

Theorem 7. *Let a and b two nodes, and assume that we have a mapping $nodesToPNPS$ which maps a node to the highest nested PNPS it is part of. Therefore, we have: $nodesToPNPS[a] = None$ or $b \in nodesToPNPS[a] \Leftrightarrow$ if Node a is intermediate node of a PNPS p then b is inside p*

Proof. \Rightarrow) if $nodesToPNPS[a] = None$ therefore a is not part of any PNPS therefore the second part of the equivalence is True.

if $b \in nodesToPNPS[a]$ therefore b is part of the most nested PNPS that contains a . This implies that b is part of any PNPS that contains a because all other PNPSs containing a contain also $nodesToPNPS[a]$.

\Leftarrow) if Node a is intermediate node of a PNPS p then b is inside p . It means that b is part of any PNPS a is intermediate node of. Therefore b is part of $nodesToPNPS[a]$ since it is a PNPS a is part of,

Theorem 8. *The update of $nodesToPNPS$ in algorithm 1 ensured that $nodesToPNPS$ maps a node to the highest nested PNPS it is part of*

Proof. What we want to prove is that if before the update of $nodesToPNPS$, $nodesToPNPS$ mapped a node to the highest nested PNPS it is part of, then after the update it will still be the case.

When adding a new branch from A to B either the subnetwork from A to B was already a PNPS therefore we should set the value for new nodes only (and that's what the update will do) or the subnetwork from A to B is a new PNPS. if the subnetwork from A to B is a new PNPS, we can distinguish two cases:

1. there is no PNPS between A and B : We are adding a new PNPS and the intermediate nodes of this new PNPS are intermediate nodes in the branch we want to add and intermediate nodes in the path from A to B before adding the new branch.
2. there are PNPS between A and B : same as previously. The only difference is that some intermediate nodes in the path from A to B before adding the new branch will be part of a more nested PNPS. However, for these nodes, $nodesToPNPS$ will not be updated because they are already in the more nested PNPS. What is important is that nodes which is not part of a more nested PNPS are updated. This will be the case because all paths from A to B (before adding the new branch) contains those nodes since they are not intermediate nodes of any PNPS between A and B .