Transfer Entropy on financial time series

Salim Aboudou - Amine Abouseir - Thomas Delrue - Mohamed El Mennaoui - Ilyass Oubaik

CentraleSupélec

Introduction

In Information theory, **Entropy** quantifies the uncertainty carried by a signal. Introduced in 2000, **Transfer Entropy** is a variation of **Shannon Entropy** meant to detect the direction of information flows between time series.

Objectives

- Apply **Transfer Entropy** to 6 ETFs
- Compare 2 approaches of **Transfer Entropy** against **Pearson Correlation**

Definitions

Let X_t be the "target" time series and Y_t the "source" time series.

Shannon Entropy

$$H(X_t) = -\sum_{x_t \in \Omega_x} p(x_t)\log(p(x_t))$$

Transfer Entropy

$$TE_{Y\to X}(t) = H(X_t \mid X_{t-1}) - H(X_t \mid X_{t-1}, Y_{t-1})$$

$$= \sum_{x_{t-1}, y_{t-1}, x_t} p(x_{t-1}, y_{t-1}, x_t)\log\left(\frac{p(x_t \mid x_{t-1}, y_{t-1})}{p(x_t \mid x_{t-1})}\right)$$

Pearson Correlation

$$P(X,Y) = \frac{\sum(x_t - \overline{x})(y_t - \overline{y})}{\sqrt{\sum(x_t - \overline{x})^2}\sqrt{\sum(y_t - \overline{y})^2}}$$

2 approaches of TE vs. Pearson

We consider the time-series of 6 North American sectors indices (Energy, Financial, Industrial, Utilities, Metals, Oil & Gas).

Approach 1 : (Re-sampling)

- Re-sample using 1min time period
- Calculate states (up or down)
- Calculate the lag-1 TE

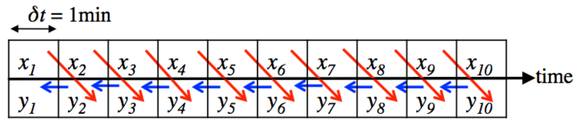

Figure 1: A representation of the lag-1 transfer entropy measure

Approach 2 : (No re-sampling)

- Calculate the lag-1 TE
- y_{t-1} corresponds to the most recent Y state before x_t.

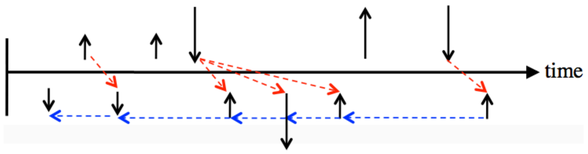

Figure 2: A representation of the asynchronous lag-1 transfer entropy measure

Pearson Correlation

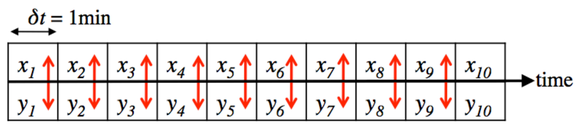

Figure 3: A representation of the Pearson correlation measure

Graphical Results

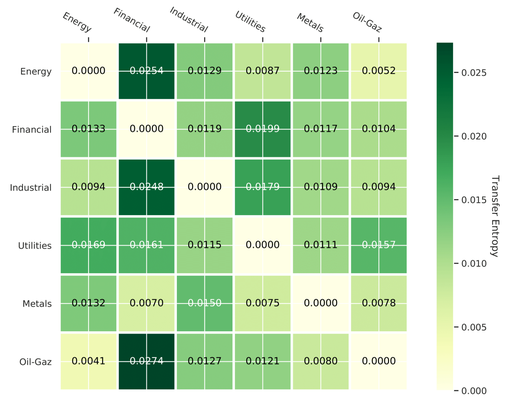

Figure 4: Transfer Entropy matrix for the different time series.

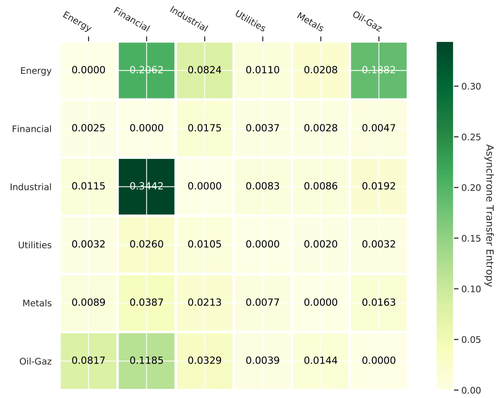

Figure 5: Asynchronous Transfer Entropy matrix for the different time series.

Figure 6: Pearson correlation matrix computed for different industrial ETF time series

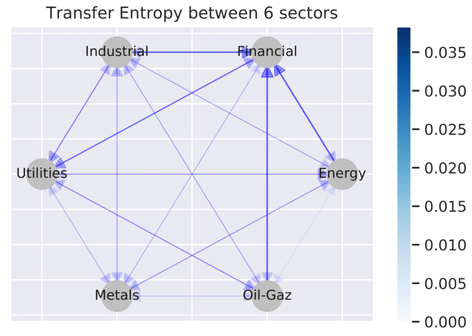

Figure 7: Transfer Entropy coefficient represented as a directed graph between all sectors

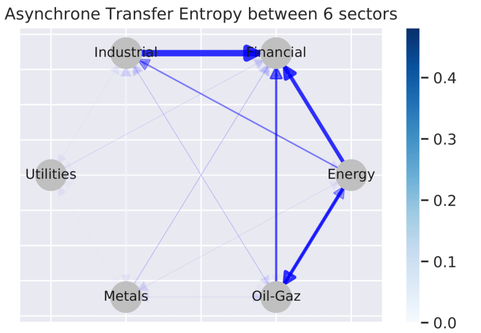

Figure 8: Asynchronous Transfer Entropy coefficient represented as a directed graph between all sectors

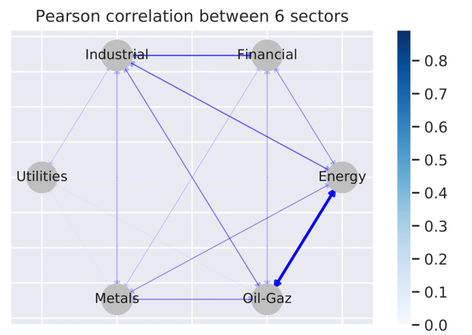

Figure 9: Pearson Correlation represented as a directed graph between all industries.

References

[1] T. **Schreiber**, *Physical Review Letters*, 2000
[2] J. **He**, P. **Shang**, *Physica A: Statistical Mechanics and its Applications*, 2017
[3] R. **Marschinski**, H. **Kantz**, *The European Physical Journal B - Condensed Matter and Complex Systemsy*, 2002