Appendix A: An illustrative example

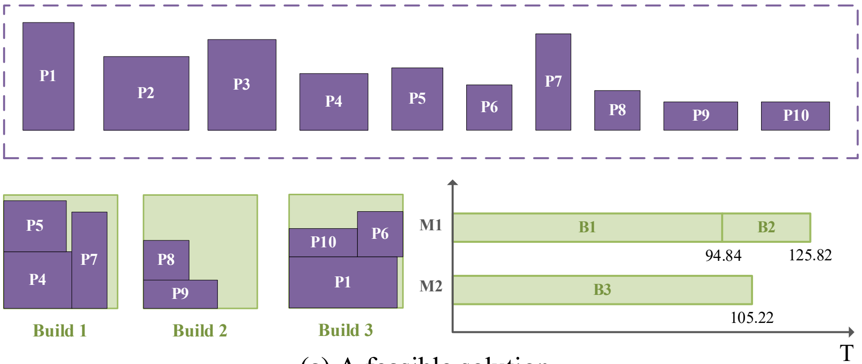
In this section, we provide an illustrative example of OAPMSAM-2DPR problem. There are 10 parts and 2 identical AM machines. The width and length of each build are both 20. The time spent per unit volume (VT), the time spent for powder-layering (HT) and the setup time (SET) are set as 0.04, 0.7 and 2, respectively. Unit production cost α is set to 1. Details of other parameters are listed in Table 5. A feasible solution and an optimal solution are presented in Figure 8.

Table 5. Parameters of parts

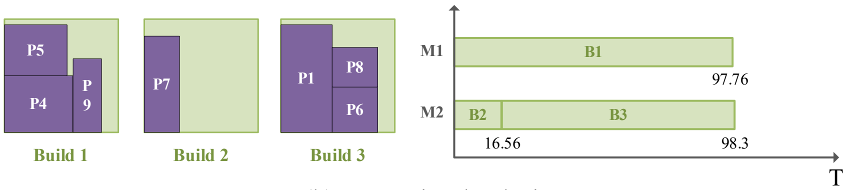
Part p	1	2	3	4	5	6	7	8	9	10
U_p	32	27	15	40	20	37	31	24	31	10
w_p	19	13	16	10	11	8	17	7	5	5
l_p	9	15	12	12	9	8	7	8	13	12
h_p	19	18	20	12	11	8	6	5	9	17
v_p	1201	3006	2224	1056	796	225	259	235	332	822

In the feasible solution, see Figure 8(a), the accepted parts are $\{1,4,5,6,7,8,9,10\}$, and the parts packed into build 1, build 2 and build 3 are $\{4,5,7\}$, $\{8,9\}$ and $\{1,6,10\}$, respectively. These parts are placed as shown in Figure 8(a), without exceeding the 2D dimensions of the build. Consequently, the processing times for build 1, build 2 and build 3 are $94.84\ (\text{i.e.}, 2 + 0.7 \times 12 + 0.04 \times (1056 + 796 + 259))$, $30.98\ (2 + 0.7 \times 9 + 0.04 \times (235 + 332))$ and $105.22\ (2 + 0.7 \times 19 + 0.04 \times (1201 + 225 + 822))$, respectively. Build 1 and build 2 are assigned to machine 1 for processing, while build 3 is assigned to machine 2 for processing. The makespan and the revenue are 125.82 and $225\ (\text{i.e.}, 32 + 40 + 20 + 37 + 31 + 24 + 31 + 10)$, respectively. Therefore, the profit of the feasible solution is $225 - 125.82 = 99.18$.

In the optimal solution, see Figure 8(b), the accepted parts are $\{1,4,5,6,7,8,9\}$, and the parts packed into build 1, build 2 and build 3 are $\{4,5,9\}$, $\{7\}$ and $\{1,6,8\}$, respectively. Similarly, we can calculate the processing times for build 1, build 2 and build 3 as 81.74, 16.56 and 97.76, respectively. The makespan and the revenue are 98.3 and 215, respectively. Therefore, the profit of the optimal solution is 116.7.



(a) A feasible solution

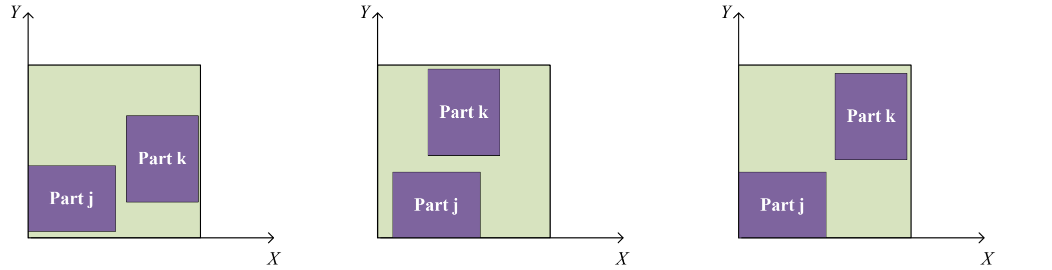


(b) An optimal solution

Figure 8. An illustrative example with 10 parts and 2 AM machines

Appendix B: Explanation of Constraints (7)–(11)

Constraints (7) – (11) define the positional relationships between two parts placed within the same build. Specifically, if Constraint (8) equals 1 and Constraint (10) equals 0, part j is positioned to the left of part k along the X-axis, ensuring no horizontal overlap (as illustrated in Figure 9(a)). If Constraint (8) equals 0 and Constraint (10) equals 1, part j is placed below part k along the Y-axis, ensuring no vertical overlap (see Figure 9(b)). If both Constraint (8) and Constraint (10) equal 1, then part j is positioned to the left and below part k along both the X- and Y-axes, maintaining complete spatial separation (see Figure 9(c)). Constraints (7) and (9) are used to determines the X-axis and Y-axis coordinates of parts. Constraint (11) ensures that Constraints (8) and (10) cannot both be zero, thereby enforcing non-overlapping placement.



(a) No overlap along the X-axis (b) No overlap along the Y-axis (c) No overlap along both X- and Y-axes

Figure 9. Positional relationship of two parts packed in the same build

Appendix C: Justification of cuts validity

The following theorems are presented to demonstrate the validity of the Benders cuts (V_4) and (V_5).

Theorem 1. Benders cuts (V_4) and (V_5) are valid.

Proof. Let the set P_b contains all the parts allocated to build b. But the allocation of parts in set P_b does not satisfy the two-dimensional constraint. This means that one needs at least two builds to pack these parts. Let set J_m^h contains all the parts that are assigned to machine m in the subsequent iteration h. Thus, we consider two cases.

Case 1: $J_m^h \cap P_b = P_b$. This case means that in the subsequent iteration h, all the parts in set P_b are assigned to machine m, i.e., $\sum_{p\in P_b} \boldsymbol{A_{pm}} = |P_b|$. By adding this into Cuts (V_4) and (V_5), we can obtain $\boldsymbol{F_m} \geq 2$ and $\boldsymbol{HM_m} \geq min_{p\in P_b}(h_p)$, which indicates that the cut is valid.

Case 2: $J_m^h \cap P_b \neq P_b$. This case means that at least one part $p^* \in P_b$ is removed from set J_m^h, i.e., $\boldsymbol{A_{p^*m}} = 0$. Therefore, we can obtain $\sum_{p\in P_b} \boldsymbol{A_{pm}} < |P_b|$. Thus, the right side of Cuts (V_4) and (V_5) becomes non-positive and we can obtain $\boldsymbol{F_m} \geq 0$ and $\boldsymbol{HM_m} \geq 0$. This indicates that Cuts (V_4) and (V_5) will not remove new feasible solutions in the subsequent iterations.

To conclude, the cuts will limit the values of variables $\boldsymbol{F_m}$ and $\boldsymbol{HM_m}$ in the subsequent iterations and will not remove new feasible solutions. Thus, Cuts (V_4) and (V_5) are valid. □

Appendix D: Detailed computational results for computation

The results presented in Tables 3 and 4 of the main text report the average objective value, average computation time, and average optimality gap across these four runs. To ensure transparency and reproducibility, the detailed computational outcomes for each individual run are provided in this appendix. Specifically, Tables 6-10 present the results of small-sized instances with $|P| = 10, 15, 20, 25$ and 30, respectively, corresponding to the aggregated outcomes summarized in Table 4. Tables 11-13 provide the detailed results for large-sized instances with $|P| = 40, 50$ and 60, respectively, corresponding to the summary in Table 5. The symbol "—" indicates that the corresponding run did not return a result within the time limit of 3600 seconds.

Table 6. Computational results for small-sized instances with |*P*|=10

Instances		Model 1			Model 2			LBBD			NLBBD_V4V5		
|*P*|	|*M*|	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
10	2	82.7	11.6	0.0	82.7	5.3	0.0	82.7	4.3	0.0	**82.7**	1.8	0.0
	2	103.0	8.4	0.0	103.0	7.8	0.0	103.0	6.8	0.0	**103.0**	2.4	0.0
	2	74.4	11.4	0.0	74.4	5.5	0.0	74.4	2.5	0.0	**74.4**	1.6	0.0
	2	84.9	3.5	0.0	84.9	1.8	0.0	84.9	0.6	0.0	**84.9**	1.3	0.0
10	3	117.5	197	0.0	117.5	64.9	0.0	117.5	9.2	0.0	**117.5**	2.2	0.0
	3	145.4	215	0.0	145.4	8.2	0.0	145.4	1.6	0.0	**145.4**	1.3	0.0
	3	110.1	56.4	0.0	110.1	21.1	0.0	110.1	13.3	0.0	**110.1**	2.7	0.0
	3	107.5	38.2	0.0	107.5	14.5	0.0	107.5	7.9	0.0	**107.5**	0.8	0.0
10	4	140.1	619	0.0	140.1	10.9	0.0	140.1	18.6	0.0	**140.1**	2.3	0.0
	4	160.4	184	0.0	160.4	6.8	0.0	160.4	4.0	0.0	**160.4**	1.4	0.0
	4	132.0	—	2.6	132.0	20.6	0.0	132.0	64.3	0.0	**132.0**	2.7	0.0
	4	134.1	—	4.5	134.1	14.9	0.0	134.1	6.0	0.0	**134.1**	3.0	0.0
Average(opt)		116.0	712	$3.5^{(10)}$	116.0	15.2	$0.0^{(12)}$	116.0	11.6	$0.0^{(12)}$	116.0	2.0	$0.0^{(12)}$

Table 7. Computational results for small-sized instances with $|P|$=15

Instances		Model 1			Model 2			LBBD			NLBBD_V4V5		
$\|P\|$	$\|M\|$	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
15	2	126.2	305	0.0	126.2	39.6	0.0	126.2	7.6	0.0	**126.2**	6.1	0.0
	2	177.7	30.2	0.0	177.7	12.0	0.0	177.7	10.0	0.0	**177.7**	3.9	0.0
	2	170.6	37.8	0.0	170.6	8.2	0.0	170.6	2.4	0.0	**170.6**	2.5	0.0
	2	103.9	69.2	0.0	103.9	6.2	0.0	103.9	1.4	0.0	**103.9**	0.4	0.0
15	3	173.9	—	1.6	178.1	2165	0.0	178.1	415	0.0	**178.1**	37.2	0.0
	3	227.7	1670	0.0	227.7	755	0.0	227.7	408	0.0	**227.7**	12.2	0.0
	3	199.5	—	1.3	199.5	1646	0.0	199.5	828	0.0	**199.5**	8.6	0.0
	3	135.3	1216	0.0	135.3	350	0.0	135.3	172	0.0	**135.3**	3.6	0.0
15	4	203.5	—	40.0	217.4	1453	0.0	217.4	—	2.1	**217.4**	127	0.0
	4	269.9	—	10.9	271.4	874	0.0	271.4	418	0.0	**271.4**	40.3	0.0
	4	230.6	—	15.5	232.9	564	0.0	232.9	680	0.0	**232.9**	29.8	0.0
	4	168.2	—	22.7	168.2	2012	0.0	168.2	913	0.0	**168.2**	56.5	0.0
Average(opt)		182.3	2077	$7.6^{(6)}$	184.1	824	$0.0^{(12)}$	184.1	621	$0.2^{(11)}$	184.1	27.3	$0.0^{(12)}$

Table 8. Computational results for small-sized instances with $|P|$=20

Instances		Model 1			Model 2			LBBD			NLBBD_V4V5		
$\|P\|$	$\|M\|$	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
20	2	123.6	—	8.9	123.6	210	0.0	123.6	37.2	0.0	**123.6**	11.6	0.0
	2	164.8	1626	0.0	164.8	—	1.4	164.8	137	0.0	**164.8**	61.7	0.0
	2	123.9	575	0.0	123.9	206	0.0	123.9	31.2	0.0	**123.9**	9.6	0.0
	2	126.0	—	11.6	126.0	571	0.0	126.0	34.9	0.0	**126.0**	14.9	0.0
20	3	183.9	—	57.7	183.5	—	5.8	184.2	—	1.5	**184.2**	180	0.0
	3	241.2	—	6.1	239.7	—	3.7	242.1	—	0.9	**242.1**	134	0.0
	3	186.4	—	2.5	185.2	—	2.4	186.4	—	2.3	**186.4**	124	0.0
	3	190.6	—	29.5	197.8	—	2.3	197.8	—	1.3	**197.8**	73.2	0.0
20	4	221.5	—	17.4	235.1	—	4.2	235.1	—	1.4	**235.1**	909	0.0
	4	283.9	—	51.5	290.8	—	4.0	291.2	—	2.4	**291.7**	—	1.8
	4	230.3	—	46.8	235.2	—	4.4	238.0	—	4.2	**238.0**	3372	0.0
	4	240.1	—	59.8	246.9	—	5.0	247.3	—	3.7	**247.8**	2690	0.0
Average(opt)		193.4	3183	$24.3^{(2)}$	196.0	2782	$2.8^{(3)}$	196.7	2420	$1.5^{(4)}$	196.8	932	$0.2^{(11)}$

Table 9. Computational results for small-sized instances with $|P|$=25

Instances		Model 1			Model 2			LBBD			NLBBD_V4V5		
$\|P\|$	$\|M\|$	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
25	2	171.0	—	64.0	178.2	—	2.3	178.5	471	0.0	**178.5**	68.5	0.0
	2	184.4	—	27.9	187.8	—	2.9	188.6	192	0.0	**188.6**	121	0.0
	2	212.4	—	57.6	214.9	—	3.1	215.6	674	0.0	**215.6**	273	0.0
	2	213.6	—	54.9	215.5	—	0.8	215.5	394	0.0	**215.5**	25.2	0.0
25	3	245.3	—	93.7	256.0	—	4.0	255.6	—	4.0	**256.8**	—	1.2
	3	267.1	—	71.6	276.8	—	3.9	277.4	—	3.4	**279.8**	—	0.5
	3	279.8	—	64.7	288.6	—	4.7	291.3	—	3.2	**291.3**	—	2.0
	3	275.5	—	66.3	277.0	—	3.1	280.6	—	1.6	**280.6**	67.3	0.0
25	4	299.2	—	67.1	309.1	—	5.2	312.1	—	4.0	**315.6**	—	0.9
	4	327.8	—	53.6	333.6	—	4.7	337.2	—	2.7	**337.8**	—	1.8
	4	335.1	—	32.5	341.8	—	4.3	344.8	—	2.5	**344.8**	—	1.1
	4	328.4	—	50.8	329.2	—	3.8	333.5	—	2.6	**333.7**	—	0.5
Average(opt)		261.6	3600	$64.5^{(0)}$	267.4	3600	$3.6^{(0)}$	269.2	2544	$2.0^{(4)}$	269.9	2146	$0.6^{(5)}$

Table 10. Computational results for small-sized instances with $|P|$=30

Instances		Model 1			Model 2			LBBD			NLBBD_V4V5		
$\|P\|$	$\|M\|$	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
30	2	195.5	—	133	206.8	—	1.8	206.8	679	0.0	**206.8**	120	0.0
	2	195.2	—	141	222.7	—	5.4	224.6	—	1.2	**224.6**	376	0.0
	2	205.8	—	163	232.7	—	5.7	235.5	—	2.3	**235.5**	767	0.0
	2	206.3	—	197	210.5	—	5.7	211.7	—	2.6	**211.8**	385	0.0
30	3	278.9	—	92.8	290.3	—	5.4	295.6	—	2.5	**295.6**	—	0.7
	3	287.9	—	92.8	307.4	—	4.2	310.8	—	2.4	**310.8**	—	0.6
	3	304.2	—	98.7	336.9	—	6.2	**338.7**	—	5.0	337.8	—	2.8
	3	270.7	—	131	279.8	—	10.8	**288.6**	—	5.7	287.5	—	2.6
30	4	339.7	—	87.6	351.2	—	6.0	354.9	—	3.4	**357.9**	—	1.3
	4	335.7	—	72.5	362.4	—	5.0	366.7	—	2.7	**366.9**	—	1.3
	4	371.1	—	74.3	405.2	—	5.4	409.0	—	3.9	**410.5**	—	1.7
	4	317.5	—	106	355.3	—	9.8	358.9	—	5.5	**359.0**	—	4.8
Average(opt)		275.7	—	$115^{(0)}$	296.8	—	$6.0^{(0)}$	300.2	3356	$3.1^{(1)}$	300.2	2537	$1.32^{(4)}$

Table 11. Computational results for large-sized instances with |*P*|=40

Instances		**Model 2**			**LBBD**			**NLBBD_V4V5**		
|*P*|	|*M*|	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
40	4	499.9	—	18.4	545.8	—	5.0	**548.3**	—	4.5
	4	394.2	—	10.5	394.1	—	7.1	**399.0**	—	5.3
	4	466.6	—	13.6	**489.2**	—	5.4	488.1	—	4.7
	4	349.7	—	27.7	400.9	—	7.1	**405.5**	—	5.8
40	5	528.9	—	25.8	**624.9**	—	3.8	624.0	—	4.4
	5	453.1	—	10.3	**466.9**	—	4.9	464.2	—	5.1
	5	475.7	—	28.8	557.4	—	6.5	**568.3**	—	5.0
	5	457.0	—	15.2	470.0	—	8.4	**483.5**	—	5.9
40	6	633.2	—	13.4	682.3	—	3.0	**683.1**	—	3.1
	6	500.1	—	8.6	515.9	—	3.0	**516.1**	—	2.9
	6	484.4	—	39.1	619.1	—	5.9	**628.6**	—	5.0
	6	462.8	—	26.4	**543.9**	—	4.2	543.2	—	5.0
Average		475.5	—	19.8	525.9	—	5.4	529.3	—	4.7

Table 12. Computational results for large-sized instances with |*P*|=50

Instances		**Model 2**			**LBBD**			**NLBBD_V4V5**		
|*P*|	|*M*|	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
50	4	442.0	—	26.7	490.4	—	12.2	**517.3**	—	5.3
	4	329.8	—	71.2	509.3	—	5.8	**518.2**	—	4.2
	4	572.2	—	15.2	593.3	—	8.2	**602.3**	—	6.5
	4	516.2	—	15.2	547.3	—	4.7	**541.7**	—	6.2
50	5	417.6	—	53.5	582.8	—	8.5	**595.4**	—	4.8
	5	340.9	—	93.6	605.9	—	4.5	**608.0**	—	5.5
	5	655.7	—	15.8	629.1	—	17.6	**706.4**	—	5.5
	5	342.1	—	101	617.7	—	7.7	**634.4**	—	5.4
50	6	536.1	—	32.1	374.5	—	87.5	**662.4**	—	4.7
	6	459.0	—	59.4	646.2	—	9.2	**686.5**	—	4.0
	6	305.7	—	173	760.8	—	7.1	**786.8**	—	4.2
	6	450.7	—	67.8	703.0	—	5.7	**714.2**	—	3.7
Average		447.3	—	60.4	588.4	—	14.9	631.1	—	5.0

Table 13. Computational results for large-sized instances with $|P|$=60

Instances		**Model 2**			**LBBD**			**NLBBD_V4V5**		
$\|P\|$	$\|M\|$	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%	Obj	Time/s	Gap/%
60	4	638.5	—	23.4	599.0	—	28.5	**723.7**	—	6.1
	4	530.9	—	34.6	646.2	—	7.2	**658.5**	—	5.6
	4	599.2	—	38.1	706.1	—	14.1	**758.1**	—	6.5
	4	558.1	—	19.9	601.2	—	10.0	**613.7**	—	6.3
60	5	259.9	—	250	564.9	—	56.6	**847.3**	—	5.3
	5	273.4	—	200	730.5	—	9.8	**762.0**	—	5.3
	5	370.3	—	156	680.6	—	35.7	**889.3**	—	4.7
	5	716.8	—	32.4	693.0	—	7.8	**708.4**	—	4.9
60	6	744.4	—	35.3	913.8	—	7.5	**943.5**	—	4.8
	6	436.4	—	105	509.2	—	73.3	**845.4**	—	3.8
	6	532.3	—	95.8	691.9	—	48.3	**989.7**	—	3.6
	6	173.2	—	379	674.2	—	21.7	**769.4**	—	6.0
Average		486.1	—	114.1	667.6	—	26.7	792.4	—	5.2