T}}\underset{52}{(\mathrm{i}S^{bb'})^\mathrm{T}}\underset{21}{(\mathcal{C}\gamma_\mu)^\mathrm{T}}\mathrm{i}\underset{14}{S}^{ac'}\underset{44_+}{\gamma^\nu\gamma^0}\gamma_5\ \underset{+45_+}{\gamma^0}\\
&=-\gamma_5\gamma^\mu\mathrm{i}S^{ca'}\gamma_\nu\mathcal{C}(\mathrm{i}S^{bb'})^\mathrm{T}\mathcal{C}\gamma_\mu\mathrm{i}S^{ac'}\gamma^\nu\gamma^5
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\underset{1}{d}^{a\mathrm{T}}\underset{12}{\mathcal{C}\gamma_\mu}\underset{2}{s}^{b}\underset{+33}{\gamma_5\gamma^\mu}\underset{3}{u}^{c}\underset{4}{\bar{u}}^{c'}\underset{44_+}{\gamma^\nu\gamma^0}\gamma_5\underset{5}{\bar{s}}^{b'}\underset{56}{\gamma_\nu\mathcal{C}}\underset{6}{\bar{d}}^{a'\mathrm{T}}\ \underset{+45_+}{\gamma^0}\\
&=(-)^{t(162534)}\times\mathrm{Tr}[\mathrm{i}S^{bb'}\gamma_\nu\mathcal{C}(\mathrm{i}S^{aa'})^\mathrm{T}\mathcal{C}\gamma_\mu]\gamma_5\gamma^\mu\mathrm{i}S^{cc'}\gamma^\nu\gamma^0\gamma^5\gamma^0\\
&=-\mathrm{Tr}[\mathrm{i}S^{bb'}\gamma_\nu\mathcal{C}(\mathrm{i}S^{aa'})^\mathrm{T}\mathcal{C}\gamma_\nu]\gamma_5\gamma^\mu\mathrm{i}S^{cc'}\gamma^\nu\gamma_5
\end{aligned}
\tag{10}
$$

注意到第一、四项是一样的，第二、三项形式上是一样的，但是由于要区分 $\mathrm{i}S_u\ or\mathrm{i}S_d$，以下 **udExchange** 表示 ud 交换对称项。因此，缩并得到

$$
\begin{aligned}
&\langle 0|\,T[J^\Lambda(x)\bar{J}^\Lambda(0)]\,|0\rangle\\
&=-\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}\{\mathrm{Tr}[\mathrm{i}S_s^{bb'}\gamma_\nu\mathcal{C}(\mathrm{i}S_u^{aa'})^\mathrm{T}\mathcal{C}\gamma_\mu]\gamma_5\gamma^\mu\mathrm{i}S_d^{cc'}\gamma^\nu\gamma_5+\gamma_5\gamma^\mu\mathrm{i}S_d^{ca'}\gamma_\nu\mathcal{C}(\mathrm{i}S_s^{bb'})^\mathrm{T}\mathcal{C}\gamma_\mu\mathrm{i}S_u^{ac'}\gamma^\nu\gamma^5+\textbf{udExchange}\}\\
&=-\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}\{\mathrm{Tr}[(\frac{\mathrm{i}\delta^{bb'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^n_{bb'}}{2}G^{n\ \mu\nu}\epsilon_{\alpha\beta\mu\nu}x^\alpha\gamma^\beta\gamma^5-\frac{\delta^{bb'}}{12}\langle\bar{s}s\rangle-\frac{m_s\delta^{bb'}}{4\pi^2x^2}+\frac{\mathrm{i}\delta^{bb'}m_s\langle\bar{s}s\rangle}{48}\not{x})\gamma_\nu\mathcal{C}\\
&\times(\frac{\mathrm{i}\delta^{aa'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^m_{aa'}}{2}G^{m\ rs}\epsilon_{pqrs}x^p\gamma^q\gamma^5-\frac{\delta^{aa'}}{12}\langle\bar{u}u\rangle)^\mathrm{T}\mathcal{C}\gamma_\mu]\gamma_5\gamma^\mu\\
&\times(\frac{\mathrm{i}\delta^{cc'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^k_{cc'}}{2}G^{k\ \rho\sigma}\epsilon_{\gamma\delta\rho\sigma}x^\gamma\gamma^\delta\gamma^5-\frac{\delta^{cc'}}{12}\langle\bar{d}d\rangle)\gamma^\nu\gamma_5\\
&+\gamma_5\gamma^\mu(\frac{\mathrm{i}\delta^{ca'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^n_{ca'}}{2}G^{n\ \mu\nu}\epsilon_{\alpha\beta\mu\nu}x^\alpha\gamma^\beta\gamma^5-\frac{\delta^{ca'}}{12}\langle\bar{d}d\rangle)\gamma_\nu\mathcal{C}\\
&\times(\frac{\mathrm{i}\delta^{bb'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^m_{bb'}}{2}G^{m\ rs}\epsilon_{pqrs}x^p\gamma^q\gamma^5-\frac{\delta^{bb'}}{12}\langle\bar{s}s\rangle-\frac{m_s\delta^{bb'}}{4\pi^2x^2}+\frac{\mathrm{i}\delta^{bb'}m_s\langle\bar{s}s\rangle}{48}\not{x})^\mathrm{T}\mathcal{C}\gamma_\mu\\
&\times(\frac{\mathrm{i}\delta^{ac'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^k_{ac'}}{2}G^{k\ \rho\sigma}\epsilon_{\gamma\delta\rho\sigma}x^\gamma\gamma^\delta\gamma^5-\frac{\delta^{ac'}}{12}\langle\bar{u}u\rangle)\gamma^\nu\gamma^5+\textbf{udExchange}\}\\
&=-\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}\{\mathrm{Tr}[(\frac{\mathrm{i}\delta^{bb'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^n_{bb'}}{2}G^{n\ \mu\nu}\epsilon_{\alpha\beta\mu\nu}x^\alpha\gamma^\beta\gamma^5-\frac{\delta^{bb'}}{12}\langle\bar{s}s\rangle-\frac{m_s\delta^{bb'}}{4\pi^2x^2}+\frac{\mathrm{i}\delta^{bb'}m_s\langle\bar{s}s\rangle}{48}\not{x})\gamma_\nu\\
&\times(\frac{\mathrm{i}\delta^{aa'}}{2\pi^2x^4}\not{x}^\mathrm{T}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^m_{aa'}}{2}G^{m\ rs}\epsilon_{pqrs}x^p(\gamma^q)^\mathrm{T}\gamma^5+\frac{\delta^{aa'}}{12}\langle\bar{u}u\rangle)\gamma_\mu]\gamma_5\gamma^\mu\\
&\times(\frac{\mathrm{i}\delta^{cc'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^k_{cc'}}{2}G^{k\ \rho\sigma}\epsilon_{\gamma\delta\rho\sigma}x^\gamma\gamma^\delta\gamma^5-\frac{\delta^{cc'}}{12}\langle\bar{d}d\rangle)\gamma^\nu\gamma_5\\
&+\gamma_5\gamma^\mu(\frac{\mathrm{i}\delta^{ca'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^n_{ca'}}{2}G^{n\ \mu\nu}\epsilon_{\alpha\beta\mu\nu}x^\alpha\gamma^\beta\gamma^5-\frac{\delta^{ca'}}{12}\langle\bar{d}d\rangle)\gamma_\nu\\
&\times(\frac{\mathrm{i}\delta^{bb'}}{2\pi^2x^4}\not{x}^\mathrm{T}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^m_{bb'}}{2}G^{m\ rs}\epsilon_{pqrs}x^p(\gamma^q)^\mathrm{T}\gamma^5+\frac{\delta^{bb'}}{12}\langle\bar{s}s\rangle+\frac{m_s\delta^{bb'}}{4\pi^2x^2}+\frac{\mathrm{i}\delta^{bb'}m_s\langle\bar{s}s\rangle}{48}\not{x}^\mathrm{T})\gamma_\mu\\
&\times(\frac{\mathrm{i}\delta^{ac'}}{2\pi^2x^4}\not{x}-\frac{\mathrm{i}g_c}{16\pi x^2}\frac{\lambda^k_{ac'}}{2}G^{k\ \rho\sigma}\epsilon_{\gamma\delta\rho\sigma}x^\gamma\gamma^\delta\gamma^5-\frac{\delta^{ac'}}{12}\langle\bar{u}u\rangle)\gamma^\nu\gamma^5+\textbf{udExchange}\}
\end{aligned}
\tag{11}
$$

以上化简用到了公式 $\mathcal{C}\gamma^\mu\mathcal{C}=(\gamma^\mu)^\mathrm{T}$，$\mathcal{C}\gamma^\mu\gamma_5\mathcal{C}=(\gamma^\mu)^\mathrm{T}\gamma_5$。费曼图有很多项，考虑各种真空凝聚量，即

$$\langle\bar{\psi}^i_\alpha\psi^j_\beta\rangle,\ \langle G^a_{\mu\nu}G^b_{\rho\sigma}\rangle,\ \langle G^a_{\mu\nu}G^b_{\rho\sigma}G^c_{\lambda\kappa}\rangle$$

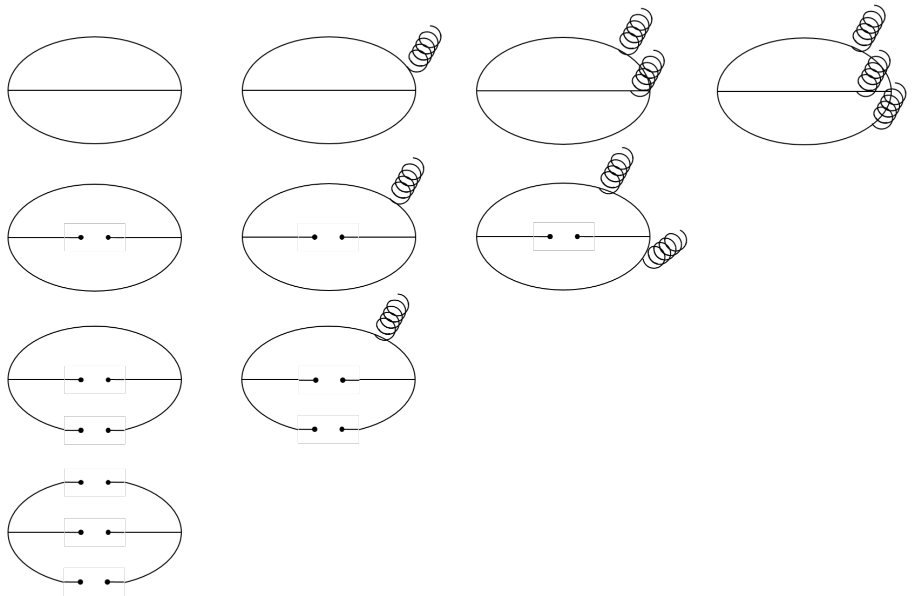

图 3: 费曼图 1

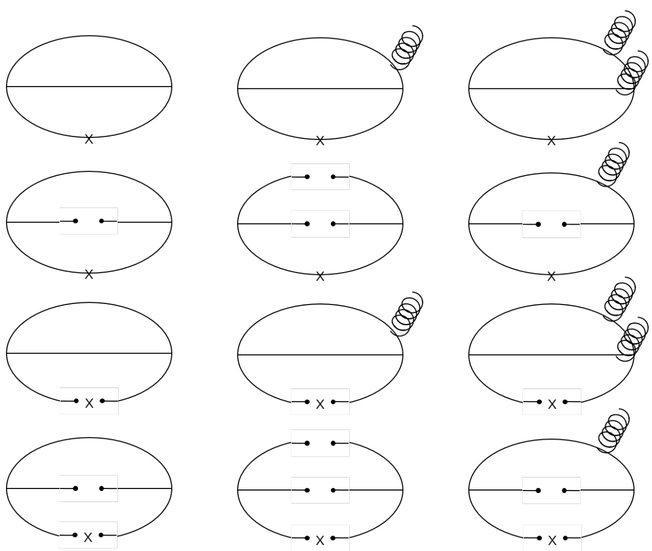

图 4: 费曼图 2

1.3 真空期待值与关联函数

$$
\frac{g_c^2 \langle \mathrm{dd}\rangle \langle G^2\rangle}{768\,\pi^4\,\bar{x}^2} + \frac{g_c^2\, m_s \langle G^2\rangle \langle \mathrm{ss}\rangle\, \bar{\gamma}^\rho.\bar{\gamma}^\lambda.\bar{\gamma}^\sigma.\bar{\gamma}^5\, \bar{\epsilon}^{\lambda\,\rho\,\sigma\,x}}{36\,864\,\pi^4\,\bar{x}^2} - \frac{g_c^2\, m_s \langle G^2\rangle \langle \mathrm{ss}\rangle\, \bar{\gamma}^\sigma.\bar{\gamma}^\lambda.\bar{\gamma}^\rho.\bar{\gamma}^5\, \bar{\epsilon}^{\lambda\,\rho\,\sigma\,x}}{36\,864\,\pi^4\,\bar{x}^2} + \frac{g_c^2 \langle G^2\rangle\, \bar{\gamma}^\rho.\bar{\gamma}^\lambda.\bar{\gamma}^\sigma.\bar{\gamma}^5\, \bar{\epsilon}^{\lambda\,\rho\,\sigma\,x}}{768\,\pi^6\,\bar{x}^6} - \frac{g_c^2 \langle G^2\rangle\, \bar{\gamma}^\sigma.\bar{\gamma}^\lambda.\bar{\gamma}^\rho.\bar{\gamma}^5\, \bar{\epsilon}^{\lambda\,\rho\,\sigma\,x}}{768\,\pi^6\,\bar{x}^6} -
\frac{m_s \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle}{24\,\pi^2\,\bar{x}^2} + \frac{m_s \langle \mathrm{dd}\rangle \langle \mathrm{uu}\rangle}{6\,\pi^2\,\bar{x}^2} - \frac{\langle \mathrm{dd}\rangle}{\pi^4\,\bar{x}^6} + \frac{i\, m_s \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{2\,\pi^4\,\bar{x}^6} - \frac{i\, m_s \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{2\,\pi^4\,\bar{x}^6} - \frac{i \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{6\,\pi^2\,\bar{x}^4} + \frac{12\, i\, \bar{\gamma}\cdot\bar{x}}{\pi^6\,\bar{x}^{10}} + \frac{1}{18} \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle
$$

图 5: 第一项（Mathematica 计算结果，未乘以 $-\frac{2}{3}$ 因子，未考虑 ud 交换项）

$$
\frac{g_c^2 \langle \mathrm{dd}\rangle \langle G^2\rangle}{1536\,\pi^4\,\bar{x}^2} + \frac{i\, g_c^2\, m_s \langle G^2\rangle \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{3072\,\pi^4\,\bar{x}^2} - \frac{g_c^2\, m_s \langle G^2\rangle}{512\,\pi^6\,\bar{x}^4} - \frac{g_c^2 \langle G^2\rangle \langle \mathrm{ss}\rangle}{1536\,\pi^4\,\bar{x}^2} + \frac{g_c^2 \langle G^2\rangle \langle \mathrm{uu}\rangle}{1536\,\pi^4\,\bar{x}^2} + \frac{i\, g_c^2 \langle G^2\rangle\, \bar{\gamma}\cdot\bar{x}}{128\,\pi^6\,\bar{x}^6} +
\frac{1}{288}\, i\, m_s \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x} - \frac{m_s \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle}{48\,\pi^2\,\bar{x}^2} + \frac{m_s \langle \mathrm{dd}\rangle \langle \mathrm{uu}\rangle}{12\,\pi^2\,\bar{x}^2} - \frac{i\, m_s \langle \mathrm{dd}\rangle\, \bar{\gamma}\cdot\bar{x}}{4\,\pi^4\,\bar{x}^6} - \frac{i \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{12\,\pi^2\,\bar{x}^4} + \frac{i \langle \mathrm{dd}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{12\,\pi^2\,\bar{x}^4} - \frac{\langle \mathrm{dd}\rangle}{2\,\pi^4\,\bar{x}^6} -
\frac{m_s \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle}{48\,\pi^2\,\bar{x}^2} + \frac{i\, m_s \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{4\,\pi^4\,\bar{x}^6} - \frac{i\, m_s \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{4\,\pi^4\,\bar{x}^6} + \frac{3\, m_s}{2\,\pi^6\,\bar{x}^8} - \frac{i \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{12\,\pi^2\,\bar{x}^4} + \frac{\langle \mathrm{ss}\rangle}{2\,\pi^4\,\bar{x}^6} - \frac{\langle \mathrm{uu}\rangle}{2\,\pi^4\,\bar{x}^6} + \frac{6\, i\, \bar{\gamma}\cdot\bar{x}}{\pi^6\,\bar{x}^{10}} + \frac{1}{36} \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle
$$

图 6: 第二项（Mathematica 计算结果，未乘以 $-\frac{2}{3}$ 因子，未考虑 ud 交换项）

$$
-\frac{2}{3}\left(\frac{g_c^2 \langle \mathrm{dd}\rangle \langle G^2\rangle}{384\,\pi^4\,\bar{x}^2} + \frac{i\, g_c^2\, m_s \langle G^2\rangle \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{768\,\pi^4\,\bar{x}^2} - \frac{g_c^2\, m_s \langle G^2\rangle}{256\,\pi^6\,\bar{x}^4} - \frac{g_c^2 \langle G^2\rangle \langle \mathrm{ss}\rangle}{768\,\pi^4\,\bar{x}^2} + \frac{g_c^2 \langle G^2\rangle \langle \mathrm{uu}\rangle}{384\,\pi^4\,\bar{x}^2} + \frac{3\, i\, g_c^2 \langle G^2\rangle\, \bar{\gamma}\cdot\bar{x}}{64\,\pi^6\,\bar{x}^6} +
\frac{1}{144}\, i\, m_s \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x} - \frac{m_s \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle}{12\,\pi^2\,\bar{x}^2} + \frac{m_s \langle \mathrm{dd}\rangle \langle \mathrm{uu}\rangle}{2\,\pi^2\,\bar{x}^2} - \frac{i\, m_s \langle \mathrm{dd}\rangle\, \bar{\gamma}\cdot\bar{x}}{\pi^4\,\bar{x}^6} - \frac{i \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{3\,\pi^2\,\bar{x}^4} + \frac{i \langle \mathrm{dd}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{6\,\pi^2\,\bar{x}^4} - \frac{2 \langle \mathrm{dd}\rangle}{\pi^4\,\bar{x}^6} -
\frac{m_s \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle}{12\,\pi^2\,\bar{x}^2} + \frac{3\, i\, m_s \langle \mathrm{ss}\rangle\, \bar{\gamma}\cdot\bar{x}}{2\,\pi^4\,\bar{x}^6} - \frac{i\, m_s \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{\pi^4\,\bar{x}^6} + \frac{3\, m_s}{\pi^6\,\bar{x}^8} - \frac{i \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle\, \bar{\gamma}\cdot\bar{x}}{3\,\pi^2\,\bar{x}^4} + \frac{\langle \mathrm{ss}\rangle}{\pi^4\,\bar{x}^6} - \frac{2 \langle \mathrm{uu}\rangle}{\pi^4\,\bar{x}^6} + \frac{36\, i\, \bar{\gamma}\cdot\bar{x}}{\pi^6\,\bar{x}^{10}} + \frac{1}{6} \langle \mathrm{dd}\rangle \langle \mathrm{ss}\rangle \langle \mathrm{uu}\rangle\right)
$$

图 7: 真空期待值最终结果

计算结果表明，三胶子凝聚没有贡献。

$$
\begin{aligned}
&\langle 0|\, T[J^\Lambda(x)\bar{J}^\Lambda(0)]\, |0\rangle \\
&= -\frac{2}{3}\{\frac{1}{6}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle(1+\frac{\mathrm{i}m_s}{24}\not{x}) \\
&\quad + \frac{1}{x^2}[\frac{g_c^2\langle\bar{s}s\rangle\langle G^2\rangle}{768\pi^4}(\mathrm{i}m_s\not{x}-1) + \frac{(6\langle\bar{u}u\rangle\langle\bar{d}d\rangle-\langle\bar{u}u\rangle\langle\bar{s}s\rangle-\langle\bar{d}d\rangle\langle\bar{s}s\rangle)m_s}{12\pi^2} + \frac{g_c^2\langle G^2\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)}{384\pi^4} \\
&\quad + \frac{1}{x^4}[-\frac{g_c^2 m_s\langle G^2\rangle}{256\pi^6} + \frac{\mathrm{i}(\langle\bar{u}u\rangle\langle\bar{d}d\rangle-2(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\langle\bar{s}s\rangle)}{6\pi^2}\not{x}] \\
&\quad + \frac{1}{x^6}[\frac{\langle\bar{s}s\rangle-2(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)}{\pi^4} + \frac{3\mathrm{i}g_c^2\langle G^2\rangle}{64\pi^6}\not{x} + \frac{\mathrm{i}m_s(\frac{3}{2}\langle\bar{s}s\rangle-\langle\bar{u}u\rangle-\langle\bar{d}d\rangle)}{\pi^4}\not{x} \\
&\quad + \frac{1}{x^8}\times\frac{3m_s}{\pi^6} + \frac{1}{x^{10}}\times\frac{36\mathrm{i}}{\pi^6}\not{x}\}
\end{aligned}
\tag{12}
$$

那么，关联函数为

$$
\begin{aligned}
\Pi(p) &= \mathrm{i}\int \mathrm{d}^4x\ \mathrm{e}^{\mathrm{i}px}\ \langle 0|\,T[J^{\Lambda}(x)\bar{J}^{\Lambda}(0)]\,|0\rangle \\
&= -\frac{2}{3}\{-\frac{g_c^2\langle\bar{s}s\rangle\langle G^2\rangle}{192\pi^2p^2} - \frac{g_c^2m_s\langle\bar{s}s\rangle\langle G^2\rangle}{96\pi^2p^4}\not{p} + \frac{(6\langle\bar{u}u\rangle\langle\bar{d}d\rangle - \langle\bar{u}u\rangle\langle\bar{s}s\rangle - \langle\bar{d}d\rangle\langle\bar{s}s\rangle)m_s}{3p^2} \\
&+ \frac{g_c^2\langle G^2\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{96\pi^2p^2} + \frac{g_c^2m_s\langle G^2\rangle\ln(-p^2)}{256\pi^4} + \frac{2\langle\bar{s}s\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) - \langle\bar{u}u\rangle\langle\bar{d}d\rangle}{3p^2}\not{p} \\
&+ \frac{[\langle\bar{s}s\rangle - 2(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)]p^2\ln(-p^2)}{8\pi^2} + \frac{3g_c^2\langle G^2\rangle[1+\ln(-p^2)]}{256\pi^4}\not{p} \\
&+ \frac{m_s(\frac{3}{2}\langle\bar{s}s\rangle - \langle\bar{u}u\rangle - \langle\bar{d}d\rangle)[1+\ln(-p^2)]}{4\pi^2}\not{p} - \frac{m_sp^4\ln(-p^2)}{64\pi^4} + \frac{p^4[1+3\ln(-p^2)]}{128\pi^4}\not{p}\} \\
&= \frac{g_c^2\langle\bar{s}s\rangle\langle G^2\rangle}{2^5\cdot3^2\pi^2p^2} + \frac{g_c^2m_s\langle\bar{s}s\rangle\langle G^2\rangle}{2^4\cdot3^2\pi^2p^4}\not{p} + 2\frac{(\langle\bar{u}u\rangle\langle\bar{s}s\rangle + \langle\bar{d}d\rangle\langle\bar{s}s\rangle - 6\langle\bar{u}u\rangle\langle\bar{d}d\rangle)m_s}{3^2p^2} \\
&- \frac{g_c^2\langle G^2\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{2^4\cdot3^2\pi^2p^2} - \frac{g_c^2m_s\langle G^2\rangle\ln(-p^2)}{2^7\cdot3\pi^4} + 2\frac{\langle\bar{u}u\rangle\langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{3^2p^2}\not{p} \\
&+ \frac{[2(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) - \langle\bar{s}s\rangle]p^2\ln(-p^2)}{2^2\cdot3\pi^2} - \frac{g_c^2\langle G^2\rangle[1+\ln(-p^2)]}{2^7\pi^4}\not{p} \\
&+ \frac{m_s(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle - \frac{3}{2}\langle\bar{s}s\rangle)[1+\ln(-p^2)]}{2\cdot3\pi^2}\not{p} + \frac{m_sp^4\ln(-p^2)}{2^5\cdot3\pi^4} - \frac{p^4[1+3\ln(-p^2)]}{2^6\cdot3\pi^4}\not{p}
\end{aligned} \tag{13}
$$

1.4 Borel 变换

接下来是对关联函数作 Borel 变换

$$
\begin{aligned}
\mathcal{B}_{M^2}(\Pi(p)) = \mathcal{B}_{M^2}\Bigg(&\frac{g_c^2\langle\bar{s}s\rangle\langle G^2\rangle}{2^5\cdot3^2\pi^2p^2} + 2\frac{(\langle\bar{u}u\rangle\langle\bar{s}s\rangle + \langle\bar{d}d\rangle\langle\bar{s}s\rangle - 6\langle\bar{u}u\rangle\langle\bar{d}d\rangle)m_s}{3^2p^2} - \frac{g_c^2\langle G^2\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{2^4\cdot3^2\pi^2p^2} \\
&-\frac{g_c^2m_s\langle G^2\rangle\ln(-p^2)}{2^7\cdot3\pi^4} + \frac{[2(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) - \langle\bar{s}s\rangle]p^2\ln(-p^2)}{2^2\cdot3\pi^2} + \frac{m_sp^4\ln(-p^2)}{2^5\cdot3\pi^4} \\
&+\frac{g_c^2m_s\langle\bar{s}s\rangle\langle G^2\rangle}{2^4\cdot3^2\pi^2p^4}\not{p} + 2\frac{\langle\bar{u}u\rangle\langle\bar{d}d\rangle - 2\langle\bar{s}s\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{3^2p^2}\not{p} - \frac{g_c^2\langle G^2\rangle[1+\ln(-p^2)]}{2^7\pi^4}\not{p} \\
&+\frac{m_s(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle - \frac{3}{2}\langle\bar{s}s\rangle)[1+\ln(-p^2)]}{2\cdot3\pi^2}\not{p} - \frac{p^4[1+3\ln(-p^2)]}{2^6\cdot3\pi^4}\not{p}\Bigg)
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
\mathcal{B}_{M^2}(\Pi(p)) = &-\frac{g_c^2\langle\bar{s}s\rangle\langle G^2\rangle}{2^5\cdot3^2\pi^2} + 2\frac{(6\langle\bar{u}u\rangle\langle\bar{d}d\rangle - \langle\bar{u}u\rangle\langle\bar{s}s\rangle - \langle\bar{d}d\rangle\langle\bar{s}s\rangle)m_s}{3^2} + \frac{g_c^2\langle G^2\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{2^4\cdot3^2\pi^2} \\
&+ \frac{g_c^2m_s\langle G^2\rangle M^2}{2^7\cdot3\pi^4} - \frac{[2(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) - \langle\bar{s}s\rangle]M^4}{2^2\cdot3\pi^2} - \frac{m_sM^6}{2^4\cdot3\pi^4} \\
&+ \not{p}\Bigg(\frac{g_c^2m_s\langle\bar{s}s\rangle\langle G^2\rangle}{2^4\cdot3^2\pi^2M^2} + 2\frac{2\langle\bar{s}s\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) - \langle\bar{u}u\rangle\langle\bar{d}d\rangle}{3^2} + \frac{g_c^2\langle G^2\rangle M^2}{2^7\pi^4} \\
&+ \frac{m_s(\frac{3}{2}\langle\bar{s}s\rangle - \langle\bar{u}u\rangle - \langle\bar{d}d\rangle)M^2}{2\cdot3\pi^2} + \frac{M^6}{2^5\pi^4}\Bigg) \\
&\propto \lambda_\Lambda^2\mathrm{e}^{-M_\Lambda^2/M^2}
\end{aligned} \tag{15}
$$

第一求和规则（$\not{p}$）：

$$
\begin{aligned}
-\frac{ba_sm_s}{144M^2} + \frac{(a_u+a_d)a_s}{9} - \frac{a_ua_d}{18} + \frac{(a_u+a_d)m_sM^2}{6} - \frac{a_sm_sM^2}{4} + \frac{bM^2}{32} + \frac{M^6}{8} &= \beta_\Lambda^2\mathrm{e}^{-M_\Lambda^2/M^2} \\
&+ \text{excited states} + \text{the continuum}
\end{aligned} \tag{16}
$$

第二求和规则（1）：

$$
\begin{aligned}
\frac{b[a_s - 2(a_u+a_d)]}{288} + \frac{a_ua_dm_s}{3} - \frac{(a_u+a_d)a_sm_s}{18} + \frac{bm_sM^2}{96} + \frac{[2(a_u+a_d) - a_s]M^4}{12} - \frac{m_sM^6}{12} \\
= \beta_\Lambda^2M_\Lambda\mathrm{e}^{-M_\Lambda^2/M^2} + \text{excited states} + \text{the continuum}
\end{aligned} \tag{17}
$$

其中,

$$
\begin{aligned}
a_q &\equiv -(2\pi)^2\langle\bar{q}q\rangle \\
b &\equiv g_c^2\langle G^2\rangle \\
\beta_\Lambda^2 &\equiv (2\pi)^4\frac{\lambda_\Lambda^2}{4}
\end{aligned}
$$

做出修正 (具体原因还不太了解, 但是**摘抄化用**论文 (DOI:10.1103/physrevd.47.3001) 中的话: To improve further the Q^2 range of the validity of the derived QCD sum rules, it is useful to incorporate the Q^2 dependence of the various terms using the renormalization-group (RG) equation. In particular, it is useful to multiply each term in the operator product expansion by a coefficient $\{[\ln(M^2/\Lambda^2)]/[\ln(\mu^2/\Lambda^2)]\}^{-2\gamma_{J^\Lambda}+\gamma_\Lambda}$, where μ is the renormalization point taken to be 0.5 GeV, Λ is the QCD scale parameter taken to be 0.1 GeV, γ_{J^Λ}, is the anomalous dimension of the current J^Λ, and γ_Λ is the anomalous dimension of the operator under consideration O_Λ(Note that after the Borel transformation, the dependence on Q^2 is translated into a dependence on the Borel mass, M.) Here the anomalous dimensions relevant in our calculation are listed below:

$$
\begin{aligned}
&J^\Lambda(x):\frac{2}{9} && \bar{q}q:\frac{4}{9} \\
&\alpha_s G^n_{\mu\nu}G^{n\ \mu\nu}:0 && m_q:-\frac{4}{9}
\end{aligned}
$$

注意,$\alpha_s=\frac{g_c^2}{4\pi}$。此时,定义 $L=\frac{\ln(M^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}$, $\Lambda=0.1GeV$, $\mu=0.5GeV$,定义 $E_0=1-\mathrm{e}^{-x}$, $E_1=1-(1+x)\mathrm{e}^{-x}$, $E_2=1-(1+x+\frac{1}{2}x^2)\mathrm{e}^{-x}$, $x=W^2/M^2$, $W^2_{p,n}=2.25GeV^2$, $b=0.474GeV^4$。此处 $W^2_{p,n}$ 我不知道是什么,暂且用质子中子的吧。另 $a_u=a_d=a_q=\frac{a_u+a_d}{2}=0.546GeV^3$, $a_s=0.8a_q$, $m_u=0.0051GeV$, $m_d=0.0089GeV$, $m_s=0.095GeV$)

第一求和规则($\not{p}$):

$$
\begin{aligned}
&-\frac{ba_sm_s}{144M^2}L^{-4/9}+\frac{(a_u+a_d)a_s}{9}L^{4/9}-\frac{a_ua_d}{18}L^{4/9}+\frac{(a_u+a_d)m_sM^2}{6}E_0L^{-4/9} \\
&-\frac{a_sm_sM^2}{4}E_0L^{-4/9}+\frac{bM^2}{32}E_0L^{-4/9}+\frac{M^6}{8}E_2L^{-4/9}=\beta_\Lambda^2\mathrm{e}^{-M_\Lambda^2/M^2}
\end{aligned}
\tag{18}
$$

第二求和规则(1):

$$
\begin{aligned}
&\frac{b[a_s-2(a_u+a_d)]}{288}+\frac{a_ua_dm_s}{3}-\frac{(a_u+a_d)a_sm_s}{18}+\frac{bm_sM^2}{96}E_0L^{-8/9} \\
&+\frac{[2(a_u+a_d)-a_s]M^4}{12}E_1-\frac{m_sM^6}{12}E_2L^{-8/9}=\beta_\Lambda^2M_\Lambda\mathrm{e}^{-M_\Lambda^2/M^2}
\end{aligned}
\tag{19}
$$

现在 Λ 粒子的质量还在指数的幂次上,可以作用算符 $M^4\frac{\partial}{\partial M^2}\ln$ 在以上两式,得到质量平方的表达式。第一求和规则($\not{p}$):

$$
\begin{aligned}
&M^4\frac{\partial}{\partial M^2}\ln\Big[-\frac{ba_sm_s}{144M^2}L^{-4/9}+\frac{(a_u+a_d)a_s}{9}L^{4/9}-\frac{a_ua_d}{18}L^{4/9}+\frac{(a_u+a_d)m_sM^2}{6}E_0L^{-4/9} \\
&-\frac{a_sm_sM^2}{4}E_0L^{-4/9}+\frac{bM^2}{32}E_0L^{-4/9}+\frac{M^6}{8}E_2L^{-4/9}\Big]=M_\Lambda^2
\end{aligned}
\tag{20}
$$

第二求和规则(1):

$$
\begin{aligned}
&M^4\frac{\partial}{\partial M^2}\ln\Big[\frac{b[a_s-2(a_u+a_d)]}{288}+\frac{a_ua_dm_s}{3}-\frac{(a_u+a_d)a_sm_s}{18}+\frac{bm_sM^2}{96}E_0L^{-8/9} \\
&+\frac{[2(a_u+a_d)-a_s]M^4}{12}E_1-\frac{m_sM^6}{12}E_2L^{-8/9}\Big]=M_\Lambda^2
\end{aligned}
\tag{21}
$$

数值分析得到 Λ 粒子的质量图如图 8

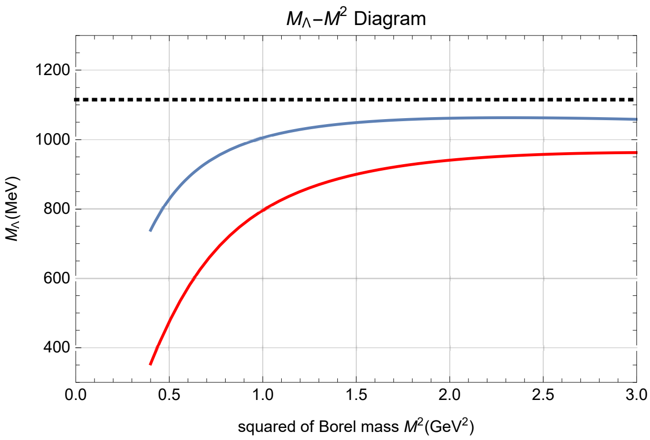

图 8: 红色线为第一求和规则对应的曲线，蓝色线为第二求和规则对应的曲线，虚线为 Λ 粒子的质量 $1115.68 MeV/c^2$

1.5 数值分析

第一求和规则得到的质量为 $940.514 MeV/c^2$，第二求和规则得到的质量为 $1061.46 MeV/c^2$，相对误差为 15.700% 与 4.860%，第一求和规则的相对误差较大。误差的来源可能是夸克传播子的项数选取不够多。

如果对于 u, d 夸克传播子都使用上述五项（即图 1与图 2），那么得到的质量和相对误差分别为 $1014.58 MeV/c^2$, 9.061%（第一求和规则）和 $1180.76 MeV/c^2$, 5.833%（第二求和规则）。

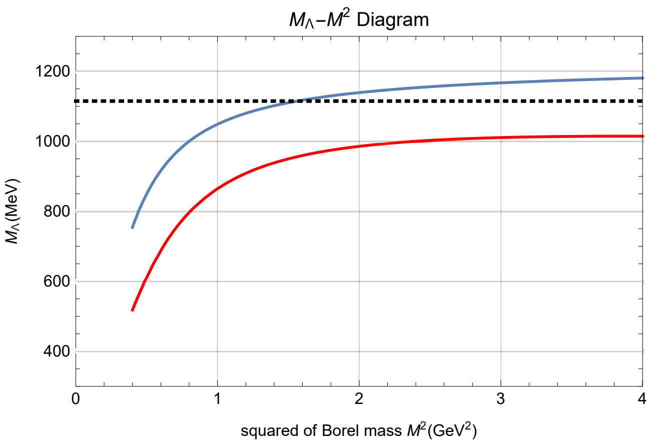

图 9: 红色线为第一求和规则对应的曲线，蓝色线为第二求和规则对应的曲线，虚线为 Λ 粒子的质量 $1115.68 MeV/c^2$

如果采用 12 项夸克传播子，即图 10和图 11所示。

那么有如图 12的结果。第一求和规则得到的质量与相对误差为 $1119.19 MeV/c^2$,0.315%，第二求和规则得到的质量与相对误差为 $1275.56 MeV/c^2$,14.330%。第一求和规则的精度更高，这与经验判断的结果一样（论文中也是这么说的）。

$$
\begin{aligned}
iS^{ab} &\equiv \langle 0|T[q^a(x)\bar{q}^b(0)]|0\rangle \\
&= \frac{i\delta^{ab}}{2\pi^2 x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2} g_c G^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu}) - \frac{\delta^{ab}}{12}\langle \bar{q}q\rangle \\
&\quad + \frac{\delta^{ab}x^2}{192}\langle g_c\bar{q}\sigma Gq\rangle - \frac{ig_c^2\langle\bar{q}q\rangle^2 x^2}{2^5\times 3^5}\delta^{ab}\hat{x} - \frac{g_c^2\langle\bar{q}q\rangle\langle G^2\rangle x^4}{2^9\times 3^3}\delta^{ab} \\
&\quad - \frac{m_q\delta^{ab}}{4\pi^2 x^2} + \frac{m_q}{32\pi^2}\lambda^n_{ab} g_c G^n_{\mu\nu}\sigma^{\mu\nu}\ln(-x^2) - \frac{\delta^{ab}\langle g_c^2 G^2\rangle}{2^9\times 3\pi^2} m_q x^2 \ln(-x^2) \\
&\quad + \frac{i\delta^{ab} m_q\langle\bar{q}q\rangle}{48}\hat{x} - \frac{im_q\langle g_c\bar{q}\sigma Gq\rangle\delta^{ab}x^2\hat{x}}{2^7\times 3^2} - \frac{g_c^2 m_q\langle\bar{q}q\rangle^2 x^4\delta^{ab}}{2^7\times 3^5}\,.
\end{aligned}
$$

图 10: 夸克传播子

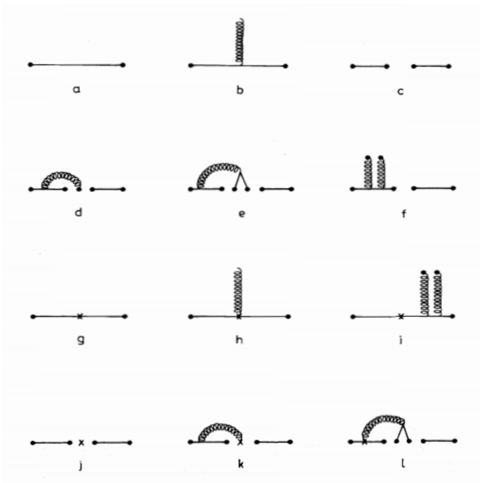

图 11: 夸克传播子费曼图

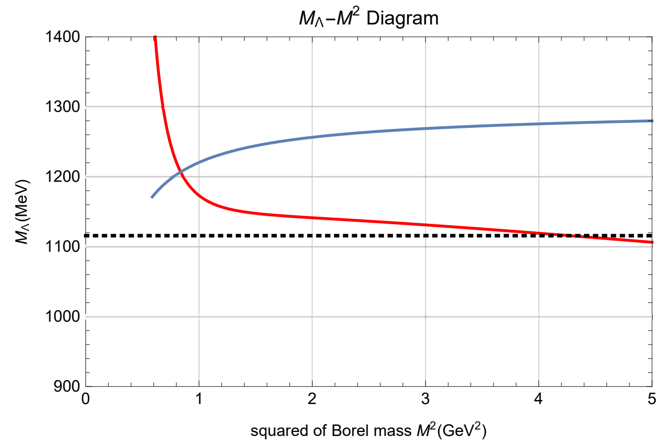

图 12: 红色线为第一求和规则对应的曲线，蓝色线为第二求和规则对应的曲线，虚线为 Λ 粒子的质量 $1115.68 MeV/c^2$